Symbiotic Competition and Intellectual Property

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Introduction

Patent History, Economics and Policy

- Intellectual property has been studied and awarded for centuries
  - First codified patent system in 15C Venice pretty much prevails
  - Smith, Jefferson, Pigou, Marshall, Arrow, etc., etc.
- Nordhaus (1969): The optimal life of a patent T^* balances
  - Benefit: Profit from a patent motivates innovation
  - Cost: Inefficiency due to monopoly
- Patents protect innovators from imitators appropriating their ideas without attribution and reducing their profit through competition, thus dissuading investment in innovation
- Current global standard: Patents last 20 years

Macro/IO View of Patents

- Main goal in this paper is to estimate T^*
- We extend Nordhaus's model to incorporate
  - Process innovations and knowledge spillovers: Virtuous cycle of productivity growth via mutual imitation and follow-on innovations
  - Macro framework: Semi-endogenous growth with a continuum of heterogeneous industries and imperfectly elastic labor supply
- New costs emerge in addition to previous benefits and costs
  - Opportunity cost of forgone symbiotic productivity growth from virtuous spillover cycles
  - Industries with higher spillovers contribute more to growth than those with lower spillovers
  - Now more spillovers may shorten patents
- We calibrate our model to US data
  - T^* lies between 8 and 14 years
  - Nordhaus's market power effect on T^* has the same magnitude as that of symbiotic competition and both effects are substantial

Model

Symbiotic Productivity Growth

- Two firms, 1 and 2; industry under patent with duration $T \in [0, \infty]$
- At time t, each firm j has the linear technology

$$y_{jt} = A_{jt}\ell_{jt}$$

  where ℓ_{jt} is labor input, $A_{jt} = \exp(Z_{jt})$, and Z_{jt} is log productivity
- During the life of the patent $(t < T)$, firm 1 is a monopolist and

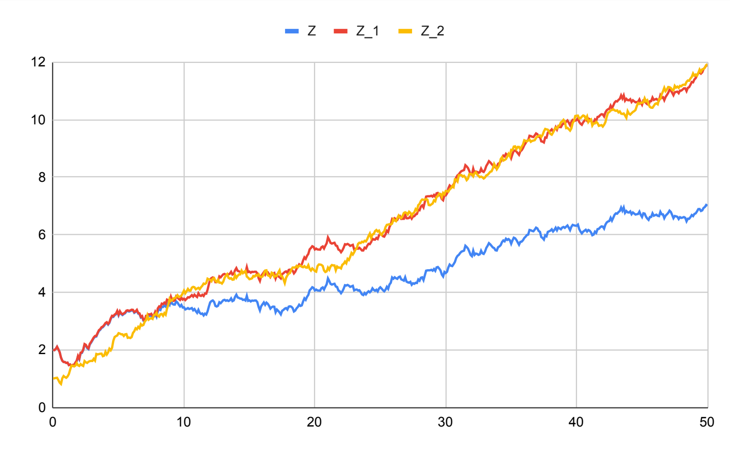
$$dZ_{1t} = \mu dt + \sigma dW_{1t}$$

- Once the patent expires $(t \geq T)$, a firm's log productivity Z_{jt} obeys

$$dZ_{jt} = \begin{cases} (\mu + \theta)dt + \sigma dW_{jt} & \text{if } Z_{jt} < Z_{-jt} \\ \mu dt + \sigma dW_{jt} & \text{if } Z_{jt} \geq Z_{-jt} \end{cases} \qquad (*)$$

  - $\mu > 0$: productivity growth due to "learning by doing"
  - $\theta > 0$: catch-up from knowledge spillovers, or "imitation"
  - W_1 and W_2 are iid, capturing "process innovations"

Productivity Sample Path



- Productivity grows faster under competition than under monopoly
- Growth is driven by the catch-up of laggards
- Firms engage in neck-and-neck competition—productivity gap hovers around 0 but sometimes veers away

Average Productivity and Productivity Gap

- Average productivity $X = \frac{1}{2}(Z_1 + Z_2)$ obeys the law of motion

$$dX_t = (\mu + \theta/2)dt + \sigma dW_{xt}$$

- Productivity gap $Y = \frac{1}{2}(Z_1 - Z_2)$ obeys the law of motion

$$dY_t = -\theta \text{sgn}(Y_t)dt + \sigma dW_{yt}$$

- Y is a stationary process with double-exponential long-run pdf

$$\frac{1}{2}(\theta/\sigma^2)e^{-(\theta/\sigma^2)|y|}$$

General Equilibrium

Households

- Continuum of identical households with mass
$$N_t = N_0 e^{gt}$$
- A household's utility over output and labor streams is
$$\int_0^\infty e^{-rt}\left[c_t - \tfrac{1}{1+1/\phi}\eta_t \ell_t^{1+1/\phi}\right]dt$$
  - c_t is consumption of final good
  - ℓ_t is labor supplied
  - η_t is used later to keep labor constant in the balanced growth path
  - ϕ is the Frisch elasticity of labor supply
- Labor is used to produce
  - New varieties of intermediate goods; "research" ℓ_{rt}
  - The intermediate goods themselves; "production" ℓ_{pt}
$$\ell_t = \ell_{rt} + \ell_{pt}$$
- No borrowing or lending, goods cannot be stored

Blueprints

- Blueprints encode new varieties of intermediate goods
- There is a continuum, with unit mass, of blueprint-producing firms
- A firm that employs $\ell_{rt} N_t$ units of labor during $[t, t + dt)$
  - Obtains a blueprint with probability $\gamma \ell_{rt} N_t dt$
  - Doesn't with probability $1 - \gamma \ell_{rt} N_t dt$
- Blueprints die of obsolescence at rate δ
- Stock of blueprints is B_t, with $B_0 > 0$ given

$$dB_t = (\gamma \ell_{rt} N_t - \delta B_t) dt$$

- Continuum of existing blueprints are labeled $i \in [0, B_t]$

Patents and Intermediate Good Production

- When a blueprint producing firm creates a blueprint $i \in [0, B_{t_0}]$ at t_0, it forms a new intermediate good producing firm $i1$ that operates with technology $y_{i1t} = A_{i1t}\ell_{i1t}$
- Blueprints become patented immediately upon creation
- Before its patent expires, firm $i1$ sets a monopoly price p_{i1t} satisfying

$$\frac{p_{i1t} - c_{i1t}}{p_{i1t}} = 1 - \alpha$$

- After $i1$'s patent expires, duopoly prices p_{i1t} and p_{i2t} satisfy

$$\frac{p_{ijt} - c_{ijt}}{p_{ijt}} = \frac{(1-\alpha)(1-\beta)}{(1-\rho_j)(1-\alpha) + \rho_j(1-\beta)}$$

  where

$$\rho_j = p_{ijt}^{-\beta/(1-\beta)} \Big/ \left[p_{i1t}^{-\beta/(1-\beta)} + p_{i2t}^{-\beta/(1-\beta)}\right]$$

  and $c_{ijt} = w_t / A_{ijt}$ is ij's marginal cost of production at time t
- Initial conditions: $(Z_{i1,t_0}, Z_{i2,t_0+T})$ match long run distribution of Y

Final Good Producers

- A continuum, with unit mass, of final good producers buy inputs (y_{ijt}) and produce output y_t with the CRS technology
$$y_t = \left[\int_0^{B_t} \left(y_{i1t}^{\beta} + \varepsilon_{it} y_{i2t}^{\beta} \right)^{\alpha/\beta} di \right]^{1/\alpha}$$
where $\alpha \leq \beta < 1$ and
$$\varepsilon_{it} = \begin{cases} 0 & \text{if blueprint } i \text{ is under patent at time } t \text{ and} \\ 1 & \text{otherwise} \end{cases}$$
- Profit-maximizing final good producers demand each y_{ijt} as a function of market prices and total quantity produced y_t according to
$$y_{i1t}^{m} = y_t p_{i1t}^{-1/(1-\alpha)}$$
for a monopolist $i1$ and
$$y_{ijt}^{d} = y_t p_{ijt}^{-1/(1-\beta)} \left[p_{i1t}^{-\beta/(1-\beta)} + p_{i2t}^{-\beta/(1-\beta)} \right]^{-(\beta-\alpha)/[\beta(1-\alpha)]}$$
for a duopolist ij, with the price of the final good normalized to 1

Dividends

- Write intermediate good-producing firms' profit flow as
  - $\Pi^m_{i1t} = \max_p\{y^m_{i1t}(p)(p - c_{i1t})\}$
  - $\Pi^d_{ijt} = \max_p\{y^d_{ijt}(p, p_{ikt})(p - c_{ijt})\}$
- Π^d_{i2t} is paid to households as a dividend; the representative household gets

$$\Pi_t = \int_0^{B_t} \varepsilon_{it}\Pi^d_{i2t}di$$

- Π^m_{i1t}, Π^d_{i1t} is paid to blueprint producers to pay for labor used to produce the blueprint in the first place
- This pins down wages

Labor Market Equilibrium

- Labor per capita is supplied by households: ℓ_{pt} and ℓ_{rt}
- Intermediate good producers demand labor at given wages

$$L_{pt} = \int_0^{B_t} (y_{i1t}/A_{i1t}) + \varepsilon_{it}(y_{i2t}/A_{i2t})di$$

  where $N_t \ell_{pt} = L_{pt}$ in labor market equilibrium
- To find ℓ_{rt}, use households' optimality condition
- To complete the model, let $\bar{\Pi}_{i1t} = EPV_{i1t}(\text{profit})$:

$$\bar{\Pi}_{i1t} = \mathbb{E}_t \left[\int_t^{t+T} e^{-(r+\delta_f)(s-t)} \Pi^m_{i1s} ds + \int_{t+T}^{\infty} e^{-(r+\delta_f)(s-t)} \Pi^d_{i1s} ds \right]$$

- Research firms: risk neutral, flow profit equals $\gamma \ell_{rt} \bar{\Pi}_{it} - w_t \ell_{rt}$, so

$$w_t = \gamma \bar{\Pi}_{i1t}$$

Equilibrium and Balanced Growth Path

- An equilibrium in this economy is standard: prices and allocations such that all households and firms optimize and markets clear
- A BGP is an equilibrium where output and the stock of varieties grow at a constant rate over time
- If η_t is such that ℓ_t is constant over time then BGP exists
- In the balanced growth path
    - Endogenous growth rate in the BGP equals the population growth rate g, as usual (Romer)
    - Productivity growth in industries affects output levels in the BGP
    - Patent policy generally exhibits inverted-U shape
- Our quantitative results below focus on the BGP

Calibration

Calibrated Parameters

Externally calibrated parameters		
ϕ	3	Frisch elasticity of labor supply
g	0.95%	Population growth rate
r	3%	Discount rate
δ_f	8%	Exit rate of firms
$[\underline{\mu}, \overline{\mu}]$	$[0, 7E[\mu]]$	Support of the distribution of μ
$[\underline{\theta}, \overline{\theta}]$	$[0, 10E[\theta]]$	Support of the distribution of productivity spillovers

Internally calibrated parameters		
$E[\mu]$	0.96%	Growth rate in productivity of a leading firm
$E[\theta]$	1.35μ	Productivity spillovers from leader to follower
σ	0.068	Size of the shocks to firms' productivity
β	0.62	Parameter for within-industry elasticity of substitution
α	0.75	Parameter for elasticity of substitution across industries
δ	2.2%	Depreciation rate of blueprints
σ_μ^2	0.20	Variance of μ
σ_θ^2	0.38	Variance of θ

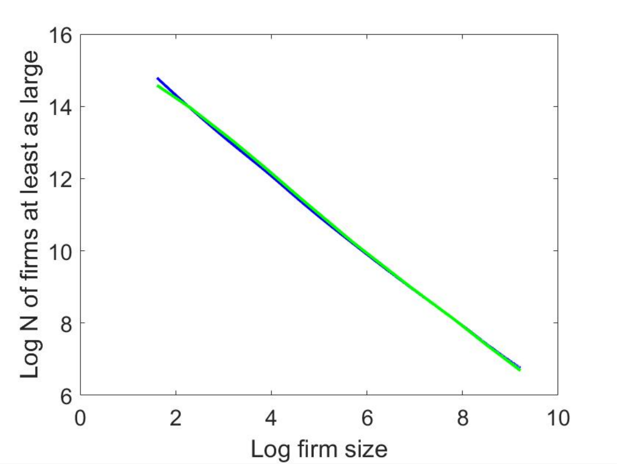
Targeted Moments and Values

	Target	Model
Annual GDP growth rate	$\approx 2.5\%$	2.48%
Average markup	≈ 0.50	0.493
Standard deviation of productivity	33%	31.9%
Price drop upon patent expiration	$\approx 35\%$	36.6%

- GDP growth rate is average for the US over the past 30 years
- Average markup is consistent with the literature (De Loecker et al. 2020, Vlokhoven 2022, Haltiwanger et al. 2022)
- Standard deviation of productivity is from OECD (2020)
- Average growth rate of firms' labor productivity is 1.6% per year
- Spillovers are set so the relative growth rate of laggard firms is between 2 and 3 times higher than leading firms
- μ and θ have a truncated log-normal distribution

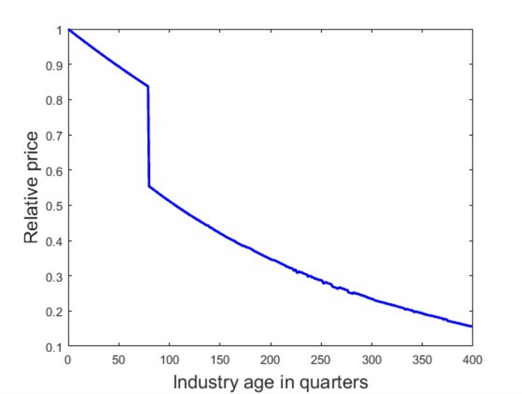
Size Distribution of Firms

We calibrate our model of heterogeneous spillovers to minimize the average error with respect to the size distribution of firms:



Price Drop upon Patent Expiration

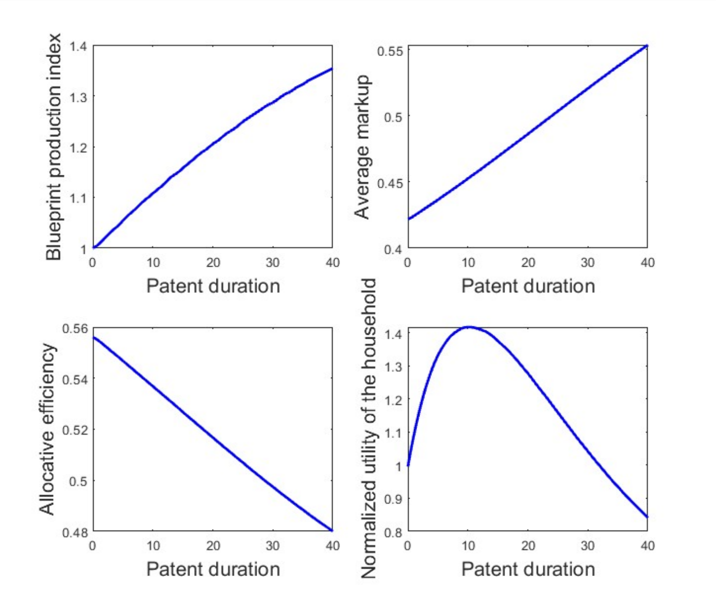
In line with Vondeling et al. (2018), our calibration implies a price drop upon the expiration of a patent of about 34%



- Example of expected prices posted across industry ages
- Price drops discontinuously when a patent expires after 80 quarters

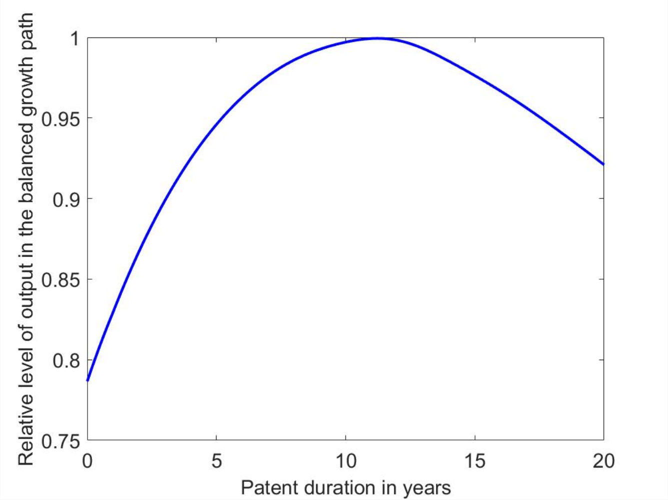
Results

Investment, Markups, Efficiency and Welfare

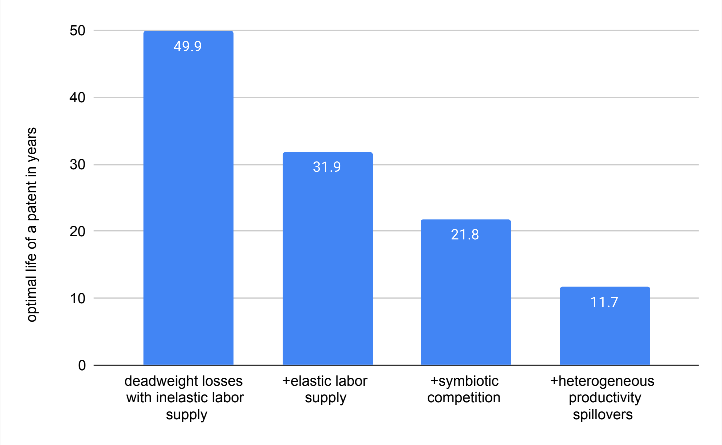


The Case Against Patents

No patents yield lower utility than the status quo policy of $T = 20$ years

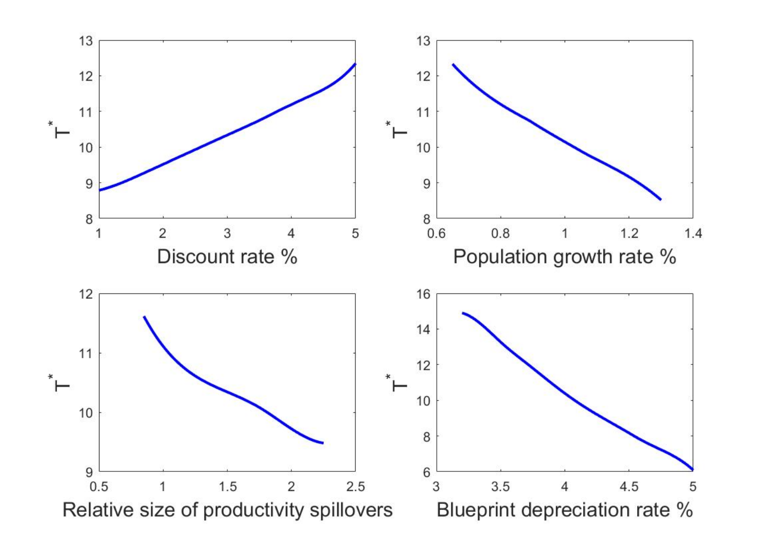


Decomposing the Optimal Life of a Patent



Nordhaus's market power effect ($\approx$ 20 years) is comparable to the symbiotic effect with spillover heterogeneity ($\approx$ 18 years)

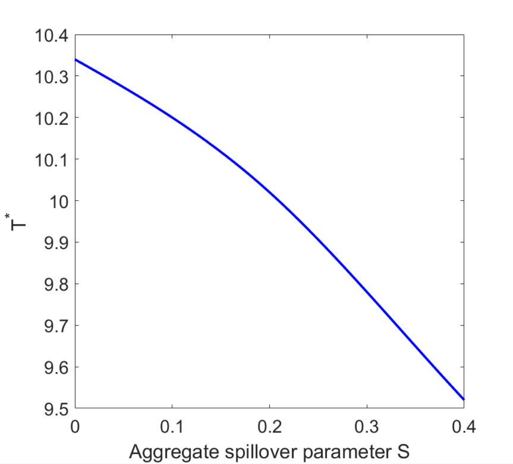
Robustness



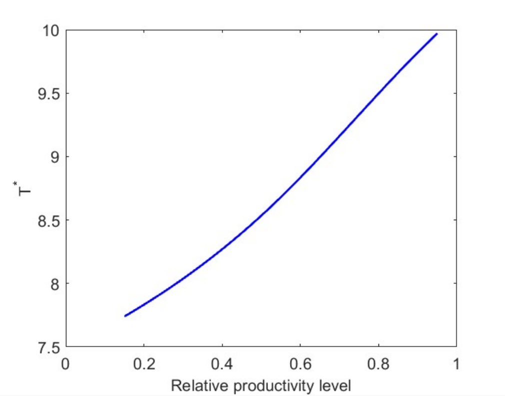
13
12
11
10
9
8
T*
1
2
3
4
5
Discount rate %
0.6
0.8
1
1.2
1.4
Population growth rate %
0.5
1.5
2
2.5
Relative size of productivity spillovers
16
14
6
3
3.5
4
4.5
Blueprint depreciation rate %

Endogenous Growth

We consider extending the model to incorporate endogenous growth à la Lucas (1988) but find a small effect, in line with Jones (1995)



Endogenous Firm-Level TFP Growth



Extending the model to allow for endogenous μ has a modest effect on T^* in our calibration (x axis is the ratio of μ in monopoly over duopoly)

Conclusion

Summary and Conclusions

- We introduce an endogenous growth model with dynamic, mutual imitation dubbed "symbiotic competition"
- $T^* \in [8, 14]$ years in our calibration
- Contra Boldrin and Levine's (2013) claim that patents are wasteful
  - We agree that profit from developing new technologies provides enough incentives for much innovation even without patents
  - But in our model intellectual property protection may improve welfare
  - Without patents, consumption is typically about 60 to 90% of the maximum level at optimal patent length in the balanced growth path
  - In our calibration, "no patents" is worse than "patents last 20 years"
- Knowledge spillovers create a counterweight to Nordhaus (1969)
  - Easier imitation can lead to shorter patents
  - Spillover heterogeneity: high-spillover industries matter more for reducing the life of a patent than low-spillover ones
  - **Symbiotic effect $\approx$ market power effect on $T^* \approx$ 20 years**