## COMPARATIVE RATIONALITY

Mauricio Ribeiro

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#### THE RATIONAL CHOICE MODEL

MOTIVATION

• Rationality: choices are rational whenever there exists a complete and transitive relation  $\succeq$  such that x is chosen from a menu A whenever  $x \succeq y$ , for all  $y \in A$ 

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- Rational choice underlies most applied work in Economics
  - ▶ i.e., people maximize preferences over different domains
- Policy and welfare implications of this work rely on the extent to which choices are rational

#### VIOLATIONS OF RATIONALITY

- Choices can fail to be rational...
  - e.g., Battalio et al. (1973), Sippel (1997), Mattei (2000), Harbaugh, Krause and Berry (2001), Février and Visser (2003), Choi et al (2007, 2014), Manzini and Mariotti (2010), Costa-Gomez et al (2019), Nielsen and Rehbeck (2020), and Boaucida (2021)

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- and for several reasons:
  - ▶ Intransitivities, incompleteness, behavioral biases, unobserved constraints, choice overload, different choice procedures

#### REACTING TO THE VIOLATIONS

MOTIVATION

#### (I)Measuring Incompatibility with Rationality

e.g., Afriat (1973), Houtman and Maks (1985), Swofford and Whitney (1986), Varian (1990), Echenique, Lee, and Shum (2011), Apesteguia and Ballester (2015), Dean and Martin (2016), Caradonna (2020), de Clippel and Rozen (2020)

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#### (II)Models of Boundedly Rational Choice

e.g., Kalai, Rubinstein and Spiegler (2002), Masatlioglu and Ok (2005), Eliaz and Ok (2006), Manzini and Mariotti (2007), Rubinstein and Salant (2008), Cherepanov, Feddersen and Sandroni (2013), Frick (2016)

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#### **Behavioral Welfare Analysis**

e.g., Bernheim and Rangel (2007, 2009), Green and Hojman (2007), Rubinstein and Salant (2012), Apesteguia and Ballester (2015), Horan and Sprumont (2016), Nishimura (2016), Caliari (2020)

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- This approach compares the rationality of choices indirectly
  - (I) What if two indices disagree?
  - (II) Completeness of the induced rationality ranking

#### THE APPROACH OF THIS PROJECT

- Introduce a criterion for **comparative judgments** of rationality that:
  - (A) Delivers intuitive comparisons
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  - (C) Compares the predictive mistakes of rationality
- (II) The criterion induces an **incomplete** rationality ordering over choices
  - (A) Some comparisons are "easy", while others are "hard"
  - (B) Indices should agree with the "easy" comparisons

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	a	b	С	С
Friend 4	а	а	Ь	С

Menus	{ <i>a</i> , <i>b</i> }	{ <i>b</i> , <i>c</i> }	$\{a,c\}$
Friend 1	а	С	С
Friend 2	а	С	С
Friend 3	Ь	С	С
Friend 4	а	b	С

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	$\{a,c\}$
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	Ь	С	С
Friend 4	а	а	Ь	С

Menus	$\{a,b,c\}$	{ <i>a</i> , <i>b</i> }	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	b	С	С
Friend 4	а	а	b	С

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	$\{a,c\}$
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Friend 2	а	а	С	С
Friend 3	а	b	С	С
Friend 4	а	а	b	С

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	$\{a,c\}$
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Friend 1 > Friend 2 > Friend 3

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Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	b	С	С
Friend 4	а	а	b	С

Friend  $1 \succ \text{Friend } 2 \succ \{\text{Friends } 3,4\}$ 

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Frien	d 1	С	а	С	С
Frien	id 2	а	а	С	С
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Friends 3 and 4 are hard to compare

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Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	Ь	С	С
Friend 4	а	а	Ь	С

#### The Violation Criterion

Friend *i* is at least as rational as Friend *j* when for each sub-collection of menus...

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	b	С	С
Friend 4	а	а	Ь	С

The Violation Criterion

If Friend i violates rationality in the sub-collection, then Friend j violates rationality in the sub-collection

# The Rationality Ordering and

Indices of Incompatibility

#### THE RATIONALITY ORDERING

#### **DEFINITION**

Given two choice correspondences  $c_1$  and  $c_2$  defined over a collection of menus A, we say that

•  $c_1$  is at least as rational as  $c_2$  if, for every  $\mathcal{B} \subseteq \mathcal{A}$ ,

 $c_1$  is not rationalizable on  $\mathcal{B} \implies c_2$  is not rationalizable on  $\mathcal{B}$ 

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- We then write  $c_1 \succsim_{\mathsf{rat}} c_2$ 
  - $ightharpoonup c_1 \succ_{\mathsf{rat}} c_2$ :  $c_1$  is more rational than  $c_2$
  - $ightharpoonup c_1 \sim_{\sf rat} c_2$ :  $c_1$  is as rational as  $c_2$
  - $ightharpoonup c_1$  and  $c_2$  are  $\succsim_{rat}$ -incomparable

# INDICES OF INCOMPATIBILITY

#### **DEFINITION**

An **index of incompatibility** I assigns numbers to choice correspondences defined over a collection of menus A in a way that I(c) = 0 if, and only if, c is rationalizable on A.

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I is consistent with ≿<sub>rat</sub> if

$$\begin{cases} c_1 \succ_{\mathsf{rat}} c_2 \text{ implies } I(c_1) < I(c_2) \\ c_1 \leadsto_{\mathsf{rat}} c_2 \text{ implies } I(c_1) = I(c_2) \end{cases}$$

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I is consistent with ≿<sub>rat</sub> if

$$\begin{cases} c_1 \succ_{\mathsf{rat}} c_2 \text{ implies } I(c_1) < I(c_2) \\ c_1 \backsim_{\mathsf{rat}} c_2 \text{ implies } I(c_1) = I(c_2) \end{cases}$$

• / is weakly consistent with \( \subseteq \text{rat} \) if

$$c_1 \succsim_{\mathsf{rat}} c_2 \mathsf{ implies } I(c_1) \leqslant I(c_2)$$

An Outline of the

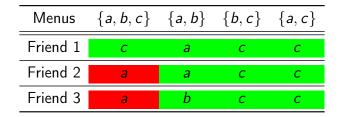
Results

Example

RESULTS 0000000

# ≿<sub>RAT</sub> AND PREDICTIVE ERRORS

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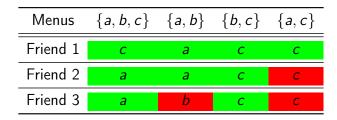


Friends 2 and 3 choosing a on  $\{a, b, c\}$  is a **predictive error** based on pairwise menus

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
Friend 2	а	а	С	С
Friend 3	а	b	С	С

What are the predictive errors

based on the collection  $\{\{a, b, c\}\}$ ?



What are the predictive errors

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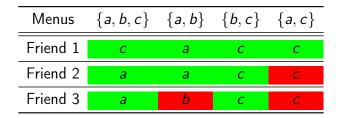
Friend 2 choosing c on  $\{a, c\}$ 

Friend 3 choosing c on  $\{a, c\}$  and b on  $\{a, b\}$ 

Menus	$\{a,b,c\}$	$\{a,b\}$	{ <i>b</i> , <i>c</i> }	{ <i>a</i> , <i>c</i> }
Friend 1	С	а	С	С
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#### The Prediction Criterion

Friend *i* is *at least as rational as* Friend *j* when for each sub-collection of menus...



#### The Prediction Criterion

If we incorrectly predict Friend i's choices based in the sub-collection, then we incorrectly predict Friend j's choices based in the sub-collection



# CONSISTENCY WITH ≿<sub>RAT</sub>

# Proposition (Characterization of $\succsim_{RAT}$ )

The Violation Criterion is equivalent to the Prediction Criterion.

# CONSISTENCY WITH ZRAT

# PROPOSITION (CHARACTERIZATION OF $\succsim_{\text{RAT}}$ )

The Violation Criterion is equivalent to the Prediction Criterion.

• Takeaway: \( \sum\_{rat} \) comparatively checks for predictive errors

## CONSISTENCY WITH $\succsim_{RAT}$

# PROPOSITION (CHARACTERIZATION OF $\succeq_{RAT}$ )

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• Takeaway: \( \sum\_{rat} \) comparatively checks for predictive errors

#### PROPOSITION (CHARACTERIZATION OF CONSISTENCY)

An index I is (weakly) consistent with  $\succsim_{rat}$  if, and only if, it is a (weakly) monotonic aggregator of predictive errors.

EXAMPLE



Results

# ZRAT AND EXISTING INDICES: AN EXAMPLE

No existing index of incompatibility is consistent with  $\succsim_{\mathsf{rat}}$ 

## ERAT AND EXISTING INDICES: AN EXAMPLE

No existing index of incompatibility is consistent with  $\succsim_{rat}$ 

The Houtman-Maks index:

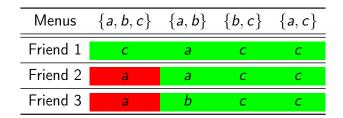
 $I_{HM}(c) := \min\{|\mathcal{B}| : \mathcal{B} \subseteq \mathcal{A} \text{ and } c \text{ is rationalizable on } \mathcal{A} \setminus \mathcal{B}\}$ 

#### ≿<sub>RAT</sub> AND EXISTING INDICES: AN EXAMPLE

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#### The Houtman-Maks index:

$$I_{HM}(c) \coloneqq \min \left\{ |\mathcal{B}| : \ \mathcal{B} \subseteq \mathcal{A} \ \text{and} \ c \ \text{is rationalizable on} \ \mathcal{A} ackslash \mathcal{B} 
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# CRAT AND WEAKLY CONSISTENT INDICES

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### ERAT AND WEAKLY CONSISTENT INDICES

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## ERAT AND WEAKLY CONSISTENT INDICES

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- The Houtman-Maks index is weakly consistent with  $\succsim_{\rm rat}$
- Method to "fix" a weakly consistent index
  - Calculate the index in each sub-collection
  - Monotonically aggregate these numbers

## ERAT AND WEAKLY CONSISTENT INDICES

- Takeaway: the Houtman-Maks index disregards evidence of incompatibility
- The Houtman-Maks index is weakly consistent with  $\succsim_{\rm rat}$
- Method to "fix" a weakly consistent index
  - Calculate the index in each sub-collection.
  - Monotonically aggregate these numbers
- The method allows that we assign different weights to different types of violations

 Applying this method to the Houtman-Maks index, I propose a new index of incompatibility: the Average Houtman-Maks index

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- (II) The Average Houtman-Maks index is (much) more discerning than the Houtman-Maks index
- (III) The average Houtman-Maks index is more responsive to the increase in different types of violations of rationality

(I)  $\succsim_{\mathsf{rat}}$  and models of boundedly rational choice

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- $({\rm II}) \,\succsim_{\rm rat}$  and the Behavioral Welfare Framework of Bernheim and Rangel

- Erat and models of boundedly rational choice
- (II)  $\succeq_{\mathsf{rat}}$  and the Behavioral Welfare Framework of Bernheim and Rangel
- (III) Using  $\succsim_{\mathsf{rat}}$  to understand violations in experimental data
  - Identifying common types of violations

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  - (V) Extensions of ≿<sub>rat</sub>