

Optimal inflation with firm-level shocks¹

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¹The views expressed here are those of the authors only and do not necessarily reflect the views of Deutsche Bundesbank, ECB or the Eurosystem.

Introduction

- Price setting models nowadays include *idiosyncratic shocks* to match micro pricing moments (Golosov-Lucas 2007)
- These models have been used widely for positive analysis, e.g. to infer the degree of monetary non-neutrality
- However, there isn't much normative analysis; e.g. not much is known about the optimal rate of inflation in these models
- “Folk wisdom” for normative conclusions: firm-level shocks do not alter the optimality of zero inflation – the logic of the standard NK model applies
- We study optimal inflation in canonical sticky price models with idiosyncratic productivity shocks
- We abstract from production networks and from shocks to market power

Preview: key friction

- Generally, efficiency requires $\frac{\text{reset price}(j)}{\text{price level}} = \frac{\text{aggregate productivity}}{\text{idiosyncratic productivity}(j)}$
- With price stickiness, reset prices are inefficiently forward-looking:

$$\frac{\text{reset price}(j)}{\text{price level}} = \frac{\text{aggregate productivity}}{\text{expected discounted idiosyncratic productivity}}$$

⇒ reset price distribution compressed relative to flexible prices

Preview: reset price compression with AR(1) shocks

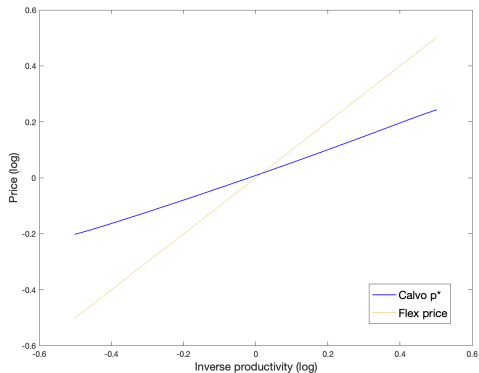


Figure: Optimal price compression in Calvo

Preview: nature of the misallocation

- Reset price of productive firms is inefficiently high, hence productive firms supply too small a share of aggregate demand relative to social optimum
- Reset price of unproductive firms is inefficiently low, hence unproductive firms supply too much of aggregate demand
- Reset price compression presents a new motive for choosing a *negative* optimal inflation rate π^*
- Negative inflation rate helps align reset prices more closely with the current idiosyncratic shock realizations.
- By speeding up the pass-through of marginal cost shocks to prices, negative inflation increases aggregate productivity

Trade-off

Two forces:

- 1 The usual distortion between reset and non-reset prices (calling for zero inflation) in standard NK models
- 2 The distortion of reset prices themselves (calling for -100% inflation).

The first-best (efficient) allocation can no longer be achieved

The optimal trade-off is resolved at inflation rates around -2% .

For plausible calibrations it may even be below the Friedman optimum!

Related work

- Large literature on money non-neutrality in pricing models with idio shocks
 - ▶ Golosov-Lucas 2007: Near-neutrality of money
 - ▶ Midrigan 2011: Simple menu cost model matches poorly histogram of price changes
 - ▶ Karadi & Reiff 2019: Large tax shocks reveal that degree of money non-neutrality is very sensitive to shape of idiosyncratic shock distribution
 - ▶ Our results rely on shocks' mean reversion – a generic feature in the literature
- Small literature on π^* in pricing models with idiosyncratic shocks
 - ▶ Burstein & Hellwig 2008: Sticky prices vs Friedman rule
 - ▶ Blanco 2019: Sticky prices vs ZLB
 - ▶ Adam, Gautier, Santoro & Weber 2021: Sticky prices and product lifecycles
 - ▶ We focus on sticky prices only and rule out non-stationary productivity

Overview - NK model with firm-level shocks

- Representative household with discount rate $\beta \in (0, 1)$
- Aggregate output is CES composite with substitution elasticity θ
- Offset effect of flexible price markup using sales subsidy τ ,

$$\frac{\theta}{\theta - 1} \frac{1}{1 + \tau} = 1$$

\Rightarrow) flexible price allocation is efficient / first best

- Markov process for idiosyncratic productivity shocks
- Aggregate productivity is endogenous and depends on inflation
- Calvo pricing, $\alpha \in [0, 1)$ denotes probability of not adjusting price

Technology of firm j and idiosyncratic productivity shocks

$$Y_{jt}Z_{jt} = L_{jt} \quad (1)$$

Inverse of idiosyncratic productivity shock is finite-state Markov

$$Z_{jt} = z^0 \xi_{jt} \quad (2)$$

- z is $K - 1$ vector with inverse level of productivity in each state
- idio state ξ_{jt} equals $K - 1$ unit vector e_j when idio state equals j
- unit unconditional mean $E(Z_{jt}) = 1$
- focus on ergodic distribution of idiosyncratic shocks, $\xi = E(\xi_{jt})$
- impose zero covariance btw idio shocks and aggregate variables

Aggregation of firms with idiosyncratic shocks

Aggregate technology

$$Y_t = \frac{1}{\Delta_t} L_t$$

Inverse aggregate productivity is output-weighted average of inverse idiosyncratic productivities,

$$\Delta_t = \int_0^1 Z_{jt} \underbrace{\left(\frac{Y_{jt}}{Y_t} \right)}_{=(P_{jt}/P_t)^\theta} dj$$

Analytical aggregation yields recursive representation of Δ_t [Details](#)

Gross inflation $\Pi_t = \frac{P_t}{P_{t-1}}$ with P_t the welfare-based price level [Details](#)

Misallocation under zero inflation: distortion # 1

- WLOG let us assume shocks are i.i.d, 2 states (j, k) and zero inflation
- Output is demand determined. A firm that adjusted when its state was j and now has state k : $Z_k Y_j = L_k$
- Demand is determined by the reset price, which is a weighted average over current and expected future idiosyncratic costs $p_j = \vartheta w \left[\left(1 - \frac{\alpha}{2}\right) Z_j + \frac{\alpha}{2} Z_k \right]$
- A fully flexible price instead is set as: $p_j = \vartheta w^e Z_j$
- So reset prices are distorted (compressed)

Misallocation under zero inflation: distortion #2

- Firms with constant prices have continuously evolving idiosyncratic costs
- Let us average labor over all firms with the same demand
$$Y_j \left[\left(1 - \frac{\alpha}{2}\right)Z_j + \frac{\alpha}{2}Z_k \right] = L^j$$
- Under rational expectations the average j -firm has the cost level that the resetting firm anticipates when in state j
- Substituting in demand and the reset price, we obtain the contribution of the average j -firm to aggregate productivity:
$$\left[\left(1 - \frac{\alpha}{2}\right)Z_j + \frac{\alpha}{2}Z_k \right]^{(1-\theta)} = \frac{L^j}{L} \Delta^{(1-\theta)}$$
- With flex prices instead, assuming same shares of firms in states j and k we obtain $\left(1 - \frac{\alpha}{2}\right)Z_j^{(1-\theta)} + \frac{\alpha}{2}Z_k^{(1-\theta)} = \frac{L^{ej}}{L^e} (\Delta^e)^{(1-\theta)}$
- With sticky prices aggregate productivity is influenced by prices that reflect averaged costs
- While with flex prices aggregate productivity is influenced by prices reflecting non-averaged idiosyncratic costs

Graphical illustration: zero inflation

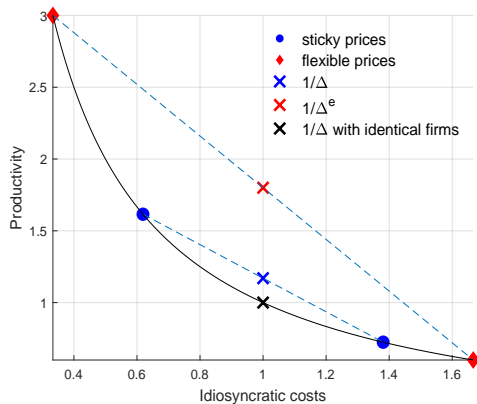


Figure: Zero inflation relative to flex price

Graphical illustration: negative inflation

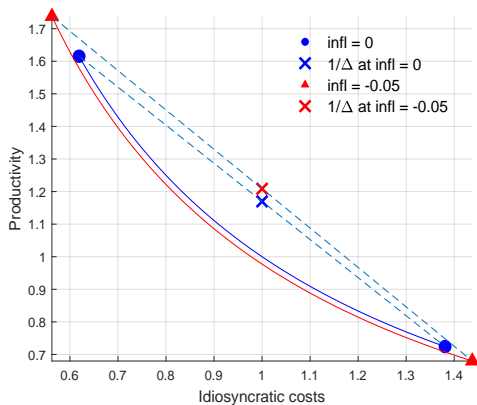


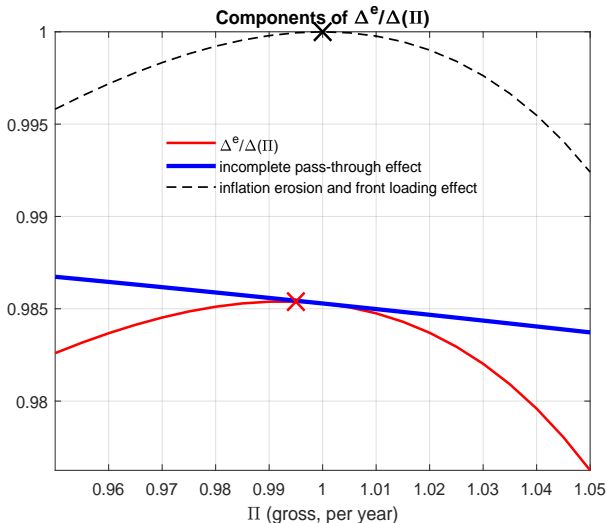
Figure: Negative inflation relative to zero inflation

Decomposing the aggregate productivity distortion

Proposition: Consider the limit $\beta \rightarrow 1$ and idiosyncratic shocks that discretize a stationary AR(1) process with persistence $\rho < 1$ using Rouwenhorst (1995). Then, the productivity distortion is given by

$$\frac{\Delta^e}{\Delta(\Pi)} = \underbrace{\left(\frac{\sum_j \xi_j (1 + (z_j - 1))^{1-\theta}}{\sum_j \xi_j \left(1 + (z_j - 1) \frac{1 - \alpha \Pi^\theta}{1 - \alpha \Pi^\theta \rho}\right)^{1-\theta}} \right)^{\frac{1}{1-\theta}}}_{\text{reset price compression (RPC)}} \cdot \underbrace{\left(\frac{1 - \alpha}{1 - \alpha \Pi^\theta} \right)^{\frac{1}{\theta-1}}}_{\text{inflation erosion effect}} \cdot \underbrace{\left(\frac{1 - \alpha \Pi^\theta}{1 - \alpha \Pi^\theta \rho} \right)}_{\text{inv front loading effect} = 1/\phi}.$$

With sticky prices, $\alpha > 0$, no inflation rate fully eliminates the productivity distortion.



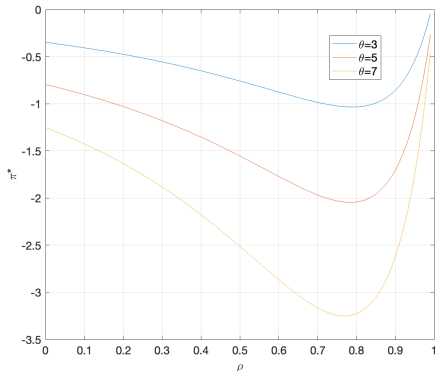
- ① $\Delta^e/\Delta(\Pi) = 1$ *infeasible* given policy tradeoff from firm-level shocks
- ② With firm-level shocks, prominent price stability result, $\Pi^* = 1$, fails

Optimal inflation in Calvo

Consider a sticky-price steady state with firm-level productivity shocks, $\beta \neq 1$ and $\frac{1}{1+\tau} \frac{\theta}{\theta-1} = 1$. Then, optimal inflation maximizing steady-state utility is negative,

$$\Pi^* < 1,$$

and does not restore efficiency, $\Delta^e / \Delta(\Pi^*) < 1$.



A generalized-hazard state-dependent pricing model

- Model by Woodford (2011). Has rational inattention microfoundations as shown by Steiner et al. (2017)
- Nests Calvo (1983) pricing and Golosov & Lucas (2007) with fixed menu costs as two polar cases

The equilibrium adjustment probability function takes the following form:

$$\lambda(G) = \frac{\bar{\lambda} \exp(\frac{G}{\xi})}{1 + \bar{\lambda} \exp(\frac{G}{\xi})} \quad (3)$$

Parameter	Description	Value	Source
β	Monthly discount	0.9984	Annual real rate of 2%
γ	Intertemporal elast. of subst.	2	Golosov-Lucas (2007)
ζ	Frisch labor supply elast.	1	Ibid
χ	Coefficient on labor disutility	6	Ibid
θ	Elasticity of subst. across varieties	7	Ibid

Idiosyncratic shocks, exogenous and calibrated parameters

Logarithm of idiosyncratic productivity shocks follows

$$a_{jt} = \rho a_{jt-1} + \sigma \epsilon_t, \quad \epsilon_t \sim N(0, 1)$$

Shock process discretized into finite-state Markov process

Table: Estimated parameters

Description	Parameter	Value
Information cost	ξ	1.8370
Menu cost	κ	0.0359
Persistence of productivity shock	ρ	0.8989
Std dev of productivity shock	σ	0.0944

Matched moments

Table: Matched moments

Moment	Data	Model
Frequency of price changes	0.10	0.10
Std of price changes	0.0557	0.0557
Kurtosis of price changes	3.86	3.86
Persistence ρ_{adj}	0.896	0.896

Stationary distribution of firms

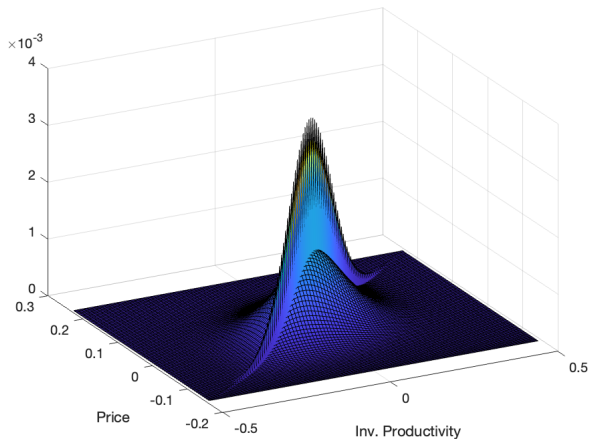


Figure: Stationary distribution of firms

Adjustment probability function

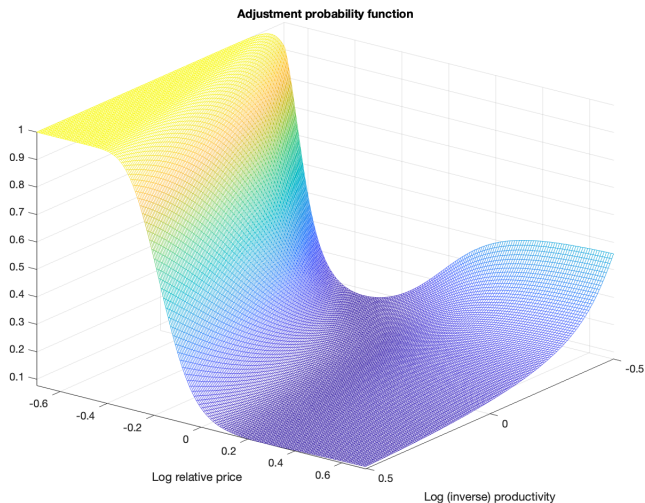


Figure: Adjustment probability function in SDP model

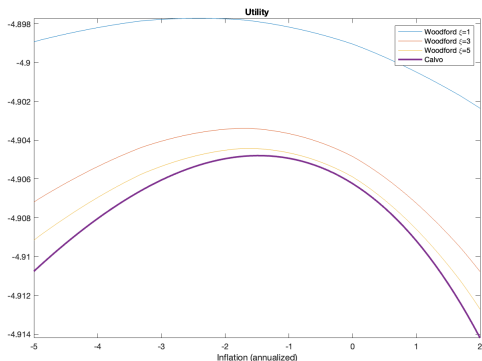


Figure: Inflation and utility in Calvo and Woodford models

- Calvo: $\pi^* = 1.4\%$
- Woodford (2011): $\pi^* = 2\%$
- 30-70 basis points productivity gain vis-a-vis targeting +4% inflation

Conclusions

Pricing models consistent with observed price change heterogeneity imply sizeable misallocation at zero inflation and so a role for monetary policy to reduce it

- Stickiness coupled with idiosyncratic productivity shocks distorts newly set prices thereby reducing aggregate productivity
- This distortion provides a new motive for choosing negative π^*

Ongoing work

- Study Ramsey optimal policy, transitional dynamics, Ramsey steady state

Recursive representation of aggregate productivity

$$\Delta_t = z^\theta V_t \quad (4)$$

$$V_t = (1 - \alpha) D_t^\theta \text{diag}(z^\theta N_t)^{-\theta} \xi + \alpha \Pi_t^\theta F V_{t-1} \quad (5)$$

$$N_t = I_K w_t + \alpha \beta F E_t[\Pi_{t+1}^\theta N_{t+1}] \quad (6)$$

$$D_t = 1 + \alpha \beta E_t[\Pi_{t+1}^{\theta-1} D_{t+1}] \quad (7)$$

- V_t is $K \times 1$, N_t is $K \times K$
- F is transition matrix of idiosyncratic shocks
- Π_t is gross inflation
- w_t is wage [▶ Back](#)

Aggregation: Price level and recursive pricing rule

Price level

$$P_t^{1-\theta} = \int_0^1 P_{jt}^{1-\theta} dj \quad (8)$$

Average reset price

$$(P_t^*)^{1-\theta} = \frac{1}{1-\alpha} \int_{J_t^*} (P_{jt}^*)^{1-\theta} dj, \quad (9)$$

where set J_t^* has mass $1-\alpha$ and contains firms that can adjust price in t

$$\frac{P_{jt}^*}{P_t} = \frac{\vartheta}{D_t} z^\theta N_t \xi_{jt}, \quad (10)$$

$$(P_t^*)^{1-\theta} = \left(\frac{\vartheta}{D_t} \right)^{1-\theta} z^\theta N_t \text{diag}(z^\theta N_t)^{-\theta} \xi \quad (11)$$

with $\vartheta = \frac{\theta}{1-\theta} \frac{1}{1+\tau}$

▶ Slides

Sketch of proof for optimal inflation being negative

Π^* maximizes steady state utility. With $\beta \neq 1$ and $\frac{\theta}{\theta-1} \frac{1}{1+\tau} = 1$, obtain:

- 1 labor is independent of Π hence maximizing output also maximizes utility
- 2 $\delta(\Pi) = \mu(\Pi)^{-1}$ hence maximizing output requires maximizing $\delta(\Pi)$
- 3 $\delta(\Pi)' = 0$ implies that Π^* fulfills

$$\begin{aligned} & \left(\frac{\Pi - 1}{\Pi} \right) \frac{1}{(1 - \alpha\Pi^{\theta-1})(1 - \alpha\Pi^\theta)} \\ &= \left(\frac{1 - \rho}{(1 - \alpha\Pi^\theta\rho)^2} \right) \frac{\sum_j \xi_j \hat{z}_j \left(1 + \hat{z}_j \frac{1 - \alpha\Pi^\theta}{1 - \alpha\Pi^\theta\rho} \right)^{-\theta}}{\sum_j \xi_j \left(1 + \hat{z}_j \frac{1 - \alpha\Pi^\theta}{1 - \alpha\Pi^\theta\rho} \right)^{1-\theta}}. \end{aligned} \tag{12}$$

4 $\text{sgn}\{\Pi^*\} = \text{sgn}\left\{ \sum_j \xi_j \hat{z}_j \left(1 + \hat{z}_j \frac{1 - \alpha\Pi^\theta}{1 - \alpha\Pi^\theta\rho} \right)^{-\theta} \right\} = -1 \quad \square$ [▶ slides](#)

References

- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12(3), 383–398.
- Steiner, J., C. Stewart, and F. Matejka (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica* 85(2), 521–553.