Organizational Change and Reference-Dependent Preferences[†]

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Abstract: Reference-dependent preferences can explain several puzzling observations on organizational change. Loss aversion clarifies why change is often stagnant or slow for long periods followed by a sudden boost in productivity during a crisis. Moreover, it accounts for the fact that different firms in the same industry can have significant productivity differences. The model also demonstrates the importance of expectation management even if all parties have rational expectations. Social preferences explain why it may be optimal to split up a firm in two different entities.

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1 Introduction

In the early 1980s the iron ore and steel industry in the Great Lakes area was hit by a severe competitive shock. For more than 100 years it was protected from foreign competition by its geographic location. Then it suddenly faced Brazilian competitors offering steel at substantially lower prices putting its own future into doubt. However, the local steel industry managed to survive and thrive by implementing cost reductions and boosting productivity. Within a few years, labor productivity doubled. This remarkable development was not due to the introduction of any new technologies, nor to major capital investments or the exit of low productivity firms. Instead, it was mainly caused by organizational improvements, in particular changes of inefficient work practices and the more efficient use of existing capital. This raises the question of why these organizational improvements were not implemented earlier. Why did it need a crisis to implement change?[1]

This example is part of a larger puzzle: the existence of substantial and persistent heterogeneity in total factor productivity among firms within narrowly defined industries. Syverson (2004) reports for the US that a firm at 90th percentile of productivity has a TFP that is 1.9 times higher than the TFP of a firm at the 10th percentile (on average in industries at the four-digit level). These differences cannot be accounted for by differences in observable inputs or heterogeneous prices. Increasing evidence suggests that differences in productivity are associated with organizational differences and differences in management practices.[2] However, if organizational and managerial practices are crucial for productivity, why don't all firms adopt best practices and use the most efficient organizational structures? What are the frictions that prevent profit maximizing firms from producing at the efficiency frontier?

In this paper, we explore the implications of reference-dependent preferences, specifically loss aversion and social comparisons, for organizational change.[3] Technological changes create profitable opportunities for the firm that require organizational changes. In the basic version of

[1] Schmitz (2007) offers a detailed and thought-provoking case study of this episode.

[2] See e.g. Bloom et al. (2014), Bloom et al. (2019).

[3] Loss aversion and social preferences are the most widely studied "biases" in behavioral economics. A recent meta study by Brown et al. (2023) reports that the average parameter of loss aversion accross 607 empirical estimates is $\lambda = 1.955$, surprisingly close to the initial estimate by Tversky and Kahneman (1992) of $\lambda = 2.25$. Fehr and Charness (2023) offer a recent survey on the vast literature on social preferences.

the model we abstract away from any informational or contractual frictions and assume that organizational change is perfectly contractible. However, change imposes costs on workers. Without loss aversion the firm will implement the materially efficient change and compensate workers for the additional effort. If workers are loss averse, there is a threshold such that for all technological changes below this threshold the firm will stick to the status quo. If the threshold is exceeded the firm will adjust, but it will adjust less than in the situation without loss aversion. Thus, as is well known, loss aversion creates inertia.

This changes in a crisis. A crisis reduces the outside options of workers as the firm is threatened by bankruptcy, which would lead to job loss and the need to accept lower-paying jobs in other industries. In order to keep their jobs, workers have to make concessions, which can take the form of wage cuts or changes in work practices. Both of these concessions are perceived as losses by workers. To induce workers to accept organizational changes the firm can now offer lower wage cuts (reduced losses), while it would have to offer higher wage increases (greater gains) in normal times. Thus, in a crisis the effect of loss aversion cancels out in the workers' marginal rate of substitution, and the firm will jump to the materially efficient organizational structure if there are sufficient worker rents to be appropriated by the firm. This also explains why wages don't fall in a recession: Workers will make concessions by working harder rather than by accepting wage cuts.

Our basic model captures the main intuition of why a crisis triggers organizational change, but it leaves several questions unanswered. The crisis has an effect only if workers enjoy a rent. Where does this rent come from? What happens if the reference point adjusts to the new contract and the new expectations? How do rational players prepare for the possibility of crisis in the initial contract?

We develop a simple infinite horizon model that answers these questions. In this dynamic model there is technological change in every period. In normal times wages and effort weakly increase in every period. There is inertia, i.e. there is either no organizational adaptation or less than material efficiency requires. If there is organizational change, workers suffer a one-period disutility due to loss aversion for which they are compensated by a permanent wage increase. This gives rise to a quasi-rent that builds up over time. If the crisis hits, workers have to concede this quasi-rent by agreeing to substantial organizational change. If parties rationally anticipate the crisis, they will agree on higher wages to compensate for the expected utility loss in the crisis and they will delay change because change is cheaper to implement in the crisis. Thus, the model explains why there is often inertia or very slow change for extended periods of time, but then a sudden jump of productivity triggered by a crisis. Furthermore, the model explains why organizational change is history dependent and may result in large productivity differences between firms that have been founded at different points in time or faced idiosyncratic crises.

So far, we assumed that change is both deterministic and perfectly contractible. In an extension of the model we consider a principal-agent problem between the owner of the firm and a manager who has to be incentivized to implement change stochastically. However, successful change always requires that workers go along with it. If the workers' reference point is (partially) determined by their expectations, an increase in the probability of successful change increases the reference point and makes it cheaper to pay workers to accept change. Thus, the owner will induce the manager to either implement the desired change with a high probability (or certainty), or not to implement it at all. This highlights the importance of expectations management, even if all parties form rational expectations. This is consistent with the emphasis on effective leadership, vision, and creating a sense of urgency for successful organizational change by practitioners and management consultants.[4]

Reference dependence is also a significant factor in social preferences, as individuals tend to assess their situation in comparison to others in their reference group. In a final version of the model we show that this phenomenon can explain effort and wage compression within organizations and that it may be optimal to split a firm into separate entities to avoid social comparisons.

Our paper is related to three strands of the literature. First, there is an empirical literature on how competitive shocks affect productivity and productivity differences between firms. Bloom et al. (2014) report empirical evidence from the World Management Survey showing that there are large and persistent productivity differences across firms.[5] Higher total factor productivity is correlated with better management practices, but also with more

[4] See e.g. Kotter (1995) and Burke (2017).

[5] See Syverson (2011) and Gibbons and Henderson (2012) for surveys of the literature on productivity differences between firms in seemingly similar enterprises.

intense competition. Performance increases in bad times, in particular for low productivity firms. Bloom et al. (2019) show that the introduction of right-to-work laws in some states in the US is associated with improved managerial practices and efficiency increases. In Bloom et al. (2017) they also show that an increase of competition is associated with the introduction of better management techniques. Holmes and Schmitz (2010) survey case studies examining the behavior of firms that experienced dramatic changes in their competitive environment. They report that nearly all studies show that competitive shocks lead to increases in industry productivity. Plants that survive the competitive shock are typically found to have large productivity gains. Furthermore, these gains often account for most of the overall industry gains. Backus (2020) finds that an increase of competition in the ready-mix concrete industry has a direct positive effect on productivity (not driven by firm selection). All of this is consistent with our model. However, none of these papers can explain, why it needs a competitive shock or a crisis to raise productivity.

Second, our paper contributes to the small but growing literature on reference-dependent preferences in dynamic models. Pagel (2017) shows that the incorporation of dynamic reference-dependent preferences into a macro model can account for empirically observed consumption-savings patterns.[6] Pagel (2016) uses reference-dependent preferences to explain the observed equity premium volatility and the equity premium puzzle. Eliaz and Spiegler (2014) develop a model of labor market dynamics in which workers have reference dependent fairness preferences which gives rise to wage stickiness in recessions, similar to the downward rigidity of wages in our model. Macera (2018) analyzes optimal incentive contracts in a dynamic moral hazard model with loss-averse agents. She shows that the principal backloads bonus compensation and pays a fixed wage in the present period if agents are sufficiently loss averse. Herweg, Karle, and Müller (2018) examine the role of loss aversion on renegotiation in a classical buyer-seller setting. They emphasize the role of expectations, showing that if buyers do not expect to renegotiation then the parties may indeed not be able to renegotiate, even if the outcome is ex-post inefficient. Karle and Schumacher (2017) analyze the incentives of a monopolist to release ex-ante product information. They show that good information gives rise to an attachment effect if consumers are loss averse and adjust their expectations. In the

[6] Relatedly, Van Bilsen, Laeven, and Nijman (2020) find that with a backward-looking form of reference dependence consumers delay painful consumption reductions.

context of auctions von Wangenheim (2021) shows that with dynamic reference-dependent preferences the English auction yields lower revenues than a Vickrey auction due to a decrease of the attachment effect in the dynamic English auction. Rosato (2023) shows that a similar effect can explain empirically observed revenue declines in sequential auctions, since remaining bidders become less optimistic. Alesina and Passarelli (2019) study the effects of loss aversion in electoral competition. If there is an exogenous shock to voter preferences, the election outcome depends on the initial status quo. Furthermore, there are long-term cycles in policies with self-supporting movements to the right or to the left. Lockwood and Rockey (2020) apply loss aversion to electoral competition in a representative democracy. They show that an incumbent reacts less strongly to a shift in voter preferences than challengers.[7] None of these papers applies dynamic reference dependence to organizational change. Furthermore, the main effect driving our results is new and does not play a role in the previous literature.

Finally, there is a large literature in management science on organizational change (see e.g. Kotter (1995) and Burke (2017). This literature emphasizes the role of expectations management, the importance of short-term wins, and the need of urgency (i.e. of a crisis). This literature documents many interesting and important insights but lacks a rigorous microeconomic foundation.

The rest of the paper is organized as follows. The next section introduces the static version of the model. The model is "Coasean" in the sense that there are no informational or contractual frictions. The only deviation from the standard model is loss aversion. The model shows that loss aversion gives rise to inertia in normal times, but that parties will adjust towards material efficiency in a crisis. Section 3 sets up the dynamic, infinite-horizon model, in which the reference point adjusts over time. The model shows that the principal offers a permanent wage increase to compensate for change in normal times which gives rise to a quasi-rent that builds up over time. When a crisis hits, this quasi-rent is expropriated in order to implement drastic change. The model also shows how companies in the same industry using the same technology can have substantial productivity differences for extended periods of time. Section 4 introduces a principal-agent problem and shows how the principal can use expectation management to reduce the cost of implementing change. Section 5 looks

[7]Lockwood, Le, and Rockey (2022) study the interaction of loss aversion and incomplete recall on dynamic electoral competition.

at reference dependent social preferences to explain wage and effort compression within a firm. Section 6 concludes.

2 A Coasean Model With Reference Dependence

There are two players, the owner of the firm (the principal, "she") and the workers, represented by a union (the agent, "he"), who negotiate on organizational change. We focus on the effect of reference-dependent preferences on the negotiation outcome, so we abstract away from any informational or contractual frictions. The parties can implement any change via Coasean bargaining.

We start out with a simple one-period model. There is a state of the world, $\theta \in \Theta \subset \mathbb{R}$, that represents the current state of technology. Workers have to take an action $x \in \mathbb{R}^+$ that adapts the organization to the state of the world. This gives rise to a gross profit $v(x, \theta)$ that accrues to the owner. Adaptation requires costly effort from workers. Without loss of generality we measure x by its cost, i.e. $c(x) = x$.

The principal's profit function is given by

$$\Pi = v(x, \theta) - w - C \tag{1}$$

where $C \geq 0$ are all costs other than wages w. We assume that the gross profit function $v(x, \theta)$ is increasing and concave in x with $\frac{\partial v(x,\theta)}{\partial x} > 0$, $\frac{\partial^2 v(x,\theta)}{\partial x^2} < 0$, $\frac{\partial v(x,\theta)}{\partial \theta} > 0$, and $\frac{\partial v(x,\theta)}{\partial x \partial \theta} > 0$. Moreover, we assume $\lim_{x \to 0} \frac{\partial v(x,\theta)}{\partial x} = \infty$, $\lim_{x \to \infty} \frac{\partial v(x,\theta)}{\partial x} = 0$, and $\frac{\partial v(x,\theta)}{\partial x \partial \theta}$ strictly bounded away from zero in order to ensure interior solutions. The interpretation of θ is that it reflects the state of technology where a higher θ makes higher effort of workers more profitable.[8]

The utility function of the agent (the workers) is given by

$$U = w - x - \lambda[w^r - w]^+ - \lambda[x - x^r]^+ \tag{2}$$

where $[\cdot]^+ = \max\{\cdot, 0\}$. The agent's utility function is reference dependent. It consists of the material payoff, $w - x$, and the perceived experience of losses if the wage, w, is smaller than

[8] Technological progress may also make work easier. However, in this case there is no conflict of interest between workers and the firm which makes it easy to implement it. This is why we focus on the case where technological change requires more effort.

the reference wage, w^r, and/or if the effort, x, is larger than the reference effort level, x^r. The parameter $\lambda > 0$ measures the degree of loss aversion.[9] The reference point (w^r, x^r) is a convex combination of the status quo, (w_0, x_0) and the rational expectation (w^e, x^e), i.e. the values of w and x that workers expect to be realized at the end of the period:

$$w^r = \alpha w_0 + (1-\alpha)w^e , \tag{3}$$

$$x^r = \alpha x_0 + (1-\alpha)x^e \tag{4}$$

where $\alpha \in [0,1]$ is the relative weight of the status quo.

For the baseline model we assume that organizational change is perfectly contractible. The principal makes a take-it-or-leave-it offer (w, x) to the agent. Based on this offer the agent forms his reference point. We assume throughout that the agent expects to accept (x, w) if - given this expectation - acceptance is optimal.[10] Then the agent decides whether to accept or reject the offer. If he rejects, the old contract (w_0, x_0) remains in place which gives positive utility $U_0 = w_0 - x_0 \geq 0$ to the agent. After observing the agent's decision, the principal and thereafter the agent have the option to terminate the relationship. If one of them does so, both parties get a utility of zero.

2.1 Inertia and Material Inefficiency in Normal Times

We consider first the *normal case* where the old contract generates positive profits for the principal ($\Pi = v(x_0, \theta) - w_0 - C \geq 0$). This implies that the parties will continue their relationship even if the agent rejected the offer of the principal (which will be different in case of a *crisis* to be discussed later). The agent correctly anticipates the negotiation outcome, so $(w^e, x^e) = (w, x)$ if he is going to accept the contract and $(w^e, x^e) = (w_0, x_0)$ otherwise.[11] Because the agent anticipates that the relationship will be maintained even if he rejects the offer, his outside option utility in the regular case is given by $U_0 = w_0 - x_0 \geq 0$.

[9] An alternative interpretation of the model is in terms of concerns for fairness. In this interpretation w^r is the "fair wage" that workers expect to get, and x^r is the "fair effort level". Workers suffer from "inequity aversion", if the wage is below the fair wage, or the requested effort above the fair effort. See Section 5.

[10] Without this assumption there are other equilibria in which the firm has to pay higher wages to implement change if $\alpha < 1$. These equilibria are sustained by the expectation of the union that it will reject any wage offer below $\underline{w}$ with $x + \alpha\lambda(x - x_0) + U_0 < \underline{w} \leq x + \lambda(x - x_0) + U_0$.

[11] The idea that the reference point (partly) adapts to the action chosen follows the logic of a choice-acclimating personal equilibrium in Köszegi and Rabin (2006).

Thus, the principal's problem is:

$$\max_{w,\ x}\{v(x,\theta) - w - C\} \tag{5}$$

subject to

$$U \;=\; w - x - \lambda[w^r - w]^+ - \lambda[x - x^r]^+ \;\geq\; U_0 \tag{6}$$

As a benchmark consider the case where there is no loss aversion ($\lambda = 0$), so workers are only concerned about their material payoff. In this case the principal offers $w = x + U_0$ and chooses the materially efficient level of x such that

$$\frac{\partial v(x^{ME},\theta)}{\partial x} = 1\ . \tag{7}$$

Note that x^{ME} is an increasing function of θ (because $v_{x\theta} > 0$).

Consider now the case with loss aversion. In order to implement x the principal has to pay

$$\begin{aligned} w &= x + \lambda[\alpha w_0 + (1-\alpha)w - w]^+ + \lambda[x - \alpha x_0 - (1-\alpha)x] + U_0 \\ &= x + \alpha\lambda[w_0 - w]^+ + \alpha\lambda[x - x_0]^+ + U_0\ . \end{aligned} \tag{8}$$

We focus on the case where the principal wants to increase x (as compared to x_0) which implies that she also has to pay a higher wage to the workers. The case where the principal wants to decrease x is symmetric but less relevant, because θ, the state of technology can only go up. Thus, the principal maximizes

$$\Pi = v(x,\theta) - [x + \alpha\lambda(x - x_0) + U_0] - C\ .$$

The first order condition of this problem is

$$\frac{\partial v(x,\theta)}{\partial x} \leq 1 + \alpha\lambda\ . \tag{9}$$

with equality if $x > x_0$. Hence, the firm finds it optimal to increase effort only if $\frac{\partial v(x_0,\theta)}{\partial x} > 1 + \alpha\lambda$.

The following proposition fully characterizes the optimal effort level x^* in normal times.

Proposition 1 (Optimal Contract, Normal Times)**.** *Suppose that the status quo contract* (w_0, x_0) *satisfies* $x_0 \leq x^{ME}(\theta)$ *and*

$$v(x_0, \theta) - w_0 - C > 0.$$

Define $\overline{x}(\theta)$ *implicitly by* $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + \alpha\lambda$*. The principal offers a contract* (x^*, w^*) *to the agent that is given by*

$$x^* = \begin{cases} x_0 & \text{if } x_0 \geq \overline{x}(\theta) \\ \overline{x}(\theta) & \text{if } x_0 < \overline{x}(\theta) \end{cases} \tag{10}$$

and

$$w^* = \begin{cases} w_0 & \text{if } x_0 > \overline{x}(\theta) \\ w_0 + (1+\alpha\lambda)[\overline{x}(\theta) - x_0] & \text{if } x_0 < \overline{x}(\theta). \end{cases} \tag{11}$$

Proof. The proof follows directly from the arguments given in the text above. □

Proposition 1 shows that x^* differs from the materially efficient x^{ME}. Loss aversion drives a wedge between the marginal cost of the workers and the marginal benefit of the owner which induces the owner to stick to the status quo even if this is materially inefficient. Let $\overline{\theta}$ be implicitly defined by $\frac{\partial v(x_0,\overline{\theta})}{\partial x} = 1 + \alpha\lambda$. For all $\theta \leq \overline{\theta}$ there is full inertia. But even if $\theta > \overline{\theta}$, x^* differs from the materially efficient x^{ME}. The next proposition shows how the distortion depends on the degree of loss aversion, reference point formation, and the status quo.

Proposition 2 (Comparative Statics)**.** *The principal always implements less change than material efficiency requires, i.e.* $x^* - x_0 < x^{ME} - x_0$*.*

Furthermore, we have

(a) If the λ *(degree of loss aversion) or* α *(the weight that workers put on the status quo in the formation of the reference point) increases, the amount of organizational change,* $x^* - x_0$*, decreases, i.e.*

$$\frac{\partial x^*}{\partial \lambda}, \frac{\partial x^*}{\partial \alpha} < 0 \quad \text{if } \theta \geq \overline{\theta} \tag{12}$$

$$\frac{\partial x^*}{\partial \lambda}, \frac{\partial x^*}{\partial \alpha} = 0 \quad \text{if } \theta < \overline{\theta} \tag{13}$$

(b) An increase in λ *or in* α *widens the range of inertia, i.e.* $\frac{\partial \overline{\theta}}{\partial \lambda}, \frac{\partial \overline{\theta}}{\partial \alpha} > 0$*.*

(c) An increase of x_0 increases $\overline{\theta}$.

Proof. See Appendix

Without loss aversion ($\lambda = 0$) or with a reference point that is fully determined by rational expectations ($\alpha = 0$) the principal will implement the materially efficient outcome x^{ME}. With $\lambda, \alpha > 0$ there is inertia. The larger λ and α, the less change will be implemented and the larger is the gap between the materially efficient level of effort x^{ME} and the implemented level of effort x^*. Furthermore, the larger λ and α, the larger is range of inertia where the principal does not adjust x^* when θ increases. Finally, the range of inertia, i.e. $\overline{\theta}$, shifts upwards if the initial effort level x_0 increases.

2.2 A Parametric Example

Let $v(x, \theta) = \theta \ln x$. Then we have $\frac{\partial v(x,\theta)}{\partial x} = \frac{\theta}{x}$ and we get:

$$x^*(\theta) \;=\; \begin{cases} x_0 & \text{if } \theta \leq (1+\alpha\lambda)x_0 \\ \frac{\theta}{1+\alpha\lambda} & \text{if } \theta > (1+\alpha\lambda)x_0 \end{cases} \tag{14}$$

Figure 1 illustrates the optimal choice of x^* by the principal for $\alpha = 0.5$, $\lambda = 1$, and $x_0 = 4$. These parameters imply $\overline{\theta} = (1+\alpha\lambda)x_0 = 6$. The blue line depicts the optimal choice of $x^*(\theta)$ by the principal. If $\theta \leq \overline{\theta} = 6$, there is complete inertia. If $\theta > \overline{\theta}$, the principal adjusts x, but at a slope that is smaller than the slope of the materially efficient $x^{ME}(\theta)$. Thus, the larger θ, the larger is the gap between $x^{ME}(\theta)$ and $x^*(\theta)$.

2.3 The Effects of a Crisis

Suppose now that there is a crisis, e.g. a sudden increase in competition, significantly higher input costs, or a demand shock that hits the industry. In all of these cases the firm's profit falls. We may conveniently think of such an exogenous decrease in profits as a shock to the firm's cost parameter C.[12] Consider a situation in which the cost parameter C is such that the

[12]For tractability, we assume that the shock to profits does not affect the productivity of the worker.

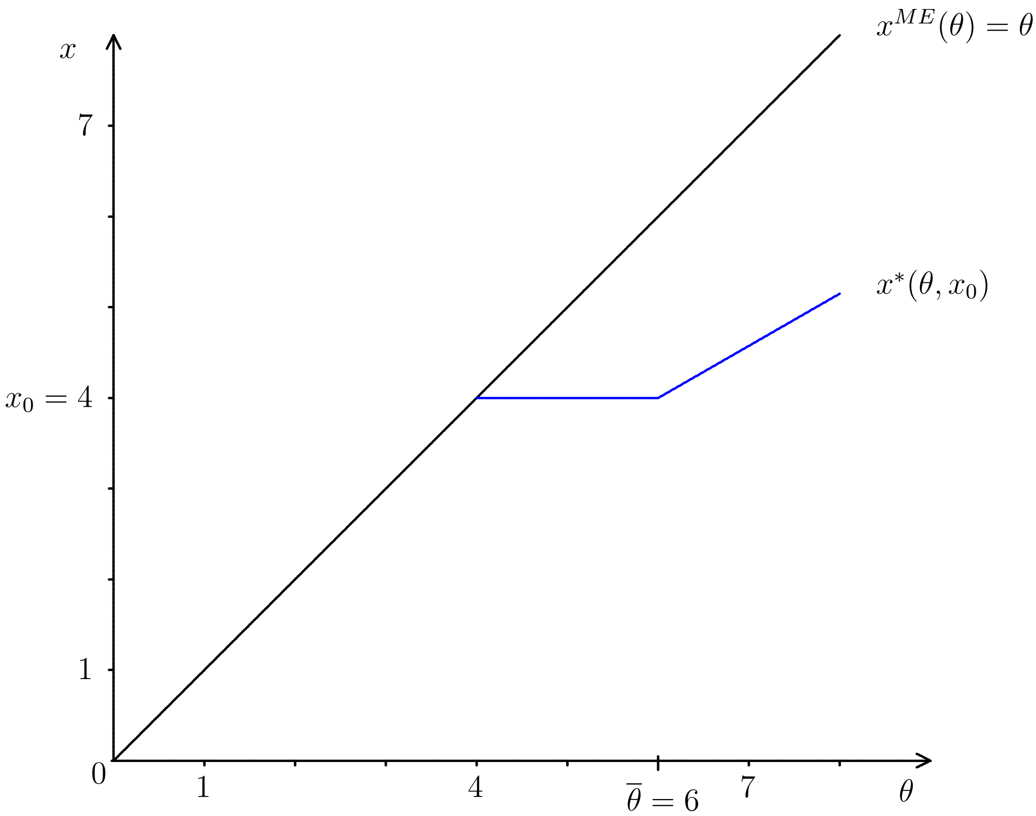

Figure 1: Organizational change as a function of θ with and without loss aversion.

status quo contract (w_0, x_0) generates negative profits. Hence, the firm would rather terminate the relationship than continue with the old contract. This changes the workers' outside option which is now given by the utility level of unemployment that is normalized to 0. The next proposition shows how the firm uses the reduced outside option of the workers to improve the terms of the contract from its perspective.

Proposition 3 (Optimal Contract in a Crises)**.** *Suppose that the status quo contract* (w_0, x_0) *satisfies* $x_0 \leq x^{ME}(\theta)$ *and*

$$v(x_0, \theta) - w_0 - C < 0.$$

Define $\hat{x}$ *implicitly by* $U(w_0, \hat{x}) = 0$ *and* $\overline{x}(\theta)$ *as in Proposition 1 by* $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + \alpha\lambda$.

1. *If* $\hat{x} \geq \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \min\{\hat{x}, x^{ME}(\theta)\}$.

2. *If* $\hat{x} < \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \overline{x}(\theta)$.

The offered wage w^* always satisfies $U(x^*, w^*) = 0$, and the union accepts the offer.

Proof. See Appendix

The workers know that they will lose their jobs if they reject the offer of the firm. Therefore, any contract that offers them at least the utility level of unemployment will be accepted. The firm can use its improved bargaining position to either reduce wages w or to increase effort x. Both of this is considered a loss by workers. Hence, an increase in effort comes at the same cost to the worker as a wage cut of equal size. Because for $x < x^{ME}$ marginal productivity satisfies $\frac{\partial v(x_0,\theta)}{\partial x} > 1$, it is more efficient to increase x rather than decrease w. Additional wage cuts will only be implemented if the materially efficient effort level still generates positive utility for the workers at the status quo wage.

The second case in Proposition 3 covers the case in which after increasing the effort to the workers' zero-utility threshold the effort is still below the optimal level $\overline{x}(\theta)$ from Proposition 1. As in Proposition 1 the firm will then implement an additional effort increase up to the threshold $\overline{x}(\theta)$, which has to be compensated at a wage rate of $1 + \alpha\lambda$ to ensure the workers' consent.

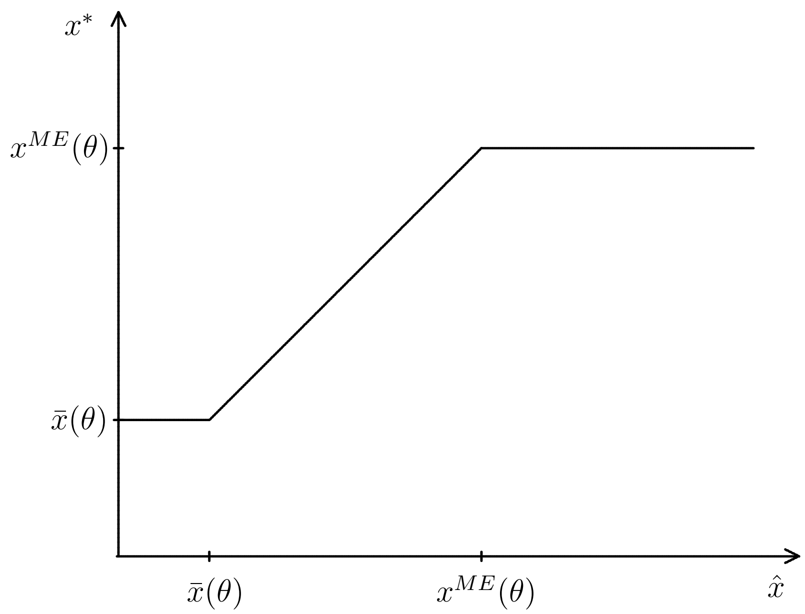

Figure 2: The Effect of a Crisis

Figure 2 illustrates Proposition 3. Note that $\hat{x}$ monotonically increases with the rent enjoyed by workers before the crisis. If this rent was small, the firm will increase x to the behaviorally efficient level $\bar{x}(\theta)$. If this rent is larger, the firm increases x^* as much as possible, i.e. it sets $x^* = \hat{x}$. If the rent is so large, that the firm could increase effort even beyond the materially efficient effort level, it will stop at x^{ME} and reduce the workers' wages rather than increase effort beyond x^{ME}.

The proposition shows that a crisis can induce a sudden jump in organizational efficiency. It also explains why wages don't fall in a recession (Bewley (1999)). In a recession, workers do make concessions, but firms and workers will not negotiate to cut wages but rather to implement more change.

The idea that a crisis has a positive effect on economic efficiency goes back at least to Schumpeter (1942). This effect is partly due to a composition effect if less efficient firms are driven out of the market and more efficient firms remain. But, as shown in many empirical studies (Holmes and Schmitz, 2010), it is also caused by a sudden increase of the efficiency of existing firm. The potential for this efficiency increase may have been there before the crisis but firms could not exploit it because of resistance to change. The effect of the crisis is to weaken this resistance of workers (and other stakeholders). This reduces the relative cost of implementing organizational change and makes change possible.

It is worth noting that while concessions increase material efficiency and may be necessary to sustain the firm, they are not "behaviorally" efficient if we incorporate the behavioral adaptation cost. Indeed, when $x_0 \in (\overline{x}, x^{ME}]$ a marginal effort increase decreases workers' utility at a rate of $1 + \alpha\lambda$, whereas firm profits only increase at a rate of $\frac{\partial v(x_0,\theta)}{\partial x} \in [1, 1 + \alpha\lambda]$. The reason is that utility is not perfectly transferable. Yet, transferring utility via increased effort is still more efficient than doing so via wage cuts.

3 Dynamic Reference Point Adjustments and Persistent Productivity Differences

The one-period model captures the main intuition for why more change is implemented in a crisis than in normal times, but there are several open questions. First, the crisis has an effect only if workers enjoy a rent, i.e. $U_0 = w_0 - x_0 > 0$. Where does this rent come from? Second, it assumes that the reference point stays fixed after a change has been implemented. However, the reference point adjusts with some delay after the new contract becomes the new status quo. If parties anticipate this, how does it change the optimal contract? Finally, rational parties anticipate that a crisis may occur. How do they prepare for it in the initial contract?

In this section we consider a dynamic model in which the reference point adjusts over time. We do not attempt to build a fully general model that allows for arbitrary cost shocks. Instead, we focus on the most interesting case where the cost shock is such that the firm may survive it only if the workers make concessions. We show that the qualitative insights of Section 2 carry over to the dynamic model. Furthermore, we show that workers accumulate over time a (quasi-)rent as a compensation for effort adjustments in the past. Parties rationally anticipate reference point adjustments and the possibility of a crisis and adjust contracts optimally which strengthens our results. We delineate the long-term dynamics, illustrate how productivity differences may persist over time, and how a crisis may increase productivity and reduce the productivity gap between comparable firms in the same industry.

We proceed in two steps. In Subsections 3.1 to 3.3 we set up the model and consider the simpler case in which workers and firm do not anticipate that a crisis may occur. We show how x is adjusted in normal times in every period (when there is no crisis), and how an unanticipated crisis affects the optimal contract. We also show how productivity differences between firms facing the same technology may arise and how they are affected by a crisis. Then, in subsection 3.4, we show that our results are even stronger with forward looking agents who form rational expectations.

3.1 A Simple Dynamic Model

Time $t = 0, 1, ...$ is discrete with an infinite horizon. We start in $t = 0$ with some state θ_0 and some contract (w_0, x_0) that satisfies $U_0 = w_0 - x_0 \geq 0$ and $x_0 \leq x^{ME}(\theta_0)$. Any contractible action must satisfy $x \leq x^{ME}(\theta)$, where $x^{ME}(\theta)$ is the materially efficient effort level as defined in (7).[13]

[13]This assumption reflects the idea that future states of technology are unpredictable and the firm cannot introduce work practices for a technology that does not yet exist. It simplifies the analysis but does not affect the qualitative results.

In every period a new state of the world (θ_t, C_t) materializes. The state of technology θ_t increases deterministically over time, i.e. $\theta_{t+1} > \theta_t$.[14] For simplicity we assume that the cost realization $C_t \in \{0, C_t^h\}$ is equal to 0 in "normal" periods and that there is at most one "crisis" with a high cost shock $C_t^h > 0$. Conditional on zero cost in all past periods, there is a crisis in the next period with probability $\mu > 0$. We are interested in the case where C_t^h is sufficiently large that the firm prefers to terminate the relationship if workers do not agree to make concessions (Assumption 1 below). A more general structure of cost shocks would give rise to many case distinctions which do not yield additional interesting insights.

Each party maximizes their expected utility, where future utility is discounted at a common discount factor $\delta < 1$. Hence, for an observed state (θ_t, C_t) and a contract (w_{t-1}, x_{t-1}) inherited from the previous period the firm maximizes

$$\Pi_{(\theta_t, C_t)}(w_t, x_t) = \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t[v(x_s, \theta_s) - w_s - C_s], \tag{15}$$

whereas the union maximizes

$$\begin{aligned} U_t(w_t, x_t | w_{t-1}, x_{t-1}) &= \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t\big[w_s - x_s - \lambda[w_s^r - w_s]^+ - \lambda\alpha[x_s - x_s^r]^+\big] \\ &= \sum_{s=t}^{\infty} \delta^{s-t} \cdot \mathbb{E}_t\big[w_s - x_s - \lambda\alpha[w_{s-1} - w_s]^+ - \lambda\alpha[x_s - x_{s-1}]^+\big]. \end{aligned} \tag{16}$$

We restrict attention to Markov perfect equilibria in the sense that any offer (w_t, x_t) and each party's subgame-perfect decision to end or continue the relationship only depend on the current state, i.e. (θ_t, C_t), and the current reference point. In particular, both parties cannot commit to any future path of actions or transfers. Moreover, we assume that if one party is indifferent in its decision it breaks ties in favor of the other party.

3.2 An Unanticipated Crisis

In this section we analyze the more tractable case where the players anticipate that the reference point will change, but they do not anticipate that a crisis occurs with positive probability.

[14] We assume implicitly that the growth in θ is bounded such that the present value of revenues $\sum_{t=0}^{\infty} \delta^t v(x_t, \theta_t)$ remains finite for all effort choices, such that present values are well defined.

The more general case in which the crisis is anticipated is dealt with in section 3.4.

The behaviorally efficient benchmark

It is useful to analyze first the behaviorally efficient contracts that a social planner would implement (ignoring participation constraints) if the cost realization was zero in each period, i.e. $\mu = 0$. The efficient contracts solve

$$\max_{(w_t, x_t)_{t \geq 1}} W = \sum_{t=1}^{\infty} \delta^{t-1} \big(v(x_t, \theta_t) - x_t - \lambda\alpha [w_{t-1} - w_t]^+ - \lambda\alpha [x_t - x_{t-1}]^+ \big).$$

Note that the wage affects efficiency only in the case of wage cuts. Falling wages create an inefficiency due to the behavioral cost. Hence, any efficient effort choice that is accompanied by a weakly increasing wage schedule is an efficient solution.

Lemma 1. *Define $\overline{x}(\theta)$ implicitly by the effort level that satisfies $\frac{\partial v(x,\theta)}{\partial x} = 1 + (1-\delta)\alpha\lambda$. For $\mu = 0$ the effort structure $(x_t^*)_{t \geq 1}$ of any behaviorally efficient contract satisfies $x_{t+1}^* \geq x_t^*$ with*

$$x_t^* = \begin{cases} x_{t-1} & \text{if } x_{t-1} \geq \overline{x}(\theta_t) \\ \overline{x}(\theta_t) & \text{if } x_{t-1} < \overline{x}(\theta_t). \end{cases}$$

Proof. See Appendix

The efficient effort is reminiscent of the effort schedule chosen by the firm in Proposition 1. Effort is weakly increasing, because θ_t is going up each period. Again, there is a region of inertia. Only if the mismatch between state θ and the associated effort is sufficiently strong it is optimal to increase the effort level. Even if effort increases, it stays below the materially efficient level of effort. However, the region of inertia and the material efficiency loss are strictly smaller than in the static benchmark. The reason is that a marginal effort increase induces a one-time marginal behavioral cost of $\lambda\alpha$ in the current period, but it reduces the behavioral cost in next period by $\delta\lambda\alpha$ because the reference point has adjusted.

The equilibrium in normal times

We are now ready to characterize the equilibrium in the benchmark case of no crisis ($\mu = 0$). Equivalently, we can think of this case as a world in which both parties do not anticipate that a cost shock may happen with positive probability. In a world without (anticipated) crisis the firm will simply implement the behaviorally efficient effort schedule and spread the necessary wage increase over all future periods.

Proposition 4. *If $\mu = 0$ the following is the unique equilibrium.*

1. *In period $t \geq 1$ the firm offers the contract (w_t^*, x_t^*), where x^* is the efficient effort level characterized in Lemma 1 and $w_t^* = w_{t-1} + (1 + (1-\delta)\alpha\lambda)(x_t^* - x_{t-1})$.*

2. *The workers accept.*

To see the intuition for this result, recall that the efficient effort is weakly increasing over time. In period t the contract (w_{t-1}, x_{t-1}) constitutes the workers' outside option, so the firm has to compensate the workers for any effort increase in order to guarantee the same utility as under contract (w_{t-1}, x_{t-1}). The compensation must cover the permanent higher cost of effort $x_t^* - x_{t-1}$ as well as the one-time behavioral adaptation cost of $\alpha\lambda(x_t^* - x_{t-1})$. The crucial observation is that the compensation for the behavioral cost must be spread evenly over all future periods. Indeed, the present value of a permanent payment $(1-\delta)\alpha\lambda(x_t^* - x_{t-1})$ is

$$\sum_{s=t}^{\infty} \delta^{s-t}(1-\delta)\alpha\lambda(x_t^* - x_{t-1}) = \alpha\lambda(x_t^* - x_{t-1}).$$

There is no other feasible compensation schedule to implement the efficient effort. Since the firm has no commitment power it cannot backload the compensation to the future. It cannot frontload the compensation either, because this would imply that wages have to fall in some future periods. But the workers can block any future decline in wages in favor of the status quo. Thus, implementing the efficient effort in this way is the best the firm can do, because it generates the highest possible joint surplus, but leaves the workers with only the utility of their outside option.

Proof. Follows directly from the text. □

Note that the optimal contract in period t results in a utility loss for workers in period t, but a permanent utility increase in all future period. Thus, starting in period $t+1$ workers enjoy a quasi-rent. Because the compensation is linear in the effort increase, the workers' quasi-rent that stems from the permanent compensations for past effort increases can easily been derived from the effort level of the previous period, i.e.

$$U_t^* = U_t^*(w_t, x_t) = (w_0 - x_0) + \sum_{s=t}^{\infty} \delta^{s-t}(1-\delta)\alpha\lambda(x_{t-1} - x_0) = (w_0 - x_0) + \alpha\lambda(x_{t-1} - x_0). \quad (17)$$

The firm's equilibrium profit in period t consists of the discounted sum of future surpluses minus the utility left to the workers, i.e.

$$\Pi_t^* = \sum_{s=t}^{\infty} \delta^{s-t}\big(v(x_s^*, \theta_s) - x_s^* - \lambda\alpha(x_s^* - x_{s-1}^*)\big) - \alpha\lambda(x_{t-1}^* - x_0). \quad (18)$$

The Effects of a Crisis

We now analyze the effects of an unanticipated crisis. Suppose that in some period t there is a cost shock that decreases the present value of the firm's expected future profits by C_t^h. If the firm's value remains positive then the shock has no impact on the firm's optimization problem. The interesting case is when the cost shock induces negative firm value for the status quo contract. Hence, the following assumption is made throughout the remainder of the paper.

Assumption 1. *In every period the potential cost shock satisfies $C_t^h > \Pi_t^*$, where Π_t^* is defined in(18).*

Assumption 1 implies that if there is a cost shock then the value of the firm becomes negative if workers receive their status quo utility, even if the most efficient contract is implemented.[15] Hence, workers are willing to make concessions to prevent the firm from closing down. Therefore, the unemployment utility of zero constitutes the new threat-point in the negotiations. Note that we allow for the possibility that the cost shock is so high that the firm cannot be rescued even if workers give up their quasi-rent. In this case the firm closes down and workers become unemployed.

The following Proposition is a straightforward generalization of Proposition 3 to the dynamic case.

Proposition 5. *Suppose that Assumption 1 holds and that there is a crisis in period t. Define $\hat{x}$ implicitly by $U_t(w_{t-1}, \hat{x}) = 0$, and $\overline{x}(\theta)$ as in Lemma 1 by $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + (1-\delta)\alpha\lambda$.*

[15] Thus, a fortiori, the value of the firm becomes negative for any other feasible contract as well.

1. *If* $\hat{x} \geq \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \min\{\hat{x}, x^{ME}(\theta)\}$.

2. *If* $\hat{x} < \overline{x}(\theta)$ *the firm offers a contract with* $x^* = \overline{x}(\theta)$.

The offered wage w^* *always satisfies* $U(x^*, w^*) = 0$*, and the union accepts the offer. The firm decides to continue the relationship if and only if its expected profits from the above contract are non-negative.*

Proof. See Appendix

Again, the firm finds it more profitable to increase effort rather than decrease wages. Decreasing the wage or increasing the effort by one unit has identical effects on the workers' expected utility. For the firm, increasing the effort is even more appealing than in the static case: since θ is growing over time, a higher effort level today avoids costly adaptations in the future.

3.3 Long Term Dynamics and Persistent Productivity Differences

We now illustrate long term dynamics and show how a crisis can help close productivity gaps across firms. We continue with our previous example of $v(x, \theta) = \theta \ln(x)$. The following Figure 3 depicts the materially efficient line $x^{ME}(\theta) = \theta$ in black and the behaviorally efficient line $\overline{x}(\theta) = \frac{1}{1+(1-\delta)\alpha\lambda}\theta$ in blue. We consider two firms that have access to the same production technology. The productivity of the technology is summarized by θ, which grows over time. Firms are founded at different points in time, when $\theta = 4$ and $\theta = 7$, respectively. Thus, firm 1 starts production in the materially efficient point A. The red line depicts the transition path as θ_t grows over time. Firm 1 is in the area of inertia until point B is reached. It does not find it optimal to increase effort, since the necessary compensation is too high.

From point B the contract follows the behaviorally efficient (blue) line. The firm implements effort increases each period, but these increases are smaller than the materially efficient effort increases. Since the compensation for the behavioral cost of effort adaptation is paid over time, the workers build up rent.

At $\theta = 7$ firm 2 is founded. Firm 2 also starts production at the materially efficient point F and follows the transition path along the yellow line. Since firm 1 has moved away from the

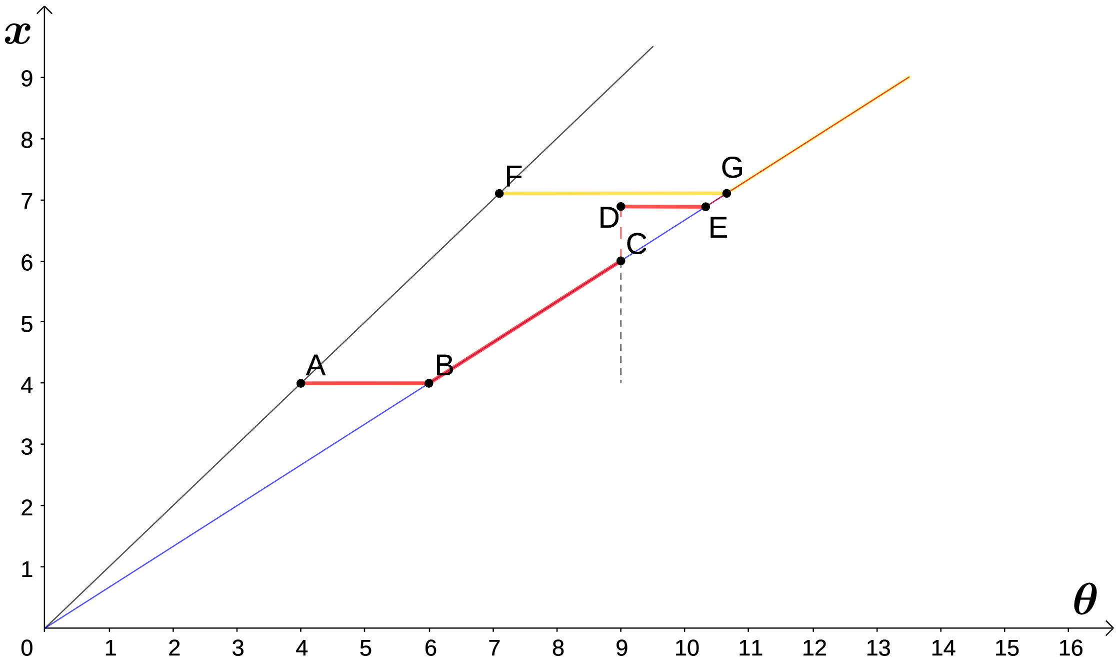

Figure 3: $v(x, \theta) = \theta \ln x$, $(1-\delta)\alpha\lambda = 0.8$, firms created at $\theta = 4$ and $\theta = 7$, crisis at $\theta = 9$

materially efficient level, there is a substantial productivity gap between the two firms. While this gap gradually closes as Firm 1 implements additional effort increases it may persist over a long time.

When $\theta = 9$ is reached, the economy is hit by a crisis that threatens the profitability of Firm 1, but not of the more efficient firm 2. By then Firm 1 is in point C, and the workers have accumulated rent of $\alpha\lambda$ times the magnitude of total effort increases, illustrated by the length of the dashed black line. Firm 1 will use this utility and the threat of unemployment to increase the effort from point C to point D.[16] If the firm is profitable in point D it will keep producing, now again in the area of inertia until it hits point E.

Notice that the more efficient firm 2 is not threatened by bankruptcy, even though it is hit by the same cost shock. Indeed, even after the effort adjustment of firm 1, firm 2 remains more profitable than firm 1. But, the productivity gap between the two firms closes discontinuously

[16]Each additional unit of effort gives rise to an immediate behavioral cost of $\alpha\lambda$ and a permanent higher material effort cost of one, the discounted present value of which is $\frac{1}{1-\delta}$. The worker has accumulated a rent of $\alpha\lambda(x_t - x_0)$. Thus, this rent can be used to "pay for" an effort increase of $\alpha\lambda(x_t - x_0) \cdot \frac{1}{\alpha\lambda + \frac{1}{1-\delta}} = \frac{(1-\delta)\alpha\lambda}{1+(1-\delta)\alpha\lambda}(x_t - x_0)$.

in a crisis, because firm 1 is able to implement more efficient work practices while firm 2 is not.

It is worth noting that even if point D was on the yellow line, i.e. both firms use the same effort after the crisis, Firm 2 remains more profitable. The reason is that it pays lower wages. Indeed, workers of Firm 2 receive no rents up to point G. Firm 1, on the other hand, has to leave some future rents to its workers in order to compensate them for the effort adjustment in the crisis. Nevertheless, the profitability gap between the two firms narrows.

The figure assumes equal levels of loss aversion of workers across firms. This does not necessarily have to be the case. Gächter, Johnson, and Herrmann (2022) find that loss aversion tends to increase in age, income and wealth. This suggests that if younger firms employ a younger workforce, their workers suffer to a lesser degree from loss aversion than their older colleagues at older firms. In this case the inertia region of old firms is larger than that of young firms since workers from older firms are more reluctant to agree to organizational change. This could be another source of persistence productivity differences across firms.

3.4 The Equilibrium with Forward-Looking Players

Now we analyze the case in which both players correctly anticipate that a crisis may happen with probability μ each period. We show that all insights continue to hold and the range of inertia even widens.

We begin the analysis by noting that under Assumption 1 the occurrence of a crisis cannot be affected by the two parties. Hence, in the contracting game, both parties treat the probability of the occurrence of a crisis, which demands concessions of the workers, as exogenous.

Second, notice that the optimal reaction to a crisis for a given status quo contract is fully analysed in Proposition 5. Indeed, due to the restriction to Markov strategies, the contract offer in a crisis only depends on the current reference point and the states θ_t and C_t^h. Hence, the problem reduces to finding the principal's optimal contract offer in normal periods, given that both parties correctly anticipate the effects of their contract on the adaptation in a potential crisis.

We start by calculating the necessary wage compensation to implement an effort increase from x_{t-1} to x_t in a normal period. The firm has to compensate the permanent effort cost and the one-time behavioral adaptation cost. Again, the compensation for the adaptation cost must be spread out equally over the current and all future normal periods. However, as compared to the case with unanticipated crises, both parties now anticipate that in a crisis the workers will receive their outside option of zero expected utility. Hence, the demanded per-period compensation for behavioral adaptation cost is higher.

To simplify notation, let

$$\gamma \equiv \big[(1 - \delta(1 - \mu)\big]\alpha\lambda.$$

As illustrated in the proof of Lemma 2, γ is the permanent per-period compensation until the crisis hits that is necessary to compensate for the behavioral cost of a one unit effort increase.

Lemma 2. *In any equilibrium the effort is weakly increasing in every normal period. In order to implement $x_t > x_{t-1}$ in period t the firm offers the contract (w_t, x_t) with*

$$w_t = w_{t-1} + (1 + \gamma)(x_t - x_{t-1}).$$

If there is no crisis until period t then

$$w_t = x_t + \gamma(x_t - x_0) + (w_0 - x_0), \tag{19}$$

and

$$U_t(w_{t-1}, x_{t-1}) = U_0(w_0, x_0) + \alpha\lambda(x_{t-1} - x_0).$$

Proof. See Appendix. □

Notice that the compensation the workers demand is strictly higher than in the scenario of unanticipated crises and is strictly increasing in μ. Since workers anticipate that their utility drops to zero if a crisis occurs they demand higher compensation in normal times.

Again, over time, as no crisis happens, the agent builds up rent that stems from compensation for past effort increases. This is utility the firm can exploit to take away in a crisis, when the threat point of the workers is given by their unemployment utility of zero. Since workers anticipate that their rent will drop to zero in a crisis, the compensation they demand increases in the probability μ that a crisis occurs.

As established in Proposition 5 in a crisis the firm will use the workers' utility to implement higher effort. The following Lemma calculates by how much the firm can at most increase the effort in a crisis when keeping the wage constant.

Lemma 3. *For a given status quo contract* (w_{t-1}, x_{t-1}) *the effort increase* Δx_t *that satisfies* $U_t(w_{t-1}, x_{t-1} + \Delta x_t) = 0$ *is given by*

$$\Delta x_t = \frac{1}{1 + (1-\delta)\alpha\lambda}\Big((w_0 - x_0) + \gamma(x_{t-1} - x_0)\Big). \tag{20}$$

Proof. See Appendix. □

The next proposition characterizes the optimal effort schedule that the firm will implement in normal times in anticipation of how the contract will change if the crisis hits.

Proposition 6. *Suppose there is no crisis until period* t*. There is a threshold* $\tilde{x}(\theta_t) \le \overline{x}(\theta_t)$ *such that the optimal effort* x_t *implemented in period* t *is given by*

$$x_t = \begin{cases} x_{t-1} & \textit{if } x_{t-1} > \tilde{x}(\theta_t) \\ \tilde{x}(\theta_t) & \textit{if } x_{t-1} \le \tilde{x}(\theta_t) \end{cases}$$

Proof. See Appendix. □

Comparing Proposition 6 to Proposition 5 shows that if parties rationally anticipate that a crisis may hit with probability $\mu > 0$, there will be even more inertia than if they do not anticipate a crisis. Because the firm anticipates that it becomes cheaper to adjust effort in a crisis, it delays the effort adaptation.

The proof of Proposition 6 requires a few case distinctions and is somewhat involved, but the intuition can be illustrated graphically with the logarithmic example that we used in Section 3.3.

Figure 4 shows the optimal effort levels chosen for the functional form and parameters in the example of Figure 3 and $\mu = 0.4$. The green line depicts the boundary of the inertia region with forward-looking players as compared to the behaviorally-efficient blue line.

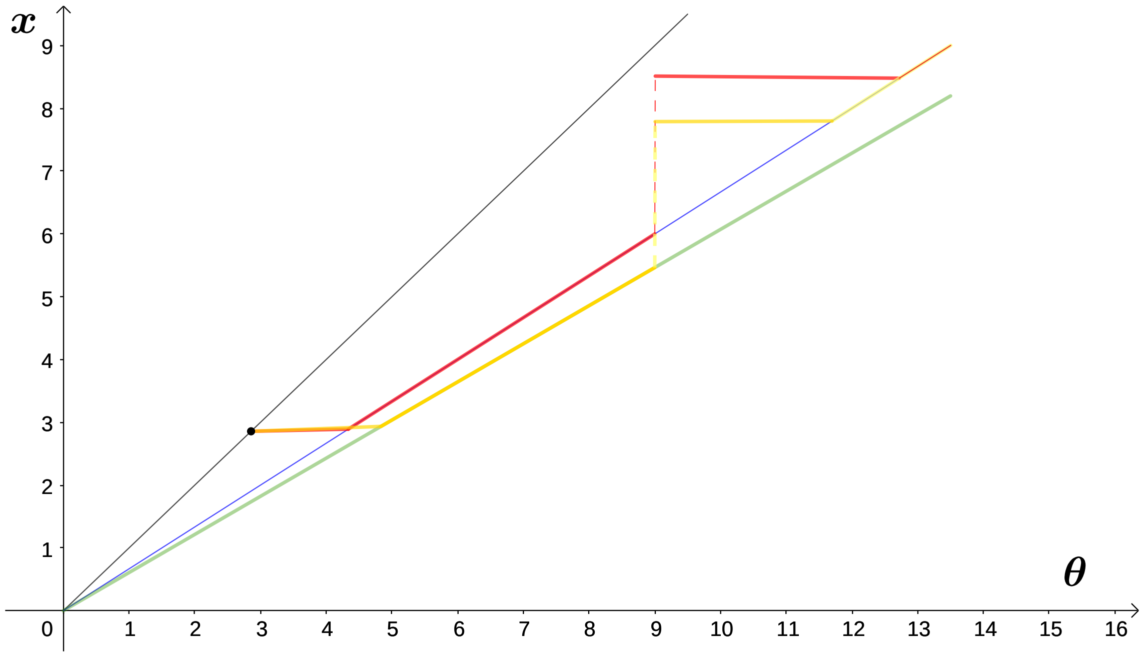

Figure 4: Logarithmic Example with Forward Looking Players

To see that the anticipation of the crisis increases inertia two cases have to be distinguished. First, if the cost shock C_t^h is sufficiently large, the workers' accumulated rent will not suffice to make the firm profitable and the principal will terminate the relationship. The anticipation of bankruptcy makes the implementation of higher effort levels less appealing, because the firm does not benefit from this effort after bankruptcy.

Second, and more interestingly, the inertia area also widens when workers' concessions suffice to keep the firm in business. There is more inertia than without anticipation of the crisis because the principal knows that effort will increase in a crisis anyway and therefore delays effort increases in normal times.[17] Intuitively, this behavior helps the principal to keep the effort level closer to the blue behaviorally efficient line $\overline{x}(\theta)$. It optimally solves a tradeoff between "too low" effort in normal periods and "too high" effort as compared to the behaviorally efficient level in a crisis.[18] The yellow line depicts the optimal transition path

[17]For the functional form of our logarithmic example the green inertia line of the bankruptcy case and the no-bankruptcy case coincide. For more general production function this will not be the case. The level of inertia then depends on whether the firm expects bankruptcy in a potential crisis.

[18]As we show in the proof of Proposition 6, the only scenario in which inertia does not strictly increase as compared to the benchmark of unanticipated crises is the rather extreme case in which θ increases so strongly between two contracting periods that even with behavioral efficient effort before the crisis and after the discontinuous effort increase in a crisis the effort is still below the behaviorally efficient level in the crisis.

of effort if a potential crisis is anticipated, as compared to the case of an unanticipated crisis (red line).

4 Expectations Management

An important aspect of organizational change is expectations management. Management books on organizational change emphasize that it is of crucial importance to convince everyone concerned that change is inevitable and that it is better to get ready for it now. To model expectation management we have to endogenize the probability of change. To keep the model tractable, we return to the one-period model and take the amount of organizational change, Δx, as exogenously given. If there is a change of $\Delta x > 0$, the owner's gross profit increases by $\Delta v > \Delta x$.[19] However, change has to be implemented by a manager who has to spend effort to increase the probability that organizational change is going to be successful. If he is unsuccessful there is no change. We assume that the manager chooses the probability of success, p, at cost $c(p)$. In order to derive a closed form solution, we consider a quadratic cost function $c(p) = \frac{c}{2}p^2$.

The owner of the firm, the principal, has to incentivize the manager to spend effort. The manager's chosen probability of success is unobservable to the principal and cannot be contracted upon. Change, however, is contractible, so the principal can offer a bonus payment b if the manager is successful in implementing it. The manager's outside option utility is normalized to 0 and he is wealth constrained, so he has to get a wage that is greater or equal than 0 in both states of the world. We assume again that only workers suffer form loss aversion (as in Section 2) and that all parties are risk neutral and maximize the expected value of their payoff.

The time structure of the model is as follows: At stage 1 the principal makes a take-it-or-leave-it contract offer to the manager. In addition she pays workers to accept the change (if the manager is successful) by making a wage offer w.[20] At stage 2 the manager chooses

This case cannot occur if the time between two contracting periods becomes sufficiently small.

[19] There is no problem to endogenize the size of Δx, but it complicates the exposition significantly without adding any new insights.

[20] Note that the principal cannot do better by offering a wage payment that is contingent on successful

the probability of success. At stage 3 nature determines whether change is successful and payments are made.

The principal's problem is a standard moral hazard problem with a risk-neutral but wealth constraint manager. It is straightforward to show that the owner will offer the manager a wage of 0 if he fails and a bonus $b \geq 0$ if he is successful.

Suppose that the principal offered a bonus b to the manager that induces him to choose a probability of success of p. What wage does the owner have to pay to workers to make them accept the change? Note that $\Delta x > 0$ implies that $w > w_0$ and that $x^r = \alpha x_0 + (1-\alpha)[x_0 + p\Delta x]$. Thus, the expected utility of workers is given by

$$\begin{aligned} U &= w - x_0 - p\Delta x - \alpha\lambda[w_0 - w]^+ - p\lambda[x_0 + \Delta x - (\alpha x_0 + (1-\alpha)(x_0 + p\Delta x))]^+ \\ &\quad -(1-p)\lambda[x_0 - (\alpha x_0 + (1-\alpha)(x_0 + p\Delta x))]^+ \\ &= w - x_0 - p\Delta x[1 + \lambda(1-(1-\alpha)p)] \geq U_0 \ . \end{aligned} \tag{21}$$

The following lemma characterizes the wage payment that workers have to be offered:

Lemma 4. *With probabilistic change the principal has to pay*

$$w = x_0 + p(1+\lambda)\Delta x - p^2(1-\alpha)\lambda\Delta x \ + U_0 \tag{22}$$

to workers. This wage function is concave in the probability of change p. It decreases in p iff

$$\frac{1+\lambda}{\lambda(1-\alpha)} < 2p \tag{23}$$

Proof. See Appendix □

Equation 23 is more likely to hold if λ is large, α is small and p is close to 1.[21] If $\alpha = 1$, i.e. if the reference point is fully determined by the status quo, the wage increase is just a linear function of p. The more likely the change the higher is the wage increase that workers demand to accept it. However, if $\alpha < 1$, i.e. if the reference point is partly shaped by rational expectations, the wage increase is a concave and possibly decreasing function of p. The higher

change because all parties are risk neutral and the reference point is a linear function of the rationally expected outcome. If workers are risk averse the principal would strictly prefer to make an unconditional wage offer in order to optimally insure workers.

[21]Note that (23) can only hold if $\lambda > 1$.

the probability of change, the higher is the reference point x^r, and the less do workers suffer from change. This has important implications for the probability of change that the principal wants to implement. If the weight of expectations for shaping the reference point is sufficiently large, i.e. if α is sufficiently small, the principal will induce either change with probability one or with probability zero. This is shown in the following proposition.

Proposition 7. *The probability of change that the principal wants to implement is characterized as follows:*

(a) *If $c < (1-\alpha)\lambda\Delta x$ the principal's problem to induce the manager to promote change is a convex problem. In this case the principal will implement a corner solution with either $p = 0$ or $p = 1$, with $p = 1$ if $\Delta v \geq (1 + \alpha\lambda)\Delta x + c$.*

(b) *If $c > (1-\alpha)\lambda\Delta x$ the principal's problem is concave. In this case the principal implements $p > 0$ if and only if $\Delta v > (1 + \lambda)\Delta x$, in which case p satisfies*

$$p = \min\left\{\frac{\Delta v - (1+\lambda)\Delta x}{2[c - (1-\alpha)\lambda\Delta x]}, 1\right\} \tag{24}$$

Proof. See Appendix □

The proposition shows that if α is small, i.e. if rational expectations have a large effect on the reference point, then the manager is induced to choose an extreme solution of either $p = 0$ or $p = 1$. Even if an interior solution is optimal, a decrease in α increases the probability of change. The reason is that if the reference point is at least partially determined by workers' expectations, then it becomes cheaper to implement change the more workers are convinced that the change is going to take place. This resonates with the advice given in the literature on organizational change that if you want to induce change, you have to set the expectation that the change is coming and unavoidable.

5 Reference Dependence in Teams

So far we assumed that people compare their current wage and effort level to the status quo and to their rational expectation of the future. In this section we consider a different reference point that is shaped by social comparisons. Each agent compares his situation to the situation of other agents in his reference group. He suffers a utility loss if his wage is lower than the reference wage and if his effort his higher than the reference effort.[22]

We restrict attention to the case of two workers, $i \in \{1, 2\}$ who receive different wages and spend different amounts of effort. Furthermore, we assume that the reference point only depends on social comparisons and not on comparing the proposed wage and effort level to the status quo and the rational expectation of the future. It is straightforward to extend the analysis to the case of N workers and to have multi-dimensional reference points, but it does not add any interesting insights.

Each worker compares his own situation to that of his colleague. The parameter $\beta \in [0, 1]$ captures how much the reference point weighs the wage and effort of his colleague as compared to his own situation:

$$\begin{aligned} w_i^r &= (1-\beta)w_i + \beta w_j \\ x_i^r &= (1-\beta)x_i + \beta x_j \end{aligned}$$

If $\beta = 0$, a worker is only interested in his own situation and does not engage in social comparisons. In this case the model boils down to a model without reference dependence. For $\beta > 0$ the reference point is given by a weighted average of his own situation and the situation of his co-worker.[23]

The following Lemma shows how social comparisons affect wages for given effort levels x_1 and x_2.

Lemma 5. *Suppose that $x_2 > x_1$. Then wages are given by*

$$\begin{aligned} w_1 &= U_0 + x_1 + \lambda\beta[x_2 - x_1] \ , \\ w_2 &= U_0 + x_2 + \lambda\beta[x_2 - x_1] \ . \end{aligned}$$

Proof. See Appendix □

[22] There are several recent empirical papers showing that workers are averse to pay inequality. See e.g. Card et al. (2012), Dube, Giuliano, and Leonard (2019), Cullen and Perez-Truglia (2022).

[23] The parameter β captures how much the worker compares his own situation with the situation of his co-worker, while the parameter λ captures the effect of reference dependence on utility. However, in this very simple version of the model, only the product $\lambda\beta$ is going to matter. This is no longer the case if the reference point becomes multi-dimensional.

Lemma 5 shows that in order to induce $x_2 > x_1$ the wages of both workers have to exceed $U_0 + x_i$ by $\lambda\beta[x_2 - x_1]$. This term is increasing in λ, i.e. the weight of the reference point, and β, the weight put in the reference point on the other agent. However, the difference in wages is just equal to the difference in effort levels: $x_2 - x_1$. Thus, if there is wage compression, it must be due to the principal inducing smaller effort differences than without social comparisons.

Suppose that worker $i \in \{1, 2\}$ generates gross profits of $v_i(x_i, \theta)$. Suppose further without loss that worker 2 is the more productive one, i.e. $\frac{\partial v_2(x,\theta)}{\partial x} > \frac{\partial v_1(x,\theta)}{\partial x}$ for all x and θ. The principal chooses x_1 and x_2 to maximize

$$\Pi = v_1(x_1, \theta) + v_2(x_2, \theta) - w_1 - w_2 - C$$

Without social comparisons the first best effort levels are given by

$$\frac{\partial v_1(x_1^{ME}, \theta)}{\partial x_1} = \frac{\partial v_2(x_2^{ME}, \theta)}{\partial x_2} = 1$$

The following proposition characterizes the optimal effort levels if workers engage social comparisons:

Proposition 8. *Let x^* be characterized by*

$$\frac{\partial v_1(x^*, \theta)}{\partial x} + \frac{\partial v_2(x^*, \theta)}{\partial x} = 2.$$

If $\frac{\partial v_2(x^*,\theta)}{\partial x} < 1 + 2\lambda\beta$ then the principal induces $x_1^* = x_2^* = x^*$. Otherwise the principal induces the unique effort pair (x_1^*, x_2^*) that satisfies

$$\begin{aligned} \frac{\partial v_1(x_1^*, \theta)}{\partial x_1} &= 1 - 2\lambda\beta, \\ \frac{\partial v_2(x_2^*, \theta)}{\partial x_2} &= 1 + 2\lambda\beta. \end{aligned}$$

Proof. See Appendix □

The principal will induce the less productive worker to work more than his efficient effort level, while the more productive worker works less hard than required by efficiency. The principal compresses the difference in effort levels which also reduces the difference in wages. If the difference in productivity between the workers is sufficiently small, the principal finds it optimal to implement the same effort level for both workers. There is wage compression as compared to the material efficient solution because of this effort compression.

This is consistent with interesting empirical evidence provided by Hjort, Li, and Sarsons (2022) showing that multinationals use wages paid at their headquarters as a reference point for wages paid to employees in other countries, even if the establishment is located in a low-wage region.

Social comparisons are costly for the firm because they increase wages and distort effort levels. One possibility to reduce social comparisons is to organizationally separate workers. For example, if some tasks are contracted out to an independent company, the workers of this other company are less likely to compare their situation to the situation of the workers who are employed by a different company, as compared to a situation where all of them work for the same firm. This gives an answer to the question why it may be profitable to split up a company in two different entities (the so called "Williamson puzzle").

6 Conclusions

We have shown that reference-dependent preferences can naturally account for several stylized facts about organizational change. Loss aversion explains why there is often no or slow change in normal times, but a sudden spur in productivity in a crisis. It explains why large productivity differences between firms can arise and persist for long periods of time if firms are founded at different points in time or face idiosyncratic shocks. Social preferences can explain why there is effort and wage compression within firms and why it may be optimal to split up a firm in order to avoid social comparisons.

Our model has several other interesting implications. For example, it implies that it is more difficult to implement change with older workers. Older workers have a shorter time-horizon until they retire. Furthermore, Gächter, Johnson, and Herrmann (2022) report that older people suffer more from loss aversion than younger people, that is there λ is larger. For both reasons they need to get a higher compensation to accept change, which makes it more costly to implement.

If the government protects workers with a generous social safety net and if it tries to prevent firm closures, it makes change more difficult to implement. For example, if firms can put their workers on short-term work rather than to lay them off, there is less need for workers to make concessions. If there is a general unemployment insurance, workers loose less in case of a crisis and are less willing to agree to changes. This resonates with the observation that European countries are often lagging behind Anglo-Saxon or Asian countries in the implementation of cutting-edge technologies.

In the model, we take the formation of the reference point as given. However, a company that wants to implement change could try to shape the reference point by reducing α, i.e. the weight that the reference point puts on the status quo. For example, it could focus attention on the future, by making it clear that change is unavoidable, that it will happen, and that it is better to embrace rather than to resist it. Or the company could hire a new manager who has a reputation for pushing change through, or a new owner could take over the firm who is committed to implement change. All of this shifts attention to the rational expectation that change will happen.

The theoretical exploration of these effects and the empirical validation of the impact of reference-dependent preferences on organizational change are interesting directions for future research.

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A Appendix

Proof of Proposition 2. If $\theta < \overline{\theta}$ we have $x^*(\theta) = x_0$ and the result is immediate. Recall that x^{ME} is fully characterized by $\frac{\partial v(x^{ME},\theta)}{\partial x} = 1$. If $\theta \geq \overline{\theta}$ the optimal $x^*(\theta)$ is characterized by

$$\phi(\cdot) = \frac{\partial v(x^*,\theta)}{\partial x} - 1 - \alpha\lambda = 0. \tag{A1}$$

Thus, by the concavity of $v(\cdot)$ in x we have $x_0 < x^* < x^{ME}$, which proves the first part of the proposition.[24]

(a) Changes in α and λ affect by how much the principal adjusts x. For $\theta > \overline{\theta}$ the optimal x^* is characterized by (A1). Using the implicit function theorem, we get

$$\frac{\partial x^*}{\partial \lambda} = -\frac{\frac{\partial \phi}{\partial \lambda}}{\frac{\partial \phi}{\partial x}} = -\frac{-\alpha}{\frac{\partial^2 v}{\partial x^2}} = \frac{\alpha}{\frac{\partial^2 v}{\partial x^2}} < 0 \tag{A2}$$

$$\frac{\partial x^*}{\partial \alpha} = -\frac{\frac{\partial \phi}{\partial \alpha}}{\frac{\partial \phi}{\partial x}} = -\frac{-\lambda}{\frac{\partial^2 v}{\partial x^2}} = \frac{\lambda}{\frac{\partial^2 v}{\partial x^2}} < 0, \tag{A3}$$

while $x^{ME}(\theta)$ is independent of λ and α.

(b) How does a change in loss aversion, λ, or in reference point formation, α, affect $\overline{\theta}$, i.e. the range in which there is complete inertia? This parameter is implicitly defined by:

$$\frac{\partial v(x_0,\overline{\theta})}{\partial x} - 1 - \alpha\lambda = 0 \tag{A4}$$

By the implicit function theorem we get

$$\frac{\partial \overline{\theta}}{\partial \lambda} = -\frac{-\alpha}{\frac{\partial^2 v(x_0,\theta)}{\partial x \partial \theta}} > 0\ . \tag{A5}$$

[24] An interesting question is whether the distortion increases as θ increases (starting from θ_0). For $\theta_0 < \theta \leq \overline{\theta}$ this is obviously the case. For $\theta > \overline{\theta}$ we have $\frac{\partial x^*}{\partial \theta} \leq \frac{\partial x^{ME}}{\partial \theta}$ iff

$$\frac{\partial x^*}{\partial \theta} = -\frac{\frac{\partial^2 v(x^*,\theta)}{\partial x \partial \theta}}{\frac{\partial^2 v(x^*,\theta)}{\partial x^2}} < \frac{\partial x^{ME}}{\partial \theta} = -\frac{\frac{\partial^2 v(x^{ME},\theta)}{\partial x \partial \theta}}{\frac{\partial^2 v(x^{ME},\theta)}{\partial x^2}}$$

Recall that $x^{ME} > x^*$. Thus, a sufficient condition for this to be the case is that

$$\frac{\partial^3 v(x,\theta)}{\partial x^2 \partial \theta} \geq 0 \quad \text{and} \quad \frac{\partial^3 v(x,\theta)}{\partial x^3} \geq 0\ .$$

Thus, an increase in λ increases $\overline{\theta}$, so the range of inertia unambiguously widens. An analog argument shows that $\frac{\partial\overline{\theta}}{\partial\alpha} > 0$, hence the same result holds for a change in α.

(c) The range of inertia also depends on x_0. Using the implicit function theorem again we get

$$\frac{\partial\overline{\theta}}{\partial x_0} = -\frac{\frac{\partial^2 v(x_0,\overline{\theta})}{\partial x^2}}{\frac{\partial^2 v(x_0,\overline{\theta})}{\partial x \partial \theta}} > 0 \ . \tag{A6}$$

Thus, an increase of x_0 increases $\overline{\theta}$.

□

Proof of Proposition 3. Note first that if the union rejected the contract offer, the firm would terminate the contract because the status quo generates negative profits. In this case workers would receive a utility of zero. Therefore, it is optimal for the workers to accept any offer that yields (weakly) positive utility. Hence, the firm's maximization problem is given by

$$\max_{x,w} \quad \Pi(x, w) = v(x, \theta) - w - C \qquad s.t. \quad U(x, w) \geq 0 \tag{A7}$$

Since $x_0 \leq x^{ME}$ we have $\frac{\partial v(x_0,\theta)}{\partial x_0} \geq 1$. Therefore, it is not optimal to choose any effort level $x < x_0$. Moreover, the constraint must bind, since otherwise the firm could profitably decrease the wage. Hence, constraint (A7) implies

$$w = x + \alpha\lambda[w_0 - w]^+ + \alpha\lambda(x - x_0).$$

Let $\hat{x}$ be defined by $U(\hat{x}, w_0) = 0$. Then for any (w, x) with $U(w, x) = 0$ we have $w > w_0$ if and only if $x > \hat{x}$. If we plug in the wage into the firm's objective function the maximization problem becomes

$$\max_{x} \quad \Pi(x) = \begin{cases} v(x, \theta) - (1 + \alpha\lambda)x + \alpha\lambda x_0 - C & \text{if } x > \hat{x}, \\ v(x, \theta) - x + \frac{\alpha\lambda}{1+\alpha\lambda}(x_0 - w_0) - C & \text{if } x \leq \hat{x}. \end{cases}$$

Notice that

$$\frac{\partial \Pi(x)}{\partial x} = \begin{cases} \frac{\partial v(x,\theta)}{\partial x} - (1+\alpha\lambda) & \text{if } x > \hat{x}, \\ \frac{\partial v(x,\theta)}{\partial x} - 1 & \text{if } x < \hat{x}. \end{cases}$$

is decreasing in x.

Now, we have to distinguish three cases.

1. If $\hat{x} \geq x^{ME}(\theta)$ then $\frac{\partial \Pi(x)}{\partial x} < 0$ for all $x > \hat{x}$. Since $\frac{\partial v(x^{ME}(\theta),\theta)}{\partial x} = 1$, the solution is given by $x = x^{ME}(\theta)$.

2. If $\hat{x} \in (\overline{x}(\theta), x^{ME}(\theta))$ then $\frac{\partial \Pi(x)}{\partial x} > 0$ for $x < \hat{x}$ and $\frac{\partial \Pi(x)}{\partial x} < 0$ for $x > \hat{x}$, and the solution is given by $\hat{x}$.

3. If $\hat{x} \leq \overline{x}(\theta)$ then $\frac{\partial \Pi(x)}{\partial x} > 0$ for all $x < \hat{x}$. Since $\frac{\partial v(\overline{x}(\theta),\theta)}{\partial x} = 1 + \alpha\lambda$, the solution is given by $x = \overline{x}(\theta)$.

□

Proof of Lemma 1. We first show that an efficient effort schedule cannot have decreasing effort. Consider an effort schedule that features $x_t < x_{t-1}$. Then

$$\frac{\partial W}{\partial x_t} = \delta^t \left(\frac{\partial v(x_t,\theta)}{\partial x_t} - 1 \right) + \delta^{t+1} \lambda \alpha \mathbb{1}_{x_{t+1} > x_t}.$$

Since, by assumption, $x_t < x^{ME}(\theta_t)$ we have $\frac{\partial v(x_t,\theta)}{\partial x_t} > 1$. This implies $\frac{\partial W}{\partial x_t} > 0$, which means that x_t cannot be optimal. Hence, optimal effort must be weakly increasing.

Suppose $(x_s)_{s \in \mathbb{N}}$ is an efficient effort schedule. We now fix some period $t \geq 1$ to prove the lemma. There are three possible cases: Either the optimal effort remains constant forever, or it remains constant for some periods and increases in period $t+s$, $s \geq 1$, or it increases in period t already. In the following, we show that the lemma holds in all three cases.

Case 1: The optimal effort is constant from x_{t-1} to infinity.

Denote with x the constant effort from period t to infinity. Then an equal marginal increase of effort in all periods starting in t must be suboptimal. Hence for $x = x_{t-1}$

$$0 \geq \frac{\partial W}{\partial x} = \delta^t \left(-\alpha\lambda + \sum_{s=0}^{\infty} \delta^s \left(\frac{\partial v(x, \theta_{s+t})}{\partial x} - 1 \right) \right) > \delta^t \left(-\alpha\lambda + \sum_{s=0}^{\infty} \delta^s \left(\frac{\partial v(x, \theta_t)}{\partial x} - 1 \right) \right),$$

where the last inequality exploits $\theta_t < \theta_{t+s}$ for $s > 0$. For $\delta > 0$ this implies

$$-\alpha\lambda + \frac{\frac{\partial v(x,\theta_t)}{\partial x} - 1}{1-\delta} \leq 0,$$

which is equivalent to

$$\frac{\partial v(x_t, \theta_t}{\partial x_t} \leq 1 + (1-\delta)\alpha\lambda.$$

Since $x_{t-1} = x_t$ it follows that in Case 1 indeed $x_{t-1} = x_t \geq \overline{x}(\theta_t)$.

Case 2: The optimal effort is constant from x_{t-1} to x_s for some $s \geq t$, but $x_{s+1} > x_s$.

Because the optimal level is unchanged in period s but goes up in period $s+1$, the necessary condition for x_s implies

$$0 \geq \frac{\partial W}{\partial x_s} = \delta^s \left(\frac{\partial v(x_s, \theta_s)}{\partial x_s} - 1 - \alpha\lambda + \delta\lambda\alpha \right).$$

Since $x_{t-1} = x_t = x_s$ and $\theta_t \leq \theta_s$ this implies

$$\frac{\partial v(x_t, \theta_t)}{\partial x_t} \leq 1 + (1-\delta)\alpha\lambda.$$

Hence, we are again in the case $x_{t-1} = x_t \geq \overline{x}(\theta_t)$.

Case 3: The optimal effort increases in period t, i.e. $x_t > x_{t-1}$.

Notice first that in this case we must also have $x_{t+1} > x_t$. To see this suppose that $x_t = x_{t+1}$. If $\frac{\partial W}{\partial x_t} \geq 0$ then $\frac{\partial W}{\partial x_{t+1}} > 0$, contradicting the optimality of x_{t+1}. Thus, it must be that $\frac{\partial W}{\partial x_t} < 0$, but this contradicts the optimality of x_t. Hence, we are at an interior solution $x_t \in (x_{t-1}, x_{t+1})$. The necessary condition for the optimal x_t is then given by

$$0 = \frac{\partial W}{\partial x_t} = \delta^t \left(\frac{\partial v(x_t, \theta)}{\partial x_t} - 1 - \alpha\lambda \right) + \delta^{t+1}\lambda\alpha,$$

which solves to

$$\frac{\partial v(x_t, \theta_t)}{\partial x_t} = 1 + (1-\delta)\alpha\lambda.$$

Hence, in this case indeed $x_{t-1} < x_t = \overline{x}(\theta)$. □

Proof of Proposition 5. Because the unemployment utility of zero is the new threat point, workers accept an offer if and only if their continuation utility is weakly positive. Hence, the firm's maximization problem is given by

$$\max_{(w_t,x_t)} \quad \Pi_t(w_t,x_t) \qquad s.t. \quad U_t(w_t,x_t|w_{t-1},x_{t-1}) \geq 0 \tag{A8}$$

Since $x_{t-1} \leq x^{ME}(\theta_s)$ for all $s \geq t$ we have $\frac{\partial v(x_{t-1},\theta_s)}{\partial x} \geq 1$ for all $s \geq t$. Hence, decreasing effort in period t is suboptimal for the present and all future periods. This implies that the optimal effort level x_t satisfies $x_t \geq x_{t-1}$.

The workers anticipate that in all future offers they will be made indifferent between the offer and their status quo. Hence, the expected utility of any contract (w_t, x_t) is the same as if working under this contract forever.

$$\begin{aligned} U_t(w_t,x_t|w_{t-1},x_{t-1}) &= \sum_{s=t}^{\infty} \delta^{s-t}(w_t - x_t) - \alpha\lambda[w_{t-1} - w_t]^+ - \alpha\lambda[x_t - x_{t-1}]^+ \\ &= \frac{w_t - x_t}{1-\delta} - \alpha\lambda[w_{t-1} - w_t]^+ - \alpha\lambda[x_t - x_{t-1}]^+ \end{aligned}$$

Again, the constraint in (A8) must bind, since otherwise the firm could profitably decrease the wage. Hence, the constraint implies

$$w_t = x_t + (1-\delta)\alpha\lambda[w_{t-1} - w_t]^+ + (1-\delta)\alpha\lambda(x_t - x_{t-1}).$$

Let $\hat{x}$ be defined by $U_t(\hat{x}, w_{t-1}) = 0$. Then for any (w_t, x_t) with $U_t(w_t, x_t) = 0$ we have $w_t > w_{t-1}$ if and only if $x_t > \hat{x}$. If we plug in the wage into the firm's objective function the maximization problem becomes

$$\max_{x_t} \quad \Pi_t(x_t) = \begin{cases} v(x_t,\theta_t) - \Big(1 + (1-\delta)\alpha\lambda\Big)x_t + (1-\delta)\alpha\lambda x_{t-1} - C_t^h + \delta\Pi_{t+1}^*(x_t) & \text{if } x_t > \hat{x}, \\ v(x_t,\theta_t) - x_t - \frac{(1-\delta)\alpha\lambda}{1+(1-\delta)\alpha\lambda}(x_{t-1} - w_{t-1}) - C_t^h + \delta\Pi_{t+1}^*(x_t) & \text{if } x_t \leq \hat{x}, \end{cases}$$

where $\Pi_{t+1}^*(x_t)$ denotes the expected profit from the solution of the firms maximization problem in period $t+1$. Hence,

$$\frac{\partial \Pi_t(x_t)}{\partial x_t} = \begin{cases} \frac{\partial v(x_t,\theta_t)}{\partial x_t} - \big(1 + (1-\delta)\alpha\lambda\big) + \delta\frac{\partial \Pi_{t+1}^*(x_t)}{\partial x_t} & \text{if } x_t > \hat{x}, \\ \frac{\partial v(x_t,\theta_t)}{\partial x_t} - 1 + \delta\frac{\partial \Pi_{t+1}^*(x_t)}{\partial x_t} & \text{if } x_t < \hat{x}. \end{cases} \tag{A9}$$

The firm expects the crisis to be a unique event. Hence, in $t+1$ it faces an equivalent optimization problem as in period 1, where it inherits a contract from the former period that constitutes the workers' outside option. Thus, the results from Proposition 4 apply, and we know that the firm expects wages and effort to weakly increase in the future. This implies that $\frac{\partial \Pi^*_{t+1}(x_t)}{\partial x_t}$ is weakly positive. Indeed, the only effect of higher effort in t on future profits is that it may reduce behavioral costs when increasing effort in the future.

Since $\frac{\partial v(x_t, \theta_t)}{\partial x_t} > 1$ this implies $\frac{\partial \Pi_t(x_t)}{\partial x_t} > 0$ for $x_t < \hat{x}$. Note, that if $\hat{x} > x^{ME}(\theta_t)$ then the second line of the right-hand side of (A9) is negative and we have a corner solution at $x^{ME}(\theta_t)$. Hence, as long as it is feasible, the firm will at least implement effort $\hat{x}$. This shows $x^* \geq \min\{\hat{x}, x^{ME}(\theta_t)\}$.

It remains to be shown under which conditions the firm finds it optimal to implement effort above $\hat{x}$. To this end observe that the incentives for an additional effort increase are the same as in the problem of implementing the optimal effort in Proposition 4. Indeed, if we look at the necessary wage compensation for an effort increase as specified in Proposition 4 we see that its marginal cost is identically given by $1 + (1 - \delta)\alpha\lambda$. Hence, as in Proposition 4, the firm will implement higher effort than $\hat{x}$ if and only if it is behaviorally efficient to do so. By Lemma 1 this is the case if and only if $\hat{x} < \overline{x}(\theta_t)$, in which case the efficient effort is indeed $\overline{x}(\theta_t)$. □

Proof of Lemma 2. Since $x_{t-1} \leq x^{ME}(\theta_{t-1}) < x^{ME}(\theta_t)$ choosing any $x_t < x_{t-1}$ decreases material efficiency. Hence, even disregarding behavioral costs from implementing change, it decreases efficiency in t and, since θ is increasing, a fortiori in all future periods. In any optimal offer the principal will make the workers indifferent to accept, i.e. $U_t(x_{t-1}, w_{t-1}) = U_t(x_t, w_t)$, because otherwise the principal could slightly decrease the wage and the worker would still accept. Hence, a decrease in efficiency would decrease the firm's profits. This implies, effort must be weakly increasing in any equilibrium.

By Proposition 5 workers' utility drops to zero when a crisis occurs. Hence, workers discount future utility by a factor $\delta(1-\mu)$. Hence,

$$
\begin{aligned}
& U_t(w_{t-1}, x_{t-1}) = U_t(w_t, x_t) \\
\Leftrightarrow\quad & \sum_{s=0}^{\infty}(\delta(1-\mu))^s(w_{t-1}-x_{t-1}) = -\alpha\lambda(x_t - x_{t-1}) + \sum_{s=0}^{\infty}(\delta(1-\mu)^s(w_t - x_t) \\
\Leftrightarrow\quad & \frac{(w_{t-1}-x_{t-1})}{1-\delta(1-\mu)} = -\alpha\lambda(x_t - x_{t-1}) + \frac{(w_t - x_t)}{1-\delta(1-\mu)} \\
\Leftrightarrow\quad & w_t = w_{t-1} + \Big[1 + \big(1-\delta(1-\mu)\big)\alpha\lambda\Big](x_t - x_{t-1}) = w_{t-1} + (1+\gamma)(x_t - x_{t-1}).
\end{aligned}
$$

Iterating this formula yields

$$w_t = w_0 + (1+\gamma)(x_t - x_0) \quad\Leftrightarrow\quad w_t = x_t + \gamma(x_t - x_0) + (w_0 - x_0).$$

The last equation of Lemma 2 holds since

$$
\begin{aligned}
U_t(w_{t-1}, x_{t-1}) &= \sum_{k=0}^{\infty}\big(\delta(1-\mu)\big)^k(w_{t-1}-x_{t-1}) \\
&= \sum_{k=0}^{\infty}(\delta(1-\mu))^k\gamma(x_{t-1}-x_0) + \sum_{k=0}^{\infty}(\delta(1-\mu))^k(x_0 - w_0) \\
&= \alpha\lambda(x_{t-1}-x_0) + U_0(x_0, w_0).
\end{aligned}
$$

□

Proof of Lemma 3. Since there is no further threat of bankruptcy after the crisis, a contract (w_{t-1}, x_t) yields a utility of $w_{t-1} - x_t$ in the current and all periods but comes at a one-time behavioral cost of $\alpha\lambda\Delta x_t$. Hence, the effort level that generates zero utility satisfies

$$\alpha\lambda\Delta x_t = \sum_{i=0}^{\infty}\delta^i(w_t - x_t - \Delta x_t) = \frac{(w_t - x_t - \Delta x_t)}{1-\delta},$$

which is equivalent to

$$\Delta x_t = \frac{w_t - x_t}{1 + (1-\delta)\alpha\lambda}.$$

Inserting $(w_t - x_t)$ as given in Equation 17 in Lemma 2 yields the result. □

Proof of Proposition 6. Consider some status quo contract (w_{t-1}, x_{t-1}) and no crisis in period t. Notice that by Lemma 2 at each point in time t the wage w_t is pinned down by the effort level x_t and the workers' initial utility level $w_0 - x_0$. Hence, we suppress the wage in the notation and denote with $W_t(x_t) \equiv W_t(x_t|x_{t-1})$ the expected welfare given that the contracted effort level at time t is x_t and future effort levels are chosen optimally given x_t.

Recall that in normal times for any implemented effort wages are set such that the workers only receive their status quo utility. Hence, the principal's expected profit at time t from implementing effort x_t is

$$\Pi_t(x_t) = W_t(x_t) - U_t(w_{t-1}, x_{t-1}).$$

Since the second term is a constant that doesn't depend on x_t, the principal aims to maximize $W_t(x_t)$ given x_{t-1}. Again, the principal does not find it optimal to lower effort.

To prove the proposition, we need to show that it is never welfare maximizing to increase the effort level x_t to a level that satisfies $x_t > \overline{x}(\theta_t)$.

Denote with $W^c_{t+1}(x_t)$ the expected welfare at time $t+1$ if the crisis occurs at $t+1$ and the effort level contracted in period t was x_t. Hence,

$$W_t(x_t) = v(x_t, \theta_t) - x_t - (\alpha\lambda)(x_t - x_{t-1}) + \delta(1-\mu)W(x^*_{t+1}|x_t) + \delta\mu W^c_{t+1}(x_t).$$

Since it is suboptimal to lower effort, a high effort choice x_t may constrain the optimal choice $x^*_{t+1}(x_t)$. The marginal benefit of effort in period t is highest if it does not constrain the choice x^*_{t+1}, i.e. if $x^*_{t+1} > x_t$. In this case we have

$$W(x^*_{t+1}|x_t) = W(x^*_{t+1}|x^*_{t+1}) - \alpha\lambda(x^*_{t+1} - x_t)$$

Hence, by taking the welfare for the case $x^*_{t+1} > x_t$ we can calculate the following upper bound on the marginal welfare effect of marginal effort increases:

$$\begin{aligned}\frac{\partial W_t(x_t)}{\partial x_t} &\leq \frac{\partial v_t(x_t, \theta_t)}{\partial x_t} - 1 - \alpha\lambda + \delta(1-\mu))\alpha\lambda + \delta\mu\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \\ &= \frac{\partial v_t(x_t, \theta_t)}{\partial x_t} - \big(1 + (1-\delta)\alpha\lambda\big) + \delta\mu\left(\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} - \alpha\lambda\right).\end{aligned}$$

To show that it is never welfare maximizing to implement $x_t > \overline{x}(\theta_t)$ it suffices to show that the right-hand side of the above equation is negative for all $x_t > \overline{x}(\theta_t)$. Since for all $x_t > \overline{x}(\theta_t)$ we have $\frac{\partial v_t(x_t,\theta_t)}{\partial x_t} < 1 + (1-\delta)\alpha\lambda$ this amounts to showing that

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} \leq \alpha\lambda \tag{A10}$$

for all $x_t > \overline{x}(\theta_t)$. The remainder of the proof is devoted to that task.

Proposition 5 fully analyzes the contract in a crisis at $t+1$ given effort x_t. The principal will use the agents lower outside option to increase the effort. The amount $\Delta x_{t+1} \equiv \Delta(x_t)$ by which the principal can at most increase effort without changing the wage is calculated in Equation 20 of Lemma 3. The periods after the crisis follow the effort path calculated in Proposition 4.

Recall first that the principal will continue the relationship if and only if her expected profits remain positive. Denote with $W^{cc}(x_t)$ the welfare without cost C^h_{t+1} if the principal continues the relationship. Hence

$$W^c(x_t) = \max\{0, W^{cc}(x_t) - C^h_{t+1}\}.$$

Notice that $W^c(x_t)$ is continuous everywhere and differentiable in all points except at the x_t that satisfies $W^{cc}(x_t) = C^h_t$. Depending on the implemented x_t the welfare $W^c(x_t)$ may be either determined by bankruptcy or by one of the three cases outlaid in the case distinctions in Proposition 5. We go through all case distinctions and show that Equation A10 holds in all of these cases.

- Case $W^{cc}_{t+1}(x_t) < C^h_{t+1}$.

  The principal terminates the relationship and $W^c(x_t) = 0$. Hence,

  $$\frac{\partial W^c_{t+1}(x_t, w_0 - x_0)}{\partial x_t} = 0 < \alpha\lambda.$$

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_t)) < \overline{x}(\theta_{t+1})$.

  The principal continues the relationship. The second condition states that if the principal implements the behaviorally efficient effort level in period t then even after the discontinuous effort increase of $\Delta(x_t)$ in a crisis at $t+1$ we are in the boundary solution of Proposition 5 in which the principal increases effort x_{t+1} on the behaviorally efficient level $x_{t+1} = \overline{x}(\theta_{t+1})$. Hence, the marginal effort choice at period t has no impact on the effort implemented in and after the crisis. The principal can implement the behaviorally efficient contract as calculated in Proposition 5. Hence, the inequality in our proposition holds with equality and inertia is the same as in the unanticipated case. We will see that in all other cases inertia is strictly larger than in the case of an unanticipated crisis.

If $\overline{x}(\theta_t) + \Delta(\overline{x}(\theta_{t+1})) > \overline{x}(\theta_{t+1})$, then a fortiori $x_t + \Delta(x_t) > \overline{x}(\theta_{t+1})$ for all $x_t > \overline{x}(\theta_t)$. In this case, following Proposition 5, the principal will implement effort $x_{t+1} = \min\{x^{ME}(\theta_{t+1}), x_t + \Delta x_t\}$, and the effort enters the region of inertia. This gives rise to the last two case distinctions

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta x_t > x^{ME}(\theta_{t+1})$.

  Then in the crisis at $t+1$ the principal implements the materially efficient effort level $x^{ME}(\theta_{t+1})$. The marginal choice of effort x_t has no impact on the effort implemented in a crisis. However, it has impact on the associated wage cut in a crisis. A higher effort x_t leads to a stronger wage decrease in the crisis. Since a wage decrease comes with a behavioral cost this implies

$$\frac{\partial W^c_{t+1}(x_t, w_0 - x_0)}{\partial x_t} < 0 < \alpha\lambda.$$

- Case $W^{cc}_{t+1}(x_t) > C^h_{t+1}$ and $x_t + \Delta(x_t) \in (\overline{x}(\theta_{t+1}), x^{ME}(\theta_{t+1})$.

  This is the most interesting case of an interior solution, in which the marginal choice of effort x_t impacts the effort path in and after the crisis. We start with deriving a closed form expression for the welfare $W^{cc}_{t+1}(x_t)$ in the crisis.

  Let $T \in \{t+1, t+2, ...\} \cup \{\infty\}$ be the last period in which $x_{t+1} = x_t + \Delta(x_t)$ satisfies $x_{t+1} < \overline{x}(\theta_T)$, i.e. the period before the effort level x_{t+1} leaves the inertia region (if ever). Then,

$$\begin{aligned} W^{cc}_{t+1}(x_t) = -\alpha\lambda\Delta(x_t) + \sum_{s=t+1}^{T} \delta^{s-t-1}\big(v(x_{t+1}, \theta_s) - x_{t+1}\big) \\ -\delta^{(T-t)}\alpha\lambda(x_T - x_{t+1}) + \delta^{(T-t)} W_T(\overline{x}(\theta_T)) \end{aligned}$$

The first term corresponds to the behavioral cost from the effort adjustment in period $t+1$. The sum is the welfare generated while effort is in the inertia region. The third term is the behavioral adjustment cost to the behaviorally efficient line after effort leaves the inertia region. The last term denotes the (discounted) welfare from the remaining game where effort follows the behaviorally efficient level and is independent of the past adjustment in the crisis.[25]

Then

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = -\alpha\lambda\frac{\partial \Delta x_t}{\partial x_t} + \sum_{s=t+1}^{T} \delta^{s-t+1}\left(\frac{\partial v(x_{t+1},\theta_s)}{\partial x_t} - \frac{\partial x_{t+1}}{\partial x_t}\right) + \delta^{(T-t)}\alpha\lambda\frac{\partial x_{t+1}}{\partial x_t}$$

Since $\frac{\partial x_{t+1}}{\partial x_t} = \frac{\partial(x_t+\Delta x_t)}{\partial x_t} = 1 + \frac{\partial \Delta x_t}{\partial x_t}$ we obtain

$$\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} = \alpha\lambda + \frac{\partial x_{t+1}}{\partial x_t}\left[-\alpha\lambda + \sum_{s=t+1}^{T} \delta^{s-t+1}\left(\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}} - 1\right) + \delta^{(T-t)}\alpha\lambda\right]$$

Since in the periods $s \in \{t+1,...,T\}$ we are in the inertia region we have for these s that $\frac{\partial v(x_{t+1},\theta_s)}{\partial x_{t+1}} < 1 + (1-\delta)\alpha\lambda$, and hence

$$\begin{aligned}
\frac{\partial W^c_{t+1}(x_t)}{\partial x_t} &< \alpha\lambda + \frac{\partial x_{t+1}}{\partial x_t}\left[-\alpha\lambda + \sum_{s=t+1}^{T} \delta^{s-t+1}(1-\delta)\alpha\lambda + \delta^{(T-t)}\alpha\lambda\right] \\
&= \alpha\lambda + \alpha\lambda\frac{\partial x_{t+1}}{\partial x_t}\underbrace{\left[-1 + (1-\delta)\sum_{k=0}^{T-t-1}\delta^k + \delta^{T-t}\right]}_{=0} \\
&= \alpha\lambda
\end{aligned}$$

□

Proof of Lemma 4. The wage payment (22) follows directly from equation (21). Differentiating (22) twice with respect to p yields:

$$\begin{aligned}
\frac{\partial w}{\partial p} &= (1+\lambda)\Delta x - 2p(1-\alpha)\lambda\Delta x \\
\frac{\partial^2 w}{\partial p^2} &= -2(1-\alpha)\lambda\Delta x < 0
\end{aligned}$$

[25]If $T = \infty$ the implicit convention is that the last two terms are zero.

Note that

$$\frac{\partial w}{\partial p} < 0 \quad \text{if and only if} \quad \frac{1+\lambda}{\lambda(1-\alpha)} < 2p\ . \tag{A11}$$

□

Proof of Proposition 7. The principal maximizes

$$E\Pi = p(v + \Delta v - b) + (1-p)v - w \tag{A12}$$

subject to (22), $b \geq 0$, and

$$p \in \arg\max \left\{ pb - \frac{c}{2}p^2 \right\} \tag{A13}$$

Since $b \geq 0$, constraint (A13) is equivalent to

$$p = \min\left\{ \frac{b}{c}, 1 \right\} \tag{A14}$$

Evidently, any bonus $b > c$ is suboptimal as induces the same probability of change but a higher manager compensation than $b = c$. By setting $p = \frac{b}{c}$ and plugging the wage from (22) into the objective, the principal's problem reduces to

$$\max_{b\in[0,c]} E\Pi = v - \left[x_0 + \frac{b}{c}\Delta x[1 + \lambda(1 - (1-\alpha)\frac{b}{c})] + U_0 \right] + \frac{b}{c}[\Delta v - b], \tag{A15}$$

which can be rewritten to

$$\max_{b\in[0,c]} v - x_0 - U_0 + \frac{b}{c}\left[\Delta v - (1+\lambda)\Delta x\right] - \frac{b^2}{c^2}\left[c - (1-\alpha)\lambda\Delta x\right]\ . \tag{A16}$$

This is a convex problem if and only if $c < (1-\alpha)\lambda\Delta x$. In the convex case we obtain a boundary solution, i.e. $b \in \{0, c\}$, or equivalently $p \in \{0, 1\}$. The boundary solution is $b = p = 0$ if and only if

$$\Delta v - (1+\lambda)\Delta x < c - (1-\alpha)\lambda\Delta x,$$

i.e., if and only if

$$\Delta v < (1+\alpha\lambda)\Delta x + c,$$

which proves (a) of the proposition.

For the concave case, the FOC for the problem is given by

$$\frac{\partial E\Pi}{\partial b} = \frac{\Delta v - (1+\lambda)\Delta x}{c} - \frac{2b[c-(1-\alpha)\lambda\Delta x]}{c^2} \leq 0 \; . \tag{A17}$$

Note that at $b = 0$, $\frac{\partial E\Pi}{\partial b} = \frac{\Delta v-(1+\lambda)\Delta x}{c}$. Thus, the principal will choose the boundary solution $b = 0$ if and only if $\Delta v \leq (1+\lambda)\Delta x$. Furthermore, if (A17) holds with equality we have

$$b = \frac{c}{2}\,\frac{\Delta v - (1+\lambda)\Delta x}{c - \lambda(1-\alpha)\Delta x} \quad \Leftrightarrow \quad p = \frac{\Delta v - (1+\lambda)\Delta x}{2[c-\lambda(1-\alpha)\Delta x]}. \tag{A18}$$

Hence, if $\Delta v > (1+\lambda)\Delta x$ the principal induces $p = \min\left\{\frac{\Delta v-(1+\lambda)\Delta x}{2[c-\lambda(1-\alpha)\Delta x]}, 1\right\}$.

□

Proof of Lemma 5. Because $x_2 > x_1$, we must have $w_2 > w_1$. Thus, the firm will offer wages that give both workers exactly their outside option utility U_0:

$$\begin{aligned} U_1 &= w_1 - x_1 - \lambda[w_1^r - w_1] = U_0, \\ U_2 &= w_2 - x_2 - \lambda[x_2 - x_2^r] = U_0. \end{aligned}$$

This implies

$$\begin{aligned} w_2 &= U_0 + x_2 + \lambda[x_2 - x_2^r] \\ &= U_0 + x_2 + \lambda\left[x_2 - (1-\beta)x_2 - \beta x_1\right] \\ &= U_0 + x_2 + \lambda\beta[x_2 - x_1]. \end{aligned}$$

This implies for w_1:

$$\begin{aligned} w_1 &= U_0 + x_1 + \lambda[w_1^r - w_1] \\ &= U_0 + x_1 + \lambda\left[(1-\beta)w_1 + \beta w_2 - w_1\right] \\ &= U_0 + x_1 + \lambda\beta[w_2 - w_1]. \end{aligned}$$

Collecting terms yields

$$\begin{aligned} [1+\lambda\beta]\, w_1 &= U_0 + x_1 + \lambda\beta w_2 \\ &= U_0 + x_1 + \lambda\beta\left[U_0 + x_2 + \lambda\beta(x_2 - x_1)\right] \\ &= (1+\lambda\beta)U_0 + \lambda\beta(1+\lambda\beta)x_2 + \left[1 - (\lambda\beta)^2\right] x_1. \end{aligned}$$

Thus, we get:

$$\begin{aligned} w_1 &= U_0 + \lambda\beta x_2 + (1-\lambda\beta)x_1 \\ &= U_0 + x_1 + \lambda\beta[x_2 - x_1]. \end{aligned}$$

□

Proof of Proposition 8. Since worker 2 is the more productive one it is evidently suboptimal to implement $x_2 < x_1$. Denote in the following $\Delta \equiv x_2 - x_1 \geq 0$. Hence, the principal's problem can be written as

$$\max_{x_1,\Delta\geq 0} \Pi(x_1,\Delta) = v_1(x_1,\theta) + v_2(x_1+\Delta,\theta) - [U_0 + x_1 + \lambda\beta\Delta] - [U_0 + x_1 + \Delta + \lambda\beta\Delta]$$

The first order conditions of a potential an inner solution, $x_1 > 0, \Delta > 0$, are

$$\frac{\partial v_1(x_1,\theta)}{\partial x_1} + \frac{\partial v_2(x_2,\theta)}{\partial x_2} = 2,$$

$$\frac{\partial v_2(x_2,\theta)}{\partial x_2} = 1 + 2\beta\lambda.$$

Our regularity conditions imply that the first FOC is always at an inner solution, i.e. $x_1 > 0$. Hence, the first FOC binds with equality. The second FOC together with the constraint that $\Delta \geq 0$ implies the boundary solution $\Delta = 0$ if and only if the effort level $x_1 = x_2$ that satisfies the first FOC features $\frac{\partial v_2(x_2,\theta)}{\partial x_2} < 1 + 2\beta\lambda$. Hence, the result. □