Slow Social Learning: Innovation Adoption under Network Externalities

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Motivation


LEAF


Z.E.
100%
ELEKTRISCH


WOB EU 447


TESLA
Electric vehicle recharging point only
At any time

Motivation

Concerns about electric vehicles in early 2010's:

- Do they work in practice? Extreme weather, durability...
- Can I find a charging station? Network effect.

This paper

Key features of the model:

— Small players choose when to adopt a new innovation.

— After adoption, get a flow payoff that depends on:

- Unknown but fixed state $\rightarrow$ Informational externality.
- Amount of adopted players $\rightarrow$ Dynamic payoff externality.

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Main insights:

- Good learning technology leads to slower learning.
  - Holds under positive payoff externalities.
  - Better learning technology $\rightarrow$ more informational free-riding.

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- Good learning technology leads to slower learning.
  - Holds under positive payoff externalities.
  - Better learning technology $\rightarrow$ more informational free-riding.
- Technical: can solve the equilibria by looking at a simple problem that ignores actions in the future.
  - Closed-form solution to a complex dynamic problem.

Literature

- Payoff externalities, no social learning:
  - Katz-Shapiro 1986, Jovanovic-Lach 1989, and Farrell-Saloner 1986... Industry equilibrium: Leahy 1993, Baldursson-Karatzas 1996.
- Social learning, no payoff externalities:
  - Large games with experimentation: Frick-Ishii 2020, Laiho-Murto-Salmi 2022.
- Two-player attrition games:
  - Decamps-Mariotti 2004, Thijssen-Huisman-Kort 2006, Akcigit-Liu 2016, Kwon-Xu-Agrawal-Muthulingam 2016, Margaria 2020.
- Tipping points (following Kemp 1976):
  - Many applications, e.g. investments in productive capital (Rob 1990) and resource consumption (Diekert 2017)...

Today

1. Model and solution concept (formal)
2. Statement of the main result (informal)
3. Argument for the main result (very informal)
4. How to make the argument formal...
   - ... and the interesting results that follow

Model

- Continuous time $t \in [0, \infty)$, discount rate r.
- A continuum of identical and risk neutral players choose when to irreversibly stop (adopt the innovation).
- Flow payoff after stopping

$$du_t^i = \pi(q_t, \omega)dt + \text{noise}$$

depends on:

  - $q_t \in [0,1]$ fraction of stopped players.
  - Unknown state of the world, $\omega \in \{H, L\}$; prior belief $Pr(\omega = H) = x_0$.

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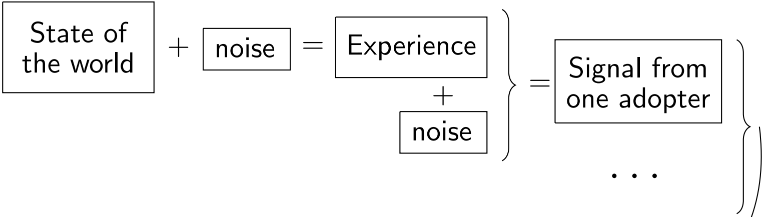
  - Unknown state of the world, $\omega \in \{H, L\}$; prior belief $Pr(\omega = H) = x_0$.
  - Assume: $\pi(q, H) > 0$, $\pi(q, L) < 0$, and π abs. cont. for all q. In this talk: $\pi_q(q, \omega) \geq 0$.

Model: learning

$$\left.\begin{array}{r} \left.\begin{array}{r} \boxed{\text{State of the world}} + \boxed{\text{noise}} = \boxed{\text{Experience}} \\ + \\ \boxed{\text{noise}} \end{array}\right\} = \boxed{\text{Signal from one adopter}} \\ \ldots \end{array}\right\}$$

Model: learning



Aggregate information in the game:

$$dU_t \sim N(q_t \pi(q_t, \omega)dt, q_t \sigma^2 dt).$$

- Interpretation: fix the total informativeness of the game and take the limit as the number of players $\to \infty$. Discrete model.
- Signal-to-noise ratio $\lambda_\sigma(q_t) := \frac{\sqrt{q_t}(\pi(q_t,H)-\pi(q_t,L))}{\sigma}$; assume $\lambda'_\sigma(q_t) > 0$.

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- Unconditional belief dynamics: $dX_t = \lambda_\sigma(q_t) x_t (1 - x_t) \, dW_t$.
  - W_t is a standard Wiener process.
  - Notation: upper case letters for random variables and lower case letters for realizations.

Markov Perfect Equilibrium

- Markov strategy $\xi_i : [0,1] \times [0,1] \to [0,1]$, from the belief and the stock of adopters to adoption probability.
- When all players follow Markov strategies, the stock Q_t is an increasing process adapted to U_t.

Player's stopping problem:

$$v(q_t, x_t) = \sup_{\tau} \mathbb{E}_{Q_t} \left[\int_{\tau}^{\infty} e^{-r(s-t)} \pi(Q_s, \omega) ds | (q_t, x_t) \right].$$

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Definition

A stock process Q_t is an equilibrium if

(I) $v(q_t, x_t) = \mathbb{E}_{Q_t} \left[\int_{\tau}^{\infty} e^{-r(s-t)} \pi(q_s, \omega) ds | (q_t, x_t) \right]$ whenever $dQ_t > 0$,

(II) $v(q_t, x_t) \geq \mathbb{E}_{Q_t} \left[\int_{\tau}^{\infty} e^{-r(s-t)} \pi(q_s, \omega) ds | (q_t, x_t) \right]$ whenever $dQ_t = 0$.

Benchmark result

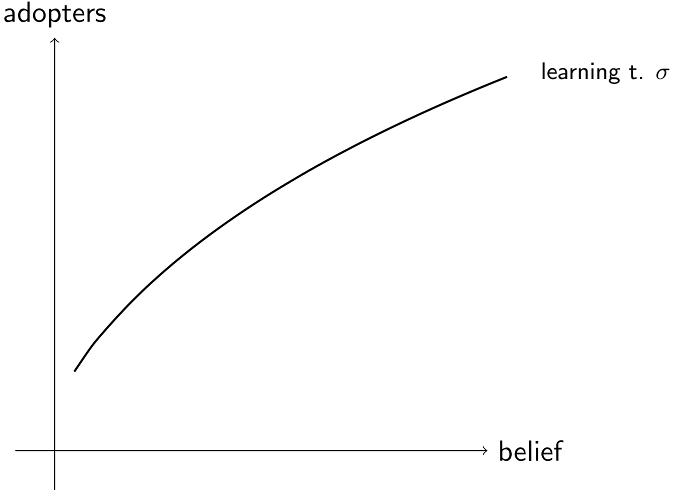
Proposition

Assume no payoff externalities: π independent of q. In equilibrium, the evolution of the belief is independent of the learning technology σ.

$\rightarrow$ Players postpone stopping under a better learning technology so much that it exactly balances out the effect of better technology.

(Almost) proof of the benchmark result

Suppose the equilibrium under learning technology σ is characterized as:



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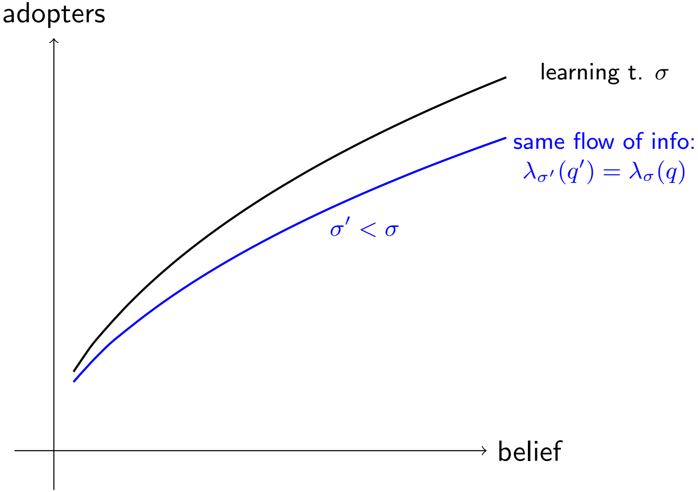

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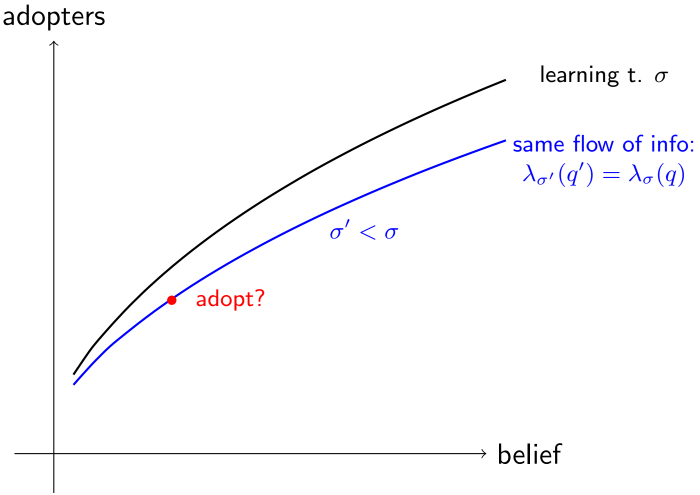

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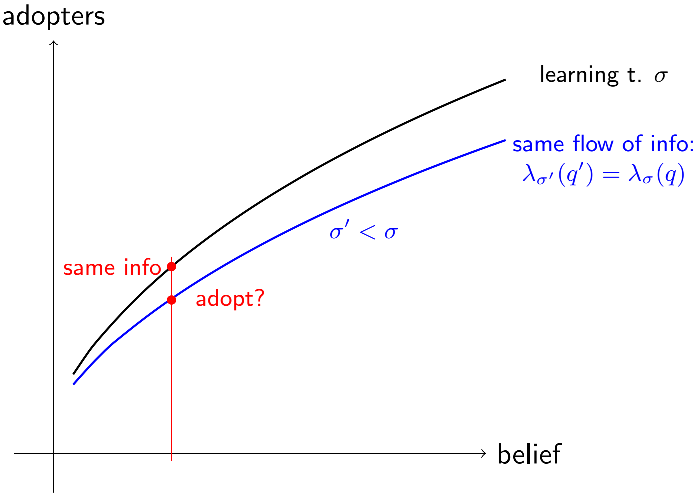

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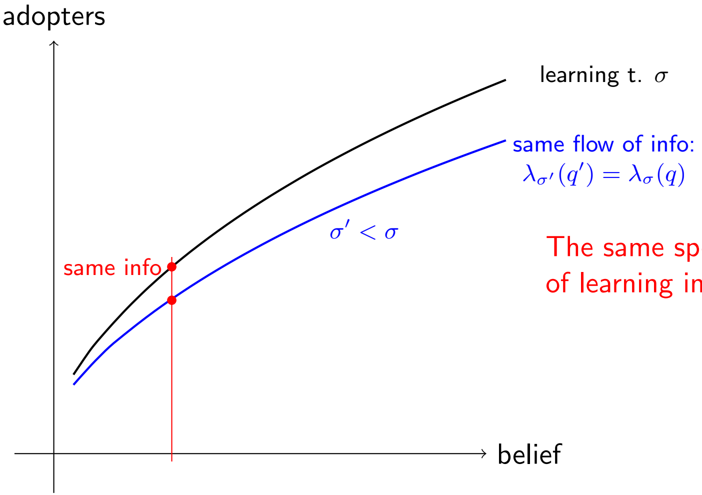


The same speed of learning in eqm

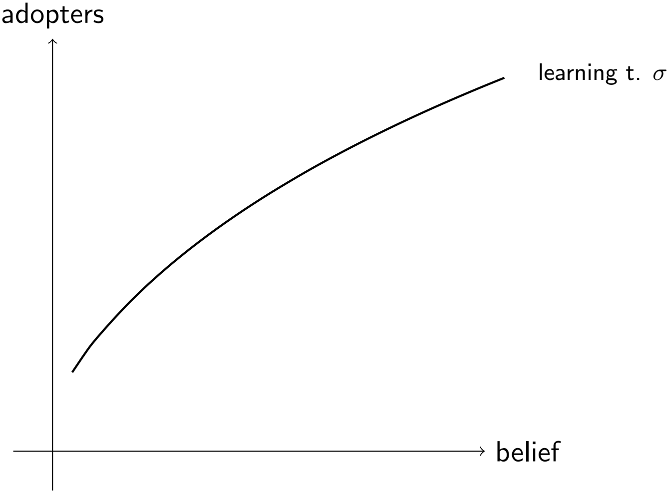
Main result

Proposition (informal Formal statement)

Assume strictly positive payoff externalities, and let $\sigma' < \sigma$*. Then, learning is strictly faster in the 'maximal equilibrium' under* σ *than in any equilibrium under* σ'*.*

$\rightarrow$ Faster learning and higher welfare under worse learning technology.

(Not a) proof of the main result



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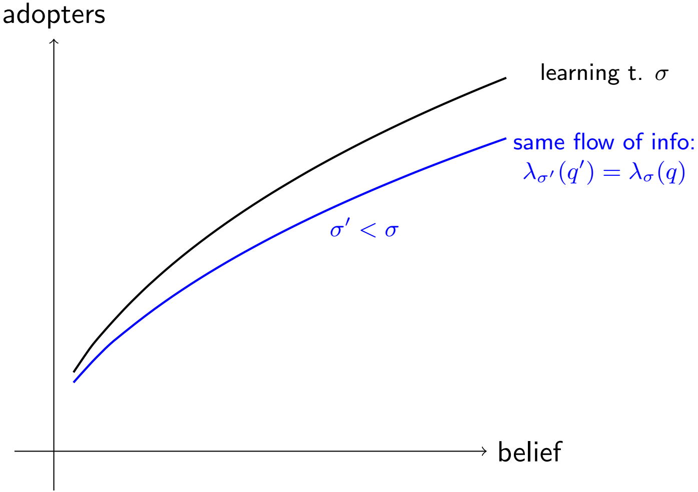

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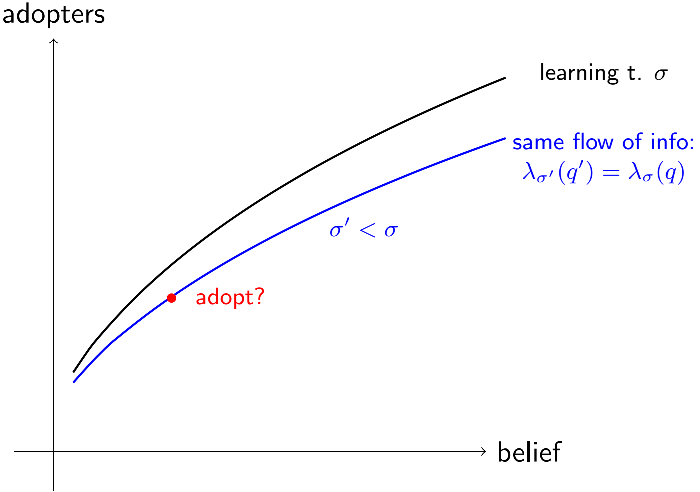

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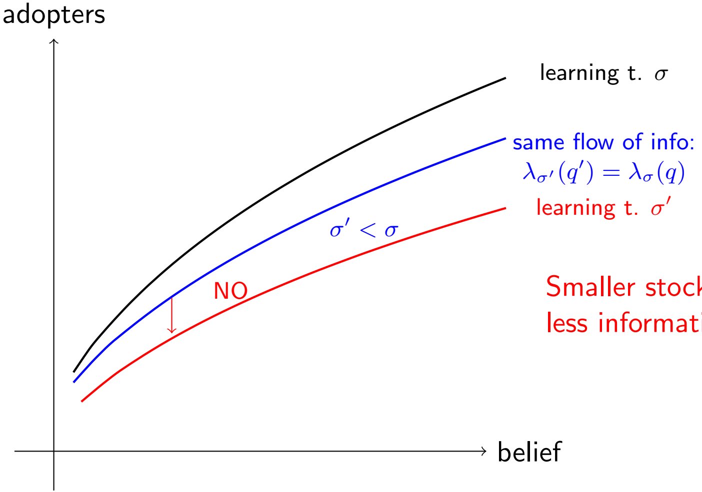
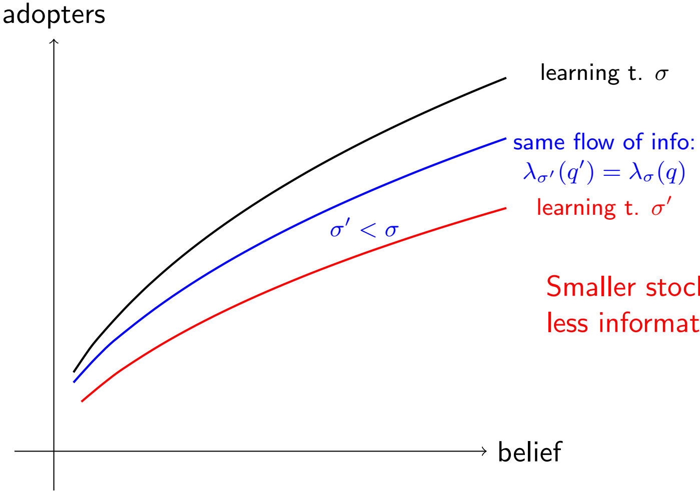


Smaller stock and less information

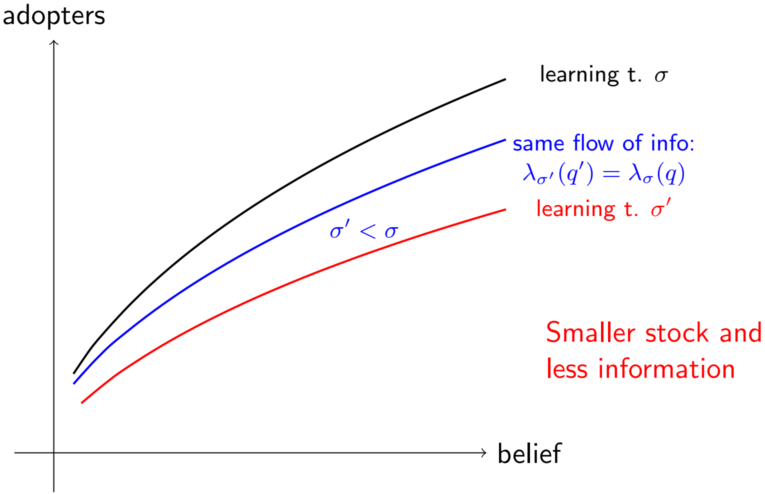
(Not a) proof of the main result



Smaller stock and
less information

→ Learning amplifies the need to subsidize new technologies with network effects.

(Not a) proof of the main result



What is missing in the argument?

- Existence and uniqueness.
- The argument is too static: ignores future adopters...

How to formalize the argument

1. Show that the equilibrium can be solved by considering a 'myopic problem' where players ignore future actions.
   - Intuition: future stopping happens only when the current stopping player would like to stop too.
   - Proof by applying the iterative elimination of strictly dominated strategies.
2. (Explicitly solve the equilibrium $\rightarrow$ cutoff rule for the 'maximal equilibrium')
3. Formalize 'faster learning': the maximal and minimal values of the belief are more extreme.
4. Complete the argument by showing that a lower cutoff implies slower learning.

Selected insights from the paper (1)

- Adoption of new technologies with positive network effects:
  - Better learning technology may slow down learning and hurt welfare.
  - Multiplicity of equilibria arises when 'coordination is more important than informational free-riding'.
- Equilibrium characterization works with any form of payoff externality.
  - Entry to a market of unknown size $\rightarrow$ negative payoff externality between the firms.

Selected insights from the paper (2)

Methodological aspect:

- Gradual learning is a good tool to analyze the joint effect of informational and payoff externalities.
- The solution technique is likely to generalize to other (endogenous) state diffusion processes.
  - E.g. the stock of adopters affects technological improvement.

Work in progress:

- Heterogeneous players
  - Inner-point optimum for learning technology.

Formal analysis: individual consumer's problem

$$\sup_{\tau} \mathbb{E}_{Q_t} \left[\int_{\tau}^{\infty} e^{-r(s-t)} \pi(Q_s, \omega) ds | (q_t, x_t) \right]$$

- Optimal stopping problem with two-dimensional state.
  - The equilibrium ties the dimensions together. illustration
- Future expectations:
  - Faster learning.
  - Payoff externality.

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- Future expectations:
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We show that the equilibria can be found by considering 'myopic' optimization.

- Find optimal stopping time with fixed $Q_t \equiv q$.

Myopic problem

Optimal stopping time with fixed $Q_t \equiv q$:

Definition (Myopic problem)

The myopic optimal stopping problem against a fixed stock q is

$$\sup_{\tau} \mathbb{E}_{Q_t \equiv q} \left[\int_{\tau}^{\infty} e^{-r(s-t)} \pi(q, \omega) ds | x_t \right],$$

where the belief evolves according to $dX_t = \lambda(q) X_t \left(1 - X_t\right) dW_t$.

- Standard one-dimensional stopping problem with parameter q.
- Solution is a cutoff rule: stop if $x \geq \bar{x}(q)$.

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- Standard one-dimensional stopping problem with parameter q.
- Solution is a cutoff rule: stop if $x \geq \bar{x}(q)$.

$$\bar{x}(q) := \frac{-\beta(q)\pi(q, L)}{(\beta(q) - 1)\pi(q, H) - \beta(q)\pi(q, L)},$$

where $\beta(q) := \frac{1}{2}\left(1 + \sqrt{1 + \frac{8r}{\lambda(q)}}\right)$.

Equivalence

- Define: $\hat{x}(q) := \max\{x \in [0,1] : x \leq \bar{x}(q') \quad \forall q' \geq q\}$.
  - The function $\hat{x}$ is the largest monotone function that has values below $\bar{x}$.

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Proposition 1

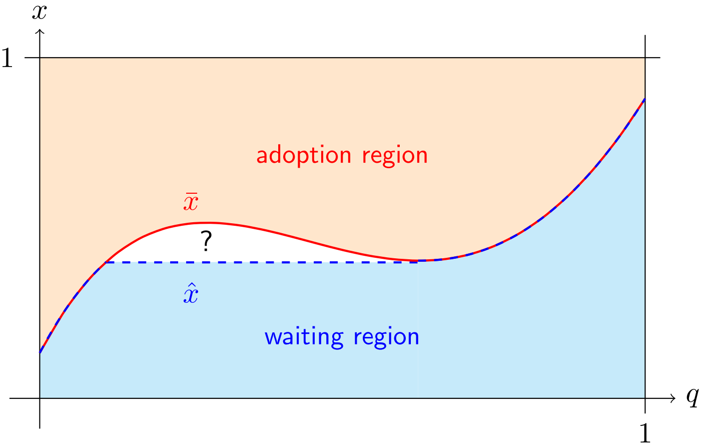
In any equilibrium, $dQ_t > 0$ if $x_t > \bar{x}(q)$ and $dQ_t = 0$ if $x_t < \hat{x}(q)$.

- In an equilibrium, players can ignore future expansions.
- Why? Because future expansions happen only when the player himself would stop too.
  - The equivalence between the actual and the myopic problem is an equilibrium property: would not hold against an arbitrary process Q_t.

Proof $\rightarrow$

Equilibrium

Proposition 1 graphically (notice that the axes are flipped!):



PROPOSITION 1: PROOF SKETCH FOR $x > \bar{x}(q)$

- Notice that the optimal stopping problem is equivalent to evaluating the following payoff:

$$\mathbb{E}_{Q_t}\left[\int_t^\tau e^{-r(s-t)}\pi(Q_s,\omega)ds|x_t,q_t\right]$$

  - Stop if positive for all τ.

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- For $x > \bar{x}(q)$, it is optimal to stop when Q is fixed and hence:

$$\mathbb{E}_{Q_t \equiv q}\left[\int_t^{\tau} e^{-r(s-t)}\pi(q,\omega)ds | x_t, q_t\right] \geq 0 \quad \text{for all } \tau.$$

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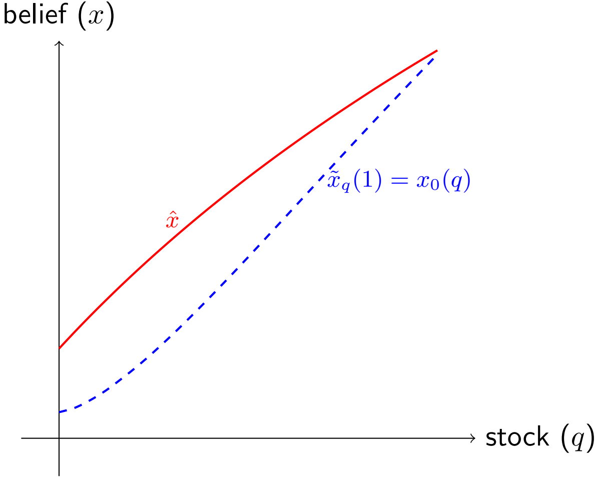
- Assume no-one stops $\rightarrow$ the optimal stopping condition is identical to the case with fixed $Q_t \equiv q$.

Proposition 1: proof sketch for $x < \hat{x}(q)$

- Iterative elimination of strictly dominated strategies:



$\tilde{x}_q(q')$: cutoff with information flow q and payoffs $\pi(q', \omega)$. Genaralization

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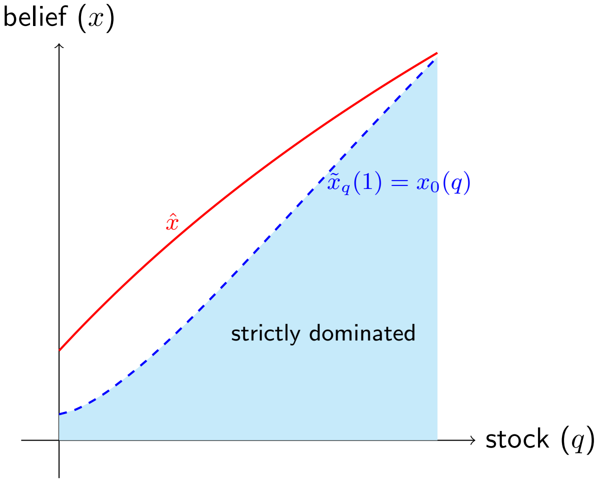

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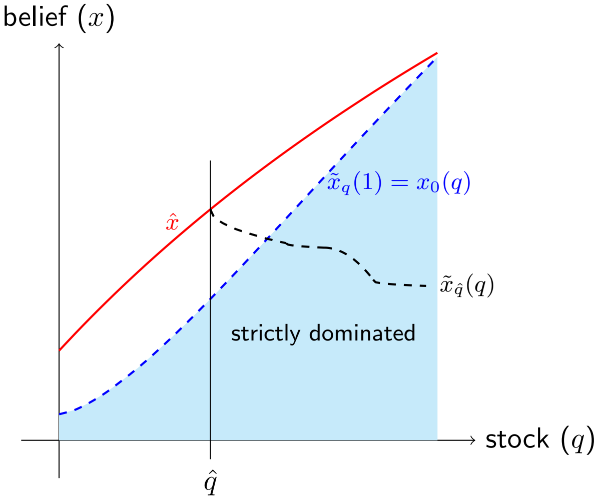

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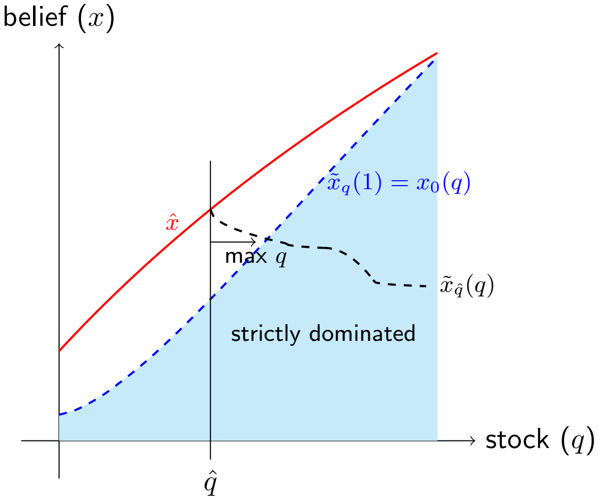

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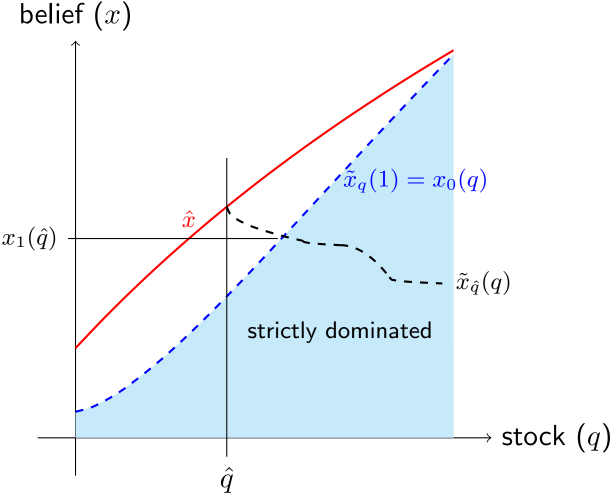

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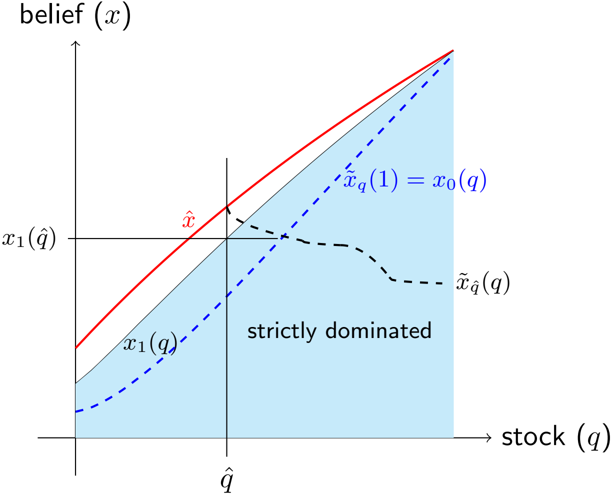

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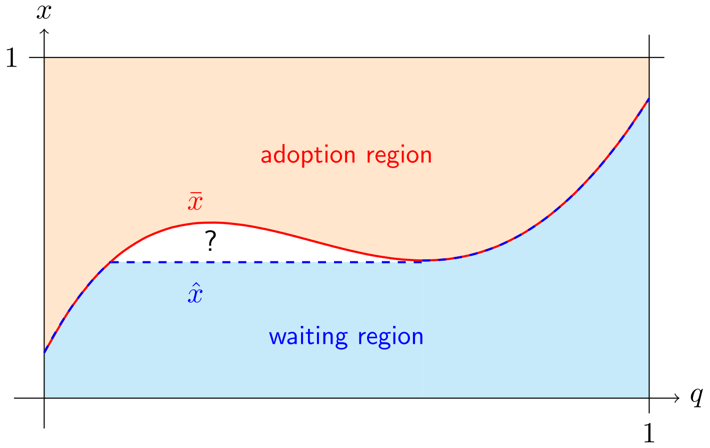
Equilibrium

Proposition 1 graphically:



$\rightarrow$ Equilibrium is unique when $\bar{x}$ is monotone.

Non-monotone cutoff belief

The myopic cutoff $\bar{x}$ is increasing in q:

— With negative and without payoff externalities.

— If the learning technology is good (noise term σ small).

- Informational externality dominates payoff externality.

The myopic cutoff $\bar{x}$ is decreasing in q if positive payoff externalities and little learning.

$\rightarrow$ Non-monotone cutoff when positive payoff externality and intermediate learning. Multiplicity of equilibria

Maximal and minimal equilibria

Definition

- The *maximal cutoff rule* is characterized by the cutoff rule $\hat{x}$: for any (x, q), $dQ = q' - q$ where $q' = \max\{s \geq q : \hat{x}(q) = x\}$.
- The *minimal cutoff rule* is characterized by the cutoff rule $\bar{x}$: for any (x, q), $dQ = q' - q$ where $q' = \min\{s \geq q : \bar{x}(q) = x\}$.

Lemma: Q_t defined by the minimal and maximal cutoff rules are equilibria.

Corollary: the maximal (minimal) cutoff equilibrium is the equilibrium with the fastest (slowest) adoptions.

Better learning technology

Recall the law-of-motion for the belief:

$$dX_t = \lambda_\sigma(Q_t)X_t(1 - X_t)dW_t, \tag{1}$$

Proposition 2 (formal)

Let $\sigma' < \sigma$. The following holds for all realization of Wiener process W_t in (1) under strictly positive payoff externalities:

(i) *Suppose the belief equals $x \in (0, \hat{x}_\sigma(1)]$ at time $t' > 0$ in the maximal equilibrium under learning technology σ'. Then the belief equals x at some $t < t'$ in the maximal equilibrium under learning technology σ.*

(ii) *Suppose the belief equals $x \in (0, \bar{x}_\sigma(1)]$ at time $t' > 0$ in the minimal equilibrium under learning technology σ'. Then the belief equals x at some $t < t'$ in the minimal equilibrium under learning technology σ.*

Proposition 2: proof sketch for maximal

— We already argued informally that the cutoff under σ' must be above the cutoff that leads to the same amount of information: $\lambda_{\sigma'}(q') = \lambda_\sigma(q)$.
  - The equivalence with the myopic problem formalized the argument.

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- Higher cutoff belief implies less variation for the belief:
  1. Static comparison yields: $\lambda_{\sigma'}(q'(x_0)) < \lambda_\sigma(q(x_0))$.

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  1. Static comparison yields: $\lambda_{\sigma'}(q'(x_0)) < \lambda_\sigma(q(x_0))$.
  2. Because x_t and x'_t reach new maxima always at the same time, the max of x_t stays above the max of x'_t as long as $\lambda'_t < \lambda_t$.
  3. λ'_t cannot reach λ_t as long as $x'_t < \hat{x}_\sigma(1)$ because then $q(x_t) > q'(x_t) > q'(x'_t)$ (the first equation need not hold if $x_t > \hat{x}_\sigma(1)$).

Better learning technology

Corollary

Assume strictly positive payoff externalities, and let $\sigma' < \sigma$.

- *The players are strictly better off in the maximal equilibrium under σ than in any equilibrium under σ' for all initial beliefs $x_0 \in (\hat{x}_\sigma(0), \hat{x}_\sigma(1)]$.*
- *The players are strictly worse off in the minimal equilibrium under σ' than in any equilibrium under σ for all initial beliefs $x_0 \in (\bar{x}_\sigma(0), \max_{q\in[0,1]} \bar{x}_\sigma(q)]$.*

Adoption patterns

Assume positive payoff externalities and intermediate learning technology:

- Belief dynamics create an S-shaped adoption curve.
- Better learning technology makes adoption more back-loaded.
  - Higher cutoff.
  - Slower learning.
- Stronger payoff externality makes adoption more back-loaded because of increasing payoff.

Heterogeneous players

Why to look at heterogeneity:

- Important in applications.
  - Robustness.
  - How to allocate subsidies.
- Some players strictly prefer waiting:
  - Direct benefit from endogenous learning in eqm.

Work in progress...

Extension: heterogeneous consumers

The same model as before, except:

- Players have types $\theta \sim U(0, 1)$.
- Flow payoff after stopping: $\pi(\theta; q, \omega)$.
    - Assume: $\pi_\theta(\theta; q, \omega) \geq 0$ for all θ, q, ω.

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— Flow payoff after stopping: $\pi(\theta; q, \omega)$.

- Assume: $\pi_\theta(\theta; q, \omega) \geq 0$ for all θ, q, ω.

— Skimming property holds: higher types stop first.

- One-to-one mapping between the stock and the type: $\theta(q) := 1 - q$.
- Flow payoff when $\theta(q)$ stops: $\pi(\theta(q); q, \omega) =: \pi_\omega(q)$.
- $\pi_\omega(q)$ can be increasing or decreasing or any mixture.

Equilibrium with heterogeneity

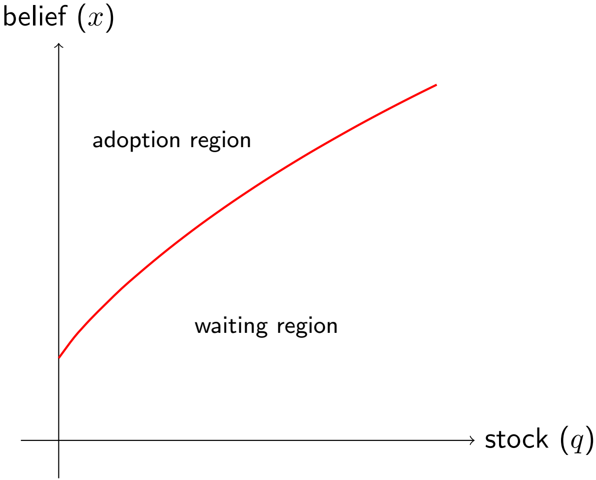
Proposition 1 generalizes: the equilibria are characterized by

$$\bar{x}(q) := \frac{-\beta(q)\pi(\theta(q); q, L)}{(\beta(q) - 1)\,\pi(\theta(q); q, H) - \beta(q)\pi(\theta(q); q, L)},$$

where $\beta(q) := \frac{1}{2}\left(1 + \sqrt{1 + \frac{8r}{\lambda(q)}}\right)$.

MPE process Q_t: illustration

Increasing boundary, discrete time illustration:



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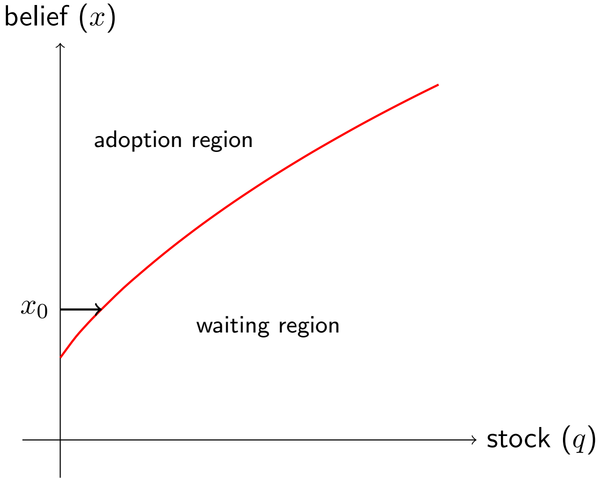

MPE process Q_t: illustration

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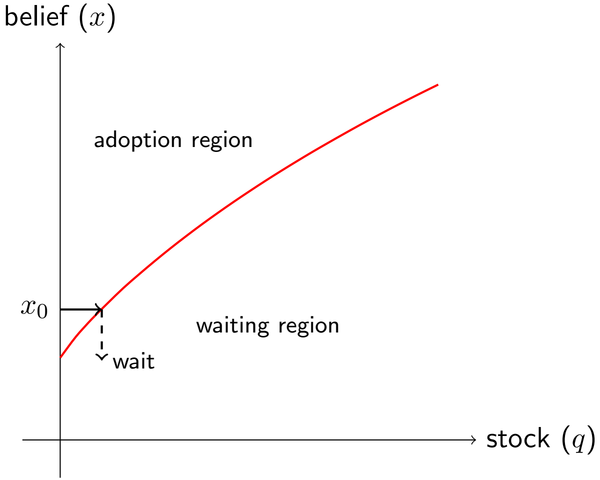
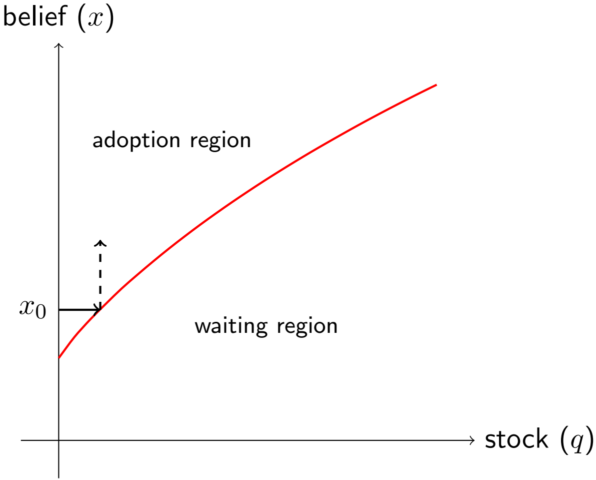


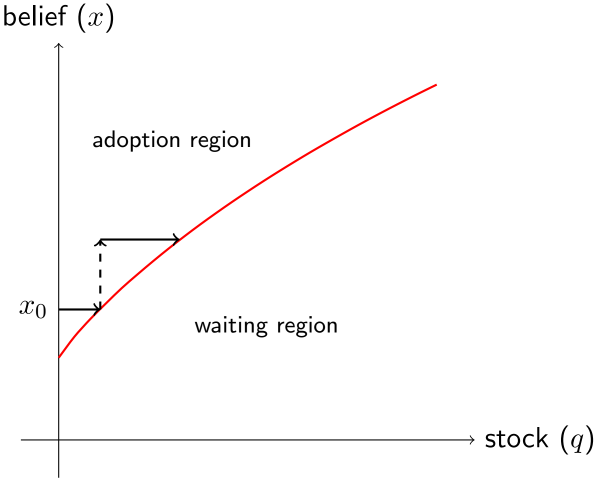
MPE process Q_t: illustration

Increasing boundary, discrete time illustration:



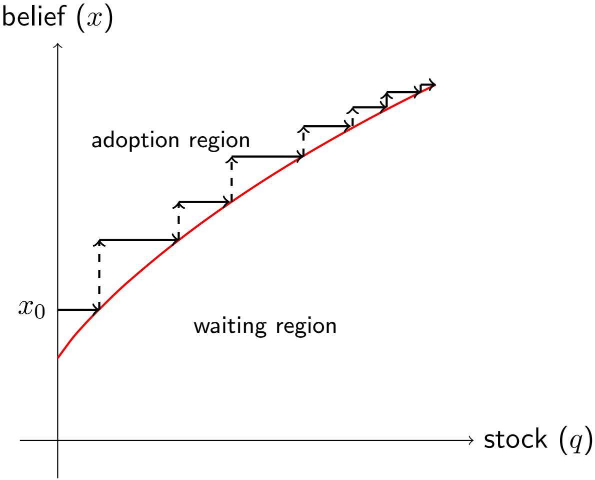
MPE PROCESS Q_t: ILLUSTRATION

Increasing boundary, discrete time illustration:



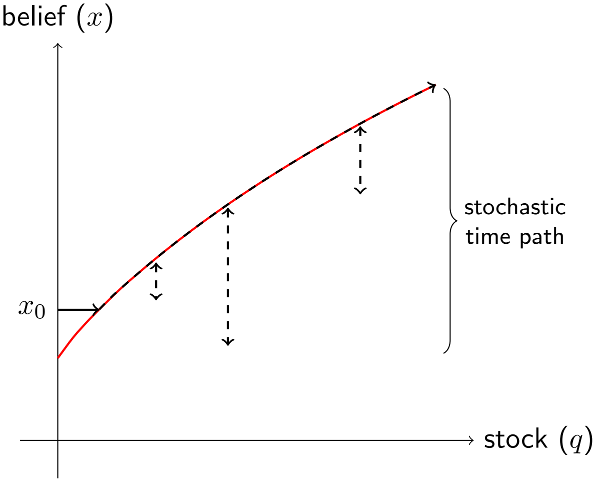
MPE process Q_t: illustration

Increasing boundary, discrete time illustration:



Expansion boundary

In continuous time model, the belief process is continuous a.s. and "jumps" are infinitesimally small.



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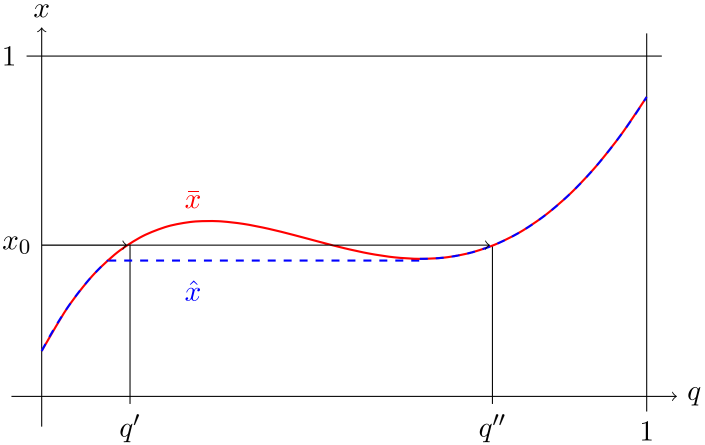
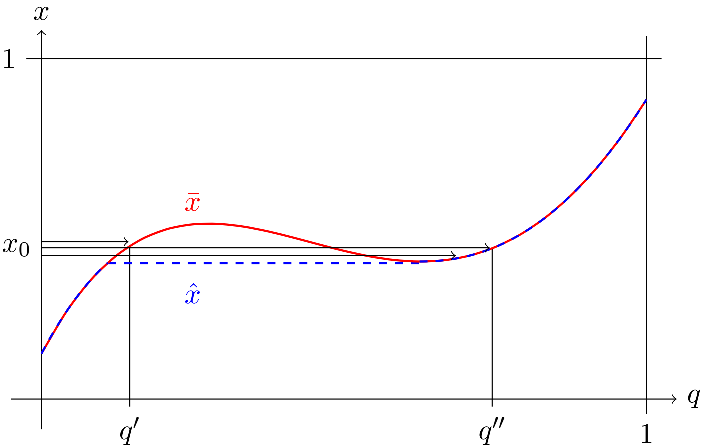
Multiplicity of equilibria

Suppose $\bar{x}$ is not monotone.



Players may coordinate to (q', x_0) or to (q'', x_0) from $(0, x_0)$.

Other equilibria



- Can construct eqm where players coordinate to (q', x_0) for beliefs just above x_0 and to (q'', x_0) for beliefs just below.
  - The number of adopters is non-monotone in the belief.

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References: Pictures of EV

- RENAULT: By M 93, Wikipedia curid=18496311
- NISSAN: By TTTNIS - Own work, Wikipedia curid=61748245
- VW: By M 93, Wikipedia curid=33508549
- TESLA: By Vauxford - Own work, Wikipedia curid=76762503

Discrete approximation

- Discrete time: $dt, 2dt, 3dt, \dots$.
- There are n players.
- In every period, each player who has stopped receives a conditionally iid. payoff:

$$u_t^i \sim N\left(\frac{\pi(q,\omega)dt}{n}, \frac{\sigma^2 dt}{n}\right).$$

- Let the current number of stopped agents be k:

$$\sum_{i=1}^{k} u_t^i \sim N\left(\pi(q,\omega)dt\frac{k}{n}, \sigma^2 dt\frac{k}{n}\right).$$

- The continuous time learning process follows as a limit when $n \to \infty$ and $dt \to 0$.
  - The stock captures the fraction of stopped players: $q = k/n$.

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Proof of Proposition 1: generalization to arbitrary externalities

— Define $\tilde{x}_q(q')$: cutoff with information flow q and payoffs $\max s \in [q, q'] \pi(s, \omega)$.

- Now, $\tilde{x}_q(q')$ is weakly decreasing in q' for all forms of π.

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Better learning technology

Recall the law-of-motion for the belief:

$$dX_t = \lambda_\sigma(Q_t) X_t (1 - X_t) dW_t, \tag{2}$$

Proposition 2

Let $\sigma' < \sigma$. The following holds for all realization of Wiener process W_t in (2) under strictly positive payoff externalities:

(i) *Suppose the belief equals $x \in (0, \hat{x}_\sigma(1)]$ at time $t' > 0$ in the maximal equilibrium under learning technology σ'. Then the belief equals x at some $t < t'$ in the maximal equilibrium under learning technology σ.*

(ii) *Suppose the belief equals $x \in (0, \bar{x}_\sigma(1)]$ at time $t' > 0$ in the minimal equilibrium under learning technology σ'. Then the belief equals x at some $t < t'$ in the minimal equilibrium under learning technology σ.*

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Better learning technology

Recall the law-of-motion for the belief:

$$dX_t = \lambda_\sigma(Q_t) X_t (1 - X_t) dW_t, \tag{3}$$

Proposition (formal)

Let $\sigma' < \sigma$. The following holds for all realization of Wiener process W_t in (3) under strictly positive payoff externalities:

- *Suppose the belief equals $x \in (0, \hat{x}_\sigma(1)]$ at time $t' > 0$ in the maximal equilibrium under learning technology σ'. Then the belief equals x at some $t < t'$ in the maximal equilibrium under learning technology σ.*

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