Comparative Rationality

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- Rational choice underlies most applied work in Economics
  - i.e., people maximize preferences over different domains
- Policy and welfare implications of this work rely on the extent to which choices are rational

Violations of Rationality

- Choices can fail to be rational...
  - e.g., Battalio et al. (1973), Sippel (1997), Mattei (2000), Harbaugh, Krause and Berry (2001), Février and Visser (2003), Choi et al (2007, 2014), Manzini and Mariotti (2010), Costa-Gomez et al (2019), Nielsen and Rehbeck (2020), and Boaucida (2021)

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- ... and for several reasons:
  - Intransitivities, incompleteness, behavioral biases, unobserved constraints, choice overload, different choice procedures

Reacting to the Violations

(I) **Measuring Incompatibility with Rationality**

- e.g., Afriat (1973), Houtman and Maks (1985), Swofford and Whitney (1986), Varian (1990), Echenique, Lee, and Shum (2011), Apesteguia and Ballester (2015), Dean and Martin (2016), Caradonna (2020), de Clippel and Rozen (2020)

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(II) **Models of Boundedly Rational Choice**

- e.g., Kalai, Rubinstein and Spiegler (2002), Masatlioglu and Ok (2005), Eliaz and Ok (2006), Manzini and Mariotti (2007), Rubinstein and Salant (2008), Cherepanov, Feddersen and Sandroni (2013), Frick (2016)

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(III) Behavioral Welfare Analysis

- e.g., Bernheim and Rangel (2007, 2009), Green and Hojman (2007), Rubinstein and Salant (2012), Apesteguia and Ballester (2015), Horan and Sprumont (2016), Nishimura (2016), Caliari (2020)

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- The **standard approach** uses indices
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- This approach compares the rationality of choices **indirectly**
  (i) What if two indices disagree?
  (ii) Completeness of the induced rationality ranking

The Approach of this Project

(i) Introduce a criterion for **comparative judgments** of rationality that:

- (a) Delivers intuitive comparisons
- (b) Improves our understanding of departures from rationality
- (c) Compares the predictive mistakes of rationality

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(ii) The criterion induces an **incomplete** rationality ordering over choices

(a) Some comparisons are "easy", while others are "hard"

(b) Indices should agree with the "easy" comparisons

Ranking the Rationality of Choices

Menus	$\{a, b, c\}$	$\{a, b\}$	$\{b, c\}$	$\{a, c\}$
Friend 1	c	a	c	c
Friend 2	a	a	c	c
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Ranking the Rationality of Choices

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The Violation Criterion

Friend i is *at least as rational as* Friend j

when for each sub-collection of menus...

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The Violation Criterion

If Friend i violates rationality in the sub-collection,

then Friend j violates rationality in the sub-collection

The Rationality Ordering

and

Indices of Incompatibility

The Rationality Ordering

Definition

Given two choice correspondences c_1 and c_2 defined over a collection of menus $\mathcal{A}$, we say that

- c_1 **is at least as rational as** c_2 if, for every $\mathcal{B} \subseteq \mathcal{A}$,

$$c_1 \text{ is not rationalizable on } \mathcal{B} \implies c_2 \text{ is not rationalizable on } \mathcal{B}$$

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  - $c_1 \succ_{\text{rat}} c_2$: c_1 is **more rational than** c_2
  - $c_1 \sim_{\text{rat}} c_2$: c_1 is **as rational as** c_2
  - c_1 and c_2 are $\succsim_{\text{rat}}$-**incomparable**

Indices of Incompatibility

Definition

An **index of incompatibility** I assigns numbers to choice correspondences defined over a collection of menus $\mathcal{A}$ in a way that $I(c) = 0$ if, and only if, c is rationalizable on $\mathcal{A}$.

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- I is **consistent with** $\succsim_{\text{rat}}$ if

$$\begin{cases} c_1 \succ_{\text{rat}} c_2 \text{ implies } I(c_1) < I(c_2) \\ c_1 \backsim_{\text{rat}} c_2 \text{ implies } I(c_1) = I(c_2) \end{cases}$$

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- I is **weakly consistent with** $\succsim_{\text{rat}}$ if

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An Outline

of the

Results

$\succsim_{\text{RAT}}$ and Predictive Errors

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Friends 2 and 3 choosing a on $\{a, b, c\}$ is a **predictive error**

based on pairwise menus

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What are the **predictive errors**

based on the collection $\{\{a, b, c\}\}$?

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The Prediction Criterion

If we incorrectly predict Friend i's choices based in the sub-collection,

then we incorrectly predict Friend j's choices based in the sub-collection

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Proposition (Characterization of $\succsim_{\text{RAT}}$)

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Proposition (Characterization of Consistency)

An index I is (weakly) consistent with $\succsim_{rat}$ if, and only if, it is a (weakly) ***monotonic aggregator*** *of predictive errors.*

$\succsim_{\text{RAT}}$ and Existing Indices: an example

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  1. Calculate the index in each sub-collection
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- Method to "fix" a weakly consistent index
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  2. Monotonically aggregate these numbers
- The method allows that we assign different weights to different types of violations

$\succsim_{\text{RAT}}$ and New Indices of Incompatibility

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(iii) The average Houtman-Maks index is more responsive to the increase in different types of violations of rationality

Other Results

(I) $\succsim_{\text{rat}}$ and models of boundedly rational choice

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(V) Extensions of $\succsim_{\text{rat}}$