Regulatory Collateral Requirements and Delinquency Rate in a Two-Agent New Keynesian Model*

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Abstract

In light of the high levels of systemic risks and the elevated probability of a crisis occurring, understanding the effectiveness of macro-prudential policies is becoming increasingly crucial. We incorporate a collateral-based macro-prudential policy into a two-agent New Keynesian model, this policy adjusts counter-cyclically to the state of the borrowing sector. We show that regulators accommodate high delinquency rates by allowing for tighter collateral requirements. An active macro-prudential policy amplifies the impact of a monetary policy shock on output and labor supply, and this policy emerges as a potential tool to prevent the risk of delinquency in the short run.

Keywords: macro prudential policies, credit supply, collateral constraint, monetary policy
JEL Codes: E32, E44, G21

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1 Introduction

The capacity of borrowers to pay back their debt is a crucial consideration for banks when approving loans for firms or individuals. Market conditions and lack of collateral are also determinants of financial institutions' credit policies. A prominent paper by Rajan (1994) provides a theory of credit policies, where regulatory intervention and demand-side conditions can have unforeseen consequences on bank credit policy. However, if banks issued debt against collateral it is theoretically plausible that credit growth is also driven by the bank's collateral policy, yet the existence of this channel is intuitive and is difficult to validate. Generally, the main purpose of collateral policy is to protect banks from risk exposure, help in dealing with delinquent debt, and guarantee that the liquidation of collateral will be sufficient to cover their losses. When a loan is classified as uncollectible, banks can proceed to the liquidation of the collateral to recover such losses. In hard times, banks often experience a high level of delinquent debt, which erodes their liquidity and forces them to tighten their collateral policy. At the very least, banks will reduce lending and heighten the economic downturn.

In this paper, we focus on the macroeconomic implications of collateral-constrained agents. We formalize the idea and assume that borrowers own specific collateral while the willingness to lend depends on lenders' preferences for collateral. This is important because the bank's collateral policy will depend mainly on regulatory decisions and demand-side conditions, specifically the state of the borrowing sector. In response to the high delinquency rate,[1] banks adopt tightened collateral conditions, leading to a contraction in loan supply. Our goal is to understand how collateral affects aggregate over time and evaluate the effectiveness of collateral-based macro-prudential policy in preventing the default risk. The idea that collateral may represent an important source of macroeconomic fluctuations is widely discussed in compelling papers by Justiniano et al. (2015), Guerrieri and Iacoviello (2017), Mendicino et al. (2020), and Becard and Gauthier (2022).

We present a two-agent New Keynesian framework where borrowers can default on their loans and face a collateral constraint. A meaningful interpretation is that a debt contract is established between borrowers and lenders. The bank lends money and the borrower pledges collateral that serves as protection for the lender in case the borrower fails to pay their loans. There are three main ingredients to this model. First, there are two types of households, borrowers, and savers. Borrowers, for instance, are assumed to be collateral constrained. Second, firms are financially constrained and face collateral constraints when borrowing from banks.[2] Finally, there are regulators in this economy who can observe the delinquency rate on business and mortgage loans and adjust the level of the collateral requirement in each period according to their observation of the deterioration of the financial situation of borrowers. This is to say that changes in collateral requirements will be driven by borrowers' conditions (demand-side) and regulators' decisions (supply-side).

We estimate the model using US aggregate data and financial data over the period 1984Q1–2021Q4. The estimated model allows us to perform counterfactual exercises to assess the impact of collateral-based macro-prudential policy. We maintain the central assumption that regulators adjust their

[1] The terms delinquency rate and charge-off rate are used interchangeably. They both mean the percentage of loans that have been classified by banks as a loss relative to total lending.

[2] We hypothesized that firm debt is mainly asset-based. We are aware that decomposing firm debt into cash flow-based debt and asset-based debt will render our analysis consistent with the empirical evidence in Lian and Ma (2021). Nevertheless, we believe our formulation of the profit-maximizing firm that faces a collateral constraint is a fair approximation. For instance, if firms that pledged future cash flows against debt cannot meet their interest payments, those firms may either reschedule the debt, or issue new equity or debt, or sell their own assets. We see the latter case applies the same logic as the asset-based debt.

collateral requirements based on the borrower's financial conditions in the baseline model and study the responses of the collateral requirements when the delinquency rate increases. We investigate the outcome of positive collateral requirements for business and mortgage loans on output and labor. We also compare the impact of a contractionary monetary policy on output and labor under the assumption that regulators can observe the charge-off rate level and hypothesize what would happen to output and labor if regulators were not aware of the delinquency rate level. We finally examine the effects of a contractionary monetary policy shock on the delinquency rate under the presence of a collateral channel.

The main results lead to four main conclusions. First, when the condition of the borrowers deteriorates, regulators have an incentive to lower their collateral requirements. Second, a macro-prudential policy that attempts to raise the collateral requirements and expand credit to businesses and households would boost aggregate demand, causing output and labor to increase. Third, an active collateral policy amplifies the responses of output and labor to a monetary policy shock. Finally, and most importantly, with the presence of a collateral channel a contractionary monetary policy can be effective in preventing the risk of delinquency in the short run, as it leads to a decline in the level of charge-off rate level on business and mortgage loans.

A large literature exists that relies on incorporating financial intermediation into mainstream macroeconomic models to improve both policy analysis and predictions of a dynamic general equilibrium model, including classic models such as Bernanke and Gertler (1989) and Bernanke et al. (1999), and Christiano et al. (2014). This work relates to several important papers that focus on financial frictions and banking intermediation. These papers include Brunnermeier and Sannikov (2014), Curdia and Woodford (2016), Jermann and Quadrini (2012), Jakab and Kumhof (2015), He and Krishnamurthy (2012), Bianchi and Saki (2022), and Akinci and Queralto (2022). This literature considers the effect of various policies on financial stability and emphasizes the importance of financial regulations and banking system efficiency. However, most of these papers only consider the impact of monetary policy and financial regulations on the economy and are abstract from the collateral channel. Instead, in our paper we focus on the effects of macro-prudential collateral policy under the assumption that households and firms are financially constrained. Our assumption of collateral-constrained agents allows us to accommodate a macro-prudential collateral policy that depends both on the regulator's decisions (supply-side) and on the borrower's conditions (demand-side). Additionally, this assumption helps us to assess the effectiveness of collateral policy in preventing the risk of delinquency.

Several influential contributions to the literature on New Keynesian models with financial frictions are pioneered by Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Iacoviello (2015), Justiniano et al. (2015), Guerrieri and Iacoviello (2017), and Gertler et al. (2017). This literature has emphasized the importance of modeling financial frictions into macroeconomic models. In this paper, we build on this large literature and we introduce the financial intermediation sector into a two-agents New Keynesian model under the assumption that borrowers in this economy are collateral-constrained and may default on their loans. We contribute to this literature by accounting for a regulator that conducts an active collateral policy that considers the changes in the severity of the borrower's condition. We demonstrate that a mix of macro-prudential collateral policy and monetary policy emerges as a potential tool to prevent the risk of delinquency on business and mortgage debt in the short run. These quantitative findings are related to a series of papers analyzing the effects of macro-prudential policies, such as Kim and Mehrotra (2018), Franz (2020), Aikman et al. (2021), Martin et al. (2021), Van der Ghote (2021), and Ottonello et al. (2022).

Relatedly, this study contributes to the literature on collateral and its macroeconomic impact. Past papers highlighting the role of collateral include Williamson (2016) and Geanakoplos and Zame (2007). Recent work on collateral shocks can be found in Becard and Gauthier (2022). They have developed a macroeconomic model with heterogeneous agents, in which both households and firms face collateral constraints. Our paper differs to a large extent because we go beyond the analysis of the macroeconomic effects of a collateral shock. We instead consider a macro-prudential collateral policy that depends both on the demand and supply-side. Indeed, regulators in our model can conduct a rigid collateral policy. They can also monitor the changes in the severity of borrowers' conditions and adjust their collateral requirement actively. Related work by Guerrieri and Lorenzoni (2017) on borrowing limits identifies the channels by which the shock to borrowing limits propagates. Several contemporaneous papers investigate optimal capital requirements Mendicino et al. (2020) and Ottonello and Song (2022). These literature also includes empirical studies, such as Christensen et al. (2009), Alpandaa and Zubairy (2017), Lambertini et al. (2013), Rubio and Carrasco-Gallego (2014), Bianchi and Mendoza (2018), and Drechsel and Kim (2022). Our contributions to this growing literature are (i) a tractable two-agent New Keynesian model with a macro-prudential collateral policy, which we (ii) use to understand the collateral channel and (iii) show that this channel can lead to the amplification of monetary policy shocks.

The paper proceeds as follows. Section 2 lays out key insights into the collateral policy of banks. Section 3 presents the theoretical model. Section 4 describes the estimation results and tests the model fit. Section 5 discusses the effects of a high delinquency rate on collateral, highlights the mechanisms that lead to a tightening of collateral requirements, and analyzes the impact of collateral requirements on the real economy. Section 6 examines the effects of monetary policy in the presence of a collateral channel. Section 7 concludes.

2 Understanding Banks' Collateral Policies

In this section, we present some stylized facts on the dynamics of credit supply and collateral liquidation in the US. We also provide empirical evidence of the inverse relationship between loan supply and loan charge-off rate. We exploit bank level data to examine the effects of tighter collateral terms on credit supply. We then investigate whether banks and/or regulators alter their collateral policies by adjusting the collateral requirement to the loan charge-off rate.

2.1 Uncollectible Debt and Collateral Liquidation: Some Stylized Facts

As noted previously, credit supply in an economy is essentially determined by the regulator's collateral policy. Changes in collateral requirements lead to either the expansion or the contraction of the credit supply. Figure 1 displays the existence of an inverse relationship between total debt with the net charge-off rate. The vertical axis measures the log of total debt. Total debt corresponds to both commercial and industrial loans and real estate loans. The horizontal axis represents the log of the net charge-off rate. The observation is quarterly and represents the aggregated data at the national level from the Federal Deposit Insurance Corporation (FDIC) data. Given the high level of default, a regulator can anticipate loan loss and change their collateral policy, which will cause a decline in credit supply to firms and individuals. Also, it is most likely that when the charge-off rate level is high, banks may experience a decline in liquidity, which will force them to adopt a tight collateral policy and thus reduce loan supply.

Considering the relationship between total debt and net charge-off rate, it is natural to think of the inverse relationship between these two variables as the result of tight credit conditions. Indeed, when banks observe the deterioration of the borrower's financial condition, the main implication is that banks will decrease the level of collateral requirement, which will translate into a contraction in credit supply.

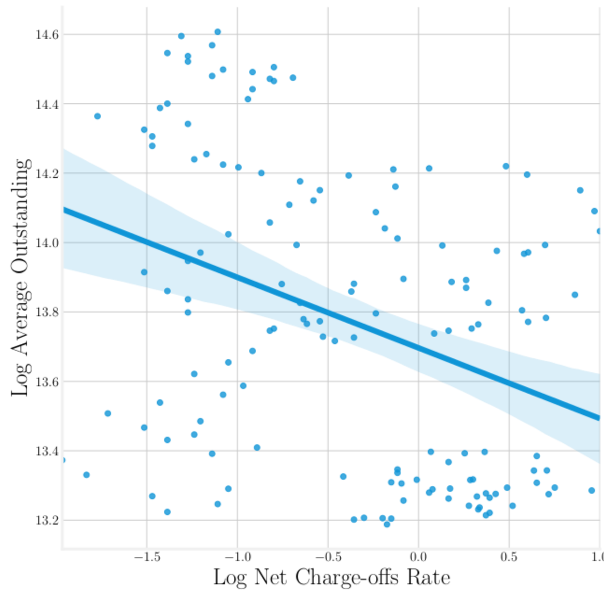


Figure 1: Correlation between Average Outstanding Debt and Net charge-off rate

Notes: Quarterly data retrieved from the Federal Deposit Insurance Corporation (FDIC); Sample: 1984Q1-2021Q4

In general, banks facing a high level of loan charge-off are also subject to a drying up of liquidity. In this situation, banks can repossess and liquidate the collateral and recover loan losses. The dynamics of collateral liquidation are shown in Figure 2. We observe that collateral liquidation lagged the delinquent business and mortgage debt. This is evident during the economic downturn, with collateral liquidation peaking two years after a higher level of bad debt. Total recoveries peaked in 2004 and 2013 and decreased in subsequent years.

2.2 Collateral Requirement Shocks: Bank-level Evidence

To show how collateral requirement tightness affects loan supply, we construct a measure of collateral requirement tightness

$$\Delta\theta_t = \theta_t - \theta_{t-1} \tag{2.1}$$

where θ_t represents the percentage of banks reporting tightness in collateral requirements. Changes in θ_t reflect relaxed or tight collateral conditions using the Small Business Lending Survey (SBLS) of the Federal Reserve Bank of Kansas City. An increase (decrease) in $\Delta\theta_t$ reflects tighter (more relaxed) collateral conditions. We linked this indicator with the Federal Deposit Insurance Corporation (FDIC) data over time and at the bank level. We then estimate the quantile model of the form

$$Q_{m_i|\Delta\theta_i}(\tau \mid \Delta\theta_i) = \alpha_\tau \Delta\theta_i^T + \epsilon_{i,t} \tag{2.2}$$

where $\Delta\theta_i$ is the observed collateral requirement tightness indicator over the period 2017Q4-2021Q3, $\tau \in (0,1)$ is the τ^{th} quantile of m_i. $Q_{m_i|\Delta\theta_i}(\tau \mid \Delta\theta_i)$ is the conditional τ^{th} quantile loan supply given collateral requirements $\Delta\theta_i$. α_τ is the coefficient that measures the impact of collateral requirements on loan supply m at the bank level i for a given τ.

Figure 3 presents evidence of heterogeneity in the banks' credit supply with respect to tightness in collateral requirements. In fact, banks in the upper tail of the conditional credit supply

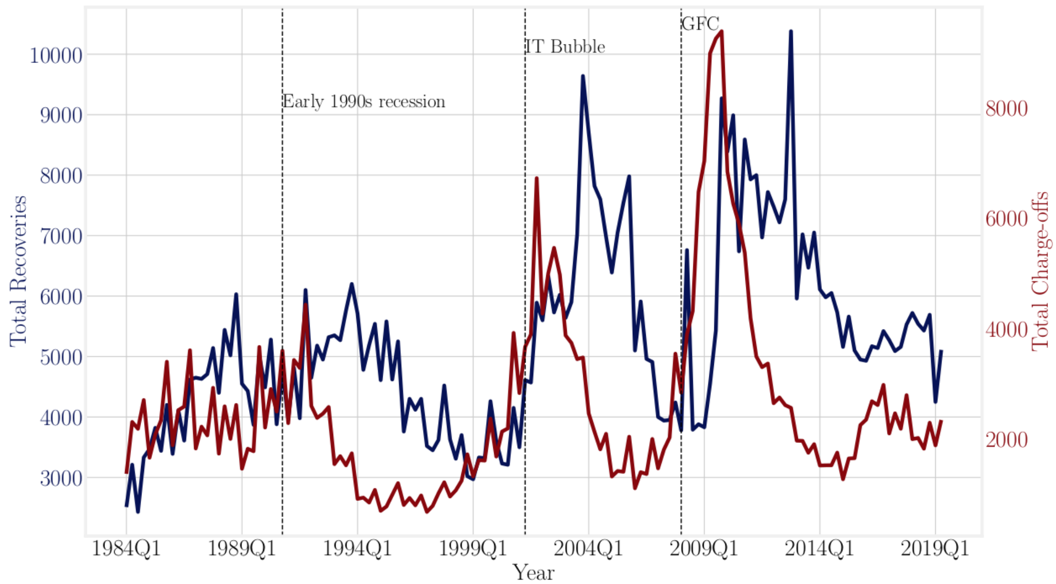


Figure 2: US Banks' Total Recoveries on CI and HH Loans

Notes: Quarterly data retrieved from Federal Deposit Insurance Corporation (FDIC); Sample: 1984Q1-2021Q4

Table 1: Quantile-regression Estimates

	Coefficient	Lower Bound	Upper Bound
Quantile-regression			
0.1	0.00	-8.82	8.82
0.2	0.00	-12.80	12.80
0.3	-26.41	-44.61	-8.20
0.4	-112.20	-136.38	-88.02
0.5	-191.28	-221.45	-161.10
0.6	-269.61	-305.54	-233.67
0.7	-343.58	-385.18	-301.99
0.8	-412.69	-460.36	-365.02
0.9	-491.66	-545.87	-437.46
Ordinary least squares			
$\Delta\theta$	-191.28	-221.45	-161.10
Observations	84004		

Note: The table shows the estimates of quantile regression: $Q_{m_i|\Delta\theta_i}(\tau \mid \Delta\theta_i) = \alpha_\tau \Delta\theta_i^T + \epsilon_{i,t}$, and the estimates of ordinary least squares: $m_{i,t} = \beta\Delta\theta_{i,t} + \epsilon_{i,t}$.

distribution face a significant decrease in credit supply after tightening collateral conditions. On the other hand, there is little effect of collateral conditions on credit supply for lower quantiles. For higher quantiles, tightness in collateral terms leads to a large decline in credit supply (varying between -200 and -600), suggesting that those banks who lend more appear to be penalized by the tightness of collateral conditions.

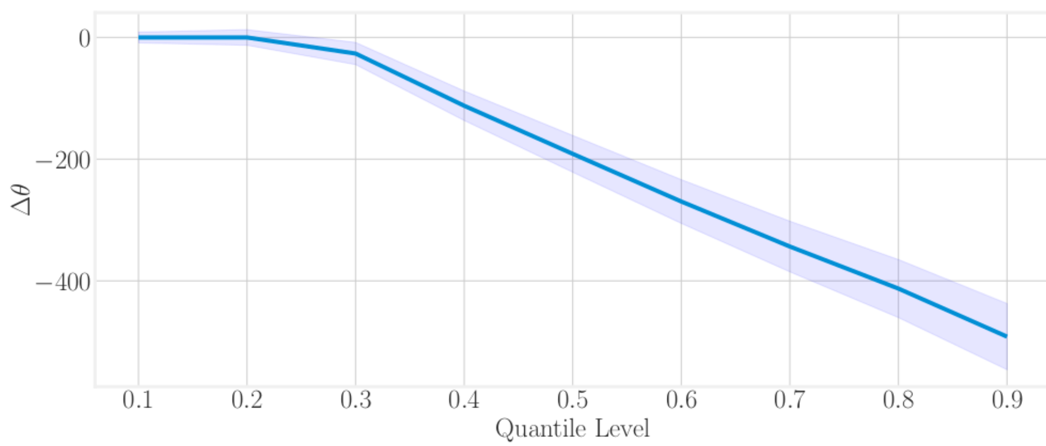


Figure 3: Quantile-regression Estimates of α_τ

Notes: Estimates of the effect of tightness in collateral conditions on bank's loan supply conditional (by quantile)

Table 1 presents empirical estimates of (2.2). The results reveal a significant negative relationship between tightness in collateral conditions and loan supply. Loan supply declines significantly when collateral conditions become tighter at the upper tail of the conditional credit supply distribution. However, loan supply is barely affected by collateral conditions at the lower tail of the conditional credit supply distribution.

Comparing the quantile regression results with the ordinary least squares regression in Table 1, we observe that the latter underestimates the effects in the higher quantiles and overestimates the effects in the lower quantile. Another advantage of quantile regression is that it is more resilient to outliers than the simple ordinary least squares method (Koenker, 2017).

2.3 Collateral Policy of Banks: An Intuitive Channel

A recent survey from the Federal Reserve Bank of Kansas City (see Figure 4) reveals that more than 25 percent of banks reported tight collateral requirements in the middle of the COVID-19 crisis, which temporarily increased the strain on borrowers, before decreasing to 5 percent at the end of 2021. This observation suggests that banks pursue a tight collateral policy during uncertain times, with collateral requirements typically varying. A bank becomes more prudent during hard times because the level of risk reaches higher levels. After observing the level of default, banks can tighten their collateral conditions to reduce the bank's exposure to risk. Whereas in normal times, banks have loose policies when the level of risk is at the lowest level.

In practice, banks can observe the delinquency rate on loans and may attempt to change or maintain their collateral requirements. We assume that the change in bank collateral policy is correlated with changes in demand-side conditions, such as the default probability of borrowers, which is significantly higher during an economic downturn. It is straightforward to engage a collateral policy rule to understand its dependence on the state of the economy, presumably the level of default rate, which can be a suitable proxy for the deterioration of borrower's conditions. We adopt the identifying assumption that collateral requirements will depend on the previous level of collateral requirements and the default rate with an exogenous process given by

$$\phi_t^h = \rho_{\phi h}\phi_{t-1}^h + (1-\rho_{\phi h})\alpha_h \mathcal{X}_{t-1}^h + \epsilon_t^{\phi h}$$

Here ϕ_t^h denotes the collateral requirement imposed on households loans, $\rho_{\phi h}$ is the collateral policy smoothing parameter, α_h denotes the collateral policy weight on the loan delinquency rate,

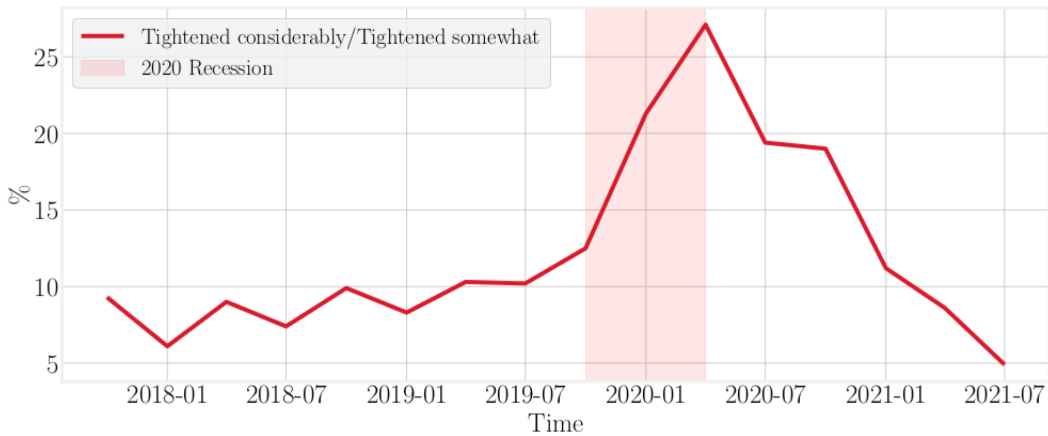


Figure 4: Percentage of Banks Reporting a Change in Collateral Requirement

Source: Federal Reserve Bank of Kansas City. Small Business Lending Survey - Aggregate Data (Section D.2)

$\mathcal{X}^h_{t-1}$ represents the delinquency rate on household loans, and $\epsilon^{\phi h}_t$ is the innovation to the collateral requirement, which is an i.i.d. with mean zero and variance $\sigma_{\phi h}$. The collateral requirement on firms is

$$\phi^e_t = \rho_{\phi e}\phi^e_{t-1} + (1-\rho_{\phi e})\alpha_e \mathcal{X}^e_{t-1} + \epsilon^{\phi e}_t$$

We assume that the capital requirement depends on the last period level and the default rate on firm loans $\mathcal{X}^e_{t-1}$. The parameter $\rho_{\phi e}$ is the collateral policy smoothing parameter, α_e denotes collateral policy weight on loan delinquency rate, and changes in collateral requirement are proxied by the innovations $\epsilon^{\phi e}_t$. The insight behind these two equations is that the collateral requirement imposed by banks on borrowers is mechanically linked to the percentage of total loans that have been charged-off.

We view our paper as complementary to previous work on the optimal loan-to-value ratio. Related work by Alpandaa and Zubairy (2017) introduced a regulatory loan-to-value ratio to address households' indebtedness and studied the optimal value for regulatory collateral requirements, whereas Lambertini et al. (2013) and Rubio and Carrasco-Gallego (2014) defined a loan-to-value ratio that responds countercyclically to credit growth. We depart from this literature and assume that collateral requirements respond to the delinquency rate on business and mortgage debt. This is an important point and hence the collateral requirements will be driven by the change in the severity of the borrowing sector (demand-side) and changes in regulators' decisions (supply-side). This assumption regarding collateral requirements helps us to assess whether a macro-prudential collateral policy could be effective in preventing the risk of delinquency.

But do collateral requirements mitigate the default risk? To answer this question, we develop a macroeconomic model to investigate the effects of collateral requirement changes on credit supply and the probability of default. The model assumes that agents are collateral constrained and that regulators will set the level of collateral requirements after observing the percentage of total loans that have been charged-off, which provides information about the deterioration of the financial situation of borrowers. We will also examine the implications of collateral requirements on the macroeconomy and whether a mix of monetary policy and collateral policy can prevent the risk of default.

3 A Model with Regulatory Collateral Requirements

The model developed in this paper shares the key features of New Keynesian models with the financial sector, including forward-looking agents who live infinitely and maximize their profits in a discrete-time economy. These agents maximize their utility and profits with the presence of nominal rigidities in prices and wages. The main agents in this framework are two types of households: constrained and unconstrained households, bankers, entrepreneurs, firms, labor contractors, regulators, and a central bank. In equilibrium, all households, firms, and banks behave optimally, and all markets are clear. To conserve space, we relegate the full derivation of the model to the Appendix A. In this setting, the key ingredients are: (i) the presence of collateral-constrained households; (ii) collateral-constrained entrepreneurs, (iii) and a regulator that can conduct a liberal or tight collateral policy that generates macroeconomic fluctuations. Below, we will illustrate how this feature is important for the central bank's monetary policy and how it mitigates the default risk.

Unconstrained Households. Consider households that maximize their lifetime utility by providing labor services, consuming the final goods, and making deposits. All households in this economy have identical preferences, which take the form

$$E_0\Sigma_{t=0}^{\infty}\beta^t\left\{\zeta_{c,t}(\log(c_{u,t}-b_u c_{u,t-1}))-\psi_l\frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l}\right\} \tag{3.1}$$

subject to $p_t c_{u,t}+d_t \leq w_t l_{u,t}+(1+r_t)d_{t-1}$, where $0<\beta<1$ is the discount factor, $c_{u,t}$ is the per capita consumption, d_t is the deposit, w_t is the hourly earnings, $l_{u,t}$ represents the hours worked, and b is the internal habit in consumption. The parameter σ_l is the curvature on the disutility of labor, and ψ_l is the disutility weight on labor. Consumers face shocks to consumption preferences with the preference shock $\zeta_{c,t}$, which is assumed to evolve as follows $\zeta_{c,t}=\rho_{\zeta_c}\zeta_{c,t-1}+\epsilon_t^{\zeta_c}$. The consumer decides on the level of consumption, deposit, and labor with the marginal utility $\lambda_{u,t}$.

Collateral-Constrained Households. Financially constrained households chooses consumption $c_{c,t}$, hours worked $l_{c,t}$, debt d_t^h and assets h_t, to maximize their expected lifetime utility

$$E_0\Sigma_{t=0}^{\infty}\beta^t\left\{\zeta_{c,t}(\log(c_{c,t}-b_c c_{c,t-1}))-\psi_l\frac{(l_{c,t})^{1+\sigma_l}}{1+\sigma_l}+\chi\log(h_t)\right\} \tag{3.2}$$

subject to the flow budget constraint $p_t c_{c,t}+q_t^h h_t+d_{t-1}^h(1+r_{t-1}^e)\leq w_t l_{c,t}+q_t^h h_{t-1}+d_t^h$, and collateral constraint $\phi_t^h q_t^h h_t \geq (1+r_t^e)d_t^h$, where ψ_l is the weight of labor in the utility function and σ_l is the inverse of the Frisch elasticity of labor supply. The parameter β denotes the discount factor, χ is the weight of assets in the utility of households, and ϕ_t^h is the collateral requirement imposed on constrained households. We assume that collateral-constrained households are aware of the price level p_t, the interest rate r_t^e and the wage rate w_t. Constrained households decide on consumption c, hours l, debt d^h, and assets h with a marginal utility $\lambda_{c,t}$. We define the probability that a debt can be charged-off $\mathcal{X}_t^h$ as $\mathcal{X}_t^h=\left(1/(1+e^{-\sigma_t^h})\right)$ for all $\sigma_t^h\in(0,+\infty)$; σ_t^h reflects the volatility that can lead to high or low probability of debt being uncollectible with no chance of being repaid. This shock follows a standard autoregressive process of order 1, $\sigma_t^h=\rho_h\sigma_{t-1}^h+\epsilon_t^h$. We further define the collateral requirement on real estate loans as $\phi_t^h\sim F\left(\phi_{t-1}^h,\mathcal{X}_{t-1}^h\right)$, where ϕ_t^h is a shock that satisfies the following exogenous process $\varphi_t=\rho_{\phi h}\phi_{t-1}^h+(1-\rho_{\phi h})\alpha_h\mathcal{X}_{t-1}^h+\epsilon_t^{\phi h}$, with $\epsilon_t^{\phi h}\sim N(0,\sigma_{\phi h}^2)$. Thus, regulators in this economy adjust the collateral requirements at time t

to the previous level of collateral requirements ϕ^h_{t-1} and to the level of the loan charge-off rate $\mathcal{X}^h_{t-1}$.
Furthermore, we assume that unconstrained households make loans to collateral-constrained households at the nominal interest rate r^e, via a financial intermediary that has a participation constraint $(1-\mathcal{X}^h_t)(1+r^e_t)d^h_{t-1} + (1-\mu^h)\mathcal{X}^h_t\phi^h_t q^h_t h_{t-1} \geq (1+r_t)d^h_{t-1}$. This condition gives an equilibrium in which banks make no loss when providing loans to collateral-constrained agents. With probability $(1-\mathcal{X}^h_t)$, the borrowers are able to pay back the debt $(1+r^e_t)d^h_{t-1}$, and with probability $\mathcal{X}^h_t$, the bank cannot collect debt and has classified it as delinquent. In the case of delinquency, this condition implies a cost $(1-\mu^h)$ associated with the repossession of the collateral $\phi^h_t q^h_t h_{t-1}$. Regardless of whether the borrower defaults on their loans, the banks will always receive a total earnings from lending equal to $(1+r_t)d^h_{t-1}$. Aggregating labor and consumption of the two types of households' yields $l_{i,t} = [l_{u,t} + l_{c,t}]$, and $c_{i,t} = [c_{u,t} + c_{c,t}]$.

Labor Market and Wage Stickiness. Labor contractors hire households $i \in \{u, c\}$ and sell homogeneous labor services to the intermediate good producers. Labor services take the form $l_t = [\int_0^1 (l_{i,t})^{1/\nu_l} di]^{\nu_l}$, subject to $\int_0^1 w_{i,t} l_{i,t} di = w_t l_t$, where ν_l denote the fixed wage markup and $w_{i,t}$ is the wage rate. There is a monopoly union that represents all workers, sets wages and faces Calvo style frictions, $w_t = [(1-\zeta_l)(w_{t-1})^{1/(1-\nu_l)} + \zeta_l(w^*_t)^{1/(1-\nu_l)}]^{1-\nu_l}$, where $1-\zeta_l$ is the probability that the monopoly reoptimizes the wages, while with probability ζ_l the monopoly cannot reoptimize.

Goods Production and Price Stickiness. The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, which is defined as $y_t = [\int_0^1 y_{j,t}^{1/(1+\nu_p)} dj]^{1+\nu_p}$. The final good is indexed by $j \in [0,1]$, and $1 \leq \nu_p < \infty$. A higher price markup ν_p implies that firms have market power and that the good is less substitutable for other goods. With capital and labor inputs, the monopolist produces the intermediate good y_t and the intermediate good production function takes the following form

$$y_t = (k_t)^\alpha (z_t l_t)^{1-\alpha} \tag{3.3}$$

where l_t is capital input, l_t is the labor input and z_t is an exogenous productivity shock $z_t = \rho_z z_{t-1} + \epsilon^z_t$. The parameter $\alpha \in (0,1)$ measures the responsiveness of output to changes in capital. We assume that prices are sticky, by using a variant of Calvo type frictions as in Christiano et al. (2014). The monopolist can set the price p_t of the good by reoptimizing the price with probability $1-\zeta_p$, or without reoptimizing the price with probability ζ_p, and p^*_t is the price level that maximizes the expected discounted value of future profits into the future. The price level is defined by $p_t = [(1-\zeta_p)(p_{t-1})^{\nu_p/(1-\nu_p)} + \zeta_p(p^*_t)^{\nu_p/(1-\nu_p)}]^{(1-\nu_p)/\nu_p}$. After setting the prices and the quantities, the monopolist minimizes the production cost $w_t l_t + r^k_t k_t$ subject to production function (3.3). The producer chooses the level of labor and capital allocation with a marginal cost μ_t.

Capital Market. In a capital market, the previous capital is combined with investment goods to produce a new capital, which is supplied to entrepreneurs with the following technology:

$$k_t = (1-\delta)k_{t-1} + \left(1 - \frac{S}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) i_t, \tag{3.4}$$

where capital decays at the fixed rate $0 < \delta \leq 1$. According to this equation, the new capital depends on the existing capital and investment good i_t. The quantity of investment at period t is proportional to the adjustment cost function S. Capital producers choose investments to maximize the present value and future operating profits, less the total investment cost, as shown below[3]: $q_t^k i_t - [1 + S/2(i_t/i_{t-1} - 1)^2]i_t$. Given by the law of motion for capital stock k_t and investment expenditures, capital producers sell capital to entrepreneurs at price q^k. Capital producers' optimal investment would equalize the marginal revenue product of capital q^k to the marginal cost of investment goods.

Entrepreneurs and Collateral Constraints. Capital producers sell capital to entrepreneurs at price q_t^k. We assume the capital flow of the entrepreneur is given by $k_t = r_{t-1}^k k_{t-1} + (1-\delta)k_{t-1}$. The provision of capital to firms at time t equals the return on renting capital services and the previous capital k_{t-1}, which decays at a fixed rate δ. The entrepreneur enjoys the average nominal rate of return on capital r_t^k and chooses the quantity of capital k_t.

Note that each entrepreneur purchases capital goods k_t at the price q_t^k using loans m_t obtained from the bank and their net worth n_t^e, i.e., $k_t q_t^k = m_t + n_t^e$ for all t. Entrepreneurs are hit by a shock $\mathcal{X}_{e,t} = 1/\left(1 + e^{\sigma_{e,t}}\right)$, which describes the probability that the entrepreneurs are unable to pay their debt. Letting $\sigma_{e,t}$ obey an AR(1) process $\sigma_{e,t} = \rho_{\sigma_e}\sigma_{e,t-1} + \epsilon_t^{\sigma_e}$, thus, the default probability increases when $\sigma_{e,t}$ goes up. The collateral constraint for entrepreneurs can be defined as $\phi_{e,t}(1 + r_t^k)q_t^k k_t \geq (1 + r_t^e)m_t^e$. The rntrepreneur's debt is charged-off with probability $\mathcal{X}_{t-1}^e$. As the entrepreneur is unable to pay the interest and principle, the pledged collateral is seized by the bank. We assume that collateral requirements $\phi_{e,t}$ evolve over time according to the following law of motion $\phi_t^e = \rho_{\phi_e}\phi_{t-1}^e + (1 - \rho_{\phi^e})\alpha_e\mathcal{X}_{t-1}^e + \epsilon_t^{\phi_e}$. This shock reveals that regulators adjust the level of collateral requirements to the previous level at $t-1$, where ρ_{ϕ_e} determine the regulator's policy weight on previous collateral requirements. Additionally, regulators can observe the charge-off rate level of business loans and adjust their collateral requirements accordingly with $(1 - \rho_{\phi^e})\alpha_e$, interpreted as the regulator's policy weight on the level of charge-off rate. The innovation $\epsilon_t^{\phi_e}$ is assumed to be an i.i.d. with mean zero and standard deviation σ^{ϕ_e}.

The entrepreneur's expected earnings are given by

$$E_t\left\{\left[(1 + r_t^k)q_t^k k_t - (1 + r_t^e)m_t^e\right](1 - \mathcal{X}_t^e)\right\}$$

where r_t^k is the rate of return on capital and r_t^e is the net interest rate paid by the entrepreneurs on their debt m_t. For the sake of simplicity, we define the probability that entrepreneurs will default on their loans $\mathcal{X}_t^e$ and the probability that they can meet the scheduled repayments on their debt $1 - \mathcal{X}_t^e$. We assume that the leverage level of entrepreneurs is defined as $Lev_t = m_t^e/n_t^e$. Thus, the entrepreneur's problem is to maximize their entrepreneurial earnings[4] subject to a participation constraint

$$E_t\left\{ (1 - \mathcal{X}_t^e)\,(1 + r_t^e)m_t + (1 - \mu^e)\mathcal{X}_t^e(1 + r_t^k)\phi_t^e q_t^k k_t \geq (1 + r_t)m_t \right\} \tag{3.5}$$

Note that the equation of the entrepreneur's bank participation (3.5) must hold with strict equality in every state of nature. The first term on the left-hand side of the equation corresponds to returns from non-defaulting entrepreneurs, and the second term corresponds to the returns from defaulting entrepreneurs whose collateral is seized by banks, net of monitoring costs μ^e. The right-hand

[3] The capital producer's profit is given by $q_t^k k_t - [1 + S/2\,(i_t/i_{t-1} - 1)^2]i_t$, where the law of motion for capital $k = k' + [1 + S/2\,(i_t/i_{t-1} - 1)^2]i_t$, given that in a steady-state, $S(.) = 0$ becomes $k - k' = I_t$. Then the capital producer's problem can be written as $q_t^k i_t - [1 + S/2\,(i_t/i_{t-1} - 1)^2]i_t$.

[4] See Technical Appendix for detailed computations.

side describes the return on loans given the interest rate demanded by banks r_{t+1}. We define the entrepreneur's expected net worth as $n^e_{t+1} = +\gamma^E \left((1 - \mathcal{X}^e_t - (1 - \mathcal{X}^e_t)\phi^e_t \right) \left(1 + R^k_t\right) k_t q^k_{t-1} + k_t (1-\delta) q^k_{t-1} \right)$.

Monetary Policy Rule We assume that the monetary policy obeys a Taylor rule. The linearized form of the monetary policy rule is given by

$$r_t - r = \rho_p (r_{t-1} - r) + (1 - \rho_p)[a_\pi (E_t p_{t+1} - p_t) + a_{\Delta y}(y_t - y)] + \epsilon_t, \tag{3.6}$$

which depends on the deviation between the central bank's inflation target and expected inflation rate and also depends on the deviation between the output and its steady-state. ϵ_t is the monetary policy shock with $\epsilon_t \in N(0, \sigma_r^2)$.

Resource Constraints and Equilibrium We complete the setup of the model by specifying the main aggregate resource constraints

$$y_t = c_t + f^e_t + f^h_t + g_t + i_t \tag{3.7}$$

The first term on the right hand-side of the resource constraint equation is aggregate consumption c_t, f^e_t represents banks spending on monitoring entrepreneurs with $f^e_t = \phi_t \mu^e \mathcal{X}^e_t (1 + r^k_t) q^k_t k_t$, the term $f^h_t = \phi_t \mu^h \mathcal{X}^h_t q^h_t h_t$ is the costs associated with the collateral liquidation of household loans, while g_t denotes government consumption and is given by $g_t = \eta_{g,t} y_t$. The share of government consumption in total output follows an AR(1) process with $\eta_{g,t} = \rho_g \eta_{g,t-1} + \epsilon^g_t$, and i_t is the aggregate investment. A competitive equilibrium consists of stochastic processes $\{p_t, p^*_t, F_{p,t}, K_{p,t}, w_t, w^*_t, F_{w,t}, k_t, i_t, \mu_t, q^k_t, r^k_t, \bar{r}^k_t, n^e_t, y_t, l_t, l_{u,t}, l_{c,t}, \lambda_t, \lambda_{u,t}, \lambda_{c,t}, c_t, c_{u,t}, c_{c,t}, d_t, h_t, d^h_t, q^h_t, r^e_t, f^e_t, f^h_t, m^e_t\}$ that evolve according to a system of equations. This model is driven by eight exogenous shocks: a government spending shock $\eta_{g,t}$, a technology shock z_t, a consumption preference shock ζ^c_t, a monetary policy shock r_t, a delinquency rate shock on firm loans $\mathcal{X}^e_t$, a delinquency rate shock on mortgage loans $\mathcal{X}^h_t$, a business loan collateral requirement shock ϕ^e_t, and a mortgage loan collateral requirement shock ϕ^h_t.

4 Estimation

In this section, we first present the solution method and the calibrated parameters of the model. Then, we estimate unknown parameters and shocks using Bayesian methods.

Methods and Data. Once we have solved the model using Lagrangian methods or by substituting the constraint, we can compute the non-stochastic steady-state properties (model derivations are reported in the technical appendix). After that, we approximate the equilibrium system by log-linearizing around the steady-state. Hereafter, all variables in log-deviations from the steady-state values are identified with a tilde (log-linearized model equations can be found in the technical appendix). The next step will be estimating the model by proceeding in three stages: (i) we construct the empirical data to combine it with the model equation, (ii) we calibrate some economic parameters, and (iii) we estimate the remaining economic parameters and shock parameters using Bayesian methods.

For estimation, we use quarterly US data, and the sample period covers 1984Q1 to 2021Q2 (see Figure 10 in the Appendix). The time series are normalized to have a mean zero. For example, data on the gross domestic product, consumption, investment, commercial industrial average outstanding, commercial industrial total charge-offs, real estate loans average outstanding, real estate loans total charge-offs, and government spending are deflated using their specific implicit price deflator. Then, we express these aggregate variables in per capita terms, we divide each of these variables by the civilian non-institutionalized population over 16, and finally we take the logarithmic first difference. The data on hours worked is normalized by population only, and then we transform it by taking the first difference of its logarithm. We also take the logarithmic first difference of the data on the delinquency rate of commercial industrial loans, and the delinquency rate of real estate loans. Finally, we end up with eleven observables:

$$[\Delta \log y_t^{obs}, \Delta \log c_t^{obs}, \Delta \log m_t^{e,obs}, \Delta \log d_t^{h,obs}, \Delta \log i_t^{obs}, \Delta \log l_t^{obs},$$
$$\Delta \log f_t^{e,obs}, \Delta \log f_t^{h,obs}, \Delta \log g_t^{obs}, \Delta \log \mathcal{X}_t^{h,obs}, \Delta \log \mathcal{X}_t^{e,obs}]$$

growth in output $\Delta \log y_t^{obs}$, growth in consumption $\Delta \log c_t^{obs}$, growth in commercial industrial $\Delta \log m_t^{e,obs}$, real estate loans $\Delta \log d_t^{h,obs}$, growth in investment $\Delta \log i_t^{obs}$, growth in hours worked $\Delta \log l_t^{obs}$, growth in commercial industrial loans total charge-offs $\Delta \log f_t^{e,obs}$, growth in real estate loans total charge-offs $\Delta \log f_t^{h,obs}$, growth in government spending $\Delta \log g_t^{obs}$, growth in the delinquency rate of real estate loans $\Delta \log \mathcal{X}_t^{h,obs}$, and growth in the delinquency rate of firm loans $\Delta \log \mathcal{X}_t^{e,obs}$. The technical appendix describes the system of measurement equations in detail.

Parameter	Description	Value
Household		
β	Discount rate	0.99
σ^l	Curvature on disutility of labor	1.00
b_u	Habit persistence parameter	0.63
b_c	Habit persistence parameter	0.63
ν^l	Steady-state markup suppliers of labor	1.0500
μ^h	Fraction of realized profits lost in bankruptcy	0.9400
χ	The utility weight on housing	1.5
ϕ^h	Steady-state collateral requirements on mortgage loan	0.98
Production		
δ	Depreciation rate on capital	0.0250
S	Adjustment cost function	5
α	Power on capital in production function	0.4000
ν^p	Steady-state markup, intermediate good firms	0.2000
Entrepreneurs		
γ^e	Percentage of entrepreneurs who survive	0.9762
μ^E	Fraction of realized profits lost in bankruptcy	0.94
ϕ^e	Steady-state collateral requirements on business loan	0.98
Policy and shocks		
$\eta_{g,ss}$	Share of government consumption	0.2000

Table 2: Calibrated Parameters

Parameter Estimates. The model is parameterized such that the key variables are matched to the US data over the sample period 1984–2021. The parameterization is described in Table 2. As a standard practice in the business cycle literature, we set the discount factor β to 0.99, the elasticity of labor supply σ^L to 1, and the consumption habit of the constrained and unconstrained households b^c and b^u to 0.63. The discount factor is set such that we obtain the central bank policy rate R of 1.01%, which is close to the Fed funds rate. We also set the wage markup ν^l to 1.05, which is fairly standard in the literature and close to the value of labor supply elasticity in Justiniano et al. (2011). We set the value of the utility weight on housing χ to 1.5. This value is similar to that in Justiniano et al. (2014). We assume that the steady-state price of housing stock q^h is equal to 1. We fix the price markup of intermediate good firms $\nu^p = 0.2$, which is in line with the range of values used in the literature. We also set the power on capital in production function α to 0.4. As for investment, we take a standard value of depreciation rate on capital δ to equal 0.025, a common value in the business cycle literature.

Afterward, to target the delinquency rate on mortgage loans $\mathcal{X}^h = 0.3\%$, as observed in FDIC data, we set the value of σ^h to equal 5.8. We choose the value of σ^e to be equal 4.18 to target a value 1.5% for the default rate $\mathcal{X}^e$, a value that is found in the FDIC data. The monitoring cost μ^e parameter is set to the value of 0.94, as in Christiano et al. (2014). We also set the monitoring cost of real estate loans μ^h to 0.94. In the steady-state, we set the value of collateral requirement coefficients ϕ^h and ϕ^e equal to 0.98 (as in Iacoviello and Pavan (2013) and Justiniano et al. (2014)). We pick the percentage of entrepreneurs who survive γ^e to equal 0.68. We set standard values for the Calvo parameters of sticky prices ζ^p and sticky wages ζ^l to 0.702 and 0.771, respectively. We choose the value of the steady-state government spending to output ratio η_g to be equal to 0.18 consistent with the US data.

Table 3: Prior and Posterior Distribution of Estimated Parameters

		Prior Distribution			**Posterior Distribution** [a]	
		Density	Mean	Std. Dev.	Mean	Std. Dev.
Macroeconomic Parameters						
b_u	Habit parameter	Beta	0.5	0.1	0.6514	0.0003
b_c	Habit parameter	Beta	0.5	0.1	0.6158	0.0004
ζ_l	Calvo wage stickiness	Beta	0.75	0.1	0.7896	0.0004
ζ_p	Calvo price stickiness	Beta	0.75	0.1	0.71	0.0004
S	Investment adjustment cost curvature	Normal	5	0.1	4.9785	0.0003
Shock Parameters: Persistence						
α^{π}	Policy weight on inflation	Normal	1.5	0.25	1.8084	0.0013
α^{Δ_y}	Policy weight on output growth	Normal	0.25	0.1	0.3118	0.0003
ρ_p	Policy smoothing parameter	Beta	0.5	0.1	0.8744	0.0004
ρ_g	Autoc. Gov. Spend. Shock	Beta	0.7	0.1	0.7097	0.0002
ρ_z	Autoc. Tech. Shock	Beta	0.7	0.1	0.9379	0.0007
ρ_{ζ_c}	Autoc. Cons. Pref. Shock	Beta	0.7	0.1	0.7189	0.0005
$\rho_{\sigma e}$	Autoc. B. Loan Delinq. Rate Shock	Beta	0.7	0.1	0.5068	0.0004
$\rho_{\sigma h}$	Autoc. R. E. Loan Delinq. Rate Shock	Beta	0.7	0.1	0.5312	0.0004
$\rho_{\phi e}$	Collateral Policy Smoothing Parameter - B. Loan	Beta	0.7	0.1	0.518	0.0005
$\rho_{\phi h}$	Collateral Policy Smoothing Parameter - R. E. Loan	Beta	0.7	0.1	0.4783	0.0006
$\alpha_{\phi e}$	Collateral Policy weight on R. E. Loan Delinq. Rate	Normal	1.5	0.1	1.8434	0.0008
$\alpha_{\phi h}$	Collateral Policy weight on B. Loan Delinq. Rate	Normal	1.5	0.1	1.8314	0.0006
Shock Parameters: Standard Deviation						
ϵ_g	St. dev. Gov. Spend. Shock	Inv. G.	0.02	Inf	0.0255	0.0013
ϵ_z	St. dev. Tech. Shock	Inv. G.	0.001	Inf	0.0253	0.0004
ϵ_{ζ_c}	St. dev. Cons. Pref. Shock	Inv. G.	0.001	Inf	0.0037	0.0045
ϵ_{σ_e}	St. dev. B. Loan Delinq. Rate. Shock	Inv. G.	0.001	Inf	1.2541	0.0275
ϵ_{σ_h}	St. dev. R. E. Loan Delinq. Rate. Shock	Inv. G.	0.02	Inf	1.4915	0.0261
ϵ_{κ_e}	St. dev. B. Loan Coll. Requir. Shock	Inv. G.	0.001	Inf	1.0233	0.0122
ϵ_{κ_h}	St. dev. R. E. Loan Coll. Requir. Shock	Inv. G.	0.002	Inf	0.0006	0.0005
ϵ_h	St. dev. R. E. Volume Shock	Inv. G.	0.001	Inf	0.0015	0.0032
ϵ_ρ	St. dev. M. P. Shock	Inv. G.	0.001	Inf	0.0647	0.0039
Measurement Errors on The Observables						
$\varrho_{y^{obs}}$	Measurement errors on y_t	Inv. G.	0.002	Inf	0.0423	0.0027
$\varrho_{i^{obs}}$	Measurement errors on i_t	Inv. G.	0.001	Inf	0.0452	0.0026
$\varrho_{l^{obs}}$	Measurement errors on l_t	Inv. G.	0.002	Inf	0.0351	0.0031
$\varrho_{f^{e,obs}}$	Measurement errors on f_t^e	Inv. G.	0.001	Inf	1.7618	0.0203
$\varrho_{f^{h,obs}}$	Measurement errors on f_t^h	Inv. G.	0.002	Inf	1.0802	0.0244

Notes: The table report the results of Bayesian estimation, posterior statistics are constructed using 10,000 draws per chain.

We turn now to describe the procedure to estimate the unknown parameters. Following a Bayesian approach, we choose prior distributions of selected parameters for estimation. Researchers typically rely on existing empirical literature or beliefs derived from macro and micro-level evidence. The posterior distribution is constructed over the period 1984Q1-2021Q2 using the Metropolis-Hastings algorithm, with 10,000 draws per chain needed to achieve convergence. Estimation results are reported in Table 3.

For habit parameters b_u and b_c, we assume that the prior obeys the beta distribution with mean 0.5 and standard deviation 0.1, e.g., Iacoviello and Pavan (2013). Including habit formation in the model allows for hump-shaped responses of macroeconomic variables to economic shocks. Calvo price stickiness parameters ζ_l and ζ_p are assumed to follow the beta distribution with mean 0.75 and standard deviation 0.1. According to our estimates, the model exhibits a high degree of price and wage rigidity and the value of these two estimates is consistent with (Guerrieri and Iacoviello, 2017). We also assume that the prior of adjustment cost function S follows a Normal distribution with mean 5 and standard deviation 0.1, e.g., (Christiano et al., 2014), and (Guerrieri and Iacoviello, 2017).

We assume that the priors are centered at 0.7 for the persistence of shock processes (government spending, technology, consumption preference, real estate delinquency rate, business loan delinquency rate, collateral requirement on business loans and collateral requirement on real estate loans) with a beta distribution that ensures that the estimates are bounded between 0 and 1 (the standard deviation of the prior is equal to 0.1). We also assume that the prior of standard deviation of structural shocks follows a beta distribution (with mean 0.01 and standard deviation ∞).

With nine orthogonal shocks and five measurement errors, we will use eleven observable variables, described in the text above, to estimate the model. Regarding the identification of the two main financial shocks in the model, the collateral requirements and delinquency rate shocks, we use aggregated mortgage and firm debt data. We also add delinquency rates on firm and mortgage loans, in addition to total charge-offs on firm and real estate debt, from the FDIC to identify these two shocks.

The model includes exogenous changes in delinquency rates to agree with the occasional crisis that occurs when the delinquency rate is substantially higher.[5] The estimated delinquency rate shock equations are given by

$$\sigma_t^h = 0.5232\sigma_{t-1}^h + \epsilon_t^h \qquad \sigma_t^e = 0.5188\sigma_{t-1}^e + \epsilon_t^e$$

where σ_t^h is the mortgage delinquency rate and σ_t^e represents the firm loan delinquency rate. The estimated coefficients suggest that the persistence of the delinquency rate shock on the mortgage debt and firm debt shock is relatively low (0.523 and 0.519, respectively). Recall that the corresponding equation for the collateral requirements on real estate loans and firm loans are defined by

$$\varphi_t^h = 0.5239\phi_{t-1}^h + (1-0.5239)1.8161\mathcal{X}_{t-1}^h + \epsilon_t^{\phi h} \qquad \varphi_t^e = 0.5268\phi_{t-1}^h + (1-0.5268)1.8046\mathcal{X}_{t-1}^e + \epsilon_t^{\phi e}$$

According to these two regressions, the reaction coefficient of collateral requirements to the mortgage delinquency rate is estimated to be relatively high (1.8161). We find that the collateral requirement on firm loans does appear to react very strongly to the firm loan delinquency rate (1.804). Our specification suggests that the magnitude of these two estimates is approximately

[5] An alternative specification would be an endogenous delinquency rate. A recent work by Candian and Dmitriev (2020) provides a coherent micro-foundation of fluctuations in bankruptcy costs and shows how the presence of endogenous liquidation costs amplifies the response of macroeconomic aggregates to financial shocks.

similar, and the mean of the coefficient on lagged collateral requirement is estimated to be 0.523 for mortgage loans and 0.526 for business loans.

A more consistent interpretation of these estimates is that banks, more specifically regulators will be more reactive to the deterioration of borrower's conditions. Under this assumption, regulators will adjust their collateral requirements as they observe the impending bad performance of the economy. A higher level of delinquency rate is more evident when an economy enters a crisis period. This peculiar behavior of delinquency rates conveys signals about the state of the borrower's financial conditions, and thus these signals influence bank credit policies through the collateral channel via a tightening of collateral terms.

5 Inspecting the Collateral Channel

Loan Supply during the IT Bubble in the 1990s and the Great Recession To support our assumption that regulators tighten their collateral requirements by decreasing the loan-to-value ratio, causing a contraction in credit supply, we present the dynamics of business credit and mortgage loans in the US between 1984 and 2021. It can be argued that changes in the delinquency rate conveyed signals about the state of the borrower's financial conditions and thus affected the credit policies of banks through the collateral channel. Indeed, a time-varying collateral requirement that adjusts to the level of charge-off rate may explain the contraction and the expansion in the supply of bank credit to individuals and the business sector. Regrading the model predictions on mortgage and business debt, we simulate the time series for real estate debt and business debt. We compare these series with the observed data, which is expressed in deflated and per capita terms.

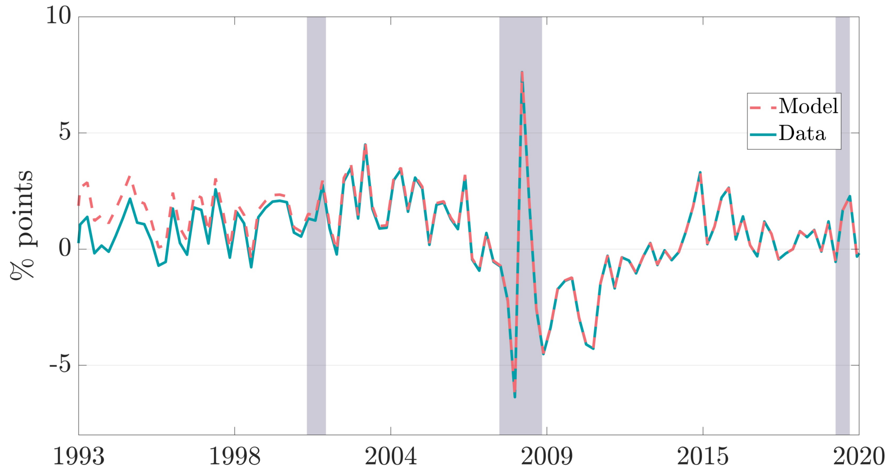
Figure 5 offers a clear summary of the antecedents to the 2001 and 2008 crises. It is evident that these crises were preceded by high credit growth and a relaxation of lending conditions. By focusing on these two remarkable recessions, starting from 1992, we can see that firm debt rose considerably, and the boom ended approximately in 1998. Following the peak, business loan growth went down, entering the bust phase and reaching the bottom in 2002. The credit cycle phases of firm debt were generally similar to the patterns observed in the growth rate of mortgage loans.

Banks that overextended credit prior to the financial crisis of 2008 and the IT bubble burst faced a higher level of charge-off rate. This faster growth in credit combined with the combination of poor performance of the economy exerted a downward impact on the willingness of banks to lend.[6] Banks typically responded to the crisis by implementing tighter collateral requirements and lending conditions on mortgage and firm loans.

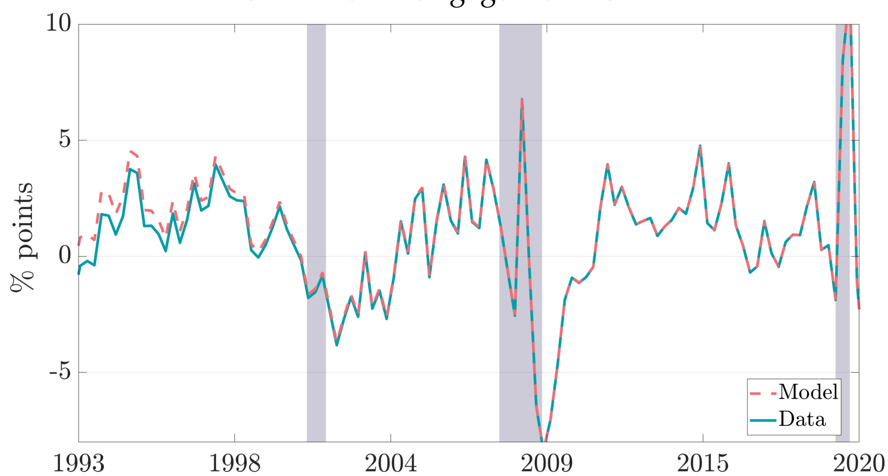
The dynamics of firm debt and the net charge-off rate is a compelling case. A visual observation shows that businesses' credit is seemingly correlated to the default rates. In Figure 6 panel A shows evidence in favor of the existence of an inverse relationship between the two. More interesting, an abnormal increase in the net charge-off rate is observed when the economy enters a crisis. Following the increase in the delinquency rate, business lending experiences a downward trend pressure until the charge-off rate starts to decrease. It is perhaps more plausible that banks tighten their collateral requirements when the level of loan charge-off rate is at a higher level and thus will cut credit to firms.

The dynamics of mortgage loans is reported in Figure 6, panel B. We find that loans to households exhibit a steady supply of credit during the crises, yet, defaults on household lending

[6] See for example, Mian et al. (2017) who show that a rise in household debt is associated with lower subsequent GDP growth.



A. Simulated Mortgage Debt vs Data



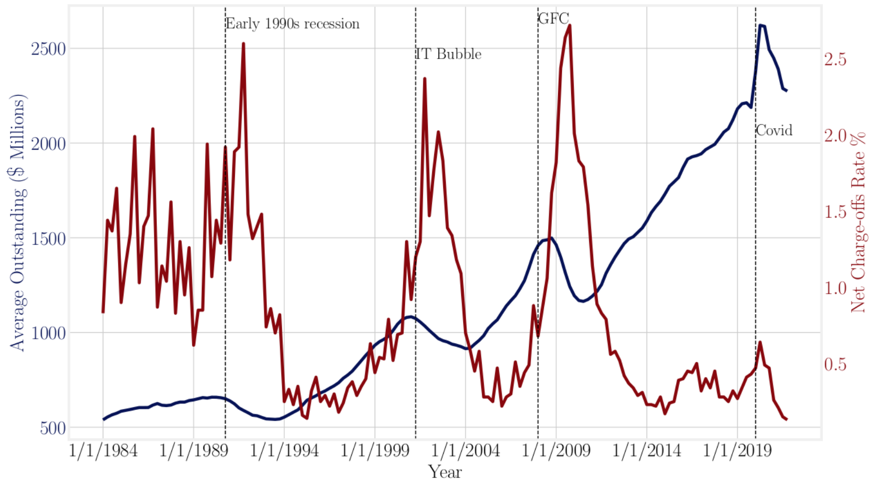
B. Simulated Firm Debt vs Data

Figure 5: Simulated Firm Loan and Mortgage Debt vs Data

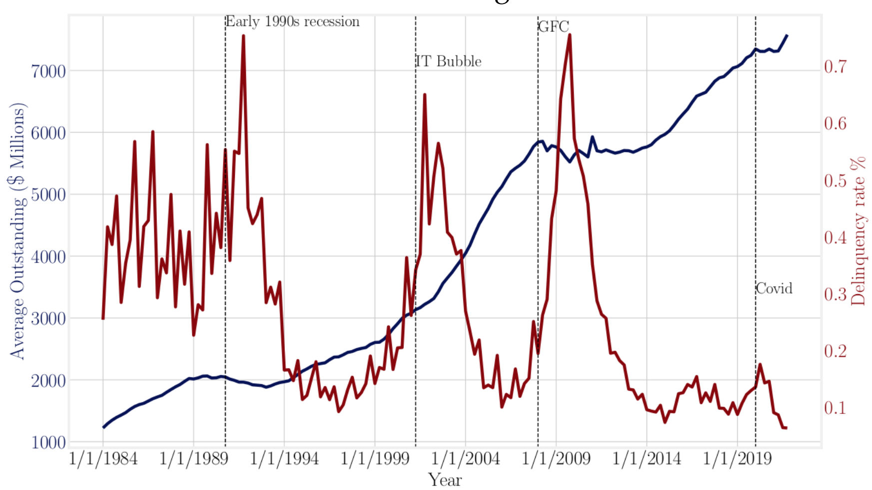
Source: FDIC; Sample: 1984Q1-2021Q2

are rising. However, before the crisis it is clear that banks had a relaxed credit policy, which is perhaps explained by the relaxation of collateral requirements before the crisis. Indeed, a soft collateral policy means an expansionary credit policy. Though, as shown in Section 3, Figure 4, it seems likely that banks relax their collateral requirements during good times and become more prudent during the economic downturn.

In summary, there is a consensus that banks relaxed lending to firms and households prior to the crisis. The expansion of credit supply conceivably means that credit is extended to borrowers who may be unable to afford the debt and consequently default on their loans. When the crisis hits the economy, a large number of firms and households default on their loans, the level of uncollectible debt becomes unsustainable and banks face higher losses. As this erodes banks' capital and induces liquidity problems for banks, banks will have an incentive to tighten their collateral policy and thus reduce lending. In our model, we hypothesized that the collateral requirements set by regulators will adjust to the level of the charge-off rate. A model that abstracts from the collateral channel would miss how the demand-side and supply-side affects the bank's



A. CI loans vs charge-off rate



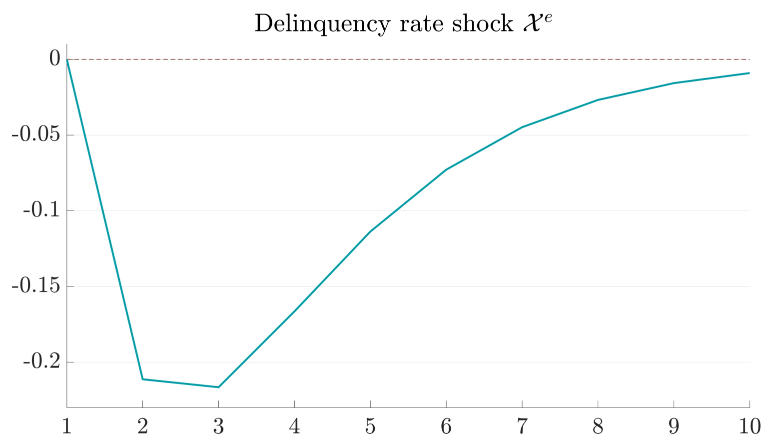
B. Mortgage Loans vs charge-off rate

Figure 6: Business and Mortgage Loans vs charge-off rate

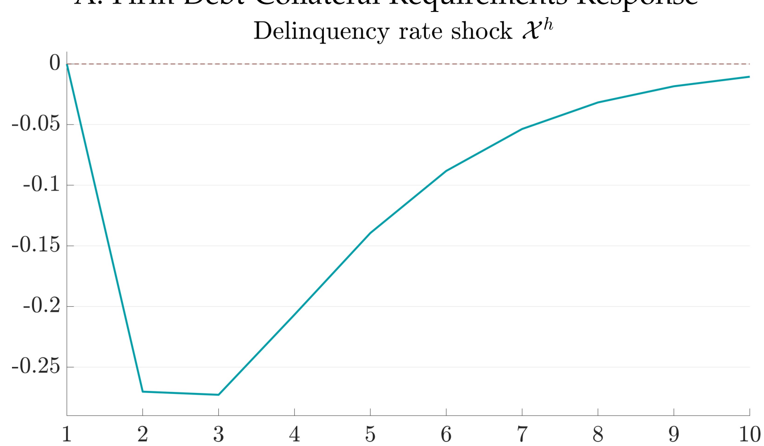
Source: FDIC; Sample: 1984Q1-2021Q2

collateral and credit policy.

Response of collateral requirements to charge-off rate The theoretical model developed in this paper assumes that the level of the delinquency rate on loans conveys information about the deterioration of the financial situation of borrowers. The interaction between the charge-off rate and collateral requirements is key: when the demand-side conditions deteriorate, banks consequently face heavy losses. Then regulators who are able to observe the impending bad performance of the economy will react by tightening their collateral requirements. However, the adoption of low collateral requirements, which was the result of high bank losses and a deterioration in the demand-side conditions, causes a reduction in bank lending and consequently affects the real economy negatively. Figure 7 displays the response of the collateral requirements to an increase in the delinquency rate. The intuition is that banks will have an incentive to reduce the collateral requirement when the delinquency rate increases. The reason is that regulators mechanically set the level of collateral requirements dependent on the charge-off rate. In this case, lenders will be more prudent during uncertain times and will reduce the credit supply. The fall in collateral requirements implies that the incidence of an increase in the charge-off rate for mortgage debt and business debt would be similar and causes the collateral constraint to be tightened.



A. Firm Debt-Collateral Requirements Response



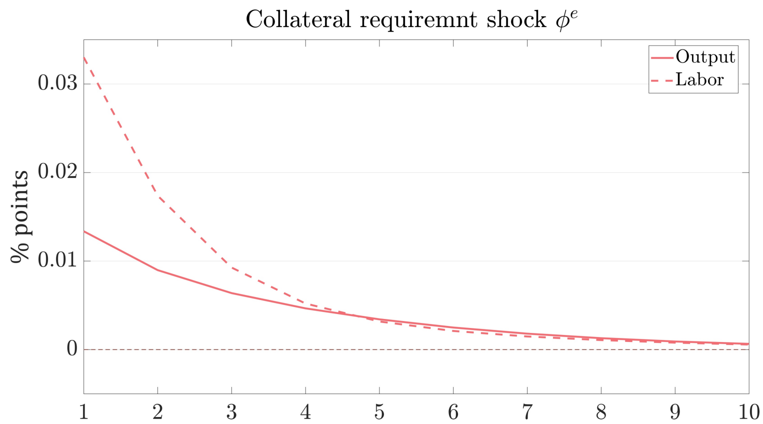
B. Mortgage Debt-Collateral Requirements Response

Figure 7: Collateral Requirements Response to an Increase in Delinquency Rates

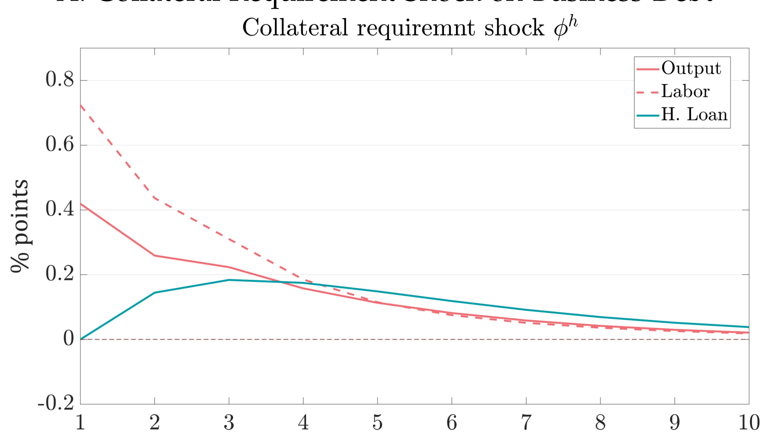
Source: FDIC; Sample: 1984Q1-2021Q2

How Does Collateral Affect the Real Economy? To understand how a collateral requirement shock propagates in the model, we first look at the response of output, and then we examine the shifts in labor.

This section considers the effect of collateral requirement friction and its implications on output. The responses to a collateral requirement shock displayed in Figure 8, panel A report a case when the economy is hit by a shock that makes business lending easier. An increase in a collateral requirement shock on business credit leads to a sharp rise in output, followed by a gradual recovery toward the steady-state level. The reason is that when collateral disturbances affect firms, the latter has enough external resources to finance their economic activity. This will eventually boost investment and thus increase total output.



A. Collateral Requirement Shock on Business Debt



B. Collateral Requirement Shock on Mortgage Debt

Figure 8: Positive Collateral Requirement Shock Output Response

Note how similar the dynamics of output are between the two cases of a rise in collateral requirements (ϕ^e, ϕ^h) for business loans and real estate loans. The response to an increase in collateral requirements on firm debt is straightforward and is displayed in Figure 8, Panel A. To have a sense of output behavior, take when the economy experiences a loosening in collateral requirements, such that collateral requirements increases. This will lead to an expansion in lending to firms. Given that banks will increase their credit supply, this will lead to a boom in the economy and an increase in demand. Firms will need to adjust and will raise their activity and thus will increase their labor supply costs to respond to the high demand. As collateral pressure subsides, output adjusts gradually to reach the steady-state level.

In addition, the case of the expansion of real estate credit supply shown in figure 8, panel B roughly exhibits a rapid response of output to a collateral requirement shock and a larger effect of collateral on labor. Figure 8, panel B has a simple interpretation. In the presence of a collateral channel, variable input costs are, by construction, affected by a collateral requirement shock and ultimately lead to an increase in output. In addition, one could assess what makes the output response rapid. Because wages are an important component of costs, firms must adjust their labor input. Given that wages are rigid, this will affect aggregate employment, which is interpreted here as total hours worked. Households can nevertheless increase their consumption, causing the aggregate output to increase further.

The main conclusion is that when credit supply is expanded, the aggregate output will be at its highest level when banks conduct a more liberal collateral policy and raise the loan supply to firms, which is similar to the case of mortgage loans.

6 Monetary Policy in the Presence of Collateral Channel

What happens to the economy when the central bank conducts a contractionary policy and raises the interest rate with the presence of a macro-prudential collateral policy? The presence of macro-prudential measures is important to better understand the propagation of a monetary policy shock and assess any potential conflicts between the two policies. In this analysis, we consider two alternative definitions of a collateral policy shock. As described in the baseline setup, we have entrepreneurs that face collateral requirements:

$$\phi_t^e = \rho^{\phi e}\phi_{t-1}^e + (1-\rho^{\phi e})\alpha^e \mathcal{X}_t^e + \epsilon_t^{\phi e}$$

The intuition behind the collateral requirements policy is that regulators that observe the level of charge-off rate will adjust the collateral requirements accordingly. This counter-cyclical response ensures that collateral requirements are driven by the demand-side, for instance, the change in the severity of the borrower's conditions.

On the other hand, we consider the variant collateral policy without accounting for the level of charge-off rates. In this case, the collateral requirement is written as

$$\phi_t^e = \rho^{\phi e}\phi_{t-1}^e + \epsilon_t^{\phi e}$$

The intuition is simple: regulators decide independently about the collateral requirement and thus the change in this collateral requirements policy is supply-driven.

Similarly, collateral-constrained households face a collateral requirement set by the regulator that takes the form of

$$\phi_t^h = \rho^{\phi h}\phi_{t-1}^h + (1-\rho^{\phi h})\alpha^h \mathcal{X}_t^h + \epsilon_t^{\phi h}$$

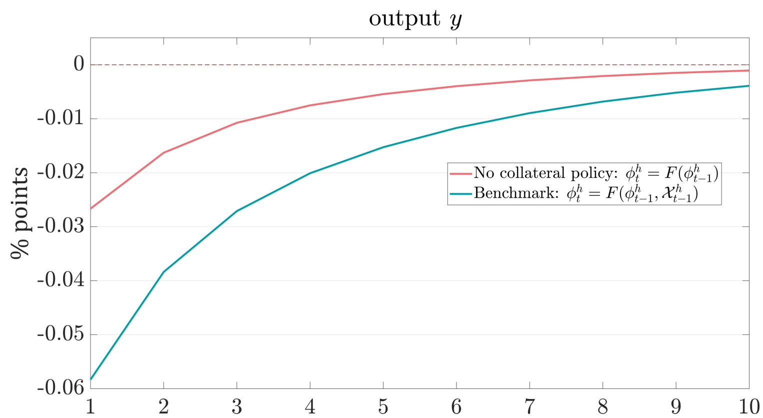
Regulators could monitor the level of the delinquency rate and thus can maintain a liberal or tight collateral policy, depending on the demand-side conditions. In addition, we consider the alternative collateral policy, which we call a less active collateral policy and is defined as

$$\phi_t^h = \rho^{\phi h}\phi_{t-1}^h + \epsilon_t^{\phi h}$$

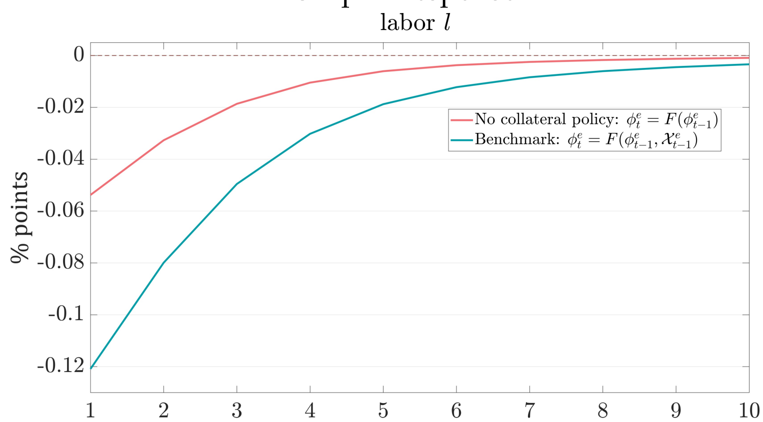
such that the decision of tightening or relaxing the collateral requirement will depend on the regulator's decisions (supply-side).

The Effects of a Contractionary Monetary Policy Shock. We perform two different exercises reflecting changes in collateral policy by analyzing the effect of a positive monetary shock when the regulator conducts a more active collateral policy ($\bar{\alpha^h} = [0, \alpha^h]$, $\bar{\alpha^e} = [0, \alpha^e]$) that corresponds to the measure of the collateral requirement response to the charge-off rate. These experiments capture the effect of a contractionary monetary policy under a fairly active collateral policy.

To proceed, we change the value for α^h and α^e to illustrate the shift in banks' collateral policies. We compute the impulse response function under the two scenarios. The intuition behind the output responses in Figure 9 is as follows. Suppose regulators implement a collateral policy that adjusts to the level of the charge-off rate. Eventually, this will influence the current total output, if firms face a contractionary monetary policy and they operate in such an environment. First, the shock will reduce the level of external finance for firms and the shock will propagate through a



A. Output Response



B. Labor Response

Figure 9: Impulse Responses to a Monetary Policy Shock: Baseline Estimates

labor supply channel by decreasing the aggregate hours demand, which influences production costs. Second, through the supply-side, firms will be forced to reduce their aggregate production when they cannot lower production costs.

Considering the presence of collateral requirements that do not adjust to the demand-side conditions implies an offset decline in total output. Monetary policy is weakened and has less of an effect on output and labor. On the other hand, an economy with an active collateral policy will be greatly affected by the shock and will experience a large drop in total output. The absence of the charge-off rate in the collateral requirements policy makes output less vulnerable to monetary policy disturbances. In this way, the shock is followed by an immediate decline in output by over -0.03% and then recovers to reach its steady-state level. Given the assumption that entrepreneurs and constrained households face a collateral requirement that determines their access to credit and depends on the net charge-off rate, a contractionary monetary policy shock can initially cause a decrease in aggregate labor by a -0.051% deviation from steady-states, and then it converges to its steady-state. However, the shock is amplified when regulators account for the delinquency rate in setting the collateral requirement, given that the collateral channel can produce a relatively amplified effect on output with a -0.035% deviation from steady-states, and labor with a -0.062% deviation from the steady-states.

Short and Long-run Effects of Monetary Policy on Delinquency Rate. Table 4 reports a scenario in which the economy is hit by a contractionary monetary policy shock with the presence of a collateral channel. If we maintain the model assumption of the collateral channel, the economy will experience a significant drop in the business loan delinquency rate following a monetary policy shock in the short run (-2.45 in the first quarter). The contraction in the business debt delinquency rate can be reverted to a sharp increase in the second quarter and then a modest rise in the fourth quarter. Clearly, the decline in delinquency rate is more pronounced, short-lived, and rapid in the short run.

Table 4: Effects of monetary policy on business loan delinquency rate $\mathcal{X}^e$

	Time			
Effects in %	**1st Quarter**	**2nd Quarter**	**3rd Quarter**	**4th Quarter**
Short run	-2.45	1.61	1.03	0.68
	8th Quarter	**12th Quarter**	**16th Quarter**	**20th Quarter**
Long run	0.06	0.02	0.01	0.01

Note: The table shows the response of the delinquency rate on business loans $\mathcal{X}^e$ to a contractionary monetary policy, percent deviation from the steady-state.

Furthermore, the long-run responses show that the business loan delinquency rate responds positively to a monetary policy shock. The general finding is that the charge-off rate responds rapidly to a collateral requirement shock in the short run. In the long run, when the monetary policy shock hits the economy and the disturbance pressure subsides, the magnitude of the decline in delinquency rate turns out to be much smaller.

Table 5: Effects of monetary policy on mortgage loan delinquency rate $\mathcal{X}^h$

	Time			
Effects in %	**1st Quarter**	**2nd Quarter**	**3rd Quarter**	**4th Quarter**
Short run	-3.20	-2.12	0.21	0.77
	8th Quarter	**12th Quarter**	**16th Quarter**	**20th Quarter**
Long run	0.75	0.28	0.09	0.03

Note: The table shows the response of the delinquency rate on mortgage loans $\mathcal{X}^h$ to a contractionary monetary policy, percent deviation from the steady-state.

We also look at an additional counterfactual to capture the response of the net charge-off rate when a contractionary monetary policy shock hits the economy with the presence of a collateral channel, Table 5. If the central bank decides to increase the interest rate, the economy can experience a decline in the mortgage loan delinquency rate in the short run with an aggregate effect of -3.20%. However, after the third quarter, the impact turns out to be positive with an increase of 0.21% in the fourth quarter. The effects appear to be positive in the long run as the disturbance becomes less strong.

In general, monetary policy can be effective in preventing high levels of default rates on real estate and business loans. For instance, if central banks decide to raise the interest rate, this will restrict money and credit supply. The consequence of this policy is that the increase in interest rate induces a decline in credit supply in the economy, thus preventing a default on business and mortgage loans in the short run.

7 Conclusion

In this paper, we estimate a two-agent New Keynesian model with collateral-constrained households and firms to analyze the impact of collateral-based macro-prudential policy and how it can prevent the risk of default. This paper has emphasized the role of collateral requirements in the context of contractionary monetary policy: output becomes less sensitive to changes in monetary policy shocks when banks conduct a less active collateral policy. However, output responds considerably to a monetary policy shock under an active macro-prudential collateral policy environment. A similar finding applies to employment. A more active collateral policy leads to a large response of employment to a monetary policy shock. Most importantly, we consider the response of the delinquency rate to the monetary policy shock under the active collateral policy. The overall conclusion that stands out from these experiments is that the transmission of a contractionary monetary policy can offset the charge-off rates in the short run, and the largest impact of monetary policy is on the path of the delinquency rate in the first quarter.

Banks should adjust their collateral requirement policies to take account of changes in the severity of borrowers' conditions. In fact, a more active collateral policy can limit banks' exposure to risk, and this will reduce the damage on external financing during the macroeconomic downturn to a certain extent. It is evident from our model that the effects of monetary policy are amplified when regulators adopt an active collateral-based macro-prudential policy, which will make banks less vulnerable to economic shocks. Therefore, banks must hold more liquid assets to serve as protection against a sudden drop in funding.

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Online Appendix

Regulatory Collateral Requirements and Delinquency Rate in a Two-Agent New Keynesian Model

Aicha Kharazi, Francesco Ravazzolo

Appendix A Model Derivations

Households optimality conditions: We start with the households utility maximization.

Unconstrained Households. The objective function is give by

$$\begin{aligned} \text{maximize} \quad & E_0\Sigma_{t=0}^{\infty}\beta^t\left\{\zeta_{c,t}(\log(c_{u,t}-b_u c_{u,t-1}))-\psi_l\frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l}\right\} \\ \text{subject to} \quad & p_t c_{u,t}+d_t\leq w_t l_{u,t}+(1+r_t)d_{t-1} \end{aligned} \tag{A.1}$$

The maximization problem yields the following first-order conditions with respect to consumption, deposit and labor:

$$\frac{\partial\mathcal{L}_t}{\partial c_{u,t}}: \quad \lambda_{u,t}p_t-\frac{\zeta_{c,t}}{c_{u,t}-bc_{u,t-1}}+b\beta E_t\frac{\zeta_{c,t+1}}{c_{u,t+1}-bc_{u,t}}=0 \tag{A.2}$$

$$\frac{\partial\mathcal{L}_t}{\partial d_t}: \quad \lambda_{u,t}-\beta E_t\lambda_{u,t+1}(1+r_{t+1})=0 \tag{A.3}$$

$$\frac{\partial\mathcal{L}_t}{\partial l_{u,t}}: \quad -\psi_l(l_{u,t})^{\sigma_l}+\mathcal{X}_t\lambda_{u,t}=0 \tag{A.4}$$

where $\lambda_{u,t}$ is the marginal utility.

Collateral Constrained Households. Financially constrained households maximize their expected lifetime utility

$$\begin{aligned} \text{maximize} \quad & E_0\Sigma_{t=0}^{\infty}\beta^t\left\{\zeta_{c,t}(\log(c_{c,t}-b_c c_{c,t-1}))-\psi_l\frac{(l_{c,t})^{1+\sigma_l}}{1+\sigma_l}+\chi\log(h_t)\right\} \\ \text{subject to} \quad & p_t c_{c,t}+q_t^h h_t+d_{t-1}^h(1+r_{t-1}^e)\leq w_t l_{c,t}+q_t^h h_{t-1}+d_t^h \\ & \phi_t^h q_t^h h_t\geq(1+r_t^e)d_t^h \end{aligned} \tag{A.5}$$

The first order condition with respect to consumption c, hours l, debt d^h, and assets h

$$\frac{\partial\mathcal{L}_t}{\partial c_t}: \quad \lambda_{c,t}p_t-\frac{\zeta_{c,t}}{c_{c,t}-bc_{c,t-1}}+b\beta E_t\frac{\zeta_{c,t+1}}{c_{c,t+1}-bc_{c,t}}=0 \tag{A.6}$$

$$\frac{\partial\mathcal{L}_t}{\partial h_t}: \quad -\frac{\chi}{h_t}+\lambda_{c,t}q_t^h-\beta E_t\lambda_{c,t+1}q_{t+1}^h-\frac{\phi_t^h q_t^h\lambda_{c,t}}{(1+r_t^e)}=0 \tag{A.7}$$

$$\frac{\partial \mathcal{L}_t}{\partial d_t^e}: \qquad \beta E_t \lambda_{c,t+1}(1+r_t^e) - \lambda_{c,t} = 0 \tag{A.8}$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{c,t}}: \qquad -\psi_l (l_{c,t})^{\sigma_l} + \mathcal{X}_t \lambda_{c,t} = 0 \tag{A.9}$$

Unconstrained households make loans to collateral constrained households at nominal interest rate r^e, via a financial intermediary that has a participation constraint

$$(1-\mathcal{X}_t^h)(1+r_t^e)d_{t-1}^h + (1-\mu^h)\mathcal{X}_t^h \phi_t^h q_t^h h_{t-1} \geq (1+r_t)d_{t-1}^h$$

which can be simplified to: $d_{t-1}^h = \frac{\phi_t^h(1-\mathcal{X}_t^h)q_t^h h_{t-1} + h_{t-1} q_t^h \phi_t^h \mathcal{X}_t^h (1-\mu^H)}{1+r_t}$. We also define households deposit condition $d_{t-1} = d_t^h$. Aggregating labor and consumption of the two type of households yields $l_{i,t} = [l_{u,t} + l_{c,t}]$, and $c_{i,t} = [c_{u,t} + c_{c,t}]$.

Labor Markets Labor contractors hire households $i \in \{u, c\}$ and sell homogeneous labor services to the intermediate good producers. Labor services takes the form

$$l_t = \left[\int_0^1 (l_{i,t})^{\frac{1}{\nu_l}} di \right]^{\nu_l}$$

subject to

$$\int_0^1 w_{i,t} l_{i,t} di = w_t l_t$$

where ν_l denote the fixed wage markup and $w_{i,t}$ is the wage rate.
There is a monopoly union that represent all workers and set wage and faces Calvo style frictions,

$$w_t = \left[(1-\zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}} + \zeta_l (w_t^*)^{\frac{1}{1-\nu_l}} \right]^{1-\nu_l} \tag{A.10}$$

Where $1-\zeta_l$ is the probability that the monopoly reoptimizes the wage, while with probability ζ_l the monopoly cannot reoptimize.

If the monopoly cannot reoptimize, then its sets the wage w_t^* according to

$$w_t^* = \left(\frac{E\Sigma_{s=0}^{\infty} (\beta\zeta_l)^s \psi_l \nu_l (l_{t,s})^{(1+\sigma_l)} \left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}} \right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty} (\beta\zeta_l)^s \lambda_{t+s} w_{t+s}^{-\frac{\nu_l}{1-\nu_l}} l_{t+s}} \right)^{\frac{\nu_l - 1}{\sigma_l \nu_l + \nu_l - 1}}$$

Labor union optimality conditions: The objective of households is to choose the wage level that maximizes the utility s periods into the future

$$\begin{aligned}\underset{w_{i,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\left[\lambda_{t+s}w_{i,t}l_{i,t+s}-\psi_l\frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l}\right]\\ \text{subject to} \quad & l_{i,t+s}=\left(\frac{w_{i,t}}{w_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}}l_{t+s},\end{aligned}$$

We first simplify the expression

$$E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\left[\lambda_{t+s}w_{i,t}l_{i,t+s}-\psi_l\frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l}\right]$$

$$E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{i,t}l_{i,t+s}-E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l}$$

$$E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{i,t}\left(\left(\frac{w_{i,t}}{w_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}}l_{t+s}\right)-E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\frac{\left(\left(\frac{w_{i,t}}{w_{t+s}}\right)^{\frac{\nu_l}{1-\nu_l}}l_{t+s}\right)^{1+\sigma_l}}{1+\sigma_l}$$

Then we derive the first order condition with respect to $w_{i,t}$:

$$\frac{\partial\mathcal{L}}{\partial w_{i,t}}:\quad E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}\frac{1}{1-\nu_l}w_{i,t}^{\frac{1}{1-\nu_l}-1}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}-E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\frac{\nu_l\left(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}}\right)^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{(1-\nu_l)w_{i,t}}$$

$$w_{i,t}^{\frac{1}{1-\nu_l}-1}=E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\frac{\nu_l\left(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}}\right)^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{(1-\nu_l)w_{i,t}}\frac{(1-\nu_l)}{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t}w_{i,t}^{\frac{1}{1-\nu_l}-1}=\frac{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\nu_l(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$\frac{w_{i,t}w_{i,t}^{\frac{1}{1-\nu_l}-1}}{(w_{i,t}^{\frac{\nu_l}{1-\nu_l}})^{(1+\sigma_l)}}=\frac{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\nu_l(l_{t,s})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t}^{\frac{\sigma_l\nu_l+\nu_l-1}{\nu_l-1}}=\frac{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\psi_l\nu_l(l_{t,s})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}\left(\beta\zeta_l\right)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t} = \left(\frac{E\Sigma_{s=0}^{\infty} (\beta\zeta_l)^s \psi_l \nu_l (l_{t,s})^{(1+\sigma_l)} \left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}} \right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty} (\beta\zeta_l)^s \lambda_{t+s} w_{t+s}^{-\frac{\nu_l}{1-\nu_l}} l_{t+s}} \right)^{\frac{\nu_l - 1}{\sigma_l \nu_l + \nu_l - 1}}$$

Now assume that $w_t^* = w_{i,t}$. The wage level according the the Calvo type frictions is given by

$$w_t = \left[(1-\zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}} + \zeta_l (w_t^*)^{\frac{1}{1-\nu_l}} \right]^{1-\nu_l} \tag{A.11}$$

Goods Production The economy is populated by a continuum of firms and operates under a monopolistic competition. Each firm has the final good stock, which writes $y_t = \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$. The final good is indexed by $j \in [0,1]$, and $1 \leq \nu_p < \infty$. A higher price markup ν_p implies that firms have market power and that the good is less substitutable for other goods.
With capital and labor inputs, the monopolist produce the intermediate good y_t, the intermediate good production function takes the following form

$$y_t = (k_t)^{\alpha} (z_t l_t)^{1-\alpha} \tag{A.12}$$

Where l_t is capital input, l_t is the labor input and z_t is an exogenous productivity shock $z_t = \rho_z z_{t-1} + \epsilon_t^z$. The parameter $\alpha \in (0,1)$ measures the responsiveness of output to changes in capital. We assume that price are sticky, by using a variant of Calvo type frictions. The monopolist can set the price p_t of the good by reoptimizing the price with probability $1 - \zeta_p$, or without reoptimzing the price with probability ζ_p, and p_t^* is the price level that maximizes the expected discounted value of future profits into the future. The price level is defined by

$$p_t = \left[(1-\zeta_p)(p_{t-1})^{\frac{\nu_p}{1-\nu_p}} + \zeta_p (p_t^*)^{\frac{\nu_p}{1-\nu_p}} \right]^{\frac{1-\nu_p}{\nu_p}} \tag{A.13}$$

After setting the prices and the quantities the monopolist minimize the production cost

$$w_t l_t + r_t^k k_t$$

subject to production function (A.12). The first-order conditions, with respect to labor and capital, yield the following:

$$\mu_t = \frac{w_t}{(1-\alpha)(k_t)^{\alpha} (z_t l_t)^{-\alpha}}, \tag{A.14}$$

$$\mu_t = \frac{r_t^k}{\alpha (z_t l_t)^{1-\alpha} (k_t)^{\alpha - 1}}. \tag{A.15}$$

We can also characterize an alternative expression for the marginal cost μ_t

$$\mu_t = \left(\frac{1}{1-\alpha} \right)^{(1-\alpha)} \left(\frac{1}{\alpha} \right)^{\alpha} (r_t^k)^{\alpha} w_t^{(1-\alpha)} \tag{A.16}$$

Monopolistic competition - optimality condition The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, which writes:

$$y_t = \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p}$$

Final good producers purchase the good and resell it to consumers. Their objective is to maximize their profits.

$$p_t y_t - \int_0^1 p_{j,t} y_{j,t} dj$$
$$p_t \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p} - \int_0^1 p_{j,t} y_{j,t} dj.$$

The first-order condition with respect to $y_{j,t}$ is given by

$$
\begin{aligned}
(\partial y_{j,t}): \qquad & p_t(1+\nu_p) \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{1+\nu_p-1} \frac{1}{1+\nu_p} y_{j,t}^{(\frac{1}{1+\nu_p})-1} - p_{j,t} = 0 \\
& \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{\nu_p} y_{j,t}^{(\frac{-\nu_p}{1+\nu_p})} = \frac{p_{j,t}}{p_t} \\
& \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj \right]^{-(1+\nu_p)} y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{\frac{1+\nu_p}{\nu_p}} \\
& y_t^{-1} y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\frac{1+\nu_p}{\nu_p}}.
\end{aligned}
$$

From the first-order condition, $y_{j,t}$ is given by

$$y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\frac{1+\nu_p}{\nu_p}} y_t.$$

To define the aggregate price of the final good p_t, I use an expression for the output, which is equal to price times quantities $p_t y_t = \int_0^1 p_{j,t} y_{j,t} dj$; then, I simplify to obtain the expression for the aggregate price of the final good:

$$p_t y_t = \int_0^1 p_{j,t} \left(\frac{p_{j,t}}{p_t} \right)^{-\frac{1+\nu_p}{\nu_p}} y_t dj$$
$$p_t = \left[\int_0^1 p_{j,t}^{-\frac{1+\nu_p}{\nu_p}} dj \right]^{-\frac{\nu_p}{1+\nu_p}}.$$

Calvo Pricing- optimality condition: We assume that prices are sticky, in this environment firms choose the price level that maximizes their profits:

$$\begin{aligned} \underset{p_{j,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty}\beta^s(1-\zeta^p)[y_{j,t}(p^*_{j,t}-\mu_{j,t+s}z_t p_{j,t+s})] \\ \text{subject to} \quad & y_{j,t} = y_t\left(\frac{p_{j,t}}{p_t}\right)^{\frac{\nu_p}{1-\nu_p}}, \end{aligned}$$

We solve the firm problem by substituting $y_{j,t}$ into the firm's profits function:

$$(\partial p^*_{j,t}): \qquad p^*_{j,t} = \nu_p \frac{E_t\Sigma_{s=0}^{\infty}\beta^s(1-\zeta^p)\left(\frac{p_{j,t}}{p_{t+s}}\right)^{\frac{\nu_p}{1-\nu_p}} y_{j,t+s}z_t\mu_{j,t+s}p_{j,t+s}}{E_t\Sigma_{s=0}^{\infty}\beta^s\zeta^p_s\left(\frac{p_{j,t}}{p_{t+s}}\right)^{\frac{\nu_p}{1-\nu_p}} y_{j,t+s}}$$

The recursive form is written as:

$$K_{p,t} = z_t\mu_t y_t + \beta(1-\zeta_p)E_t K_{p,t+1} \tag{A.17}$$

$$F_{p,t} = \mu_t y_t + \beta(1-\zeta_p)E_t\frac{p_{t-1}}{p_t}F_{p,t+1}, \tag{A.18}$$

Capital Market In capital market, the previous capital is combined with investment goods to produce a new capital, which is supplied to entrepreneurs with the following technology:

$$k_t = (1-\delta)k_{t-1} + \left(1-\frac{S}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2\right)i_t, \tag{A.19}$$

where capital decays at the fixed rate $0 < \delta \leq 1$. According to this equation, the new capital depends on the existing capital and investment good i_t. The quantity of investment at period t is proportional to the adjustment cost function S.

Capital producers choose investments to maximize the present value and future operating profits, less the total investment cost, as shown below:[7]

$$q^k_t i_t - [1+\frac{S}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2]i_t$$

Given by the law of motion for capital stock k_t and investment expenditures, entrepreneurs sell capital to firms at price q^k. Capital producers' optimal investment policy would equalize the marginal revenue product of capital q^k to the marginal cost of investment goods, as shown below:

$$\frac{\partial\mathcal{L}_t}{\partial i_t}: \qquad (-\mu_t)\,q^k_t + \mu_t\left(1+\frac{S}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2 + \left(\frac{i_t}{i_{t-1}}-1\right)S\frac{i_t}{i_{t-1}}\right)$$

[7] The capital producer profit is given by $q^k_t k_t - [1+S/2\,(i_t/i_{t-1}-1)^2]i_t$, where the law of motion for capital $k = k' + [1+S/2\,(i_t/i_{t-1}-1)^2]i_t$, given that in a steady state, $S(.) = 0$ becomes $k - k' = I_t$. Then the capital producer problem can be written as $q^k_t i_t - [1+S/2\,(i_t/i_{t-1}-1)^2]i_t$.

$$- S\beta\mu_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2\left(\frac{i_{t+1}}{i_t}-1\right)=0 \tag{A.20}$$

Entrepreneurs and Introducing Collateral Requirement Shock. Capital producers sell capital to entrepreneurs at price q_t^k. We assume the capital flow of the entrepreneur is given by

$$k_t = r_{t-1}^k k_{t-1} + (1-\delta)k_{t-1} \tag{A.21}$$

The provision of capital to firms at time t equals the return on renting capital services and the previous capital k_{t-1} which decays at fixed rate δ. The first order condition with respect to capital gives

$$\mu_t = E_t\beta\mu_{t+1}r_t^k + E_t\beta\mu_{t+1}(1-\delta) \tag{A.22}$$

the entrepreneur enjoys the average gross nominal rate of return on capital $(1+r_t^k)$.

Note that each entrepreneur purchase capital good k_t at price q_t^k using loans m_t obtained from bank and net worth n_t^e. Then,

$$k_t q_t^k = m_t + n_t^e \tag{A.23}$$

Entrepreneurs are hit by a shock $\mathcal{X}_{e,t} = 1/\left(1+e^{\sigma_{e,t}}\right)$ which describe the probability that entrepreneurs are unable to pay their debt. Letting $\sigma_{e,t}$ obey to an AR(1) process $\sigma_{e,t} = \rho_{\sigma_e}\sigma_{e,t-1} + \epsilon_t^{\sigma_e}$, thus, the default probability increases when $\sigma_{e,t}$ goes up. The collateral constraint for entrepreneurs can be defined as

$$\phi_{e,t}(1+r_t^k)q_t^k k_t \geq (1+r_t^e)m_t^e$$

Entrepreneur's debt is charged-off with probability $\mathcal{X}_{t-1}^e$, as the entrepreneur is unable to pay the interest and principle, as a result the pledged collateral is sized by the bank. we assume that collateral requirements $\phi_{e,t}$ evolve over time according to the following law of motion $\phi_t^e = \rho_{\phi_e}\phi_{t-1}^e + (1-\rho_{\phi^e})\alpha^e\mathcal{X}_{t-1}^e + \epsilon_t^{\phi_e}$. This shock reveals that regulators adjust the level of collateral requirement to the previous level at $t-1$ where ρ_{ϕ_e} determine the regulator policy weight on previous collateral requirement. Additionally, regulators can observe the level of charge-offs rate of business loans and sdjust collateral requirement accordingly with $(1-\rho_{\phi^e})\alpha^e$ interpreted as the regulator policy weight on the level of charge-offs rate. The innovation $\epsilon_t^{\phi_e}$ is assumed to be an i.i.d. with mean zero and standard deviation σ^{ϕ_e}.

The entrepreneur expected earnings is given by

$$E_t\left\{\left[(1+r_t^k)q_t^k k_t - (1+r_t^e)m_t^e\right](1-\mathcal{X}_t^e)\right\}$$

where r_t^k is the rate of return on capital and r_t^e is the net interest rate paid by entrepreneurs on their debt m_t. For the sake of simplicity, we define the probability that a entrepreneurs will default on their loans $\mathcal{X}_t^e$, and the probability that they can make meet scheduled repayment on their debt $1-\mathcal{X}_t^e$. We assume that the leverage level of entrepreneurs is defined as $Lev_t = m_t^e/n_t^e$. Thus, the problem of the entrepreneur is to maximize the entrepreneurial earnings subject to participation constraint

$$E_t\left\{ (1-\mathcal{X}_t^e)\,(1+r_t^e)m_t + (1-\mu^e)\mathcal{X}_t^e(1+r_t^k)\phi_t^e q_t^k k_t \geq (1+r_t)m_t \right\} \tag{A.24}$$

Note that the equation of entrepreneur bank participation (A.24) must hold with strict equality in every state of nature. The first term on the left-hand side of the equation corresponds to returns from non-defaulting entrepreneurs, and the second term corresponds to the returns from defaulting entrepreneurs whose collateral is sized by banks and net of monitoring cost μ^e. The right-hand side describes the return on loans given the interest rate demanded by banks r_{t+1}. We define the entrepreneur expected net worth as $n_{t+1}^e = +\gamma^E\left((1-\mathcal{X}_t^e-(1-\mathcal{X}_t^e)\,\phi_t^e)\left(1+R_t^k\right)k_t q_{t-1}^k + k_t\,(1-\delta)\,q_{t-1}^k\right)$.

Appendix B Steady states.

After calibrating model parameters, we obtain the steady state of the model. First, we solve for r^k_{ss}

$$\begin{aligned}
\lambda_t &= E_t \beta \lambda_{t+1} r^k_t + E_t \beta \lambda_{t+1}(1-\delta) \\
\lambda_{ss} &= \beta \lambda_{ss} r^k_{ss} + \beta \lambda_{ss}(1-\delta) \\
r^k_{ss} &= \frac{1}{\beta} - (1-\delta).
\end{aligned}$$

We pin down the value of r_{ss} in the steady state

$$\begin{aligned}
\lambda_t - \beta E_t \lambda_{t+1}(1 + r_{t+1}) &= 0 \\
\lambda_{ss} - \beta \lambda_{ss}(1 + r_{ss}) &= 0 \\
r_{ss} &= \frac{1}{\beta} - 1,
\end{aligned}$$

we also compute the value of q^k_{ss}

$$\begin{aligned}
& q^k_t \lambda_t - \lambda_t [1 + S\left(\zeta_{i,t} \frac{i_t}{i_{t-1}}\right) + \zeta_{i,t} \frac{i_t}{i_{t-1}} S'\left(\zeta_{i,t} \frac{i_t}{i_{t-1}}\right)\Big] + \beta \lambda_{t+1} \zeta_{i,t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 S''\left(\zeta_{i,t+1} \frac{i_{t+1}}{i_t}\right) = 0 \\
& q^k_{ss} = 1,
\end{aligned}$$

and p^*_{ss} is

$$\begin{aligned}
p_t &= \left[(1-\zeta_p)(p_{t-1})^{\frac{\nu_{p,t}}{1-\nu_{p,t}}} + \zeta_p (p^*_t)^{\frac{\nu_{p,t}}{1-\nu_{p,t}}}\right]^{\frac{1-\nu_{p,t}}{\nu_{p,t}}} \\
p_{ss} &= \left[(1-\zeta_p)(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}} + \zeta_p (p^*_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}}\right]^{\frac{1-\nu_{p,ss}}{\nu_{p,ss}}} \\
p^*_{ss} &= \left(\frac{(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}} - (1-\zeta_p)(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}}}{\zeta_p}\right)^{\frac{1-\nu_{p,ss}}{\nu_{p,ss}}}.
\end{aligned}$$

The steady state value of w^*_{ss} is given by

$$\begin{aligned}
w_t &= \left[(1-\zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}} + \zeta_l (w^*_t)^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l} \\
w_{ss} &= \left[(1-\zeta_l)(w_{ss})^{\frac{1}{1-\nu_l}} + \zeta_l (w^*_{ss})^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l} \\
w^*_{ss} &= \left(\frac{(w_{ss})^{\frac{1}{1-\nu_l}} - (1-\zeta_l)(w_{ss})^{\frac{1}{1-\nu_l}}}{\zeta_l}\right)^{1-\nu_l},
\end{aligned}$$

and the marginal cost of production μ_{ss} is

$$p_t = (1 + \nu_{p,t})\mu_t$$

$$\mu_{ss} = \frac{1}{(1+\nu_{p,ss})} p_{ss}.$$

Given that $q^k_{ss} k_{ss} = m_{ss} + n^e_{ss}$ and $Lev_{ss} = m_{ss}/n^e_{ss} = (q^k_{ss} k_{ss} - n^e_{ss})/n^e_{ss}$. We use the participation constraint and the expected net worth to solve simultaneously for k_{ss} and n^e_{ss}

$$\begin{aligned}
((1-\mathcal{X}^e_t) + (1-\mu^e)\mathcal{X}^e_t)\,\phi^e_t(1+r^k_t)q^k_t k_t &\geq (1+r_t)(q^k_t k_t - n^e_t) \\
((1-\mathcal{X}^e_{ss}) + (1-\mu^e)\mathcal{X}^e_{ss})\,\phi^e_{ss}(1+r^k_{ss})q^k_{ss} k_{ss} &\geq (1+r_{ss})(q^k_{ss} k_{ss} - n^e_{ss}),
\end{aligned}$$

$$\begin{aligned}
n^e_t &= (1-\mathcal{X}^e_t) - \phi^e_t(1-\mathcal{X}^e_t))(1+r^k_t)(Lev_{ss}+1)n^e_t \left(\frac{1+r_t}{1+r^k_t}\right) + (1-\delta)q^k_t k_t \\
n^e_{ss} &= (1-\mathcal{X}^e_{ss}) - \phi^e_{ss}(1-\mathcal{X}^e_{ss}))(1+r^k_{ss})(Lev_{ss}+1)n^e_{ss} \left(\frac{1+r_{ss}}{1+r^k_{ss}}\right) + (1-\delta)q^k_{ss} k_{ss}.
\end{aligned}$$

We then solve for investment i_{ss}

$$\begin{aligned}
k_t &= (1-\delta)k_{t-1} + \left(1 - S\big(\zeta_{i,t}, \frac{i_t}{i_{t-1}}\big)\right) i_t \\
k_{ss} &= (1-\delta)k_{ss} + \left(1 - S\big(\zeta_{ss}, \frac{i_{ss}}{i_{ss}}\big)\right) i_{ss} \\
k_{ss} &= (1-\delta)k_{ss} + i_{ss} \\
i_{ss} &= k_{ss} - (1-\delta)k_{ss} \\
i_{ss} &= \delta k_{ss}.
\end{aligned}$$

We also calculate m_{ss}

$$\begin{aligned}
m_t &= q_t k_t - n^e_t \\
m_{ss} &= q_{ss} k_{ss} - n^e_{ss}.
\end{aligned}$$

Next we find, the value of labor in steady state l_{ss}

$$\begin{aligned}
\mu_t &= \frac{r^k_t}{\alpha \gamma_t (z_t l_t)^{1-\alpha} (k_t)^{\alpha-1}} \\
l_t &= \left(\frac{r^k_t}{\alpha \gamma_t \mu_t (k_t)^{\alpha-1}}\right)^{\frac{1}{1-\alpha}} \frac{1}{z_t} \\
l_{ss} &= \left(\frac{r^k_{ss}}{\alpha \gamma_{ss} \mu_t (k_{ss})^{\alpha-1}}\right)^{\frac{1}{1-\alpha}} \frac{1}{z_{ss}},
\end{aligned}$$

and subsequently solve for output y_{ss}

$$\begin{aligned}
y_t &= \gamma_t (k_t)^{\alpha} (z_t l_t)^{1-\alpha} \\
y_{ss} &= \gamma_{ss} (k_{ss})^{\alpha} (z_{ss} l_{ss})^{1-\alpha}.
\end{aligned}$$

We obtain the steady state value of the government spending g_{ss}

$$\begin{aligned} g_t &= \eta_g y_t \\ g_{ss} &= \eta_g y_{ss}. \end{aligned}$$

We compute r^e_{ss}

$$\begin{aligned} \beta E_t \lambda_{t+1}(1 + r^e_t) - \lambda_t &= 0 \\ \beta \lambda_{ss}(1 + r^e_{ss}) - \lambda_{ss} &= 0 \\ \beta \lambda_{ss}(1 + r^e_{ss}) &= \lambda_{ss} \\ \beta(1 + r^e_{ss}) &= 1 \\ (1 + r^e_{ss}) &= \frac{1}{\beta} \\ r^e_{ss} &= \frac{1}{\beta} - 1, \end{aligned}$$

we also obtain the steady state value of d^h_s

$$\begin{aligned} (1 - \mathcal{X}^h_{ss})\phi_{ss} q^h_{ss} h_{ss} + (1 - \mu^h)\mathcal{X}^h_{ss}\phi_{ss} q^h_{ss} h_{ss} &= (1 + r_{ss}) d^h_s \\ d^h_s &= \frac{(1 - \mathcal{X}^h_{ss})\phi_{ss} q^h_{ss} h_{ss} + (1 - \mu^h)\mathcal{X}^h_{ss}\phi_{ss} q^h_{ss} h_{ss}}{(1 + r_{ss})}, \end{aligned}$$

we compute the value of f^e_{ss}

$$\begin{aligned} f^e_t &= \phi_t \mu^e \mathcal{X}^e_t (1 + r^k_t) q^k_t k_t \\ f^e_{ss} &= \phi_{ss} \mu^e \mathcal{X}^e_{ss} (1 + r^k_{ss}) q^k_{ss} k_{ss}. \end{aligned}$$

Finally, we solve for f^h_{ss}

$$\begin{aligned} f^h_t &= \phi_t \mu^h \mathcal{X}^h_t q^h_t h_t \\ f^h_{ss} &= \phi_{ss} \mu^h \mathcal{X}^h_{ss} q^h_{ss} h_{ss}. \end{aligned}$$

Appendix C Log-Linearized Model

Business Loan Delinq. Rate Shock $\tilde{\sigma}^e_t = \epsilon_{\sigma_e,t} + \rho^e \tilde{\sigma}^e_{t-1}$

Real Estate Loan Delinq. Rate Shock $\tilde{\sigma}^h_t = \epsilon_{\sigma_h,t} + \rho^h \tilde{\sigma}^h_{t-1}$

Business Loan Coll. Req. $\frac{1}{\mathcal{X}^e}\tilde{\mathcal{X}}^e_t - \tilde{\sigma}^e_t \sigma^e_{ss} = 0$

Real Estate Loan Coll. Req. $\frac{1}{\mathcal{X}^h}\tilde{\mathcal{X}}^h_t - \tilde{\sigma}^h_t \sigma^h_{ss} = 0$

Business Loan Coll. Req. Shock $\tilde{\phi}^e_t = \epsilon_{\phi_e,t} + \rho^e \tilde{\phi}^e_{t-1} + \tilde{\mathcal{X}}^e_t(1-\rho^e)\alpha^e$

Real Estate Loan Coll. Req. Shock $\tilde{\phi_t}^h = \epsilon_{\phi_h,t} + \rho^h \tilde{\phi}^h_{t-1} + \tilde{\mathcal{X}}^h_t(1-\rho^h)\alpha^h$

Gov. Spend. Shock $\tilde{\eta}_{g,t} = \epsilon_{g,t} + \rho^g \tilde{\eta}_{g,t-1}$

Tech. Shock $\tilde{z}_t = \epsilon_{z,t} + \rho^z \tilde{z}_{t-1}$

Cons. Pref. Shock $\tilde{\zeta}^c_t = \epsilon_{\zeta_c,t} + \rho^{\zeta c}\tilde{\zeta}^c_{t-1}$

Price according to Calvo $\zeta_p \tilde{p}^*_t = \tilde{p}_t - (1-\zeta_p)\tilde{p}_{t-1}$

Price Recursive $\tilde{F}_{p,t} = \tilde{\mu}_t + \tilde{y}_t + \left(1-\zeta_p\right)\beta\tilde{p}_{t-1} - \left(1-\zeta_p\right)\beta\tilde{p}_t + \left(1-\zeta_p\right)\beta\tilde{F}_{p,t+1}$

Price Recursive $\tilde{K}_{p,t} = \tilde{\mu}_t + \tilde{y}_t + \tilde{z}_t + \left(1-\zeta_p\right)\beta\tilde{K}_{p,t+1}$

F.O.C. w.r.t. Price $-\left(1+\nu_p\right)\tilde{\mu}_{t+1} + \tilde{p}^*_t - \tilde{p}_{t+1} = 0$

Wage according to Calvo $\zeta_l \tilde{w}^*_t = \tilde{w}_t - (1-\zeta_l)\tilde{w}_{t-1}$

Wage Recursive $\tilde{\mu}_t \frac{1}{\nu_l} - \tilde{w}^*_t + \frac{1}{\nu_l}\tilde{l}_t + \beta\left(1-\zeta_l\right)\tilde{F}_{w,t+1} - \tilde{F}_{w,t} = 0$

Wage Recursive $\tilde{F}_{w,t}\frac{1-\zeta_l}{1+\frac{1+\nu_l}{\nu_l}+\sigma^l} + \tilde{w}^*_t \frac{1-\zeta_l}{1+\frac{1+\nu_l}{\nu_l}+\sigma^l} + \zeta_l \tilde{w}_{t-1} - \tilde{w}_t = 0$

Capital Law of Motion $\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \delta\tilde{i}_t - \tilde{i}_t S + S\tilde{i}_{t-1}$

F.O.C. w.r.t. Investment $-\tilde{\mu}_t - \tilde{q}^k_t + \tilde{\mu}_t + S\tilde{i}_t - S\tilde{i}_{t-1} + S\tilde{i}_t$

$-S\tilde{i}_{t-1} + S\tilde{i}_t - S\tilde{i}_{t-1} - S\beta\tilde{\mu}_{t+1} - S\beta * \tilde{i}_{t+1} + S\beta * \tilde{i}_t + S\beta\tilde{i}_{t+1} - S\beta * \tilde{i}_t$

Producer Euler Equation $\tilde{\mu}_{t+1} = \alpha\tilde{r}^k_t + (1-\alpha)\tilde{w}_t$

F.O.C. w.r.t Capital-Capital Producer $-\tilde{R}^k_{t+1} + \tilde{r}^k_t + (1-\delta)\tilde{q}^k_{t+1} - \tilde{q}^k_t = 0$

F.O.C. w.r.t Capital-Firm $-\tilde{r}^k_t + (1-\alpha)\tilde{z}_t + \tilde{l}_t(1-\alpha) - (1-\alpha)\tilde{k}_t = 0$

F.O.C. w.r.t Labor-Firm $-\tilde{l}_t - \frac{1}{(-\alpha)}\tilde{z}_t + \frac{1}{(-\alpha)}\tilde{w}_t - \alpha\tilde{k}_t = 0$

Entrepreneur Optimality Condition $\tilde{\phi}^e_t + \tilde{\mathcal{X}}^e_t + \tilde{R}^K_{t+1} - \tilde{R}_t - \tilde{n}^e_t = 0$

Entrepreneur Net Worth Definition $-\tilde{n}^e_t - \tilde{\phi}^e_t + \tilde{R}^k_t - \tilde{r}_{t-1} + \tilde{k}_{t-1} + \tilde{q}^k_{t-1} + \tilde{n}^e_{t-1} = 0$

Production Function $(1-\alpha)\tilde{l}_t + (1-\alpha)\tilde{z}_t + \alpha\tilde{k}_t - \tilde{y}_t = 0$

Monetary Policy Shock $\tilde{r}_t - \rho^p\tilde{r}_{t-1} - (1-\rho^p)\,\alpha_\pi\tilde{p}_t - (1-\rho^p)\,\alpha_y\tilde{y}_t - \epsilon_t = 0$

Government Spending Share $\tilde{y}_t - \tilde{g}_t + \tilde{\eta}_{g,t} = 0$

Labor Supply-unconstrained Households $\tilde{l}_{u,t} = \tilde{l}_t$

Labor Supply-constrained Households $\tilde{l}_{c,t} = \tilde{l}_t$

F.O.C. w.r.t. Labor-unconstrained Households $\sigma^l\tilde{l}_{u,t} = \tilde{w}_t + \tilde{\lambda}_{u,t}$

F.O.C. w.r.t. Labor-constrained Households $\sigma^l\tilde{l}_{c,t} = \tilde{w}_t + \tilde{\lambda}_{c,t}$

F.O.C. w.r.t. Consumption-unconstrained Households $\tilde{p}_t + \tilde{\lambda}_{u,t} - \tilde{\zeta}_{c,t} + \tilde{c}_{u,t} - b_u\tilde{c}_{u,t-1} + \beta b_u\tilde{\zeta}_{c,t+1} - \tilde{p}_t = 0$

F.O.C. w.r.t. Consumption-constrained Households $\beta b_c\tilde{\zeta}_{c,t+1} + \tilde{p}_t + \tilde{\lambda}_{c,t} - \tilde{\zeta}_{c,t} + \tilde{c}_{c,t} - b_c\tilde{c}_{c,t-1} - \tilde{p}_t = 0$

Reals Estate Volume Shock $\tilde{h}_t = \epsilon_{h,t} + rho^q\tilde{h}_{t-1}$

Budget constraint-unconstrained Households $\tilde{c}_{u,t} = \tilde{r}_{t-1} + \tilde{w}_t + \tilde{l}_{u,t} - \tilde{d}_t$

Households Deposit Defintion-unconstrained Households $-\tilde{d}_{t-1} + \tilde{d}^h_t = 0$

Participtaion Constraint-unconstrained Households $-\tilde{d}^h_{t-1} + \tilde{\mathcal{X}}^h_t + \tilde{\phi}^h_t + \tilde{q}^h_t + \tilde{h}_{t-1} + \tilde{q}^h_t + \tilde{h}_{t-1} - \tilde{r}_t = 0$

Housholds Budget Constraint-unconstrained Households $\tilde{c}_{c,t} = \tilde{d}^h_t + \tilde{h}_{t-1} + \tilde{q}^h_t + \tilde{w}_t + \tilde{l}_{c,t} - \tilde{q}^h_t + \tilde{h}_t - \tilde{d}^h_{t-1} + \tilde{r}^e_t$

Business Loan Total Charge-offs $\tilde{q}^k_{t-1} + \tilde{k}_{t-1} - \tilde{f}^e_t + \tilde{\mathcal{X}}^e_{t-1} + \tilde{\phi}^e_{t-1} + \tilde{R}^K_{t-1} = 0$

Real Estate Loan Total Charge-offs $\tilde{h}_{t-1} - \tilde{f}^h_t + \tilde{\mathcal{X}}^h_{t-1} + \tilde{\phi}^h_{t-1} + \tilde{q}^h_{t-1} = 0$

Business Loan Total Definition $\tilde{m}_t = \tilde{k}_t + \tilde{q}^k_t - \tilde{n}^e_{t+1}$

Total Consumption $\tilde{c}_t = \tilde{c}_{u,t} + \tilde{c}_{c,t}$

Total Output $\tilde{c}_t + D\tilde{I}FL_t = \tilde{y}_t - \tilde{i}_t - \tilde{f}^e_{t-1} - \tilde{f}^h_{t-1} - \tilde{g}_t$

Firm Value $\tilde{v}_t = \tilde{k}_{t-1} + \tilde{q}^k_{t-1} + \tilde{n}^e_{t-1} + \tilde{k}_{t-1} + \tilde{q}^k_{t-1} + \tilde{R}^K_t - \tilde{\phi}^e_t$

Fundamnetal Value and Average Capital Price $\tilde{\mathcal{S}}_t = \tilde{k}_t + \tilde{q}^k_t - \tilde{v}_t$

Housing Stock Fundamnetal Value $Q\tilde{h}F_t = -\chi^h \tilde{h}_t - (\chi^h + \beta)\tilde{\lambda}_{c,t} + \tilde{\phi}^h_t + \tilde{q}^h_t - \tilde{r}^e_t + \beta\tilde{\lambda}_{c,t+1} - \beta\tilde{q}^h_{t+1}$

Fundamnetal Value and Average Housing Stock Price $r\tilde{esi}_t = Q\tilde{h}F_t - \tilde{q}^h_t$

Measurement Equations:

Gross Domestic Product	$\text{gdpobs}_t = \tilde{y}_t + \varrho_{y^{obs},t}$
Consumption	$\text{consumptionobs}_t = \tilde{c}_t$
CI Loans	$\text{CIloanobs}_t = \tilde{m}_t$
Real Estate Loans	$\text{Hloanobs}_t = \tilde{d}^h_t$
Investment	$\text{investobs}_t = \tilde{i}_t + \varrho_{i^{obs},t}$
Total Hours	$\text{hoursobs}_t = \tilde{l}_t + \varrho_{l^{obs},t}$
CI Charge-offs	$\text{CIchargoffobs}_t = \tilde{f}^e_t + \varrho_{f^{e,obs},t}$
Real estate Charge-offs	$\text{Hcharfoffobs}_t = \tilde{f}^h_t + \varrho_{f^{h,obs},t}$
Gov. Spending	$\text{govobs}_t = \tilde{g}_t$
Delinquency Rate-Real Estate Loans	$\text{defaulthobs}_t = \tilde{\mathcal{X}}^h_t$
Delinquency Rate-Business Loans	$\text{defaulteobs}_t = \tilde{\mathcal{X}}^e_t$

Appendix D Data.

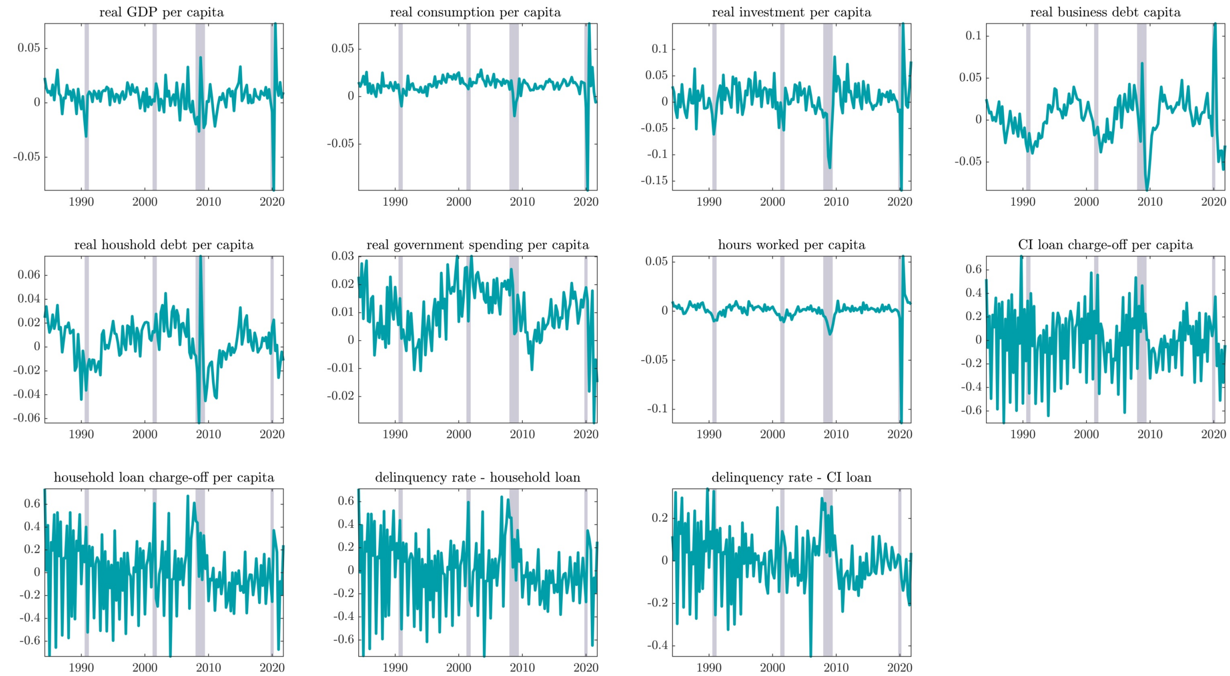

Figure 10: US Data

Note: Quarterly data retrieved from Federal Reserve Bank of St. Louis Database (FRED) and Federal Deposit Insurance Corporation (FDIC)

Appendix E Equilibrium Definition

A competitive equilibrium consists of stochastic processes $\{p_t, p_t^*, F_{p,t}, K_{p,t}, w_t, w_t^*, F_{w,t}, k_t, i_t, \mu_t, q_t^k, r_t^k, \bar{r}_t^k, n_t^e, y_t, l_t, l_{u,t}, l_{c,t}, \lambda_t, \lambda_{u,t}, \lambda_{c,t}, c_t, c_{u,t}, c_{c,t}, d_t, h_t, d_t^h, q_t^h, r_t^e, f_t^e, f_t^h, m_t^e\}$ that evolve according to a system of equations. This model is driven by 8 exogenous shocks: government spending shock $\eta_{g,t}$, technology shock z_t, consumption preference shock ζ_t^c, monetary policy shock r_t, delinquency rate shock on firm loan $\mathcal{X}_t^e$, delinquency rate shock on mortgage loan $\mathcal{X}_t^h$, business loan collateral requirement shock ϕ_t^e, and mortgage loan collateral requirement shock ϕ_t^h.

- *In each period t, firms maximize their profits by converting intermediate goods to final goods, setting prices and wages under nominal inertia.*

- *Entrepreneurs maximize profits by providing capital services and borrowing from banks. Entrepreneurs are collateral constrained and can default on their loans.*

- *Unconstrained households maximize their lifetime utility by providing labor services, consuming the final good, and making a deposit.*

- *Constrained households maximize their lifetime utility by providing labor services, consuming the final good, purchasing real estate assets, and obtaining a loan to finance their purchase. Collateral constrained households face collateral constraints and can default on their loans.*
- *Banks intermediate the flow of deposits and loans.*
- *A macroprudential collateral policy is conducted by a regulator who adjusts the collateral requirement to the level of the charge-off rate.*
- *A central bank set the policy rate depending on expected inflation and the deviation of output from its steady state.*