On the Importance of Information Quality and Returns

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Abstract. This article highlights the significance of information quality and returns in resolving the conflict between information spreading and acquisition for explaining the impact of information acquisition on welfare. In a market with complete information, the act of disclosing information can either enhance or deteriorate overall welfare, depending on certain factors. Firstly, welfare may be negatively affected by information purchasing behaviors that are subject to market risk adjustments. Secondly, the welfare also depends on the evaluation of the benefits versus the costs of utilizing the information. This evaluation can either offset or reinforce the welfare losses caused by information purchasing, depending on whether the returns outweigh the costs. Additionally, the aggregate and individual welfare effects are stylized in response to each fundamental of risk. In a market with asymmetric information, welfare cuts are caused, but as the intensity of information acquisition weakens, the negative effect becomes milder, and the overall welfare can be Pareto-improved.

Keywords: information acquisition, information quality, market fundamentals, welfare, asymmetric information.
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1 Introduction

Information acquisition is prime facie attractive in the sense that the information usually facilitates the deployment of advanced trading strategies. The welfare effect for a representative trader whose information acquisition decision is flexibly adjusted due to erratic informational shocks and market-level risks is, however, beset because the net benefits and costs of being in certain market states as well as factors related to the welfare effects have not been adequately described.

This paper studies the impact of costly information acquisition on welfare. Market participants, in practice, face information frictions when they trade with others. Information acquisition is a useful way to identify the frictions. Given that information is costly and noisy, traders, however, do not always choose to acquire it since they have to weigh the benefits against costs from utilizing the information. Hence, the equilibrium number of informed traders should vary according to the content of information available for sale. In theory, as a result of information frictions, market equilibrium must not be equivalent to market efficiency; thus some efficiency losses are common regardless of which equilibrium is reached. However, this paper investigates more than the common. Many papers have already discussed the welfare effect regarding utilizing information, *e.g.* Hirshleifer (1971), Laffont (1985), Morris and Shin (2002), Cornand and Heinemann (2008) and Hu and Qin (2013). In contrast, this paper recovering more welfare patterns typically addresses the following questions: what trader's behaviors explain the welfare effect of information acquisition? How do the welfare patterns respond to fundamentals about uncertainty? And what roles are the asymmetric information playing in shaping the welfare function? These questions are nontrivial because information acquisition could either improve or compromise market efficiency, and the reasons behind this dichotomy as well as the welfare patterns in different market states deserve more research attention.

We attempt to answer these questions by constructing a continuous rational expectation equilibrium (REE) model. Suppose that there are infinite trading days, similar to Kyle (1985)'s continuous trading game. Informational shocks of trading day T happen to affect the formation of market risk fundamentals of trading day T+1. Following the practices of Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015), we assume that these information shocks of period T remain too rigid to affect the informational states of period T+1. Rather, they contribute to the informational states of period T+2 through the effect on market risk fundamentals of period T+1. This information structure makes it easy to investigate how the welfare patterns respond to exogenous informational shocks. In this study, Grossman and Stiglitz (1980)'s model is adapted to solve the intraperiod problem. Furthermore, the welfare function is defined as the expected payoffs of the representative trader. For the market of complete information, the welfare effect is measured by calculating the welfare difference before and after information acquisition. In a market of incomplete information, the welfare effect function is continuous in the intensity of information acquisition.

We find that the aggregate welfare effect can be decomposed and thus accrued to two individual effects, *i.e.* the behavior of information purchasing (IP, henceforth) and the behavior of information evaluation (IE, henceforth). First, the purchasing decision itself is made based on the pecuniary cost and may have a welfare effect; therefore, it is supposed to face systematic risk adjustments. These adjustments are justified because purchasing the same information can surely have different welfare consequences at different market states, even though the information price is fixed. Second, the IE reflects the benefits and costs of acquiring information. The benefits are how much of projected returns they gain from utilizing the information, while the costs are the losses of market efficiency due to revealing information *per se*. The benefits can be escalated by utilizing information of better quality or by improving market randomness whereas the costs are only associated with information quality.

Given that by simultaneously considering the quality of information and the market randomness can the conflict between information spreading and information acquisition be tactically resolved, this paper goes to assess the conditions at which information acquisition brings in extra market efficiency or inefficiency, jointly and individually. First, the IP effect always causes welfare losses which are affected by risk adjustments. The risk adjustments are, in turn, directly affected by information advantages (τ_ε) and informativeness (τ_s); they are also indirectly affected by returns to information (τ_u) as a substitute for τ_ε and as a complement for τ_s, as well as by the degree of risk-aversion (ρ) as a substitute for τ_s and as a complement for τ_ε. More importantly, this paper shows that as long as the summation of the returns to information and the relative information quality is smaller than a cutoff value, the IE effect brings in welfare gains. This is the most crucial finding since it raises the possibility of sign flipping of the value previously found in the literature (*e.g.* Hirshleifer (1971), Laffont (1985), Morris and Shin (2002) and Hu and Qin (2013)). Otherwise, the IE reinforces the welfare cuts caused by the IP. As market states continuously evolve over time, the welfare displays stylized patterns in each fundamental parameter. Once the states were fixed, the overall effect, collectively, depends on the IP and IE effects. Hence, the welfare patterns are driven by different shocks to different fundamentals. Despite of having measurement errors, the main findings are still robust for some well-tuned parameters.

We also investigate the welfare effect when information acquisition is continuous and the asymmetric information problem turns on. Because of the strategic substitution in information acquisition, the asymmetric information problem is supposed to become less severe as more traders become informed. As known, asymmetric information tends to cause efficiency losses, which has been theoretically implied by Akerlof (1970), Rothschild and Stiglitz (1976) and Laffont (1985) and also empirically implied by Einav, Finkelstein, and Schrimpf (2010). Likewise, the strategic substitution assumption implies that information acquired plays a role similar to an insurance for uninformed traders since as price becomes more informative, the marginal benefits from not utilizing the information increase while the marginal benefits from utilizing the information decrease. Meanwhile, the marginal cost of information goes down. Thus, as the proportion of informed traders increases, the welfare patterns should be convex individually and collectively. By performing some simulations in the present study, the conjectures are confirmed. As long as the fundamentals are sensitive to the asymmetric information, the overall welfare displays a convex pattern with respect to any of these fundamentals. This convexity can be driven either by the slowdown in the welfare cuts caused by the IP and the IE or by the welfare increase due to the IE.

Related literature. This paper contributes to the literature studying the role of information acquisition on welfare, given that heterogeneous market participants behave differently towards information frictions. Below, we discuss a few related studies to which this paper intensively responds.

In many existing studies, the analysis of such information issues are conducted under REE setting. Some of the analytical models are static *e.g.* Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985), whereas like Kyle (1985), Wang (1993), Campbell, Grossman, and Wang (1993), Wang (1994), He and Wang (1995), Brennan and Cao (1997), and Llorente, Michaely, Saar, and Wang (2002) construct dynamic REE models. In contrast, this paper constructs a continuous REE model fitting the continuous fashion of Kyle (1985) to Grossman and Stiglitz (1980)'s model. With respect to the information rigidities, they are introduced by Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015) and these rigidities should be assumed in each period since the instant information spreading is almost impossible due to high adjustment costs. This assumption is similar to the claim of Reis (2006) who sets forth that firms face huge costs of utilizing new information.

The welfare is defined as the expected utility for the representative agent following Arrow (2012). Among others (*e.g.* Amador and Weill (2010), Colombo, Femminis, and Pavan (2014)), he stresses that the social welfare is a social ordering that maximizes the social utility according to which the individual values in the community on alternative social states are aggregated. Colombo et al. (2014) do find a wedge between choosing a social optimal precision of private information and an equilibrium precision of private information, and they also look at how the social value of public information is affected by the inefficiency in acquiring private information. Similarly, we are aware of the wedge between equilibrium and efficiency assuming that the information is homogeneous under a given market state. However, this is not a weak assumption compared to some others who consider differential information (*e.g.* Hellwig (1980), Diamond and Verrecchia (1981),Verrecchia (1982)).

The discrete case of the welfare effect considers the welfare change after private information is purchased by all traders and becomes public information analogous to market prices. This corresponds to the other strand of literature regarding the welfare impact of public information. Morris and Shin (2002) believe that public information can harm social welfare once agents have some private information while Cornand and Heinemann (2008) argue that precise public information improves social welfare in some certain situations. In this paper, we respond to this debate by finding that welfare gains or cuts fully rely on the realization of market states. Hirshleifer (1971) attributes the losses of welfare by introducing more public information to the losses of risk-sharing opportunities. We will critically expound the downside of the “Hirshleifer effect” if the information quality and the returns to information are collectively considered. Rahi and Zigrand (2018) allow for the learning externalities and show that refraining information gathering can enhance welfare because in this case, agents can learn more from prices, given noisy private signals. Once we allow for the interdependence between valuations of different agents, we can see similar results. An earlier study by Stein (1987) is about the role of imputing speculators on destabilizing prices which causes welfare reduction. The speculators *de facto* materialize the noise trading. It is consistent with us since price destabilization means that price is more informative. A close paper is from Hu and Qin (2013) stating that more informed traders reduce *ex ante* utility. They *de facto* follow Laffont (1985) focusing on the fully revealed REE where the computation is more tractable. Complementary to these two papers that entail a fully revealing market as a prior, we generalize the model to develop the full picture of welfare patterns and allow for some partial revealing.

Last but not least, the main contributions lie in five. First, the proposed model is minimally deviated from the classic ones but derives meaningful implications about welfare. Second, this paper emphasizes the importance of information quality and returns on resolving Grossman and Stiglitz’s conflict, and in turn, also collectively driving the welfare effect of information acquisition. Third, this paper finds that the welfare effect can be either positive or negative; welfare improvements are rarely reported in the literature. Fourth, a full descriptive account of how the welfare effect responds to market-level risk fundamentals is developed. Fifth, the role of asymmetric information on the continuous welfare effect as information acquisition becomes more intensive is shown to become milder, in line with the strategic substitution assumption.

The rest of the paper is organized as follows: Section 2 will introduce the setting of the model, solve the REE and derive the cost cutoffs. In Section 3, the welfare will be calculated for both the discrete case and the continuous case, and based on the decomposition, the welfare patterns with respect to the fundamentals are expounded case by case and at the end, the role of asymmetric information is discussed by running some simulations. Section 4 concludes the paper.

2 The Model

Setup. The model is based on Grossman and Stiglitz (1980), which employs competitive rational expectation equilibria (REE) with information frictions as the primary setup. Similar to Kyle (1985)’s one-shot auctions, the model extends the static Grossman and Stiglitz’s model over an infinite period, although the market is still governed by demand schedules. Market states can vary from day to day as the fundamentals and informational states change over time due to informational shocks occurring in each period. Traders respond differently to varying market states, but optimally, as the evaluation of the benefits and costs of information acquisition and the market consequences on efficiency differ. As a result, welfare patterns are influenced by market states and become stylized in each fundamental.

The states of information are represented by $\mathbf{\Psi}_{i,t} = (\theta_t, s_t, \varepsilon_t)'$, where θ_t denotes the liquidation parameter that captures the depth of the market and can be interpreted as a measure of future asset returns, s_t corresponds to the signals received at period t, and ε_t captures the imprecision of the signals. The market risk fundamentals, denoted as $\mathbf{\Psi}_{m,t} = (\boldsymbol{\tau}'_t, \rho_t)'$, include $\boldsymbol{\tau}'_t = \{\tau_{s,t}, \tau_{\varepsilon,t}, \tau_{u,t}\}$ that captures the informativeness of the signals, the informational advantages and the returns to information, respectively, which explain the only sources of uncertainty in this economy, along with the CARA coefficient ρ_t. A high $\tau_{\varepsilon,t}$ disciplines precise signals by screening out much useless information, or even mistakes. Conversely, a low $\tau_{s,t}$ implies informative signals as they are drawn from a dispersed distribution containing much information. A low $\tau_{u,t}$ implies high returns to information, as noise trading introduces significant randomness into the market.

Assume that informational shocks are representative for all other market-level shocks within a trading day, and they are realized before becoming shocks to the fundamentals of the next period. As shown in Figure 1, standing at period t, the market states of period $t-1$ have been realized whereas the market states of period $t+1$ onwards are projected, expressed with a top dot. At period $t-1$, $\mathbf{\Upsilon}_{i,t-1}$ represents the settled informational shocks before approaching period t. Hence, the fundamentals at period t are the sum of the previous period's fundamentals and the aggregate informational shocks, denoted as, $\mathbf{\Psi}_{m,t-1}+\mathbf{\Upsilon}_{i,t-1}$, where the shocks $\mathbf{\Upsilon}_{i,t-1}$ are mutually independent, expressed as $(\upsilon_{s,t-1}, \upsilon_{u,t-1}, \upsilon_{\varepsilon,t-1}, \upsilon_{\rho,t-1})'$. Assume that the shocks $\mathbf{\Upsilon}_{i,t} \sim \boldsymbol{N}(\boldsymbol{\xi}, \mathbf{\Sigma}_{\boldsymbol{\upsilon}})$ where $\mathbf{\Sigma}_{\upsilon}$ is the variance-covariance matrix with all off-diagonal elements equaling zero. Hence, the fundamentals follow random walks with deterministic drifts $\boldsymbol{\xi}$ and assume that the realizations in these fundamentals are an order-1 Markov process. If the informational shocks have no real effect *i.e.* $\boldsymbol{\xi} = \mathbf{0}$, there is $\mathbb{E}(\mathbf{\Psi}_{m,t}|\mathbf{\Psi}_{m,t-1}) = \mathbf{\Psi}_{m,t-1}$. Standing at period $t-1$, $\mathbf{\Upsilon}_{t-1}$ are to be determined. Otherwise, if there is a positive $\boldsymbol{\xi}$, then $\mathbb{E}(\mathbf{\Psi}_{m,t}|\mathbf{\Psi}_{m,t-1}) = \mathbf{\Psi}_{m,t-1} + \boldsymbol{\xi}$. Once the market opens, all market participants have access to the predetermined $\mathbf{\Psi}_{m,t}$, which is common knowledge. As for $\mathbf{\Psi}_{i,t}$, θ_t and ε_t are actually realized at the end of period $t-1$ while s_t arrives at the current date. The question is whether to purchase s_t to predict $\dot{\theta}_{t+1}$ based on the fundamentals $\mathbf{\Psi}_{m,t}$, despite the imprecision of the signals $\dot{\varepsilon}_{t+1}$. In line with the assumptions made by Coibion and Gorodnichenko (2012) and Coibion and Gorodnichenko (2015), we assume that any current informational shocks cannot be realized until the end of each trading day. So the informational shocks $\dot{\mathbf{\Upsilon}}_{i,t-1}$ can only affect the current fundamentals $\mathbf{\Psi}_{m,t}$ and then further have an effect on the projection of $\dot{\mathbf{\Psi}}_{i,t+1}$, irrespective of $\dot{\mathbf{\Psi}}_{i,t}$. Hence, the informational shocks at period $t-1$ are information rigidities for projecting the informational states at period t. Traders are not able to utilize any new information before the role of new information on the fundamentals is all set. This phenomenon can be explained by the concept of inattentiveness presented in Reis (2006), where firms are unable to react to new information due to the fixed costs associated with attention. In this case, the most likely explanation for the information rigidities is the presence of significant adjustment costs that hinder the rapid dissemination of new information. So the beliefs are based on yesterday's situation $\mathbf{\Psi}_{m,t-1}$. For instance, if $\tau_{\varepsilon,t} \nearrow \infty$, this predicts $\dot{\varepsilon}_{t+1} \searrow 0$. Unless such extreme cases, $\dot{\varepsilon}_{t+1}$ can only be observed when

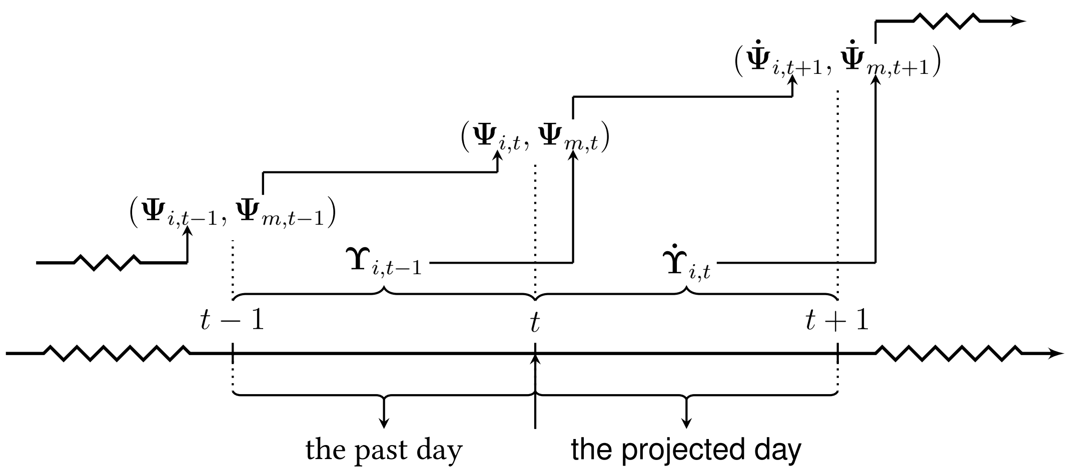


Figure 1: The trading timeline.

the t+1 period ends and $\dot{\theta}_{t+1}$ will be also known at the moment. $\dot{\theta}_{t+1} - \theta_t$ is the *ex post* returns to the asset but the more interesting perspective is the *ex ante* returns as the distributions of $\{s_t, u_t, \varepsilon_t\}$ are often non-degenerated. Although Veldkamp (2006) assumes that information about the next period's persistent payoff is revealed, apart from prices, we share the same assumption that the acquired information pertains to the current period, which can help predict the next period's payoffs. However, we differ in our approach, as we restrict instant information spreading.

Intraperiod Trader's Problem. The model adopts a CARA-Gaussian framework for each trading day in a market that includes one risky and one riskless asset. There is a continuum of risk-averse traders. Noise trading is necessarily incorporated for implementing an REE, but it is not studied, although it could be trading due to sentiments, hedging or liquidity demand. For each trader i, her CARA utility is determined by her demand schedules X_{it}, given the opening price p_t, the *ex post* closing price $\dot{\theta}_{t+1}$, CARA coefficient ρ_{it} and information price k_t.

$$U(X_{it}; p_t, \dot{\theta}_{t+1}, k_t) = -e^{-\rho_{it}[(\dot{\theta}_{t+1} - p_t)X_{it} - k_t]} \tag{1}$$

The variability of the CARA coefficient captures the diversity of attitudes toward risks. Traders evaluate the random variable $\dot{\theta}_{t+1}$ with respect to the risky asset, which represents market liquidity. When transactions result in a narrow price spread, the market tends to be deep and liquid. It follows a normal distribution $N(\bar{\theta}_{t+1}, \sigma_{\theta t+1}^2)$. There are three types of traders in the market: informed traders, uninformed traders, and noise traders. Among the continuum of traders, μ_t are informed and the remaining $1 - \mu_t$ are uninformed. To avoid the confusion of "schizophrenic" traders, here μ_t comprises of a large number of traders (each potentially infinitesimal in theory) such that each trader's behavior has no price impact, which is in the spirit of Hellwig (1980) and abstracts from Kyle (1989). Traders are considered informed once they pay a one-time cost of k_t for a signal s_{it}, while uninformed traders do not possess any signal. The behavior of noise traders is agnostic and the aggregate noise trading u_t is uncorrelated with any random variables. The signal s_{it} delivers some information about $\dot{\theta}_{t+1}$, albeit with measurement error $\dot{\varepsilon}_{i,t+1}$. Specifically, we have $\dot{\theta}_{t+1} = s_{it} + \dot{\varepsilon}_{it+1}$, where s_{it} and $\dot{\varepsilon}_{it+1}$ are mutually independent, and follow distributions $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon t}^2)$ and $s_{it} \sim N(\bar{\theta}_{t+1}, \sigma_{st}^2)$. Hence,

$\{\dot{\theta}_{t+1}, s_{it}, \dot{\varepsilon}_{it+1}\}$ are random variables such that first, $\dot{\theta}_{t+1}$ has the same mean as s_{it} because of $\int_0^{\mu_t} s_{it} di = \mu_t \dot{\theta}_{t+1}$ and $\int_0^{\mu_t} \dot{\varepsilon}_{it+1} di = 0$, and second, the variance of asset return is $\sigma_{\theta t+1}^2 = \sigma_{st}^2 + \sigma_{\varepsilon t+1}^2$. The information set of informed traders is $\mathcal{F}_I = \{s_t, p_t\}$, while the information set of uninformed traders and noise traders is $\mathcal{F}_U = \{p_t\}$. In general, the REE entails the following optimal demand of informed and uninformed traders and market clearance condition:

$$X_I(s_{it}, p_t) \in \arg\max_z \mathbb{E}_t[\dot{U}(z)|\mathcal{F}_I],\ i \in [0, \mu_t] \tag{2}$$

$$X_U(p_t) \in \arg\max_z \mathbb{E}_t[\dot{U}(z)|\mathcal{F}_U],\ j \in [\mu_t, 1] \tag{3}$$

$$\int_0^{\mu_t} X_I(s_{it}, p_t) di + \int_{\mu_t}^1 X_U(p_t) dj + u_t = 0 \tag{4}$$

To adhere to Grossman and Stiglitz (1980), it is assumed that all informed traders observe the same signal s_t, which makes s_t a sufficient statistic and p_t an excess information. This means that the heterogeneity in signals among informed traders is ignored, in contrast to the approach taken by Hellwig (1980) and Kyle (1989) (even though the latter assumes an identical distribution for the different signals). As a result, it holds that $\mathbb{E}_t(\dot{\theta}_{t+1}|s_t) = s_t$ and $\mathbb{V}_t(\dot{\theta}_{t+1}|s_t) = \sigma_{\varepsilon t}^2$.[1] The market clearance condition is then given by:

$$\mu_t X_I(s_t, p_t) + (1 - \mu_t) X_U(p_t) + u_t = 0 \tag{5}$$

in which the informed demand and uninformed demand are:

$$X_I(s_t, p_t) = a_t(s_t - p_t) \tag{6}$$

$$X_U(p_t) = \frac{\mathbb{E}_t[\dot{\theta}_{t+1}|\mathcal{F}_U] - p_t}{\rho_t \mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_U]} \tag{7}$$

where $a_t = \rho_t^{-1} \tau_{\varepsilon,t}$ denote risk-adjusted information advantage, where ρ_t^{-1} represents risk tolerance (Vives (2010)). The market prices contain as much information as the term $\omega_t = s_t + u_t/\mu_t a_t$. Following Vives (2010), the noise trading follows $N(-1, \sigma_{ut}^2)$, which implies an average supply of one share. When the distribution of noise trading collapses, say $\sigma_{ut}^2 \searrow 0$, the average supply becomes one share. The negative mean ensures that trades are still implementable, even when the distribution of noise trading collapses[2]. Assuming all traders are equally risk-averse, say $\rho_{It} = \rho_{Ut} = \rho_t$, the conditional expected utility for uninformed traders and informed traders are:

$$\mathbb{E}_t[\dot{U}(X_U(p_t))|\mathcal{F}_U] = -exp\left\{-\frac{(\mathbb{E}_t[\dot{\theta}_{t+1}|\mathcal{F}_U] - p_t)^2}{2\mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_U]}\right\} \tag{8}$$

[1] To explain this, note that $\mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_I) = \mathbb{E}_t(\dot{\theta}_{t+1}|s_t, s_t + \frac{u_t}{\mu_t a_t}) = \mathbb{E}_t(\dot{\theta}_{t+1}|s_t) = \mathbb{E}_t(s_t + \varepsilon_t|s_t) = s_t$. Similarly, $\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_I) = \mathbb{V}_t(\dot{\theta}_{t+1}|s_t, s_t + \frac{u_t}{\mu_t a_t}) = \mathbb{V}_t(\dot{\theta}_{t+1}|s_t) = \mathbb{V}_t(s_t + \varepsilon_t|s_t) = \sigma_{\varepsilon t}^2$. If s_t is known, then we know other than s_t, prices only contain noise. However, if only prices are known, then we cannot tease out the information in s_t from the noise. Hence, $\mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_I) \neq \mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_U)$ implies that signals contain more information than prices.

[2] In this case, informed traders and uninformed traders submit identical market orders since market prices fully reveal private information. The negative mean guarantees that the "noise" traders can still be the net suppliers of asset shares.

$$\mathbb{E}_t[\dot{U}(X_I(s_t,p_t))|\mathcal{F}_I] = -e^{\rho_t k_t} exp\left\{-\frac{(s_t-p_t)^2}{2\sigma_{\varepsilon t}^2}\right\} \quad (9)$$

In addition, if the transition to the uninformed information set solely involves considering the price, then the informed traders' conditional expected utility is[3]:

$$\mathbb{E}_t[\dot{U}(X_I(s_t,p_t))|\mathcal{F}_U] = e^{\rho_t k_t}\sqrt{\frac{\mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_I]}{\mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_U]}}(\mathbb{E}_t[\dot{U}(X_U(p_t))|\mathcal{F}_U]) \quad (10)$$

Equilibrium. The REE involves solving a Bayesian game with two stages. In the first stage, traders must decide whether to purchase a signal or not, based on their own information set and by maximizing their expected utility. In the second stage, an exogenous price k_t hits the market, resulting in $\mu_t^*(k_t)$ informed traders and $1-\mu_t^*(k_t)$ uninformed traders, where $\mu_t^*(k_t)$ represents the best response that balances the unconditional expected payoffs.

The REE can take on two different types, depending on whether $\mu_t^*(k_t)$ can balance the payoffs. If it can, then an interior equilibrium is reached, meaning that $EU_U(\mu_t^*(k_t)) = EU_I(\mu_t^*(k_t))$, where $EU_I(\mu_t^*(k_t)) = \mathbb{E}_t[\dot{U}(X_I(s_t,p_t))]$ and $EU_U(\mu_t^*(k_t)) = \mathbb{E}_t[\dot{U}(X_U(p_t))]$. However, if either $\mu_t^*(k_t) = 0$ or $\mu_t^*(k_t) = 1$, then the equilibrium is at a corner. In a strict corner equilibrium, $EU_U(0) > EU_I(0)$ and $EU_U(1) < EU_I(1)$. At weak corner equilibria, the payoffs are also at break-even. The asymmetric information problem is resolved at any corner equilibria, as all traders base their behavior on an identical information set in every state. However, if $0 < \mu_t^*(k_t) < 1$ is reached, information asymmetry emerges. In any case, information frictions remain, as the signals do not directly indicate future returns, but are instead amplified by their own randomness σ_{st} and by mistakes $\sigma_{\varepsilon t}$, and also affected by market randomness σ_{ut}. Even in the absence of noise trading, signals still do not equal future returns due to their own variability and imprecision. As a result, the REE does not imply social optimality, nor does social optimality imply the REE.

Now let's examine the borderline equilibrium, where the first and last marginal trader are indifferent between being informed and uninformed. In this case, the ratio of the unconditional expected utility of informed traders to that of uninformed traders is:

$$\frac{EU_I[\mu_t^*(k_t)]}{EU_U[\mu_t^*(k_t)]} = \phi[\mu_t^*(k_t)] = e^{\rho_t k_t}\sqrt{\frac{\tau_{\varepsilon,t}^{-1}}{\{\tau_{s,t} + [\mu_t^*(k_t)a_t]^2\,\tau_{u,t}\}^{-1} + \tau_{\varepsilon,t}^{-1}}} \quad (11)$$

The equilibrium decision rule can be expressed as $\phi(\mu_t^*(k_t)) = 1$, where k_t and μ_t are inversely related. This relationship can be justified by the fact that a lower cost attracts more people to buy the signal. This inverse relationship can be interpreted, from a pecuniary motivation perspective, as the reason for the convex welfare cuts that arise from purchasing the information. In terms of welfare analysis, as μ_t changes, the exogenous cost is observationally equivalent to the endogenous information price in Veldkamp (2006), assuming strategic substitution. Here, the border equilibrium is defined for determining the cost thresholds.

[3]See Vives (2010) Section 4.6 Appendix for the proof.

Definition 1. Borderline Corner Equilibrium

- ▷ Lower Corner Equilibrium (hereafter, LCE): the corner equilibrium satisfies $\phi(0) = 1$. The second-stage best response of $\mu_t^*(k_t)$ is 0, and the net benefits of purchasing signals break even the net benefits of not purchasing signals simultaneously.
- ▷ Higher Corner Equilibrium (hereafter, HCE): the corner equilibrium satisfies $\phi(1) = 1$. The second-stage best response of $\mu_t^*(k_t)$ is 1, and the net benefits of purchasing signals break even the net benefits of not purchasing signals simultaneously.

Cost Thresholds. $\phi[\mu_t^*(k_t)]$ illustrates the relationship between two dimensions, where an increase in $\mu_t^*(k_t)$ leads to an increase in the function, while an increase in k_t results in a shift upwards. At a borderline equilibrium, $\phi(0) = 1$ or $\phi(1) = 1$ is prominent, Thus, the costs can be expressed as a function of the fundamentals and the CARA coefficient, denoted as $k(\boldsymbol{\tau}_t'; \rho_t)$. The costs exhibit a clear dichotomy between a high cutoff, $k^H(\boldsymbol{\tau}_t'; \rho_t)$ and a low cutoff $k^L(\boldsymbol{\tau}_t'; \rho_t)$. If $\phi(0) = 1$, it implies that traders remain uninformed when the cost, k_t, is high enough, traders are still all uninformed. However, they feel indifferent about purchasing or not purchasing the signals. On the other hand, if $\phi(1) = 1$, it means that all traders have purchased the signals when the cost is low enough, despite feeling indifferent. In both cases, traders benefit equally. The cutoffs can be determined using the following two equations:

$$e^{\rho_t k^H(\boldsymbol{\tau}_t';\rho_t)}\sqrt{\frac{\tau_{\varepsilon,t}^{-1}}{\tau_{s,t}^{-1}+\tau_{\varepsilon,t}^{-1}}} = 1 \Rightarrow k^H(\boldsymbol{\tau}_t';\rho_t) = \frac{1}{2\rho_t}\ln\left(\frac{\tau_{\varepsilon,t}}{\tau_{s,t}}+1\right) \tag{12}$$

$$e^{\rho_t k^L(\boldsymbol{\tau}_t';\rho_t)}\sqrt{\frac{\tau_{\varepsilon,t}^{-1}}{(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}}} = 1 \Rightarrow k^L(\boldsymbol{\tau}_t';\rho_t) = \frac{1}{2\rho_t}\ln\left(\frac{\tau_{\varepsilon,t}}{\tau_{s,t}+\frac{\tau_{\varepsilon,t}^2\tau_{u,t}}{\rho_t^2}}+1\right) \tag{13}$$

Hence, the best response of $\mu_t^*(k_t)$ can be written like:

$$\mu_t^*(k_t) = \begin{cases} 0 & \text{, if } k_t \geqslant k_t^H \\ a_t^{-1}\sqrt{\frac{(e^{2\rho_t k_t}-1)^{-1}\tau_{\varepsilon,t}-\tau_{s,t}}{\tau_{u,t}}} & \text{, if } k_t^L < k < k_t^H \\ 1 & \text{, if } k_t \leqslant k_t^L \end{cases} \tag{14}$$

When the market fundamentals change, the cost cutoffs for achieving equilibrium also change. The two following remarks discuss the impact of each market-level fundamental in determining the cost cutoffs.

Remark 1.1 $k^L(\boldsymbol{\tau}_t'; \rho_t)$ is monotonically decreasing in $\tau_{s,t}$ and $\tau_{u,t}$ while it is not monotonic in $\tau_{\varepsilon,t}$ and ρ_t. $k^H(\boldsymbol{\tau}_t'; \rho_t)$ is decreasing in ρ_t and $\tau_{s,t}$, increasing in $\tau_{\varepsilon,t}$, and independent of $\tau_{u,t}$.

The relationship between $k^H(\boldsymbol{\tau}_t'; \rho_t)$ and ρ_t is negative, as more risk-averse traders prefer to start buying information at a lower initial price. Then it increases in $\tau_{\varepsilon,t}$ and decreases with $\tau_{s,t}$ because high $\tau_{\varepsilon,t}$ and low $\tau_{s,t}$ indicate that the information is of high quality and informative, leading to expensive prices. One more difference between these two cost cutoffs lies in $\tau_{u,t}$. Specifically, $k^H(\boldsymbol{\tau}'_t;\rho_t)$ is not affected by $\tau_{u,t}$, while $k^L(\boldsymbol{\tau}'_t;\rho_t)$ is indeed affected by it. This is primarily due to the fact that at $k^H(\boldsymbol{\tau}'_t;\rho_t)$, market prices simply follow a mean-preserving pattern[4] and there is no information revealed and neither is there learning information from the market prices, and thus the signals should not be marked up. When $\mu^*(k_t)=0$, the channels of increasing price informativeness via $\tau_{u,t}$ and a_t are cut off. However, $k^L(\boldsymbol{\tau}'_t;\rho_t)$ should decrease in the returns to information since $\mu^*(k_t)=1$ amplifies the channels of $\tau_{u,t}$ and a_t the most. As $\tau_{s,t}$ rises, the signals are charged flatter due to the same reason as previously. $k^L(\boldsymbol{\tau}'_t;\rho_t)$ is not monotonic in $\tau_{\varepsilon,t}$ and ρ_t because of the potential counter forces. On one hand, $\tau_{\varepsilon,t}$ increases the price informativeness, but on the other hand, ρ_t cuts it. Traders learn more information from more informative prices, which as a substitution, dampens the motivation to purchase the information. With less demand, the signals are marked down. However, high informational advantages should be sold expensively. When facing less informative prices, traders' demand for signals rises, thereby raising the information price. However, more risk-averse traders want cheaper information. Thus, the non-monotonicity arises due to the competing forces. The learning behavior is the underlying reason for the non-monotonicity. The intensity of learning is assessed in $(\tau_{s,t}+\mu_t a_t^2\tau_{u,t})^{-1}$, which is actually $\mathbb{V}_t(s_t|p_t)$. It measures the impreciseness of inferring the signals without purchasing, based solely on market prices.

■ **Remark 1.2** The learning intensity is strengthened in μ_t, $\tau_{s,t}$, $\tau_{\varepsilon,t}$, and $\tau_{u,t}$ and weakened in ρ_t. (i) $\tau_{s,t}\nearrow\infty$ directly causes $\mathbb{V}_t(\dot{\theta}_{t+1}|p_t)=\mathbb{V}_t(\dot{\theta}_{t+1}|s_t)$ and $\dot{\theta}_{t+1}\sim N(s_t,\sigma_{\varepsilon,t})$, irrespective of μ_t. (ii) For $\mu_t>0$, if either $\tau_{u,t}\nearrow\infty$, $\tau_{\varepsilon,t}\nearrow\infty$, or $\rho_t\searrow 0$, the same situation is reached.

Remark 1.2 emphasizes two methods of rendering information acquisition futile, both of which result in perfect learning. This futility arises from the cost cutoffs that are inhibited to zero (Remark 1.3). To address this issue, the model assumes imperfect learning. Conversely, if any of the parameters $\tau_{s,t}\searrow 0$, $\tau_{u,t}\searrow 0$, $\tau_{\varepsilon,t}\searrow 0$, or $\rho_t\nearrow\infty$, information outsiders are unable to glean anything from signals that are excluded from market prices. Consequently, learning becomes ineffective.

■ **Remark 1.3** $k^L(\boldsymbol{\tau}'_t;\rho_t)$ has the following limits:

$$\lim_{\rho_t\nearrow\infty}k^L(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\rho_t\searrow 0}k^L(\boldsymbol{\tau}'_t;\rho_t)=0$$

$$\lim_{\tau_{\varepsilon,t}\nearrow\infty}k^L(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\tau_{\varepsilon,t}\searrow 0}k^L(\boldsymbol{\tau}'_t;\rho_t)=0$$

$$\lim_{\tau_{s,t}\nearrow\infty}k^L(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\tau_{s,t}\searrow 0}k^L(\boldsymbol{\tau}'_t;\rho_t)=\frac{1}{2\rho_t}\ln\left(\frac{\rho_t^2}{\tau_{\varepsilon,t}\tau_{u,t}}+1\right)$$

$$\lim_{\tau_{u,t}\nearrow\infty}k^L(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\tau_{u,t}\searrow 0}k^L(\boldsymbol{\tau}'_t;\rho_t)=k^H(\boldsymbol{\tau}'_t;\rho_t)$$

[4]Recall that at the LCE, prices equal to $\bar{\theta}_{t+1}+\rho_t\sigma_{\theta t+1}^2u_t$, meandering around $\bar{\theta}_{t+1}-\rho_t\sigma_{\theta t+1}^2$ with a dispersion of $\rho_t^2\sigma_{\theta t+1}^4\sigma_{u t}^2$. In general, market prices contain the amount of information as much as $s_t+u_t/\mu_t a_t$. When $\mu_t\searrow 0$, the randomness from u_t will be enlarged significantly so market prices are just random. Market clearance implies that $X_U(p_t)+u_t=0$, which further implies that prices can be expressed by a function of u_t, indicating that they are simply random.

With respect to $k^H(\boldsymbol{\tau}'_t;\rho_t)$, the limits are:

$$\lim_{\rho_t\nearrow\infty} k^H(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\rho_t\searrow 0} k^H(\boldsymbol{\tau}'_t;\rho_t)=\infty$$
$$\lim_{\tau_{\varepsilon,t}\nearrow\infty} k^H(\boldsymbol{\tau}'_t;\rho_t)=\infty \text{ and } \lim_{\tau_{\varepsilon,t}\searrow 0} k^H(\boldsymbol{\tau}'_t;\rho_t)=0$$
$$\lim_{\tau_{s,t}\nearrow\infty} k^H(\boldsymbol{\tau}'_t;\rho_t)=0 \text{ and } \lim_{\tau_{s,t}\searrow 0} k^H(\boldsymbol{\tau}'_t;\rho_t)=\infty$$

When individuals are extremely risk-averse, $k^L(\boldsymbol{\tau}'_t,\rho_t)$ becomes zero because the high demand resulting from low informativeness is dominated by the need for low prices due to excessive risk aversion. However, the hyper informative signals cannot be read from prices and the limit of the cost is then positive. As the returns to information rise, the maximum $k^L(\boldsymbol{\tau}'_t,\rho_t)$ cannot exceed $k^H(\boldsymbol{\tau}'_t,\rho_t)$. With respect to $k^H(\boldsymbol{\tau}'_t,\rho_t)$, the limits all square with the monotone relationship between it and each fundamental.

For the strict corner equilibria, Equation (11) can still be used for deriving the break-even cost cutoffs. To do so, assume that there exists a positive state-contingent number φ^s_t such that $\phi(0)=1+\varphi^H_t$ for any $k(\boldsymbol{\tau}'_t,\rho_t)>k^H(\boldsymbol{\tau}'_t,\rho_t)$ and $\phi(1)=1-\varphi^L_t$ for any $k(\boldsymbol{\tau}'_t,\rho_t)<k^L(\boldsymbol{\tau}'_t,\rho_t)$. Hence, given the fundamental parameters, the break-even cost cutoffs $k^\dagger(\boldsymbol{\tau}'_t;\rho_t)$ can still be expressed by using φ which measures the distance away from the cost cutoffs. Hence, the admissible sets are $\varphi^H_t\in\mathbb{R}_+$ and $\varphi^L_t\in\left(0,1-\sqrt{\frac{\tau^{-1}_{\varepsilon,t}}{(\tau_{s,t}+a^2_t\tau_{u,t})^{-1}+\tau^{-1}_{\varepsilon,t}}}\right]$. In effect, there are some approaches to filling the wedges in φ^H_t and φ^L_t by possibly adjusting the quadruple $\{\boldsymbol{\tau}'_t,\rho_t\}$ and consequently, the market goes back to the HCE and the LCE, respectively. In line with Remark 1.3, the wedge φ^H_t can be filled if $\tau_{\varepsilon,t}$ is high while $\tau_{s,t}$ and ρ_t are low. The wedge φ^L_t can be filled by raising $\tau_{u,t}$ or $\tau_{s,t}$, even though the way in adjusting ρ_t and $\tau_{\varepsilon,t}$ is not monotonic. Hence, the focus on the borderline corner equilibrium does not lose much generality and as will be seen later, the welfare patterns are unambiguous for strict corner equilibria.

$$k^\dagger(\boldsymbol{\tau}'_t,\rho_t)=\begin{cases}\frac{1}{2\rho_t}\ln\left[(1+\varphi^H_t)^2\left(\frac{\tau_{\varepsilon,t}}{\tau_{s,t}}+1\right)\right], & \text{if } k^\dagger>k^H_t\\ \frac{1}{2\rho_t}\ln\left[(1-\varphi^L_t)^2\left(\frac{\tau_{\varepsilon,t}}{\tau_{s,t}+\frac{\tau^2_{\varepsilon,t}\tau_{u,t}}{\rho^2_t}}+1\right)\right], & \text{if } 0\leqslant k^\dagger<k^L_t\end{cases} \tag{15}$$

3 Welfare

Setup. The *ex ante* welfare will be expounded from a more tractable but less practical case (a discrete case) to a more practical but less tractable case (a continuous case). It represents the expected welfare before market traders finish submitting their trading orders, or alternatively, it is the conjectured welfare based on the rational behavior of different types of traders possessing different information. The focus on *ex ante* welfare is canonical in the literature [Amador and Weill (2010), Colombo et al. (2014), Hendren (2021), Hu and Qin (2013), Morris and Shin (2002), Rahi and Zigrand (2018)]. This section will address several key questions, including: (i) the factors that explain the welfare effects of information acquisition, (ii) the welfare patterns that vary across each fundamental, and (iii) the role of asymmetric information in shaping these welfare patterns.

The conjectured welfare patterns depend upon some conflicting forces in the battlefield, as the impact of welfare is not linear in the acquisition of information. Thus, the crux of the paper goes to decompose the aggregate welfare and analyze the specific factors that are actually driving the total welfare effect. Each of these factors offers an avenue for explanation and the resulting welfare patterns are dependent on how these individual factors conflict or complement each other, based on their reactions to varying market conditions. Additionally, the paper places more emphasis on comparing welfare between boundary equilibria. First, this is because the calculation is less cumbersome and the decomposition can only be possibly more tractable once we abstract from information asymmetry. Second, when the boundary equilibrium is reached, two different equilibrium conditions are met simultaneously and the decision rules do follow the case with asymmetric information. Therefore, focusing on the discrete case helps to isolate the individual welfare effects that should also apply to the continuous case. But again, the inclusion of asymmetric information could contaminate the welfare patterns under complete information, as an extra effect but in a disciplined fashion.

First of all, consider a Bergsonian welfare function $\mathscr{W}(\mu^*(k_t);\boldsymbol{\tau}'_t,\rho_t)$ that captures a representative trader's welfare[5](Bergson (1938); Chavas, Menon, Pagani, and Perali (2018)):

$$\mathscr{W}(\mu^*(k_t);\boldsymbol{\tau}'_t,\rho_t)=\int_0^1 \mathbb{E}_t\{\mathbb{E}_t[\dot{U}(X_i(\mathcal{F}_i)|\mathcal{F}_U)]\}di$$

$$=\int_0^{\mu^*(k_t)} \mathbb{E}_t\{\mathbb{E}_t[\dot{U}(X_i(\mathcal{F}_I))|\mathcal{F}_U]\}di+\int_{\mu^*(k_t)}^1 \mathbb{E}_t\{\mathbb{E}_t[\dot{U}(X_i(\mathcal{F}_U))|\mathcal{F}_U]\}di \tag{16}$$

Due to the homogeneity in each type of traders, the welfare function can be simply rewritten as $\sum_i^{\mathcal{I}}\mu'_{it}(k_t)\mathbb{E}_tU_i$. The Bergsonian welfare function is weighted against the unconditional expected utility and $\mu'_{it}(k_t)$ is welfare weights satisfying $\partial\mathscr{W}(\mu^*(k_t);\boldsymbol{\tau}'_t,\rho_t)/\partial U_i=\mu'_{it}(k_t)$. The weights are normalized as ${\mu_I}'_t(k_t)=\mu^*_t(k_t)$ for informed traders, ${\mu_U}'_t(k_t)=1-\mu^*_t(k_t)$ for uninformed traders, and ${\mu_u}'_t(k_t)=1$ for noise traders where $\mu^*_t(k_t)$ is the actual best response function of the REE.

Noise traders. While noise traders have real consequences on welfare, the welfare function abstracts from these consequences for a couple of reasons[6]. First, the

[5]Here is a mini literature review with respect to measuring the welfare. Vives (2014) and Vives (2010) capture welfare in the expected total surplus. In Grossman and Stiglitz (1980)'s setup, some traders are asset buyers while some other traders (say, noise traders) are asset sellers in the economy, so the total surplus should be the addition of the surplus for these buyers and sellers. Any unexplained changes in the total surplus due to information disclosure are the sources of deadweight losses. Stein (1987) defines social welfare as the summation of expected consumer surplus and expected speculator's utility. Rahi and Zigrand (2018) assume that traders' objective functions are their welfare. Morris and Shin (2002) capture the social welfare by averaging individual utilities. The welfare measured in Amador and Weill (2010) and Colombo et al. (2014) is the *ex ante* expected utility of a representative agent. In a word, the definition of welfare is standard in the literature, which captures the spirit of the Bergsonian.

[6]The expected utility of noise traders can be calculated directly as $EU_u=-exp\{\rho_t(\bar{\theta}_{t+1}-p_t)+[\rho_t^2(\bar{\theta}_{t+1}-p_t)^2{\sigma_u}_t^2]/2\}$. As can be easily seen, the expected utility still depends on market prices. So the distribution of noise traders' welfare is not fixed.

problem of noise traders is not studied, as Goldstein and Yang (2019) preclude the welfare analysis of noise traders, since noise traders achieve various purposes, such as hedging or purely sentimental reasons, and are exogenous to the model, which goes against the main purpose of this paper that focuses on the trade-off between benefits and costs based on a typical REE setup, where everyone should have well-defined behaviors. To eschew these complications, we assume that noise traders only have random liquidity demand to resolve the Grossman-Stiglitz paradox. Hence, the welfare of this paper only captures a subset of the economy composed of utility-maximizing traders only, rather than social welfare. In contrast, García and Strobl (2010) cannot ignore noise traders because they affect the average wealth in the economy and therefore, the relative wealth, as a key choice variable of their concern. Second, there are no welfare externalities among traders so the welfare of noise traders is independent of the welfare analysis based on strictly well-defined behaviors. The role of noise trading on non-noisy traders, however, should be unambiguous. The noise traders sever the perfect correlation between the signals and market prices and therefore, affect the welfare of informed and uninformed traders. Rather, the welfare of noise traders does not affect how they work on the market. Hence, noise traders are important for maintaining the operations of the market whereas their welfare does not align with the main interests of this paper.

3.1 Discrete Case

3.1.1 Welfare Representation

The discrete case captures the welfare transition from $\mu_t^*(k_t) = 0$ to $\mu_t^*(k_t) = 1$ in cases where information is complete and there is only one type of trader in each state. First, for any LCE where $\phi(\mu_t^*(k_t)) \geqslant 1$[7], the uninformed demand becomes $X_U(p_t) = (\bar{\theta}_{t+1} - p_t)/\rho_t\sigma_{\theta_t}^2$ because the market clearing condition entails $X_U(p_t) = -u_t$. This is a special state in which no one attempts to disclose information, resulting in market prices that are as random as noise trading, and traders cannot gain any information from prices. Hence, market prices can be written as $\mathscr{P}_t(u_t) = \bar{\theta}_{t+1} + \rho_t\sigma_{\theta_{t+1}}^2 u_t$. Assuming $\mathbb{E}_t[X_U(p_t)] = 1$, the expected profits $\bar{\theta}_{t+1} - \mathbb{E}_t[\mathscr{P}_t(u_t)]$ are therefore $\rho_t(\tau_{s,t}^{-1} + \tau_{\varepsilon,t}^{-1})$. In the LCE, uninformed traders are uncertain about random transactions made by noise traders and can earn higher profits if they are more risk-averse or if the capital market is less liquid, meaning that traders have coarser information about private information ($\tau_{s,t} \downarrow$ implies that the signals are informative) and have less informational advantages ($\tau_{\varepsilon,t} \downarrow$). If the evaluation on $\dot{\theta}_{t+1}$ is perfectly precise ($\tau_{\theta_{t+1}}^{-1} \searrow 0$), or if traders become risk neutral ($\rho_t \searrow 0$), their expected profits shrink to zero. In such cases, purchasing signals is equivalent to knowing future returns, causing all uninformed traders to act in the same manner immediately, resulting in a perfectly liquid market without any profitable opportunities. This zero-sum game is also possibly because risk-neutral traders can preclude themselves from being disturbed by the noise trading completely. The borderline corner equilibrium facing no information disclosure yields the following wel-

[7] This directly means that $EU_I \leqslant EU_U$ since the CARA utility has an upper bound of zero. All traders feel better to stay uninformed. The same interpretation also works for the HCE.

fare.

Lemma 1. *The conditional expected utility of uninformed traders at the LCE can be shown equal to* $\mathbb{E}_t[\dot{U}_U(X_U(p_t))|\mathscr{P}_t(u_t)] = -exp\left[-\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})u_t^2/2a_t^2\tau_{s,t}\right]$. *Denote* $\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{lce}$ *as the representative welfare of the LCE that equals to*

$$\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{lce} = -\frac{exp\left[-\frac{1}{2}\frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}\right]}{\sqrt{1+\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}}} \tag{17}$$

The proof is in the Appendix Page 44.

Second, for any HCE where $\phi(\mu_t^*(k_t)) \leqslant 1$, the informed demand becomes $X_I(s_t,p_t) = (s_t - p_t)/\rho_t\sigma_{\varepsilon t}^2$, and further becomes $-u_t$ if market is clear. This is also a special case where market prices now reflect public information related to signals beyond noise trading. As a result, market prices can be written as $\mathscr{P}_t(s_t,u_t) = s_t + u_t/a_t$. The discrepancy between market prices and the signals (the error of inferring signals from prices) fully rests on the uncertainty from the noise trading. The conditional expected profit $\mathbb{E}_t(\dot{\theta}_{t+1}|\mathscr{P}_t(s_t,u_t)) - \mathscr{P}_t(s_t,u_t)$ as per Equation (26) also varies in s_t and u_t, considering the underlying fundamentals $(\boldsymbol{\tau}'_t;\rho_t)$. However, the unconditional expected profit $\bar{\theta}_{t+1} - \mathbb{E}_t[\mathscr{P}_t(s_t,u_t)]$, equals $\rho_t\tau_{\varepsilon,t}^{-1}$. Compared to the corresponding moment in the LCE, it is reduced since the role of s_t in predicting $\dot{\theta}_{t+1}$ has been captured by current prices, but not *perfectly*. Likewise, if the signals are perfectly precise ($\tau_{\varepsilon,t}^{-1} \searrow 0$) or traders become risk-neutral, the *imperfection* is resolved completely and the market becomes perfectly liquid. The borderline corner equilibrium facing full information disclosure yields the following welfare.

Lemma 2. *The conditional expected utility of uninformed traders* $\mathbb{E}_t[\dot{U}_U(X_U(p_t)|\mathscr{P}_t(s_t,u_t)]$ *now becomes*

$$-exp\left\{-\frac{\left\{\frac{\left[\tau_{s,t}\left(\bar{\theta}_{t+1}-s_t-\frac{u_t}{a_t}\right)+a_t\tau_{u,t}\right]}{(a_t^2\tau_{u,t}+\tau_{s,t})}\right\}^2}{2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}\right\}$$

which is equal to the expected utility of the informed traders conditional on prices

$$\mathbb{E}_t[\dot{U}_I(X_I(s_t,p_t))|\mathscr{P}_t(s_t,u_t)]$$

, given that $e^{\rho_t k_t^L}\sqrt{\mathbb{V}_t[\dot{\theta}_{t+1}|s_t]/\mathbb{V}_t[\dot{\theta}_{t+1}|\mathscr{P}_t(s_t,u_t)]} = 1$ *in equilibrium. Define* $\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$ *as the representative welfare of the HCE which is*

$$\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{hce} = -\frac{exp\left\{-\frac{1}{2}\frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}}\right\}}{\sqrt{1+\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}}} \tag{18}$$

The proof is in the Appendix Page 44.

For the ease of computation, the welfare functions can be decomposed by applying some simple approximation. The welfare functions at the corners can be rewritten as follows.

Lemma 3. *Assume that* $\ln\left[\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})\right] / \left[a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})\right] \approx 1$ *and* $\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})/a_t^2\tau_{s,t}\tau_{u,t}$ *is fairly large. The welfare function at the LCE and the welfare function at the HCE can be approximately written as:*

$$\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{lce}\right] \propto -\frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})} - \ln\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}} - \kappa_l \tag{19}$$

$$\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{hce}\right] \propto -\frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}} - \ln\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})} - \kappa_h \tag{20}$$

κ_l and κ_h are defined in Lemma 4. The proof is in the Appendix Page 45.

Definition 2. Define the following functions that are representative for each of the decomposed terms after the designated linearization and approximation:

$$\begin{aligned}
\mathscr{D}_{lce}(\boldsymbol{\tau}'_t;\rho_t) &= \frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}\\
\mathscr{D}_{hce}(\boldsymbol{\tau}'_t;\rho_t) &= \frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}\\
\mathscr{N}_{lce}(\boldsymbol{\tau}'_t;\rho_t) &= \frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}\\
\mathscr{N}_{hce}(\boldsymbol{\tau}'_t;\rho_t) &= \frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}}
\end{aligned}$$

In general, a higher $\mathscr{D}$ tends to increase the welfare and a higher $\mathscr{N}$ also tends to increase the welfare. Hence, the logarithm linearized welfare function at the LCE and the HCE can be denoted as $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce}$ and $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$ where $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce} = -\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{lce}\right]$ and $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce} = -\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{hce}\right]$.

Definition 3. Define the welfare effect as $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ such that

$$\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \left[\ln\frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})} - \ln\frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\epsilon,t}}\right] + \left[\ln\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}} - \ln\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}\right]$$

where the first term can be approximated as $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and the second term can be approximated as $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$.

Note that $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < 0$ represents welfare improvements due to information disclosure, while $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ represents welfare cuts[8]. However, the above analysis is subject to some measurement errors arising from the approximating process. To address this issue, we separate the IP effect from the measurement errors since the already-derived IP effect is meaningful. Our results are robust to these measurement errors, as the patterns of welfare gains and losses are almost identical to the case where the measurement errors are negligible. The only differences are the changes in cutoff values and the switch in dominance in explaining the welfare losses, which will be illustrated in the following graphs.

[8] It is worth to emphasize that the welfare change is henceforth calculated from the HCE to the LCE. So information disclosure can cause welfare gains if the welfare effect is negative. The negative value means that the welfare at HCE is higher than it is at LCE. The case of welfare cuts follows the same logic.

■ **Lemma 4.** The measurement errors of the IP effect is:

$$\Delta\kappa = \kappa_l - \kappa_h = \frac{\mathscr{D}_{hce} - \mathscr{D}_{lce}}{\mathscr{D}_{hce}(1 + \mathscr{D}_{hce})} \tag{21}$$

The measurement errors of the IE effect is zero:

$$\psi = 0 \tag{22}$$

The proof is in the Appendix Page 46.

Information quality is an important feature that affects both effects, thereby working on the overall welfare patterns. The definition of information quality is compared to the definition of price informativeness proposed by Dávila and Parlatore (2021) and Grossman and Stiglitz (1980). The former defines the absolute price informativeness following Vives (2010) that is $\mathbb{V}_t(s_t|p_t)^{-1} = \tau_{s,t} + (\mu_t a_t)^2 \tau_{u,t}$ and meanwhile the latter defines the relative price informativeness that is $(1+m_t)^{-1}$ where $m_t = \rho_t^2 \tau_{s,t} / \mu_t^2 \tau_{\varepsilon,t}^2 \tau_{u,t}$. Likewise, the analog of the absolute information quality $\mathbb{V}_t(\dot{\theta}_{t+1}|s_t) = \tau_{\varepsilon,t}$ versus the relative information quality $\tau_{\varepsilon,t}/(\tau_{\varepsilon,t} + \tau_{s,t})$ is defined. Denote $n_t = \tau_{\varepsilon,t}/\tau_{s,t}$ as the proxy of the relative information quality that can therefore, be written as $n_t/(1 + n_t)$, denoted as $\mathcal{Q}_t$. The relative information quality contains an intensive margin governed by $\tau_{\varepsilon,t}$ and an extensive margin governed by $\tau_{s,t}$. Hence, to improve it, we can directly improve the absolute information quality, say this *ceteris peribus* entails a high $\tau_{\varepsilon,t}$. Alternatively, a lower $\tau_{s,t}$ can also be, all else constant, an option, which means that the signals are more informative since they are drawn from a more dispersed distribution.

More importantly, the change in term $\mathscr{D}$ can be attributed to an intensive effect governing the behavior of IP, while the change in term $\mathscr{N}$ is, however, associated with an extensive effect governing the importance of both the returns to information and the information quality, amid the process of IE. The first intuition is from plugging the factor of the pecuniary cost factor $e^{2\rho_t k_t^L}$ into the change in $\mathscr{D}$ for teasing out the IP effect. The spending on acquiring the information can have a negative impact on welfare, which needs to be adjusted from actual pecuniary cost to welfare effect measured in utils. The adjustments are contingent on the fundamentals reflecting day-to-day fluctuations of the risks, which can arise from the information and market sides. Revealing high-quality and informative information *ceteris paribus* raises the risks because of high losses of market efficiency. Also, when information disclosure is done in a less random market, the problem will be exacerbated by amplifying the information-side impact. When people are more risk-averse, the risks caused by the information side tend to be dampened at the micro level.

The second intuition is rooted in an evaluation process about what information is disclosed to have welfare consequences. It is based on the benefits and costs of utilizing the information. The costs, in this vein, are only the efficiency losses due to information revealing, given that the IP effect has already separated the effect of pecuniary costs. Meanwhile, the benefits are the potential returns that traders can earn by utilizing the information. Traders can realize greater profits by utilizing market information in two situations. The first is when the market is more random due to more dispersed noise trading, which puts outsiders at a disadvantage and makes it difficult for them to infer the information from market prices. The second is when the relative information quality is high. However, these two cases have different implications for welfare losses. The efficiency losses tend to be positively related to the degree of the relative information quality. In the first scenario, where low-quality information is revealed in a highly random market, the benefits of utilizing the information may not come at too great a cost. In contrast, the second scenario, where benefits are derived from high-quality information, is likely to be more painful because the benefits are driven by the information side. Hence, this evaluation process considers a dynamic balancing between benefits and costs and there are multiple possibilities for the welfare consequences.

■ **Proposition 1. Welfare in the Capital Market with Complete Information**

The representative welfare can be decomposed into two effects: $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ reflects the IP effect and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ reflects the IE effect:

$$\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \underbrace{n_t\left(1+n_t\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right)e^{2\rho_t k_t^L}}_{\zeta_1(\boldsymbol{\tau}'_t;\rho_t)} + \underbrace{n_t\left(1+\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right)}_{\zeta_0(\boldsymbol{\tau}'_t;\rho_t)} \tag{23}$$

$$\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \frac{1}{1+\frac{\rho_t^2}{\tau_{\varepsilon,t}(1-\mathcal{Q}_t)}}\left[\frac{\tau_{u,t}}{1-\mathcal{Q}_t}-1\right] \tag{24}$$

Evidently, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ implies undoubted welfare losses so the overall welfare patterns depend on $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$.

- ▷ **Claim 1.1** *When $\tau_{u,t}+\mathcal{Q}_t \geqslant 1$, a welfare cut happens to both effects, thereby seeing a strictly overall welfare cut caused by information disclosure.*
- ▷ **Claim 1.2** *When $\tau_{u,t}+\mathcal{Q}_t < 1$, the IE effect offers welfare improvements, which struggles with the IP effect in the battlefield: (i) if $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$, then the former effect dominates, thereby causing an overall welfare cut, say $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce} > \widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$; (ii) if $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$, then the latter effect dominates, thereby causing an overall welfare improvement, say $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce} < \widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$; (iii) if $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$, then the two effects break even, thereby causing an unchanged welfare, say $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce} = \widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$.*

The proofs are in the Appendix Page 47.

The above discussion has been nothing more than the welfare of the borderline corner equilibria. However, what if is the feed of the information price higher than the high-cost cutoff or lower than the low-cost cutoff? In both scenarios, the IE effect is unaffected, as k_t is exogenous. In the first case, the welfare remains constant as it is not impacted by a signal markup. The second case is, however, different as further reductions in cost are expected to improve welfare. But the welfare improvement due to the low cost cannot counteract the IP effect. Even with free signals being revealed, informational impact still results in a loss of welfare. The only difference is that the size of the IP effect is reduced due to zero pecuniary burden, making it easier for the welfare gains generated from the conflicting effect to overturn it. Define $\mathscr{D}_{k>k_t^H}(\boldsymbol{\tau}_t'; \rho_t)$ as the IP term for a corner equilibrium at any $k > k_t^H$ and the other $\mathscr{D}_{k<k_t^L}(\boldsymbol{\tau}_t'; \rho_t)$ for a corner equilibrium at any $k < k_t^L$. Apply Equation (15) and the difference $\Delta\mathscr{D}_{h^+ \rightsquigarrow l^-}(\boldsymbol{\tau}_t'; \rho_t)$ can be written as $\zeta_1(\boldsymbol{\tau}_t'; \rho_t)\frac{e^{2\rho_t k_t}}{(1-\varphi^L)^2} + \zeta_0(\boldsymbol{\tau}_t'; \rho_t)$. The welfare change $\Delta\widetilde{\mathscr{W}}_{h^+ \rightsquigarrow l^-}(\boldsymbol{\tau}_t'; \rho_t) \approx \Delta\mathscr{D}_{h^+ \rightsquigarrow l^-}(\boldsymbol{\tau}_t'; \rho_t)$.

■ **Proposition 2.** *Given the fundamentals $(\boldsymbol{\tau}_t'; \rho_t)$, the following statements hold: (i) the IP effect and therefore the welfare cuts are decreasing as k_t decreases for any $k_t < k_t^L$; (ii) The welfare cuts due to free information acquisition are non-negative; (iii) the IP effect is independent to k_t for any $k > k_t^H$; (iv) the welfare cuts become even much milder disproportionately as k_t falls if traders have constrained risk tolerance and become more risk-averse, or if market is more random, or if the signals are of better quality; The proofs are in the Appendix Page 48.*

3.1.2 Information Purchasing Effect

The IP effect always causes welfare losses and is linear in the cost factor $e^{2\rho_t k_t^L}$ where the risk adjustments consist of a slope $\zeta_1(\boldsymbol{\tau}_t'; \rho_t)$ and a drift $\zeta_0(\boldsymbol{\tau}_t'; \rho_t)$ that vary monotonically with each market risk fundamental. The graphical representation of the IP effect is a straight line in the cost factor, which can be rotated counterclockwise by increasing the slope of the risk adjustments and shifted upwards by increasing the intercept of the risk adjustments (See Figure 2). Determining the slope and intercept involves considering the direct and indirect effects of market risk fundamentals on risk adjustments. The direct effect is related to the quality of information and amplifies the pecuniary burden of high-quality information. Recall that high quality means high $\tau_{\varepsilon,t}$ or small $\tau_{s,t}$. The indirect effect is the amplification/compression of the direct effect through $\tau_{u,t}$ and ρ_t. Low returns to information mean low market randomness, thereby amplifying the welfare cuts caused by high-quality information disclosure. In contrast, high returns to information tend to compress. Likewise, the low risk-aversion tends to amplify the welfare cuts through the degree of the relative information quality because a low ρ_t raises the demand functions and traders will trade more aggressively, thereby causing more welfare cuts. In contrast, a high risk aversion tends to compress. In fact, the indirect effect reflects the relationship among the fundamentals for adjusting market risks (Remark 2.2). First, $\tau_{\varepsilon,t}$ and $\tau_{u,t}$ are substitutes since for any given $\boldsymbol{\zeta}(\boldsymbol{\tau}_t'; \rho_t)'$, an increase in $\tau_{u,t}$ increases the overall market risks and takes away the room allowing for a large $\tau_{\varepsilon,t}$. $\tau_{\varepsilon,t}$ and ρ_t are, however, complements because a high ρ_t offers the room allowing for a larger $\tau_{\varepsilon,t}$. Second, $\tau_{s,t}$ are complements for $\tau_{u,t}$, and substitutes for ρ_t since $\tau_{s,t}$ and $\tau_{\varepsilon,t}$ are opposite to each other for contributing the relative information quality.

■ **Remark 2.1** The risk adjustment vector $\boldsymbol{\zeta}(\boldsymbol{\tau}_t'; \rho_t)' = (\zeta_0(\boldsymbol{\tau}_t'; \rho_t), \zeta_1(\boldsymbol{\tau}_t'; \rho_t))$ can be affected by the fundamentals directly and indirectly, given that the fundamentals governing the information quality are non-separable from other fundamentals.

Direct effect:

- $\frac{\partial \boldsymbol{\zeta}(\boldsymbol{\tau}_t'; \rho_t)'}{\partial \tau_{\varepsilon,t}} = \left(2\tau_{u,t}\tau_{\varepsilon,t}\rho_t^{-2}\tau_{s,t}^{-1} + \tau_{s,t}^{-1}, 3\tau_{\varepsilon,t}^2\tau_{u,t}\tau_{s,t}^{-2}\rho_t^{-2} + \tau_{s,t}^{-1}\right)$ is semi-positive definite.

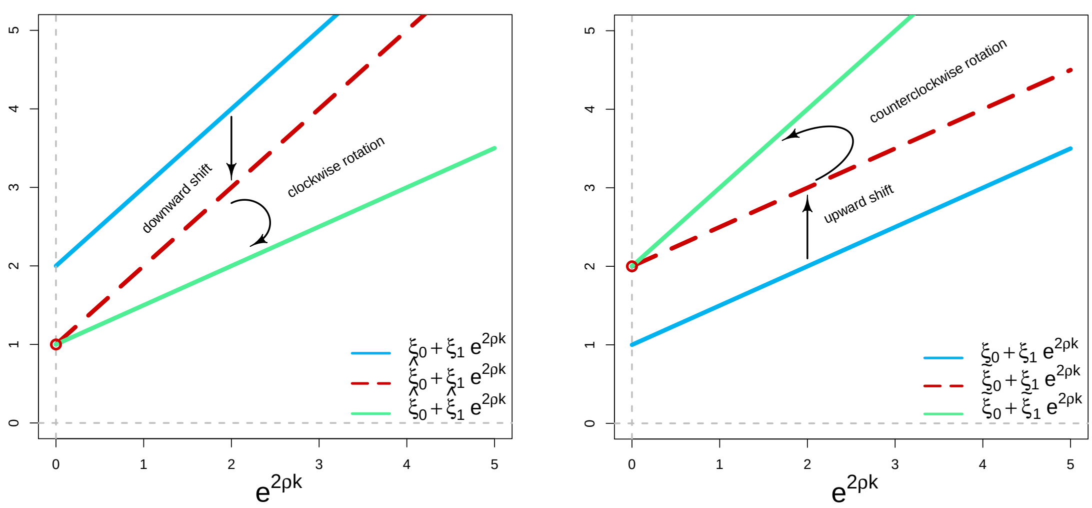
– $\frac{\partial \zeta(\boldsymbol{\tau}_t';\rho_t)'}{\partial \tau_{s,t}} = \left(-\tau_{\varepsilon,t}\tau_{s,t}^{-2} - \tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^{-2}\tau_{s,t}^{-2}, -\tau_{\varepsilon,t}\tau_{s,t}^{-2} - \tau_{\varepsilon,t}^3\tau_{u,t}\rho_t^{-2}\tau_{s,t}^{-2}\right)$ is semi-negative definite.

Indirect effect:

– $\frac{\partial \zeta(\boldsymbol{\tau}_t';\rho_t)'}{\partial \tau_{\varepsilon,t}\partial \tau_{u,t}} = \left(2\tau_{\varepsilon,t}\rho_t^{-2}\tau_{s,t}^{-1}, 3\tau_{\varepsilon,t}^2\tau_{s,t}^{-2}\rho_t^{-2}\right)$ and $\frac{\partial \zeta(\boldsymbol{\tau}_t';\rho_t)'}{\partial \tau_{s,t}\partial \rho_t} = \left(2\tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^{-3}\tau_{s,t}^{-2}, 2\tau_{\varepsilon,t}^3\tau_{u,t}\rho_t^{-3}\tau_{s,t}^{-2}\right)$ are semi-positive definite.

– $\frac{\partial \zeta(\boldsymbol{\tau}_t';\rho_t)'}{\partial \tau_{s,t}\partial \tau_{u,t}} = \left(-\tau_{\varepsilon,t}^2\rho_t^{-2}\tau_{s,t}^{-2}, -\tau_{\varepsilon,t}^3\rho_t^{-2}\tau_{s,t}^{-2}\right)$ and $\frac{\partial \zeta(\boldsymbol{\tau}_t';\rho_t)'}{\partial \tau_{\varepsilon,t}\partial \rho_t} = \left(-4\tau_{u,t}\tau_{\varepsilon,t}\rho_t^{-3}\tau_{s,t}^{-1}, -6\tau_{\varepsilon,t}^2\tau_{u,t}\tau_{s,t}^{-2}\rho_t^{-3}\right)$ are semi-negative definite.

■ **Remark 2.2** For contributing to the risk adjustments, $\tau_{\varepsilon,t}$ and $\tau_{u,t}$ are substitutes, and $\tau_{\varepsilon,t}$ and ρ_t are complements; $\tau_{s,t}$ and ρ_t are substitutes, and $\tau_{s,t}$ and $\tau_{u,t}$ are complements.



(a) $\hat{\zeta}_0 < \zeta_0$ and $\hat{\zeta}_1 < \zeta_1$.

(b) $\tilde{\zeta}_0 > \zeta_0$ and $\tilde{\zeta}_1 > \zeta_1$.

Figure 2: Risk adjusted coefficients.

3.1.3 Information Evaluation Effect

In this section, we emphasize the significance of the IE effect, which exhibits more nuanced distinctions than the IP effect. Before delving into the IE effect, it is crucial to know the role of information quality and the returns on resolving the conflict between information spreading and information acquisition.

As mentioned earlier, decreasing $\tau_{s,t}$ or increasing $\tau_{\varepsilon,t}$ are two methods that can enhance signal quality. However, they have phenomenally different market consequences. A low $\tau_{s,t}$ reduces price informativeness, limiting the potential positive effects of learning. On the other hand, a high $\tau_{\varepsilon,t}$ increases price informativeness, expanding the potential positive effects. In the first case, the signals are excludable and this excludability allows only the informed traders to increase own informativeness. For instance, as $\tau_{s,t} \searrow 0$ and $\mu^*(k_t) \nearrow 1$, consider three numerical moments: (i) $\mathbb{E}_t(\dot{\theta}_{t+1}|\mathscr{P}(s_t,u_t)) = s_t + (u_t+1)\rho_t^2/\tau_{\varepsilon,t}$; (ii) $\bar{\theta}_{t+1} = \bar{s}_t$; (iii) $\mathbb{E}_t(\dot{\theta}_{t+1}|s_t) = s_t$; and their variances: (i) $\mathbb{V}_t(\dot{\theta}_{t+1}|\mathscr{P}(s_t,u_t)) = \tau_{\varepsilon,t}/\rho_t^2\tau_{u,t} + \tau_{\varepsilon,t}^{-1}$; (ii) $\sigma_{\theta_{t+1}}^2 = \infty$; (iii) $\mathbb{V}_t(\dot{\theta}_{t+1}|s_t) = \tau_{\varepsilon,t}^{-1}$. Hence, we find that without knowing highly informative signals, traders indeed face more uncertainty and cannot mimic informed trading. In contrast, the example for the second case is that when $\tau_{\varepsilon,t} \nearrow \infty$ and $\mu^*(k_t) \nearrow 1$, the previous numerical evidence will be not there since $\mathbb{E}(\dot{\theta}_{t+1}|\mathscr{P}(s_t, u_t)) = \mathbb{E}(\dot{\theta}_{t+1}|s_t) = s_t$ and $\mathbb{V}(s_t|\mathscr{P}(s_t, u_t)) = \mathbb{V}(\dot{\theta}_{t+1}|s_t) = 0$ imply that the signals are fully revealed by market prices and the learning activity will be perfect. Thus, the information purchasers are struggling because information spreading dampens the motivation for information acquisition.

Grossman and Stiglitz (1980) wrote "*[t]here is a fundamental conflict between the efficiency with which markets spread information and the incentives to acquire information*". This paper is a touchdown to this point, viewing information quality as a crucial extension. In fact, conflict is inherent and persists. The strong conflict necessarily entails that the information precision exceeds a cutoff value and the degree of noise trading is not too weak to preclude the rise in price informativeness by offering compensation in randomness. If traders actually reveal some signals of which the information quality is below the cutoff, the information spreading is confined and the degree of information spillovers is capped, creating an incentive for traders to acquire signals. The worries about the conflict are shied away. Even though traders face low-quality signals, they still purchase them as long as $EU_I \geqslant EU_U$. Certainly, the restriction on information spreading can also be done by dispersing noise trading since this can preclude the increase in price informativeness effectively. In Proposition 1, a low $\tau_{u,t}$ and a low $\mathcal{Q}_t$ imply negative $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)$ that means welfare improvements. Hence, once the information spread problem is constrained, there could be net welfare gains. The next question is how low $\tau_{u,t}$ and $\mathcal{Q}_t$ can be at most.

The IP effect suggests that the Hirshleifer effect (Hirshleifer (1971)) is plausible, as it suggests that revealing too much information can lead to a loss of welfare due to the loss of valuable insurance opportunities. Additionally, Laffont (1985) has shown that a fully revealing Rational Expectations Equilibrium (REE) may not be Pareto optimal. In the uninformative state, welfare relies on the market-making term $\bar{\theta}_{t+1} - \mathscr{P}_t(u_t)$ (See Appendix.), where $\bar{\theta}_{t+1}$ provides the insurance to average out the uncertainty of the error $\dot{\varepsilon}_{t+1}$ and the economy seems to reach an interim Pareto optimum and information frictions are temporarily out. Otherwise, if the signal is revealed, welfare hinges on the speculative term $s_t - p_t$, which is subject to uncertainty regarding future returns because s_t is only a draw from its distribution. The Hirshleifer effect, however, may not fully explain the situation. The evidence presented in this paper contradicts it, likely because the welfare effect is caused by reasons other than insurance opportunities, indicating that something important may have been overlooked.

- **Proposition 3 The Hirshleifer Effect Revisited.** *Given nonzero and non-infinite ρ and τ_ε, as $\tau_{s,t} \nearrow \infty$, the signals are fully insured, say $\bar{\theta}_{t+1} = s_t$. the IP effect converges to zero, say $\lim_{\tau_{s,t}\nearrow\infty} \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = 0$. The IE effect converges to $\lim_{\tau_{s,t}\nearrow\infty} \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \frac{\tau_{\varepsilon,t}(\tau_{u,t}-1)}{\tau_{\varepsilon,t}+\rho_t^2} \neq 0$. If $\tau_{u,t} \leqslant 1$, it causes no welfare cuts. If $\tau_{u,t} > 1$, it, however, causes welfare cuts. The Hirshleifer effect falls short if $\tau_{u,t} \leqslant 1$ while it fails if $\tau_{u,t} > 1$. The proof is in the Appendix Page 48.*

With full insurance, it is not accurate to attribute the Hirshleifer effect to the IP effect that exhibits no variation. The only relevant factor is the IE effect, which presents the downside of the Hirshleifer effect. The main reason is the omission of information quality and returns. The paper argues that the diversity in welfare patterns is mainly driven by the behavior of IE. High-quality information takes away much market efficiency due to either high informational advantages or high informativeness. Meanwhile, the benefits are produced by taking advantage of a better signal or by using the information in a more agnostic market environment. Also, the IE effect is affected by the degree of risk aversion. It scales down the size of the welfare effect, irrespective of welfare gains or losses. The analysis is based on the limited ρ and τ_ε to avoid approximation imprecision. When τ_u is small, the approximation holds well. It is worth noting that the irregular case where $\rho \nearrow \infty$ can also make the approximation good. But, the IE converges to τ_u that is always positive and thereby welfare cuts. The Hirshleifer effect always fails.

Knowing the sources of benefits and costs, we can understand why the welfare effects of IE depend on $\tau_{u,t} + \mathcal{Q}_t$. When $\tau_{u,t}$ is low, the market is too unpredictable for outsiders to imitate informed trading, resulting in large profits for informed traders. When $\tau_{u,t}$ is high, outsiders can easily mimic informed trading, resulting in small profits for informed traders. The relative information quality $\mathcal{Q}_t$ is positively tied to the ratio $\tau_{\varepsilon,t}/\tau_{s,t}$ and has an upper bound of 1. When $\mathcal{Q}_t$ is evenly contributed by $\tau_{\varepsilon,t}$ and $\tau_{s,t}$, it defines an intermediate information quality where $\mathcal{Q}_t = 1/2$. Thus, a high quality information entails $\mathcal{Q}_t > 1/2$, *i.e.* $\tau_{\varepsilon,t} > \tau_{s,t}$ and a low quality information, however, needs $\mathcal{Q}_t < 1/2$, *i.e.* $\tau_{\varepsilon,t} < \tau_{s,t}$. Revealing high-quality information loses a large portion of market randomness, as the increase in price informativeness resulting from the relatively large $\tau_{\varepsilon,t}$ outweighs the decrease in price informativeness caused by the relatively small $\tau_{s,t}$. Consequently, the welfare cuts will be large. In contrast, revealing low-quality information, nevertheless, bears less welfare losses. Hence, welfare consequences are closely related to both $\tau_{\varepsilon,t}$ and $\mathcal{Q}_t$ collectively.

Proposition 1 demonstrates that $\tau_{u,t} + \mathcal{Q}_t \geqslant 1$ implies that both effects will likely result in decreased welfare. Given that $\mathcal{Q}_t \leqslant 1$, $\tau_{u,t} > 1$ will enforce the IE effect to cause welfare cuts, regardless of the information quality. This is because the returns to information are too low to avoid welfare cuts by even revealing the poorest signals. But when $\tau_{u,t} < 1$, the results still depend on $\tau_{u,t} + \mathcal{Q}_t$ but the difference is that welfare improvements become possible, leading to a dynamic balancing process. The more random the market environment is, the higher the returns to information, and the greater the likelihood that disclosing information will result in welfare gains, as long as $\tau_{u,t} + \mathcal{Q}_t < 1$ and the information spreading is capped.

3.1.4 Market State-contingent Welfare Patterns

In general, the outcomes of welfare gains and losses depend on a specific realization of $\{\boldsymbol{\tau}'_t; \rho_t\}$. However, it is still unclear how changes in fundamental parameters affect welfare patterns. To address this issue, we consider the case where only one fundamental parameter changes due to a streak of exogenous shocks, while all others remain constant. These shocks affect the fundamentals determined by a Markov process over time. The imputation of shocks introduces a positive $\boldsymbol{\xi}$ with respect to certain parameters, where the expected market-fundamentals for period $t+n$ are given by $\boldsymbol{\Psi}_{m,t}+n\cdot\boldsymbol{\xi}$[9]. When $\boldsymbol{\xi}$ is infinitesimal, the fundamentals can change smoothly. Thus, for any market risk fundamental x_t, the instantaneous response of the welfare effect $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_{t+1};\rho_{t+1})$ with respect to this market risk fundamental x_{t+1} is $\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_{t+1};\rho_{t+1})/\partial x_{t+1}$ that can be equal to $\left[\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_{t+1};\rho_{t+1})-\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_{t};\rho_{t})\right]/\xi_x$. Under this informational structure, the welfare patterns can be seen by simply looking at the first-order derivative of the welfare effect with respect to each market risk fundamentals. In the following study, we need a common assumption.

■ **Assumption:**

- ▷ *The informational shocks to the market risk fundamentals other than x_t have no real effect for projecting the future moments of the fundamentals, say $\xi_{-x}=0$ and $\xi_x>0$;*
- ▷ *There exists an infinitesimal positive number δ_x such that $\xi_i\in(0,\delta_x)$;*
- ▷ $x\in\{\boldsymbol{\tau}'_t;\rho_t\}$.

Shocks to the Returns to Information. $x=\tau_u$. Recall that the higher $\tau_{u,t}$ is, the lower returns the information has. The patterns are driven by imputing a streak shocks in which only $\upsilon_{u,t}$ has the real effect, on average. Hence, the expected returns to information are continuously reduced. Hence, from period t onwards, the Day t+n's returns to information equals $\mathbb{E}(\dot{\tau}_{u,t+n}|\tau_{u,t})=\tau_{u,t}+n\cdot\xi_u$ where $\xi_u\searrow 0$. For all other fundamentals, the expected values equal to $(\tau_{\varepsilon,t},\tau_{s,t},\rho_t)$ because the shocks $(\upsilon_{\varepsilon,t},\upsilon_{s,t},\upsilon_{\rho,t})$ impose the neutral effect. The smooth streak of averagely positive shocks in $\tau_{u,t}$ can be regarded as a continuous increase. The welfare patterns with respect to the shocks to the returns to information rest on four distinct cases in which two constraints are binding.

■ **Definition 4.** *The first $\tilde{\tau}_u$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ when the IP produces welfare cuts while the IE produces welfare gains; The second $\tau^*_{u,t}$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ when both effects are welfare cuts; The last $\tau^{\dagger}_{u,t}$ is an upper bound that prevents $\tau_{u,t}$ from diverging to ∞ at which there are no closed-form solutions.*

■ **Proposition 4.** *(i) When noise trading is perfectly random, say $\tau_{u,t}\searrow 0$, the IE effect implies welfare gains of $\tau_{s,t}\tau_{\varepsilon,t}/\left[(\tau_{s,t}+\tau_{\varepsilon,t})\rho_t^2+\tau_{s,t}\tau_{\varepsilon,t}\right]$ while the IP effect implies welfare cuts of $\left[2\tau_{s,t}\tau_{\varepsilon,t}+\tau^2_{\varepsilon,t}\right]/\tau^2_{s,t}$; (ii) (Useless information disclosure) when market prices fully reveal the signals, say $\tau_{u,t}\nearrow\infty$, there is a net welfare cut of $e^{\rho_t k_t}$ for the waste of money; (iii) both effects are increasing in $\tau_{u,t}$; the absolute responsiveness is whatsoever constant at $\gamma^a_{\tau_{u,t}}$; the relative responsiveness $\gamma^r_{\tau_{u,t}}$ is insofar falling if there are net welfare cuts and rising if there are net welfare gains; (iv) the welfare patterns display four distinct styles:*

[9] This is an iterated result. For $n=1$, there is $\mathbb{E}(\dot{\boldsymbol{\Psi}}_{m,t+1}|\boldsymbol{\Psi}_{m,t})=\boldsymbol{\Psi}_t+\boldsymbol{\xi}$. For $n=2$, it is $\mathbb{E}(\dot{\boldsymbol{\Psi}}_{m,t+2}|\boldsymbol{\Psi}_{m,t+1})=\boldsymbol{\Psi}_{m,t+1}+\boldsymbol{\xi}$. For $n=N$, it is $\mathbb{E}(\dot{\boldsymbol{\Psi}}_{m,t+N}|\boldsymbol{\Psi}_{m,t+N-1})=\boldsymbol{\Psi}_{t+m-1}+\boldsymbol{\xi}$. Hence, for any positive n, there must be the case where $\mathbb{E}(\dot{\boldsymbol{\Psi}}_{m,t+n}|\boldsymbol{\Psi}_{m,t})=\boldsymbol{\Psi}_{m,t}+n\cdot\boldsymbol{\xi}$. $\boldsymbol{\xi}>\mathbf{0}$ if the shocks $\boldsymbol{\Upsilon}_{i,t}$ have real effect on average. Otherwise, $\boldsymbol{\xi}=\mathbf{0}$.

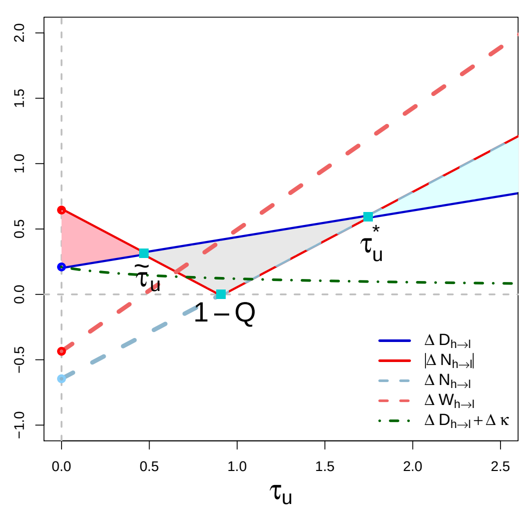
▷ **Claim 4.1** *If $\tilde{\tau}_u > 0$ and $\tau^*_{u,t} > 0$, the information disclosure makes net welfare improvements for small $\tau_{u,t} \in (0, \tilde{\tau}_u)$ and otherwise causes net welfare detriments for large $\tau_{u,t} \in (\tilde{\tau}_u, \tau^\dagger_{u,t})$ (A.1). For $\tau_{u,t} \in (1 - \mathcal{Q}_t, \tau^*_{u,t})$, the IP effect is more important for the welfare cuts and becomes otherwise less important than the IE effect for $\tau_{u,t} \in (\tau^*_{u,t}, \tau^\dagger_{u,t})$ (A.2).*

▷ **Claim 4.2** *If only $\tau^*_{u,t} > 0$, the information disclosure exactly results in (A.2), but (A.1) fails. Rather, it always triggers welfare cuts.*

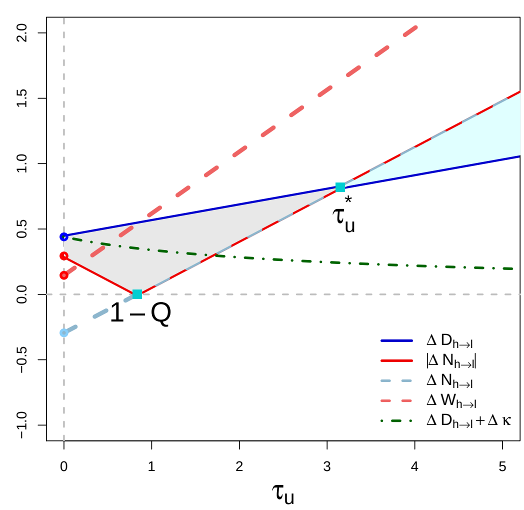
▷ **Claim 4.3** *If only $\tilde{\tau}_u > 0$, the information disclosure exactly results in (A.1), but (A.2) fails. Rather, the IP effect whatsoever dominates for $\tau_{u,t} \in (1 - \mathcal{Q}_t, \tau^\dagger_{u,t})$.*

▷ **Claim 4.4** *If neither $\tilde{\tau}_u > 0$ nor $\tau^*_{u,t} > 0$, the information disclosure whatsoever trigger welfare cuts, driven more disproportionately by the IP effect.*

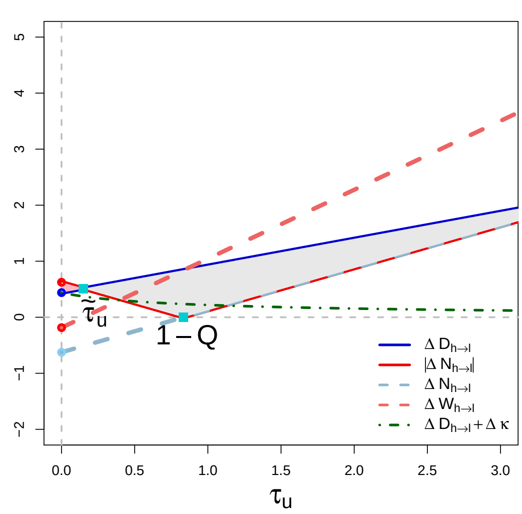
The proofs are in the Appendix Page 49.



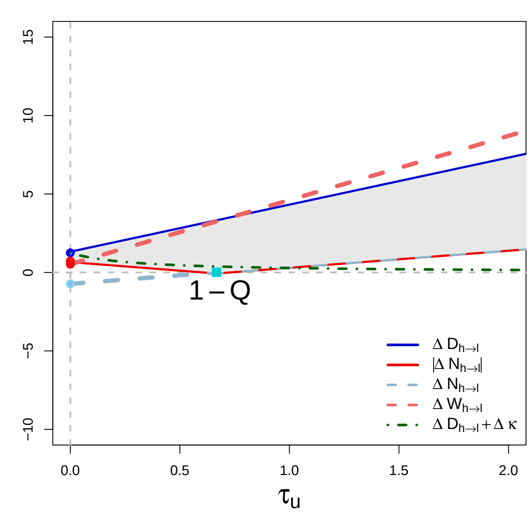
(a) $\tilde{\tau}_{u,t} > 0$ and $\tau^*_{u,t} > 0$.



(b) $\tau^*_{u,t} > 0$ and $\tilde{\tau}_{u,t} < 0$.



(c) $\tau^*_{u,t} < 0$ and $\tilde{\tau}_{u,t} > 0$.



(d) $\tilde{\tau}_{u,t} < 0$ and $\tau^*_{u,t} < 0$.

Figure 3: The welfare patterns in the returns to information $\tau^{-1}_{u,t}$.

Note: parameter selection: (a) $\{\tau_{\varepsilon,t}, \tau_{s,t}, \rho_t\} = \{0.5, 5, 0.5\}$; (b) $\{\tau_{\varepsilon,t}, \tau_{s,t}, \rho_t\} = \{0.5, 2.5, 1\}$; (c) $\{\tau_{\varepsilon,t}, \tau_{s,t}, \rho_t\} = \{2, 10, 1\}$; (d) $\{\tau_{\varepsilon,t}, \tau_{s,t}, \rho_t\} = \{1, 2, 0.5\}$.

The first statement pertains to a situation where the noise trading is extremely random, leading to maximum returns to information. $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$ is true due to $\mathcal{Q}_t \leqslant 1$, resulting in welfare improvements that offset the cuts from the IP. But if the returns reduce to zero, the market becomes fully revealing and outsiders can fully comprehend signals from market prices. Useless information disclosures lead to a waste of money, causing welfare cuts equivalent to $e^{\rho_t k_t}$ utils. This contrasts with the belief of Choi and Liang (2021) that useless information disclosure can improve welfare if people's beliefs in asset quality remain unchanged. Abstracting from the positive information price, it is at best innocuous but it is hard to see any welfare improvements once the returns to information fade away. In Figure 3, some welfare improvements can be evidently seen only for small $\tau_{u,t}$, as this scenario confines information spreading and makes information acquisition profitable. As $\tau_{u,t}$ increases, the returns from information decrease, resulting in higher welfare cuts and lower gains, assuming the welfare effect driven by information quality remains constant. Hence, the key takeaway is that welfare improvements necessarily entail a low $\tau_{u,t}$, whatever information quality is.

Furthermore, following the practice of Johnson, Boone, Breach, and Friedman (2000), we define the absolute responsiveness $\gamma^a_{\tau_{u,t}}$ and the relative responsiveness $\gamma^r_{\tau_{u,t}}$ (See Appendix), which reflect the overall slope and percentage responsiveness of welfare, respectively. $\gamma^r_{\tau_{u,t}} > 0$ can either capture a upward welfare relative to welfare cuts or a downward welfare relative to welfare gains. $\gamma^r_{\tau_{u,t}} < 0$, however, captures either a upward welfare relative to welfare gains or a downward welfare relative to welfare cuts. In this case, $\gamma^a_{\tau_{u,t}}$ is constant and positive, as both effects monotonically increase in $\tau_{u,t}$ (See Figure (7a)). It captures the constant speed of changing welfare cuts or gains. When $\gamma^r_{\tau_{u,t}} > 0$, a large welfare cut leads to a lower $\gamma^r_{\tau_{u,t}}$ and thus, a relatively slower increase in welfare cuts. When $\gamma^r_{\tau_{u,t}} < 0$, a more negative welfare improvement leads to a larger $\gamma^r_{\tau_{u,t}}$ and thus, a relatively slower reduction in welfare gains. Figure (7b) show that if there are welfare gains, $\gamma^r_{\tau_{u,t}}$ will get large with $\tau_{u,t}$ approaching to zero and drop with $\tau_{u,t}$ rising. If welfare cuts were the case, $\gamma^r_{\tau_{u,t}}$ is monotonically declining as $\tau_{u,t}$ increases. In Panel (3a), the red region is the cumulative welfare improvements for $\tau_{u,t} \in (0, \tilde{\tau}_u)$ and the size gets smaller as $\tau_{u,t}$ increases. The gains fade away if the returns are lower than the returns implied by $\tilde{\tau}_u$. Rather, the information disclosure starts off with welfare cuts. In the grey region, IP dominates but the welfare reduction from the IE effect is more sensitive in percentage to the reduction in returns to information and eventually, it exceeds the IP effect at $\tau^*_{u,t}$ and becomes the major reason for welfare cuts (the blue region). In Panel (3b), there is no room for welfare improvements since the welfare cuts caused by information quality have dominated the benefits even when $\tau_{u,t}$ is low. In fact, the relative information quality is 17% versus 9% of the case in Panel (3a). In either of these two cases, the IP effect is flatter because compared to the IE, it is relatively less sensitive to $\tau_{u,t}$ than the other two cases of Panel (3c) and Panel (3d). Likewise, in Panel (3c), there is room for net welfare gains since the relative information quality is only 13% versus 50% in Panel (3d). However, the gains fade away soon as $\tau_{u,t} > \tilde{\tau}_u$. Then the welfare cuts contributed by both effects are dominated by the steeper IP effect.

Shocks to the informational advantages. $x = \tau_\varepsilon$. The welfare patterns must also be sensitive to the absolute information quality $\tau_{\varepsilon,t}$ since it captures trader's own informational advantages that point out why information acquisition is, in essence, implementable. The precision of signals improves with higher values of $\tau_{\varepsilon,t}$. The counterfactual shocks contain the only real effect on $\tau_{\varepsilon,t}$. Similarly, there is $\mathbb{E}(\dot{\tau}_{\varepsilon,t+n}|\tau_{\varepsilon,t}) = \tau_{\varepsilon,t} + n \cdot \xi_\varepsilon$ where $\xi_\varepsilon \searrow 0$. Other fundamentals remain unchanged when neutral shocks $(\upsilon_{u,t}, \upsilon_{s,t}, \upsilon_{\rho,t})$, are imposed on average. The smooth streak of positive shocks in $\tau_{\varepsilon,t}$ can be viewed as a continuously increasing input. However, three constraints bind in four distinct cases.

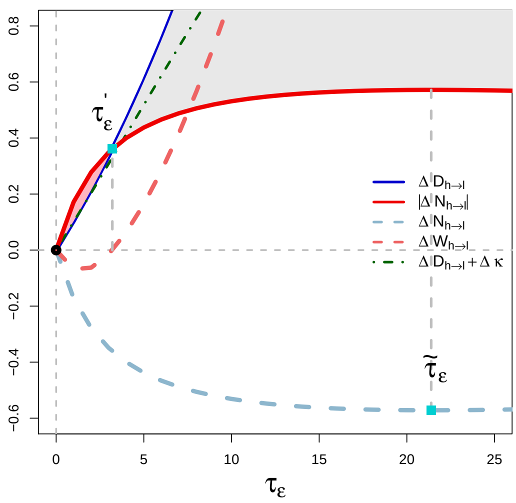
■ **Definition 5.** *The first cutoff $\tau'_{u,t}$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ when the IE produces welfare gains; the second cutoff $\tau^*_{u,t}$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ when both effects are welfare cuts; the third cutoff $\hat{\tau}_{\varepsilon,t}$ enforces $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = 0$; the last cutoff $\tilde{\tau}_{\varepsilon,t}$ minimizes $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ when it is negative.*

■ **Proposition 5.** *(i) The welfare will not be whatsoever affected by information disclosure once the informational advantages are zero; (ii) the welfare will be jeopardized by information disclosure infinitely as the informational advantages become infinitely large, in whichever individual effect; (iii) the IP effect increases as the informational advantages increase. When the returns to information are low, say $\tau_{u,t} \geqslant 1$, the IE effect shows an increasing trend and implies positive welfare cuts in all cases. When $\tau_{u,t} < 1$, there is a small room for welfare gains, say within $0 < \tau_{\varepsilon,t} < \tilde{\tau}_{\varepsilon,t}$. Otherwise it bounces to increase. The absolute responsiveness $\gamma^a_{\tau_{\varepsilon,t}}$ and relative responsiveness $\gamma^r_{\tau_{\varepsilon,t}}$ are non-linear; (iv) the welfare patterns display four distinct styles:*

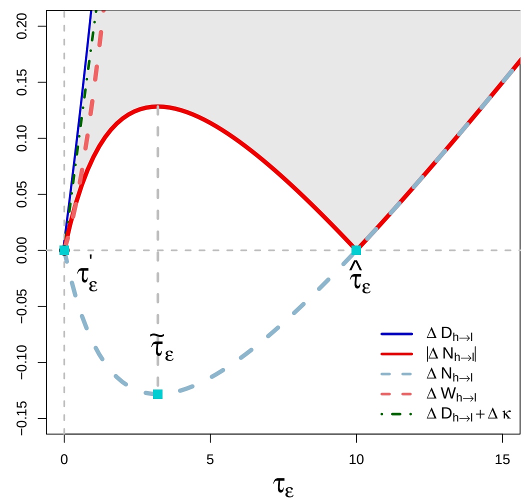
- ▷ **Claim 5.1** *If $\tau_{u,t} < 1$ and $\tau'_{\varepsilon,t} > 0$, the IE effect produces welfare gains when $0 < \tau_{\varepsilon,t} < \hat{\tau}_{\varepsilon,t}$ and the gains culminate at $\tilde{\tau}_{\varepsilon,t}$. When $0 < \tau_{\varepsilon,t} < \tau'_{\varepsilon,t}$, net welfare gains are retained and when $\tau_{\varepsilon,t} > \tau'_{\varepsilon,t}$, net welfare cuts are retained.*
- ▷ **Claim 5.2** *If $\tau_{u,t} < 1$ and $\tau'_{\varepsilon,t} \leqslant 0$, the only difference is that the IP effect always dominates so do the welfare cuts, all else constant.*
- ▷ **Claim 5.3** *If $\tau_{u,t} \geqslant 1$ and $\tau^*_{\varepsilon,t} > 0$, both effects cause welfare cuts, but before the IE goes to $\tau^*_{\varepsilon,t}$ either concavely or convexly, it dominates. After that, the dominance switches to the IP effect.*
- ▷ **Claim 5.4** *If $\tau_{u,t} \geqslant 1$ and $\tau^*_{\varepsilon,t} \leqslant 0$, the only difference is that the IP effect always dominates, all else constant.*

The proofs are in the Appendix Page 51.

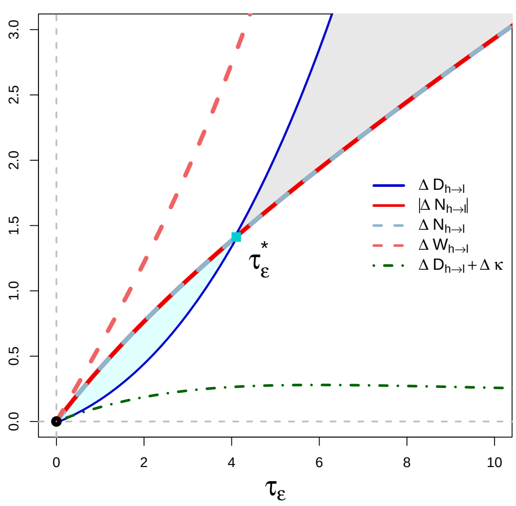
When there are no informational advantages, two welfare effects are zero. Despite the cost of information, the risk adjustment parameters are zero, resulting in traders purchasing pure noise in the market. The welfare impact is zero because of this, and a risk premium exactly offsets the paid information price, resulting in a net zero IP effect. The zero IE effect directly implies that the benefits of utilizing noise equal to



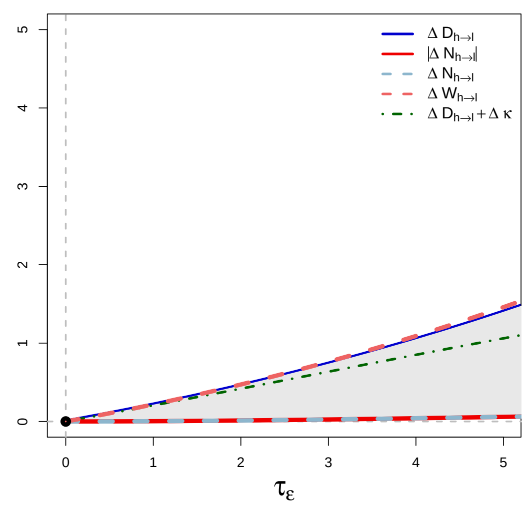
(a) $\tau_{u,t} < 1$ and $\tau'_{\varepsilon,t} > 0$.



(b) $\tau_{u,t} < 1$ and $\tau'_{\varepsilon} \leqslant 0$.



(c) $\tau_{u,t} \geqslant 1$ and $\tau^*_{\varepsilon,t} > 0$.



(d) $\tau_{u,t} \geqslant 1$ and $\tau^*_{\varepsilon,t} \leqslant 0$.

Figure 4: The welfare patterns in the informational advantages $\tau_{\varepsilon,t}$.

Note: parameter selection: (a) $\{\tau_{u,t}, \tau_{s,t}, \rho_t\} = \{0.1, 20, 2\}$; (b) $\{\tau_{u,t}, \tau_{s,t}, \rho_t\} = \{0.5, 10, 2\}$; (c) $\{\tau_{u,t}, \tau_{s,t}, \rho_t\} = \{5, 15, 3\}$; (d) $\{\tau_{u,t}, \tau_{s,t}, \rho_t\} = \{1, 10, 5\}$.

the costs of revealing noise. As informational advantages accumulate, the IP implies that the welfare detriment increases due to penalty parameters $\zeta_0(\boldsymbol{\tau}_t; \rho_t)$ and $\zeta_1(\boldsymbol{\tau}_t; \rho_t)$ increasing in $\tau_{\varepsilon,t}$. High-quality information benefits traders by offering more information, but it also boosts information spreading and loses market efficiency. This contrasts with a market with a low $\tau_{u,t}$, wherein even revealing low-quality information can make profits but the losses of market efficiency do not *pro-rata* rise. As $\tau_{\varepsilon,t}$ increases, the welfare losses quickly exceed the welfare gains due to the limited returns to information. Moreover, when $\tau_{\varepsilon,t}$ goes *ad infinitum*, the market becomes fully revealing, rendering information disclosure useless again. Regarding individual effects, the IE effect *must* be convex and the welfare gains will increase, culminate, decline and finally revert to become welfare cuts as $\tau_{\varepsilon,t}$ increases, if the returns to information are large ($\tau_{u,t} < 1$). Graphically, the IE effect drops for low $\tau_{\varepsilon,t} < \tilde{\tau}_{\varepsilon,t}$ and then increases due to more costly welfare impact. The red region conveys a cumulative net welfare gain, when $\tau_{\varepsilon,t} < \tau'_{\varepsilon,t}$ (Panel 4a). But it will vanish soon since the welfare cuts made by the IP increase quickly due to responsive risk adjustments. For any $\tau_{\varepsilon,t} > \tau'_{\varepsilon,t}$, the cumulative welfare cuts are shown in grey region where the IP is more important. Of course, the welfare gain may not exist for some $\tau_{\varepsilon,t}$ such that welfare cuts are disproportionately explained by the IP (Panel 4b). In the other two cases where $\tau_{u,t} \geqslant 1$, there is no room for welfare gains since the returns to information are too small to produce welfare improvements by the IE. In Panel (4c), the IE can be either concave or convex but it produces more welfare cuts than the IP if $\tau_{\varepsilon,t} < \tau_{\varepsilon}^{*}$ and it produces less welfare cuts if $\tau_{\varepsilon,t} > \tau_{\varepsilon}^{*}$. The switch in dominance, however, does not exist in Panle (4d). In light of responsiveness, in Panel (7c), the blue line shows that $\gamma^{a}_{\tau_{\varepsilon,t}}$ is initially negative and then increases to become positive soon and in this process, the overall welfare initially faces welfare gains that then shrink to zero and eventually become welfare cuts. The sign flipping is driven by the convexity of the IE effect. For other cases, the coast-to-coast welfare losses witness upward sloping overall welfare and $\gamma^{a}_{\tau_{\varepsilon,t}}$ is always positive. The relative responsiveness $\gamma^{r}_{\tau_{\varepsilon,t}}$ is drawn in Panel (7d) and likewise, the only special case is when welfare gains are seen. $\gamma^{r}_{\tau_{\varepsilon,t}} > 0$ and the welfare effect becomes less sensitive in percentage before stretching out to the maximum welfare gains, and then $\gamma^{r}_{\tau_{\varepsilon,t}} < 0$ and the welfare change becomes more sensitive in percentage when the overall welfare further reduces to zero, but once the overall welfare becomes cuts, $\gamma^{r}_{\tau_{\varepsilon,t}} > 0$ and the welfare change becomes less sensitive in percentage to $\tau_{\varepsilon,t}$. Since for any other cases where the possibility of welfare gains shuts down, the welfare change is always less sensitive in percentage to $\tau_{\varepsilon,t}$, which implies that the relative increase in welfare cuts drops. In general, with $\tau_{\varepsilon,t}$ increasing, the IP effect is more responsive than the IE effect, probably because it captures the risks that are more sensitive to the market-level risk adjustments.

Shocks to the Informativeness of Information. $x = \tau_s$. A higher value of $\tau_{s,t}$ implies less informative signals. Counterfactual shocks only guarantee a real expected impact on $\tau_{s,t}$. Hence, we have $\mathbb{E}(\dot{\tau}_{s,t+n}|\tau_{s,t}) = \tau_{s,t} + n \cdot \xi_s$ where $\xi_s \searrow 0$. Other fundamentals are not affected by neutral shocks. The smooth streak of positive shocks in $\tau_{s,t}$ can be regarded as a continuous increase input. There are only two distinct cases and only one constraint binds.

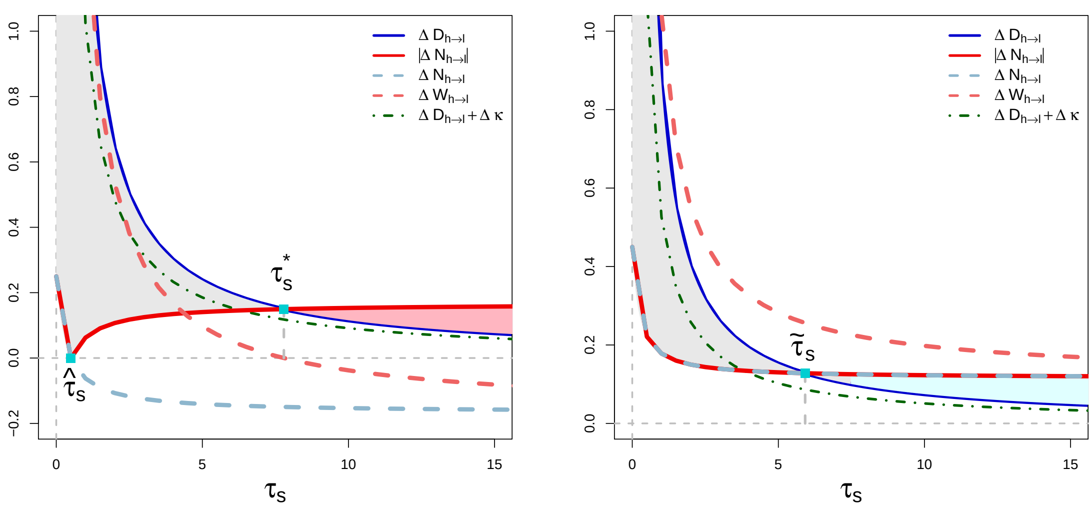
■ **Definition 6.** *The first cutoff $\tilde{\tau}_{s,t}$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ when both effects cause welfare cuts; the second cutoff $\tau^{*}_{s,t}$ poises $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ when the IE produces welfare gains; the third cutoff $\hat{\tau}_{s,t}$ enforces $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = 0$.*

■ **Proposition 6.** *(i) When the signals are extremely informative, the welfare cuts caused by the IP are infinitely large but the welfare cuts caused by the IE are limited to $\tau_{\varepsilon,t}\tau_{u,t}/\rho_t^2$; (ii) when the signals are uninformative, the overall welfare effect can be a cut if the returns to information are low, $\tau_{u,t} > 1$ and also can be an improvement if the returns are high, $\tau_{u,t} < 1$; (iii) as information becomes less informative, the welfare cuts caused by both effects will drop and it leaves more room for the IE to create welfare improvements. The absolute responsiveness $\gamma^{a}_{\tau_{s,t}}$ is negative and rising and $\gamma^{r}_{\tau_{s,t}}$ depends on the possibility of welfare gains; (iv) the welfare patterns display two distinct styles:*

▷ **Claim 6.1** *If $\tau_{u,t} > 1$ and $0 < \tau_{s,t} < \tilde{\tau}_{s,t}$, the welfare cuts are reinforced by* *both effects and the IP dominates. If $\tau_{s,t} > \tilde{\tau}_{s,t}$, the dominance switches to the IE.*

▷ **Claim 6.2** *If $\tau_{u,t} < 1$ and $0 < \tau_{s,t} < \hat{\tau}_{s,t}$, both effects cause welfare cuts and the IE starts to produce welfare gains when $\tau_{s,t} > \hat{\tau}_{s,t}$ and the welfare gains will become the net overall welfare gains if $\tau_{s,t} > \tau^*_{s,t}$.*

The proofs are in the Appendix Page 54.



(a) $\tau_{u,t} < 1$. (b) $\tau_{u,t} > 1$.

Figure 5: The welfare patterns in the informational informativeness $\tau_{s,t}$.

Note: parameter selection: (a) $\{\tau_{u,t}, \tau_{\varepsilon,t}, \rho_t\} = \{0.5, 0.5, 1\}$; (b) $\{\tau_{u,t}, \tau_{\varepsilon,t}, \rho_t\} = \{1.5, 0.3, 1\}$;

The first statement emphasizes the potential for welfare improvements only if $\tau_{u,t} < 1$. A smaller $\tau_{s,t}$ means more informativeness, which leads to improved information quality and upward adjustments for the IP effect. However, revealing higher-quality signals results in greater efficiency losses from the IE, making it more likely for overall welfare to experience net losses. As informativeness decreases, the information rapidly becomes of low or poor quality. The high returns on information create a driving force for the IE to bring about welfare improvements. In the diagram, $\tau^*_{s,t}$ captures the critical state where the net welfare change is zero. After that, the net welfare gains are phenomenal, even though the individual welfare gains already happen when $\tau_{s,t} > \hat{\tau}_{s,t}$. However, once $\tau_{u,t} > 1$, the potential for welfare gains diminishes as returns to information become too low to surpass the minimal welfare losses incurred from revealing poor information. The welfare cuts made by the IE are systematic so they are substantial as long as the returns to information are not large enough. In contrast, the welfare cuts made from the IP are flexible because they can be eliminated completely by charging zero risk adjustments. As the signals become fully uninformative, the penalty parameters go to zero. Therefore, when $\tau_{s,t} > \tilde{\tau}_{s,t}$, the welfare losses are almost because of the systematic welfare cuts inferred from the IE (Panel (5b)). The absolute responsiveness for each case is increasing and negative, indicating a slowdown in declining the welfare effect. The relative responsiveness for the second case is always increasing and negative while for the first case, it is humped due to the discontinuity at zero welfare effect and also faces a sign flip due to the possibility of net welfare gains. See Figure (7e) and Figure (7f).

Shocks to the Degree of Risk Aversion. $x = \rho$. The last thing to know is how welfare responds to the shocks in the degree of risk aversion. Similarly, the shocks only promise that ρ_t has a real expected effect, on average. We have $\mathbb{E}(\dot{\rho}_{t+n}|\rho_t) = \rho_t + n \cdot \xi_\rho$ where $\xi_\rho \searrow 0$. The expectations for all other fundamentals remain constant at $(\tau_{\varepsilon,t}, \tau_{u,t}, \tau_{s,t})$. The smooth streak of positive shocks in ρ_t can be regarded as a continuous increase input. There are four distinct cases and three restrictions bind.

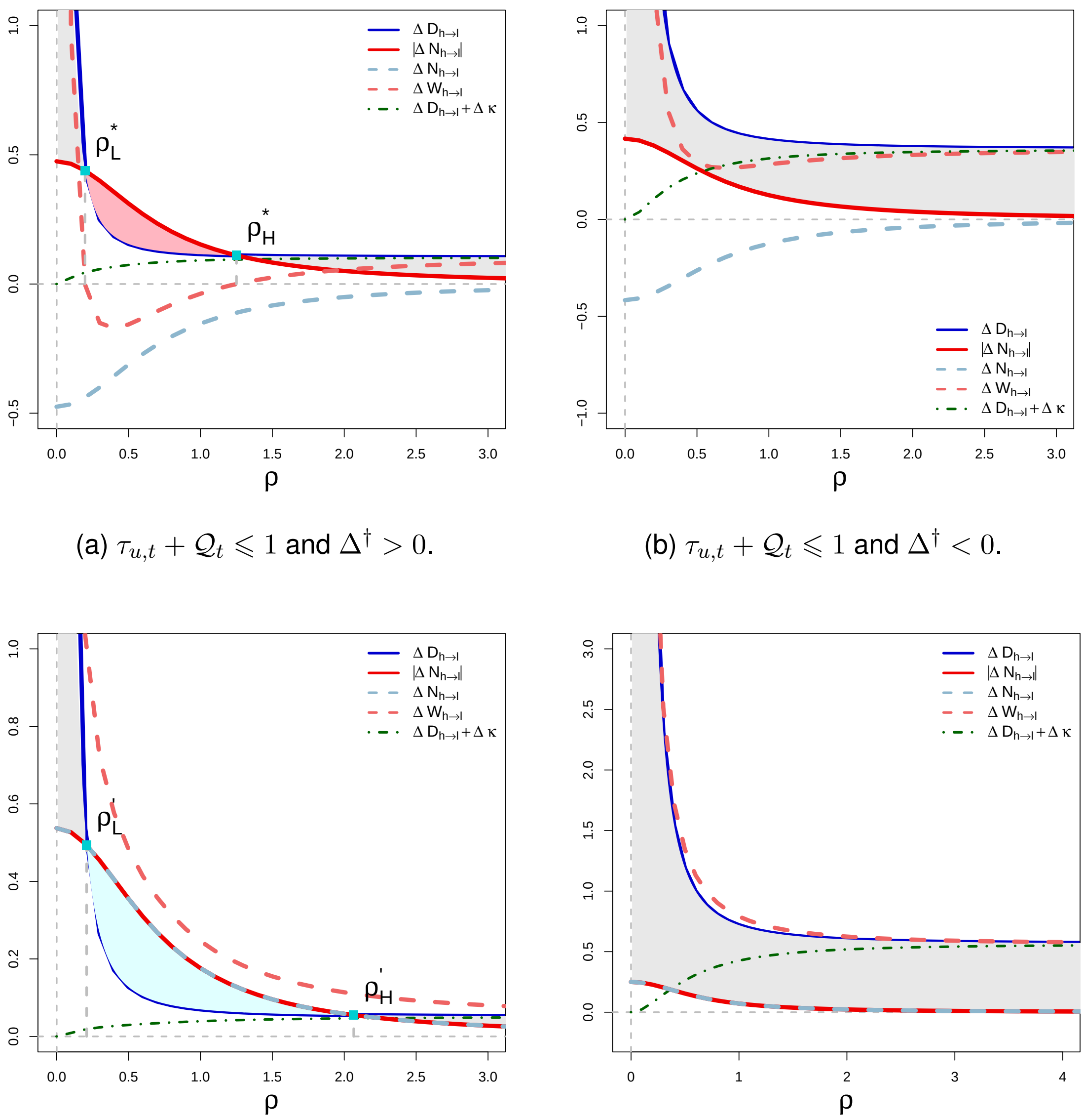
■ **Definition 7.** *The cutoffs ρ_L^* and ρ_H^* poise $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)|$ when the latter yields welfare improvements; the other two cutoffs ρ_L' and ρ_H' poise $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ when both effects yield welfare cuts.*

■ **Proposition 7.** *(i) When traders become risk neutral, the welfare losses caused by the IP goes to infinity whereas the IE effect is $[\tau_{u,t}\tau_{\varepsilon,t} + \tau_{s,t}(\tau_{u,t} - 1)] / \tau_{s,t}$ that can be welfare losses or gains, contingent on the evaluation procedure; (ii) when traders become extremely risk averse, the welfare cuts caused by the IP is $\left[\tau_{\varepsilon,t}^2 + 2\tau_{s,t}\tau_{\varepsilon,t}\right] / \tau_{s,t}^2$ whereas the welfare effect of the IE is zero; (iii) if there are overall welfare gains, the IE is concave. But for overall welfare cuts, it is convex. The IP effect is always convex. The overall welfare responds to risk aversion at $\gamma_{\rho_t}^a$ and $\gamma_{\rho_t}^r$, absolutely and relatively. (iv) the welfare patterns display four distinct styles:*

- ▷ **Claim 7.1** *If $\tau_{u,t} + \mathscr{Q}_t \leqslant 1$ and $\rho_L^* > 0$ and $\rho_H^* > 0$, we see net welfare gains for ρ_t lies inside the two cutoffs. Otherwise, it will be the net welfare cuts.*
- ▷ **Claim 7.2** *If $\tau_{u,t} + \mathscr{Q}_t \leqslant 1$ but there are no either ρ_L^* or ρ_H^*, we see net welfare cuts.*
- ▷ **Claim 7.3** *If $\tau_{u,t} + \mathscr{Q}_t > 1$ and $\rho_L' > 0$ and $\rho_H' > 0$, we see net welfare cuts and the IE dominates the IP in size when ρ_t lies inside the two cutoffs.*
- ▷ **Claim 7.4** *If $\tau_{u,t} + \mathscr{Q}_t > 1$ but there are no either $\rho_L' > 0$ or $\rho_H' > 0$, we see net welfare cuts and the IP always dominates in size.*

The proofs are in the Appendix Page 56.

Risk-neutral traders face unitary cost factor $e^{2\rho_t k_t} \searrow 1$, while their infinite risk adjustments $\{\zeta_0, \zeta_1\}$ go *ad infinitum*, resulting in infinitely large welfare cuts. The risk neutrality also drives $k_t^L \searrow 0$ and the information price is zero, leading to an overuse of information, which worsens the welfare cuts due to the already high levels of risk. As $\rho_t \nearrow \infty$, the IP effect drops to a positive limit, provided that the risk adjustments affected by other fundamentals do not reduce to zero. Depending on whether $\tau_{u,t} + \mathscr{Q}_t$ is greater than one, the IE can be either positive or negative. As the risk aversion goes up, the welfare effect shrinks to null, à la whatever welfare gains or losses. The reason is intuitive. The welfare decomposition shows that the IE is associated with demand function. Thus, a significant welfare effect entails substantial demand but, $\rho_t \nearrow \infty$



(a) $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$ and $\Delta^{\dagger} > 0$.

(b) $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$ and $\Delta^{\dagger} < 0$.

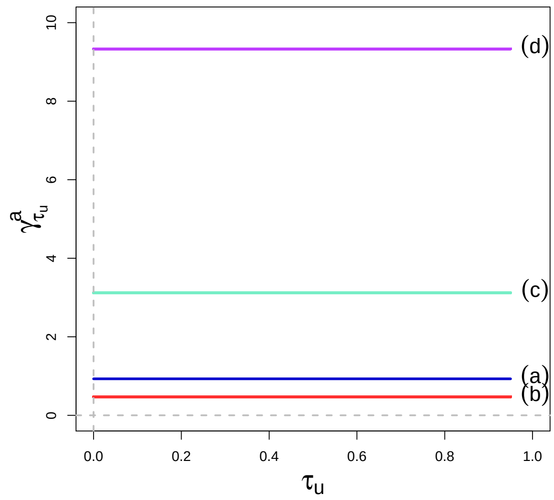
(c) $\tau_{u,t} + \mathcal{Q}_t > 1$ and $\Delta^{\ddagger} > 0$.

(d) $\tau_{u,t} + \mathcal{Q}_t > 1$ and $\Delta^{\ddagger} < 0$.

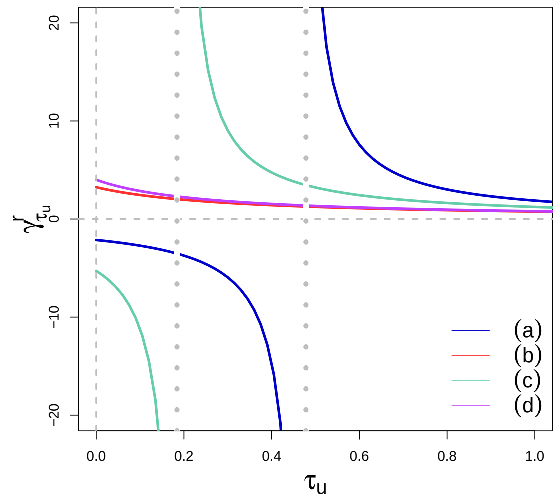
Figure 6: The welfare patterns in risk aversion - ρ_t.

Note: parameter selection: (a)$\{\tau_{u,t}, \tau_{\varepsilon,t}, \tau_{s,t}\} = \{0.5, 0.5, 10\}$; (b)$\{\tau_{u,t}, \tau_{\varepsilon,t}, \tau_{s,t}\} = \{0.5, 0.5, 3\}$; (c)$\{\tau_{u,t}, \tau_{\varepsilon,t}, \tau_{s,t}\} = \{1.5, 0.5, 10\}$; (d)$\{\tau_{u,t}, \tau_{\varepsilon,t}, \tau_{s,t}\} = \{1, 0.5, 2\}$; $\Delta^{\dagger}$ and $\Delta^{\ddagger}$ are defined in the appendix.

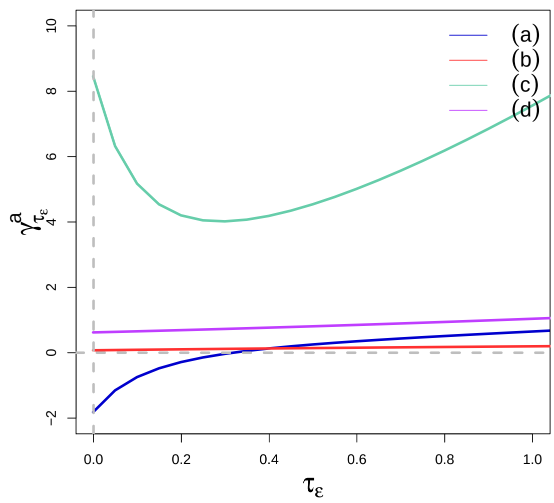
causes zero demand, compressing the IE effect to zero. In Panel (6a) and Panel (6b), we expect to see welfare improvements as $\tau_{u,t} + \mathcal{Q}_t < 1$. However, only for a small degree of risk aversion $\rho_L^* < \rho_t < \rho_H^*$, the net welfare effect produces gains. In Panel (6c) and Panel (6d) where $\tau_{u,t} + \mathcal{Q}_t > 1$ is met, we expect to see welfare cuts. Likewise, for some small $\rho_t \in (\rho_L', \rho_H')$, the welfare cuts are disproportionately made by the IE. Regarding responsiveness, the absolute responsiveness $\gamma_{\rho,t}^a$ converges to zero as traders become more risk-averse, even though the slopes of welfare are non-monotonic once the IE ever yields welfare gains. The relative responsiveness $\gamma_{\rho,t}^r$ of the first case shows that the welfare is less sensitive in percentage when approaching to the maximum net welfare gains, and is more sensitive in percentage when approaching to zero welfare. After the net welfare turns cuts, the welfare becomes less sensitive in percentage in ρ_t. For other cases, the welfare changes are less sensitive in percentage to an increasing ρ_t. See Figure (8a) and Figure (8b).



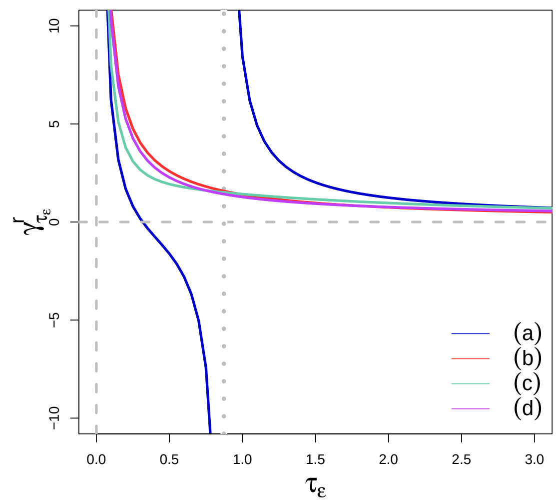
(a) The absolute responsiveness to $\tau_{u,t}$.



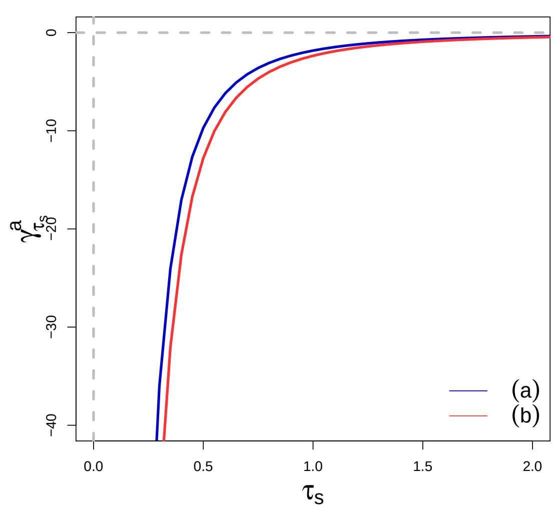
(b) The relative responsiveness to $\tau_{u,t}$.



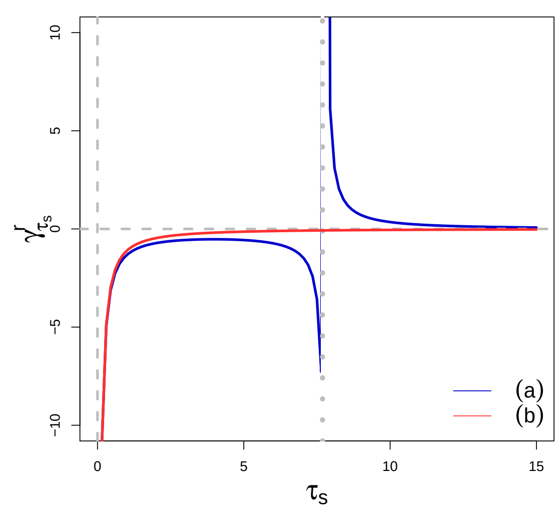
(c) The absolute responsiveness to $\tau_{\varepsilon,t}$.



(d) The relative responsiveness to $\tau_{\varepsilon,t}$.

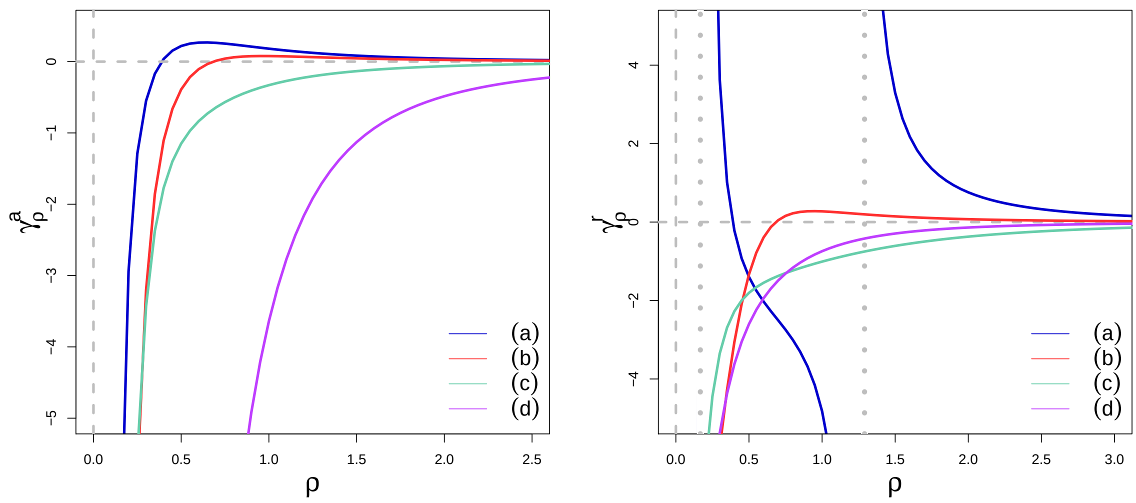


(e) The absolute responsiveness to $\tau_{s,t}$.



(f) The relative responsiveness to $\tau_{s,t}$.

Figure 7: The responsiveness in absolute and relative terms - Part I.



(a) The absolute responsiveness to ρ_t. (b) The relative responsiveness to ρ_t.

Figure 8: The responsiveness in absolute and relative terms - Part II.

Measurement Errors. As previously stated, the presence of measurement errors does not affect our findings. Namely, regarding τ_u, the measurement errors allow for flexibility in the cutoffs of τ_u. But, the welfare gains become more possible and in Panel (3c) and Panel (3d), the dominance in the welfare cut can be switched between IP plus errors and IE effect one more time, compared to the IP itself. In terms of τ_ε, other than Panel (4c) which is subject to large measurement errors, the other cases are very precise. Even with large measurement errors, the only difference is that the welfare cut is always more explained by the IE effect, compared to IP itself which crosses IE once. In terms of τ_s, the measurement errors are not an issue. For low ρ, the measurement errors are substantial but they converge to zero very soon. The welfare gains become more possible. Overall, the measurement errors appear not to matter.

3.2 Continuous Case

3.2.1 Welfare Representation

In the continuous case, both uninformed traders and informed traders coexist, leading to the problem of asymmetric information. This problem arises because uninformed traders have a disadvantage in information and face the risk of being outperformed by more knowledgeable traders. As a result, in the second stage of the Bayesian game, information acquisition becomes endogenous and is affected by asymmetric information. We regard $\mu_t^*(k_t)$ as the intensity of information acquisition and assume strategic substitution.

The earliest proof of the negative impact of asymmetric information on welfare can be found in Akerlof (1970), while Einav et al. (2010) recently estimated the welfare losses resulting from asymmetric information in the UK annuity market. Rothschild and Stiglitz (1976) provides some interpretive insights on this issue. Informed traders are similar to high-risk individuals who are more likely to purchase insurance because they require information to secure their trading payoffs. If no one buys the signals, traders can actually benefit, as information disclosure often leads to welfare cuts. Hence, the externality is negative, although it could potentially be positive if information disclosure results in welfare gains, which contrasts with their verdict. However, asymmetric information does cause welfare cuts, and this section examines the welfare dynamics associated with varying degrees of asymmetric information. Information acquisition can be viewed as insurance, as uninformed traders feel more secure as they learn more from informative market prices, despite the fact that this reduces the motivation and effectiveness of costly information acquisition. The implicit assumption behind is strategic substitutability[10]. Hence, the severity of asymmetric information gets milder as $\mu_t^*(k_t)$ rises and the overall welfare should be convex. This convexity should not be confused with the classic bid-ask spread model (Glosten and Milgrom (1985)). In that model, the specialist can certainly infer information from the history of transaction prices, even though he quotes the prices for arriving traders. However, the specialist, to avoid losses, immediately sets a higher spread if more insiders trade with him or if insiders have better information. As trades continue and more insiders arrive, the inside information will assimilated, thereby narrowing down the spread.

An interior equilibrium is achieved when the two types of unconditional expected utility are equal. The representative welfare of such an equilibrium can be expressed as:

$$\mathscr{W}_{int}[\mu_t^*(k_t)] = \mu_t^*(k_t) EU_I + (1-\mu_t^*(k_t)) EU_U = \mathbb{E}_t\left\{-exp\left[-\frac{(\mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_U)-p_t)^2}{2\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_U)}\right]\right\} \tag{25}$$

At an interior equilibrium,

$$EU_I = e^{\rho_t k_t}\sqrt{\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_I)/\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_U)}EU_U$$

in which $e^{\rho_t k_t}\sqrt{\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_I)/\mathbb{V}_t(\dot{\theta}_{t+1}|\mathcal{F}_U)} = 1$ must hold, in which $\mu^*(k_t)$ is endogenously determined and k_t is exogenous. Then the market price contains the same information as $s_t + [\mu_t^*(k_t)a_t]^{-1} u_t$ so market price predicts the signal s_t with disturbance of noise trading. In line with Vives (2010), we define $\omega_t = s_t + u_t/\mu_t^*(k_t)a_t$. Consequently, the first order and second order of moments of $\dot{\theta}_{t+1}$ conditional on $\mathcal{F}_U$ are

$$\mathbb{E}_t[\dot{\theta}_{t+1}|\mathcal{F}_U] = \frac{\tau_{s,t}}{\tau_{s,t}+(\mu_t^*(k_t)a)^2\tau_{u,t}}\bar{\theta}_{t+1} + \frac{(\mu_t^*(k_t)a_t)^2\tau_{u,t}}{\tau_{s,t}+(\mu_t^*(k_t)a_t)^2\tau_{u,t}}\left(s_t + \frac{u_t+1}{\mu_t^*(k_t)a_t}\right) \tag{26}$$

$$\mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_U] = [\tau_{s,t}+(\mu_t^*(k_t)a_t)^2\tau_{u,t}]^{-1} + \tau_{\varepsilon,t}^{-1} \tag{27}$$

where $\mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_U) = \mathbb{E}_t(\dot{\theta}_{t+1}|\omega_t) = \mathbb{E}_t(s_t|\omega_t)$ and $\mathbb{V}_t[\dot{\theta}_{t+1}|\mathcal{F}_U] = \mathbb{V}_t[\dot{\theta}_{t+1}|\omega_t] = \mathbb{V}_t[s_t|\omega_t] + \tau_{\varepsilon,t}^{-1}$. For easing presentation, denote $\tau_{s,t} + (\mu_t^*(k_t)a_t)^2\tau_{u,t}$ as $\mathcal{B}$. Thus, Grossman and Stiglitz (1980) affirm that there exists a unique linear market price such that the market can be clear.

[10]For the case of strategic complementarity, there are multiple equilibria (Ganguli and Yang (2009)) and the asymmetric information problem tends to be more severe as more informed trading is placed because in this process, uninformed traders face more uncertainty of the insider trading patterns.

■ **Lemma 5.** *If $\{s_t, \varepsilon_t, u_t\}$ follow a non-degenerate joint normal distribution and mutually independent. Namely, recall that $s_t \sim N(\bar{\theta}_{t+1}, \tau_{s,t}^{-1}), \varepsilon_t \sim N(0, \tau_{\varepsilon,t}^{-1}), u_t \sim N(-1, \tau_{u,t}^{-1})$. There is a unique linear market price clearing the market, which is $\mathscr{P}(s_t, u_t) = \alpha_1 + \alpha_2\omega_t$ where the parameters are constants:*

$$\alpha_1 = \frac{\rho_t(\tau_{\varepsilon,t}\tau_{u,t}\mu_t^*(k_t) + \bar{\theta}_{t+1}\tau_{s,t}\rho_t)(1-\mu_t^*(k_t))}{\tau_{\varepsilon,t}^2\tau_{u,t}\mu_t^*(k_t)^2 + \tau_{\varepsilon,t}\rho_t^2\mu_t^*(k_t) + \tau_{s,t}\rho_t^2} \tag{28}$$

$$\alpha_2 = \frac{\mu_t^*(k_t)(\tau_{\varepsilon,t}^2\tau_{u,t}\mu_t^*(k_t) + \tau_{\varepsilon,t}\rho_t^2 + \tau_{s,t}\rho_t^2)}{\tau_{\varepsilon,t}^2\tau_{u,t}\mu_t^*(k_t)^2 + \tau_{\varepsilon,t}\rho_t^2\mu_t^*(k_t) + \tau_{s,t}\rho_t^2} \tag{29}$$

The proofs are in the Appendix Page 58.

For uninformed investors, the expected profits are

$$\Pi = \mathbb{E}_t(\dot{\theta}_{t+1}|\mathcal{F}_U) - \mathscr{P}_t = \frac{\tau_{s,t}\bar{\theta}_{t+1} + \mu_t^*(k_t)a_t\tau_{u,t}}{\mathcal{B}} + \left[\frac{\mathcal{B} - \tau_{s,t}}{\mathcal{B}} - \alpha_2\right]\omega_t - \alpha_1$$

This means that the uninformed investors' *ex ante* expected profit only rests on the uncertainty of ω_t that determines the *ex post* clearing price. In this case, the *ex post* market price depends on the realization of the signal and the noise trading from known distributions. Hence, the welfare function in Equation (25) can be written as

$$\mathscr{W}_{int}[\mu_t^*(k_t)] = \mathbb{E}_t\left\{-exp\left[-\frac{\Pi^2}{2\left(\mathcal{B}^{-1} + \tau_{\varepsilon,t}^{-1}\right)}\right]\right\} = \mathbb{E}_t\left\{-exp\left[-\mathcal{X}_t^2\right]\right\} \tag{30}$$

Therefore, the distribution of ω_t should be known as a priori since the welfare fully rests on this distribution. Given the distributions of $s_t \sim N(\bar{\theta}_{t+1}, \tau_{s,t}^{-1})$ and

$$u_t/\mu_t^*(k_t)a_t \sim N\left(-\left[\mu_t^*(k_t)a_t\right]^{-1}, \left[\tau_{u,t}\mu_t^*(k_t)^2a_t^2\right]^{-1}\right)$$

, ω_t is also normal distributed with the mean of $\bar{\theta}_{t+1} - [\mu_t^*(k_t)a_t]^{-1}$ and the variance of $\tau_{s,t}^{-1} + [\tau_{u,t}\mu_t^*(k_t)^2a_t^2]^{-1}$. Hence, the welfare can be calculated in the following proposition.

■ **Proposition 8. Welfare in the Capital Market with Incomplete Information**

$\mathcal{X}_t$ follows a normal distribution:

$$\mathcal{X}_t \sim N\left(\frac{1}{\sqrt{2}} \cdot \frac{\bar{\theta}_{t+1}(1-\alpha_2) + \alpha_2\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}} - \alpha_1}{\sqrt{\mathcal{B}^{-1} + \tau_{\varepsilon,t}^{-1}}}, \frac{1}{2}\frac{\tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B} - \tau_{s,t}\right]^2}{\tau_{s,t}(\mathcal{B} - \tau_{s,t})\left(\mathcal{B} + \tau_{\varepsilon,t}\right)}\right)$$

The linearized welfare of an interior equilibrium $\widetilde{\mathscr{W}}_{int}[\mu_t^*(k_t)]$ can be approximated to:

$$\widetilde{\mathscr{W}}_{int}[\mu_t^*(k_t)] \approx \mathscr{D}_{int}(\mu_t^*(k_t)) + \mathscr{N}_{int}(\mu_t^*(k_t))$$

where

$$\mathscr{N}_{int}(\mu_t^*(k_t)) = \frac{\left[\bar{\theta}_{t+1}(1-\alpha_2) + \alpha_2\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}} - \alpha_1\right]^2 \mathcal{B}\tau_{\varepsilon,t}\tau_{s,t}(\mathcal{B}-\tau_{s,t})}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})\left(\mathcal{B}+\tau_{\varepsilon,t}\right) + \tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B}-\tau_{s,t}\right]^2} \tag{31}$$

$$\mathscr{D}_{int}(\mu_t^*(k_t)) = \frac{\tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B}-\tau_{s,t}\right]^2}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})\left(\mathcal{B}+\tau_{\varepsilon,t}\right)} \tag{32}$$

The proofs are in the Appendix Page 58.

3.2.2 Simulation

This section will conduct several simulations aimed at providing concrete examples for easy understanding, rather than exhaustively covering all possibilities. This is particularly important because the continuous case is not as direct as the discrete case. The purpose of the simulations is to examine the patterns of welfare and determine whether they align with our theoretical conjectures. According to theory, the problem of asymmetric information should impact both IE and IP, and ultimately shape the overall welfare.

First, the IP effect is primarily determined by the cost k_t and then is subject to risk adjustments. Unlike the discrete case, the risk adjustments cannot be written as a linear function of the cost factor. As the asymmetric information problem wanes, we can expect the cost k_t to drop. There are two conjectures about the IP patterns:

- ▷ Conjecture 1: $\frac{\partial D_{int}(\mu_t)}{\partial \mu_t} < 0$*: as more traders purchase the information, the total pecuniary costs accumulate and the total welfare tends to drop.*
- ▷ Conjecture 2: $\frac{\partial D^2_{int}(\mu_t)}{\partial \mu_t \partial k_t} > 0$*: due to the declining information price, the welfare drops slowly as the information acquisition becomes more intensified. Hence, the IP effect is convex.*

Second, the IE involves the benefits of utilizing the information and the costs of losing market efficiency. The benefits of information utilization are influenced by the problem of asymmetric information. Strategic substitution suggests that an increase in price informativeness can provide insurance for uninformed traders. Therefore, as the asymmetric information problem decreases, the benefits of utilizing information decrease while the benefits of inferring information from prices increase. As more traders reveal information, total efficiency losses are expected to increase. However, the marginal cost remains constant given the fixed quality of information governed by fixed parameters. There are two possible assumptions regarding the patterns of IE:

- ▷ Conjecture 3: *if IE produces welfare gains, then for any* $0 < \mu_t < 1$*, the IE always produces welfare gains. Then there is* $\frac{\partial N_{int}(\mu_t)}{\partial \mu_t} > 0$*. As* μ_t *increases, the marginal benefits from not utilizing the information increase while the marginal* *benefits from utilizing the information decrease. Hence, the IE effect is convex and* $\frac{\partial N_{int}^2(\mu_t)}{\partial \mu_t^2} > 0$.

- Conjecture 4: *if IE produces welfare cuts, then for any* $0 < \mu_t < 1$*, the IE always produces welfare cuts. Then there is* $\frac{\partial N_{int}(\mu_t)}{\partial \mu_t} < 0$*. There is still the convexity* $\frac{\partial N_{int}^2(\mu_t)}{\partial \mu_t^2} > 0$.

The simple simulations abstract from those extreme risk fundamentals. For instance, either $\rho_t, \tau_{\varepsilon,t}, \tau_{u,t}, \tau_{s,t}$ is too large to allow for enough variation. The benchmark simulation assumes that the market fundamentals are unitary, which represents intermediate intensities. In each simulation, only one parameter is altered. If a parameter is larger (resp. smaller) than unitary, it means a high (resp. low) intensity of the specific market feature governed by that parameter. At each time, the market state draws an evident dichotomy between high intensity and low intensity, instead of a streak of shocks. Importantly, the convexity in welfare is evident and consistent, even if the simulations may fail to generate net welfare gains. In other possible scenarios, differences between patterns are dependent on the specific realizations of the fundamentals and reflect in different locations of the curves, without significantly altering the shapes.

The Returns to Information. We opt for $\tau_{u,t} = 0.01$, which implies high returns on information. The simulation in Panel (9a) demonstrates consistency with prior conjectures about IP, where welfare cuts decrease more slowly as the asymmetric information problem becomes less severe. The behavior of IE displays coast-to-coast convex welfare improvement, which aligns with conjectures about IE. The overall welfare pattern is convex, initially declining before bouncing back. IE generates welfare gains by balancing information quality with returns. Similar to the discrete case, welfare gains are only possible with high information returns and limited information quality. This example aligns with Conjecture 3. In turn, in Panel (9b) where we, however, choose $\tau_{u,t} = 2$ representing low information returns, the IE causes small welfare cuts caused by low returns to information. This example aligns with Conjecture 4. We evidently see that the welfare patterns are convex in μ_t individually and collectively.

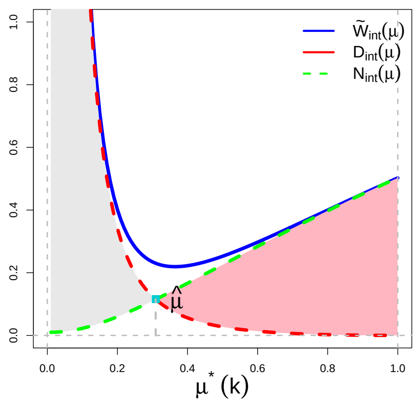
The Informational Advantages. We choose $\tau_{\varepsilon,t} = 3$, which represents a high degree of informational advantage, or equivalently, a high $\mathcal{Q}_t$ *ceteris peribus*, as in Panel (9c), where we observe that as more traders become informed, the welfare patterns take on a convex shape. Given the intermediate returns to information and high quality of the information, the IE tends to decrease when μ_t is not high, as in the discrete case. But, the difference is evident that the IE reverts to increase when μ_t becomes large enough and this is because of the waning asymmetric information problem. As conjectured, when the IE produces welfare detriments, the net marginal benefits of not utilizing the information are rising as μ_t rises. Given the initial welfare cuts, the welfare tends to drop slowly and then revert to increase. The bouncing-back right tail exactly captures the overturn rise when the marginal benefits from not utilizing the information exceed the marginal benefits from utilizing the information minus the marginal cost of information revealing. In Panel (9d), the IE effect does not show significant patterns once the informational advantages are low, because the net benefits are almost balanced by information acquisition and information spreading due to limiting information quality.

The Informational Informativeness. This is an analog to the welfare patterns in informational advantages since the high informativeness in information raises an upward impetus to the relative information quality $\mathcal{Q}_t$ while the low informativeness, nevertheless, imposes a downward stress on it. Hence, in Panel (9e), the patterns are U-shape and in Panel (9f), it becomes kind of flatter due to imposing poor information quality. Again, the welfare patterns are still convex in μ_t individually and collectively.

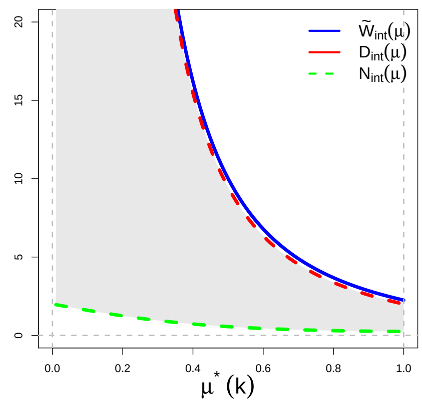
The Degree of Risk Aversion. For low risk-aversion $\rho_t = 0.5$, the welfare patterns of the IE effect looks like Panel (9c) and actually that is because of the indirect effect of ρ_t, which means that a low risk-aversion can affect the welfare in the manner of informational advantages throughout increasing the risk-tolerance-adjusted informational advantages a_t. Consequently, after adjusting for risk attitude, low risk-aversion, in effect, strengthens the effect driven by informational advantages *per se*. That is why Panel (9g) can be alike as Panel (9c), even though $\tau_{\varepsilon,t} = 1$ versus $\tau_{\varepsilon,t} = 3$. In the other way around, a high risk-aversion, say $\rho_t = 1.5$, the real effect of informational advantages will be scaled down by adjusting for the risk tolerance. So Panel (9h) also displays a flat pattern as Panel (9d), even though $\tau_{\varepsilon,t} = 1$ versus $\tau_{\varepsilon,t} = 0.5$. The real effect of informational advantages for a high risk-aversion is 2/3 and for a low risk-aversion is 2, which are similar in magnitude to 0.5 and 3 as in Panel (9c) and (9d). Given that ρ_t affects the welfare only through a_t, there is no direct effect. Since $\tau_{\varepsilon,t}$ has direct effect, the welfare patterns will bear some slight differences, even though the risk-tolerance-adjusted informational advantages are hypothetically equalized. Hence, the indirect effect of ρ_t through a_t only partially scales up and down the effect of informational advantages. This amplification/compression mechanism can be justified because less risk-averse traders tend to trade more aggressively and therefore, the welfare tends to be more sensitive whereas more risk-averse traders are supposed to be subject to less sensitive welfare patterns.

4 Conclusion

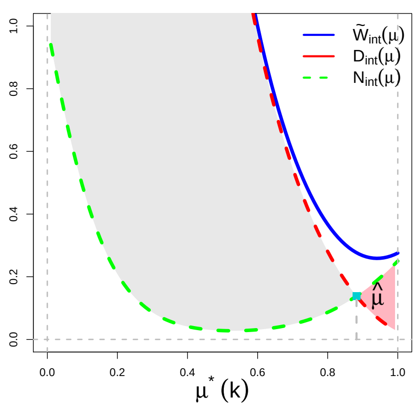
This paper minimally deviates from the classic model and requires parsimonious assumptions. However, it delivers the important analysis of welfare from the perspective of information quality and the returns to information, as a key complementarity of Grossman and Stiglitz (1980). Focusing on these perspectives resolves the uncomfortable conflict between information spreading and information acquisition since information quality and the returns to information can successfully confine the information spreading and mitigate the frustration in information acquisition. The paper clearly figures out the benefit and cost sources of acquiring information. Importantly, the conclusions are general, rather than being only valid for specific cases. The findings are counterintuitive since we find the possibility of welfare gains from information acquisition. This is restricted by the information quality and returns collectively in a dynamic trade-off.



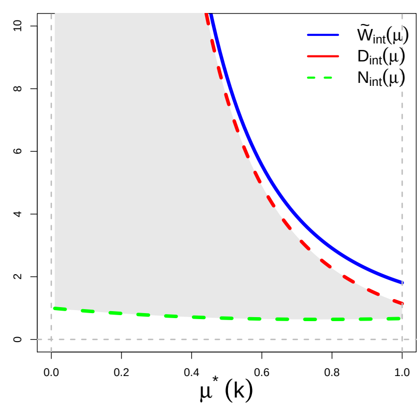
(a) High returns to information: $\tau_{u,t} = 0.01$.



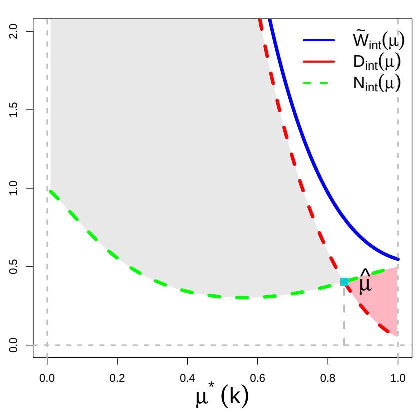
(b) Low returns to information: $\tau_{u,t} = 2$.



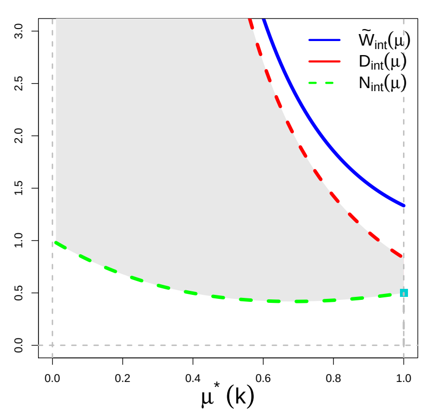
(c) High informational advantages: $\tau_{\varepsilon,t} = 3$.



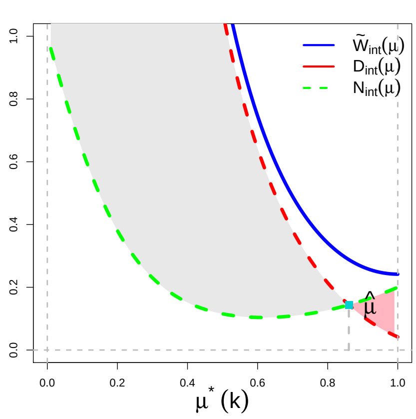
(d) Low informational advantages: $\tau_{\varepsilon,t} = 0.5$.



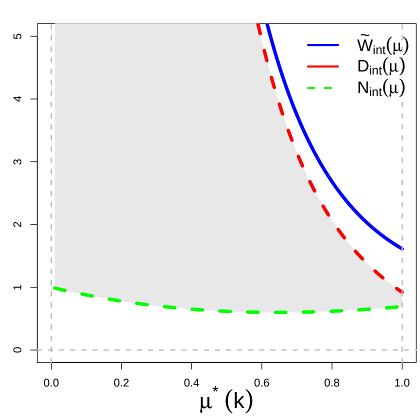
(e) High informativeness of information: $\tau_{s,t} = 0.1$.



(f) Low informativeness of information: $\tau_{s,t} = 10$.



(g) Low risk aversion: $\rho_t = 0.5$.



(h) High risk aversion: $\rho_t = 1.5$.

Figure 9: Simulated interior welfare patterns.

This finding considers the reason from the signals *per se* and the reason from the market. Moreover, this paper retrieves the full picture of welfare patterns in the fundamentals. Last but not least, this paper confirms that the asymmetric information actually causes welfare cuts that weaken as price informativeness increases.

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Supplementary Appendix

■ ***Lemma 1.*** First, the conditional expected utility of uninformed traders is:

$$\begin{aligned}\mathbb{E}_t[\dot{U}(X_U(\mathscr{P}_t(u_t)))|\mathscr{P}(u_t)] &= -exp\left\{-\frac{(\mathbb{E}_t[\dot{\theta}_{t+1}|\mathscr{P}_t(u_t)]-\mathscr{P}_t(u_t))^2}{2\mathbb{V}_t[\dot{\theta}_{t+1}|p_t]}\right\} \\ &= -exp\left\{-\frac{(\bar{\theta}_{t+1}-\mathscr{P}_t(u_t))^2}{2\sigma_{\theta t}^2}\right\} \\ &= -exp\left(-\frac{\rho_t^2\sigma_{\theta t+1}^2 u_t^2}{2}\right) \\ &= -exp\left[-\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})u_t^2}{2a_t^2\tau_{s,t}}\right] \qquad \text{(A.1)}\end{aligned}$$

where according to Equation (26) and Equation (27), there are

$$\mathbb{E}_t(\dot{\theta}_{t+1}|\mathscr{P}(u_t)) = \bar{\theta}_{t+1}$$

$$\mathbb{V}_t(\dot{\theta}_{t+1}|\mathscr{P}(u_t)) = \sigma_{st}^2 + \sigma_{\varepsilon t+1}^2 = \sigma_{\theta t+1}^2$$

once $\mu_t^*(k_t) = 0$. Formally, define the linear market price in the Grossman and Stiglitz's fashion as $\mathscr{P}(u_t) = \beta_1 + \beta_2 u_t$. According to the market clearance condition, there is

$$\mathscr{P}(u_t) = \bar{\theta}_{t+1} - \rho_t\sigma_\theta^2 u_t$$

where $\beta_1 = \bar{\theta}_{t+1}$ and $\beta_2 = -\rho_t\sigma_{\theta t+1}^2$.

Second, following the generalized Rao's formula in Demange and Laroque (1995) and Vives (2010): in this case, define $\mathcal{Y} \sim N(-\frac{\rho_t\sigma_{\theta t+1}}{\sqrt{2}}, \frac{\rho_t^2\sigma_{\theta t+1}^2\sigma_{ut}^2}{2})$ and thus, the welfare $-\mathbb{E}_t\left\{exp\left[-\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})u_t^2}{2a_t^2\tau_{s,t}}\right]\right\}$ can be written as

$$\begin{aligned}\mathscr{W}(\boldsymbol{\tau}_t'; \rho_t)_{lce} &= -\frac{exp\left(-\frac{\rho_t^2\sigma_{\theta t+1}^2}{2(1+\rho_t^2\sigma_{\theta t+1}^2\sigma_{ut}^2)}\right)}{\sqrt{1+\rho_t^2\sigma_{\theta t+1}^2\sigma_{ut}^2}} \\ &= -\frac{exp\left(-\frac{1}{2}\frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}\right)}{\sqrt{1+\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}}} \qquad \text{(A.2)}\end{aligned}$$

□

■ ***Lemma 2.*** First, the conditional expected utility of uninformed traders is:

$$\begin{aligned}\mathbb{E}_t\{\dot{U}[X_U(\mathscr{P}(s_t,u_t))]|\mathscr{P}(s_t,u_t)\} &= -exp\left\{-\frac{(\mathbb{E}_t[\dot{\theta}_{t+1}|\mathscr{P}(s_t,u_t)]-\mathscr{P}(s_t,u_t))^2}{2\mathbb{V}_t[\dot{\theta}_{t+1}|\mathscr{P}(s_t,u_t)]}\right\} \\ &= -exp\left\{-\frac{\left[\frac{\tau_{s,t}\left(\bar{\theta}_{t+1}-s_t-\frac{u_t}{a_t}\right)+a_t\tau_{u,t}}{a_t^2\tau_{u,t}+\tau_{s,t}}\right]^2}{2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}\right\} \qquad \text{(A.3)}\end{aligned}$$

Assume that market prices follow $\mathscr{P}(s_t,u_t) = \gamma_1 + \gamma_2\omega_t$ where $\omega_t = s_t + \frac{u_t}{\mu_t^*(k_t)a_t}$ is the informational proxy that contains exactly the same information as market prices, repeatedly used in this paper. At the HCE, the market price is

$$\mathscr{P}(s_t, u_t) = s_t + \frac{u_t}{a_t}$$

so the result follows since $\gamma_1 = 0$ and $\gamma_2 = 1$. Then the following conditional expectation and variance also follow Equation (26) and Equation (27).

$$\mathbb{E}_t(\dot{\theta}_{t+1}|\mathscr{P}(s_t, u_t)) - \mathscr{P}(s_t, u_t) = \bar{\theta}_{t+1} + \frac{\mathbb{C}(s_t + \dot{\varepsilon}_{t+1}, s_t + \frac{u_t}{a_t})}{\mathbb{V}_t(s_t + \frac{u_t}{a_t})}\left(s_t + \frac{u_t}{a_t} - \bar{\theta}_{t+1} + \frac{1}{a_t}\right) - s_t - \frac{u_t}{a_t}$$

$$= \frac{\tau_{s,t}\left(\bar{\theta}_{t+1} - s_t - \frac{u_t}{a_t}\right) + a_t\tau_{u,t}}{a_t^2\tau_{u,t} + \tau_{s,t}} \tag{A.4}$$

$$\mathbb{V}_t(\dot{\theta}_{t+1}|\mathscr{P}(s_t, u_t)) = (\tau_{s,t} + a^2\tau_{u,t})^{-1} + \tau_{\varepsilon,t}^{-1} \tag{A.5}$$

The unconditional expected utility of informed traders satisfies

$$\mathbb{E}_t\{\dot{U}[X_I(s_t, \mathscr{P}(s_t, u_t))]|\mathscr{P}(s_t, u_t)\} = e^{\rho_t k_t^L}\sqrt{\frac{\mathbb{V}_t[\dot{\theta}_{t+1}|s_t]}{\mathbb{V}_t[\dot{\theta}_{t+1}|\mathscr{P}(s_t, u_t)]}}\mathbb{E}_t\{\dot{U}[X_U(\mathscr{P}(s_t, u_t))]|\mathscr{P}(s_t, u_t)\}$$

$$= -exp\left\{-\frac{\left[\frac{\tau_{s,t}\left(\bar{\theta}_{t+1} - s_t - \frac{u_t}{a_t}\right) + a_t\tau_{u,t}}{a_t^2\tau_{u,t} + \tau_{s,t}}\right]^2}{2\left[(\tau_{s,t} + a^2\tau_{u,t})^{-1} + \tau_{\varepsilon,t}^{-1}\right]}\right\} \tag{A.6}$$

as in equilibrium $e^{\rho_t k_t^L}\sqrt{\frac{\mathbb{V}_t[\dot{\theta}_{t+1}|s_t]}{\mathbb{V}_t[\dot{\theta}_{t+1}|\mathscr{P}(s_t,u_t)]}} = e^{\rho_t k_t^L}\sqrt{\frac{\tau_{\varepsilon,t}^{-1}}{(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}}} = 1.$

Second, apply the generalized Rao's formula again, after defining $\mathcal{Y}' = \frac{\frac{\tau_{s,t}\left(\bar{\theta}_{t+1} - s_t - \frac{u_t}{a_t}\right) + a_t\tau_{u,t}}{a_t^2\tau_{u,t} + \tau_{s,t}}}{\sqrt{2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}}$. Given that the informational proxy ω_t follows $N(\bar{\theta}_{t+1} - \frac{1}{a_t}, \tau_{s,t}^{-1} + (a_t^2\tau_{u,t})^{-1})$, the distribution of $\mathcal{Y}'$ is also normal and follows

$$\mathcal{Y}' \sim N\left(\frac{\tau_{s,t}a_t^{-1} + a_t\tau_{u,t}}{(a_t^2\tau_{u,t} + \tau_{s,t})\sqrt{2\left[(\tau_{s,t} + a_t^2\tau_{u,t})^{-1} + \tau_{\varepsilon,t}^{-1}\right]}}, \frac{\tau_{s,t} + \tau_{s,t}^2(a_t^2\tau_{u,t})^{-1}}{2(a_t^2\tau_{u,t} + \tau_{s,t})^2\left[(\tau_{s,t} + a_t^2\tau_{u,t})^{-1} + \tau_{\varepsilon,t}^{-1}\right]}\right)$$

and the welfare of the HCE is

$$\mathscr{W}(\boldsymbol{\tau}_t'; \rho_t)_{hce} = -\mathbb{E}_t\left[e^{-\mathcal{Y}'^2}\right] = -\frac{exp\left\{-\frac{\frac{\left(\tau_{s,t}a_t^{-1} + a_t\tau_{u,t}\right)^2}{2(a_t^2\tau_{u,t}+\tau_{s,t})^2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}}{1+\frac{\tau_{s,t}+\tau_{s,t}^2(a_t^2\tau_{u,t})^{-1}}{(a_t^2\tau_{u,t}+\tau_{s,t})^2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}}\right\}}{\sqrt{1+\frac{\tau_{s,t}+\tau_{s,t}^2(a_t^2\tau_{u,t})^{-1}}{(a_t^2\tau_{u,t}+\tau_{s,t})^2\left[(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}\right]}}}$$

$$= -\frac{exp\left(-\frac{1}{2}\frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}}\right)}{\sqrt{1+\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}}} \tag{A.7}$$

□

■ ***Lemma 3.*** The simple procedure follows with the first step in which we take a simple logarithm on both sides of each welfare function (Equation (A.8) and Equation (A.11)); the second step is to extract the proportional terms (Equation (A.9) and

Equation (A.12)).

$$\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{lce}\right] = -\frac{1}{2}\frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})} - \frac{1}{2}\ln\left[1+\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}\right] \tag{A.8}$$

$$\propto \frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})} + \ln\left[1+\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}\right] \tag{A.9}$$

$$= \frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})} + \ln\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}} + \kappa_l \tag{A.10}$$

$$\ln\left[-\mathscr{W}(\boldsymbol{\tau}'_t;\rho_t)_{hce}\right] = -\frac{1}{2}\frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}} - \frac{1}{2}\ln\left[1+\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}\right] \tag{A.11}$$

$$\propto \frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}} + \ln\left[1+\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}\right] \tag{A.12}$$

$$= \frac{\tau_{u,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}} + \ln\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})} + \kappa_h \tag{A.13}$$

where κ_l and κ_h capture measurement errors such that:

$$\kappa_l = \ln\left[1+\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}\right] - \ln\frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}} \tag{A.14}$$

$$\kappa_h = \ln\left[1+\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}\right] - \ln\frac{\tau_{s,t}\tau_{\varepsilon,t}}{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})} \tag{A.15}$$

□

■ ***Lemma 4.*** First, we express the IP as the change rate as in the Proposition 1, so we have the approximated IP:

$$\ln\mathscr{D}_{lce} - \ln\mathscr{D}_{hce} = \ln\left(\frac{\mathscr{D}_{lce}}{\mathscr{D}_{hce}}\right) = \frac{\mathscr{D}_{lce}}{\mathscr{D}_{hce}} - 1 \tag{A.16}$$

Thus, the actual IP is:

$$\ln(1+\mathscr{D}_{lce}) - \ln(1+\mathscr{D}_{hce}) = \frac{1+\mathscr{D}_{lce}}{1+\mathscr{D}_{hce}} - 1 \tag{A.17}$$

Thus, the measurement error is:

$$\kappa_l - \kappa_h = \frac{\mathscr{D}_{hce}-\mathscr{D}_{lce}}{\mathscr{D}_{hce}(1+\mathscr{D}_{hce})} \tag{A.18}$$

Second, following the same procedure, the approximated IE is:

$$\ln\left(\frac{\mathscr{N}_{lce}}{\mathscr{N}_{hce}}\right) = \frac{\mathscr{N}_{lce}}{\mathscr{N}_{hce}} - 1 \tag{A.19}$$

Meanwhile, the actual IE is:

$$\frac{\mathscr{N}_{lce}}{\mathscr{N}_{hce}} - 1 \tag{A.20}$$

Thus, the measurement error is: $\psi = 0$. □

■ ***Proposition 1.*** First, the ratio of $\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ equals to

$$\frac{\mathscr{D}_{lce}(\boldsymbol{\tau}'_t;\rho_t)}{\mathscr{D}_{hce}(\boldsymbol{\tau}'_t;\rho_t)} = \frac{\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}\tau_{u,t}}\frac{a_t^2\tau_{u,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t}+\tau_{s,t})}{\tau_{s,t}\tau_{\varepsilon,t}}$$

$$= 1+\frac{\tau_{s,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{s,t}^2} \tag{A.21}$$

Hence, the change rate is $\frac{\mathscr{D}_{lce}(\boldsymbol{\tau}'_t;\rho_t)}{\mathscr{D}_{hce}(\boldsymbol{\tau}'_t;\rho_t)}-1 = \frac{\tau_{s,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{s,t}^2} \geqslant 0$ and there is always $\mathscr{D}_{lce}(\boldsymbol{\tau}'_t;\rho_t) \geqslant \mathscr{D}_{hce}(\boldsymbol{\tau}'_t;\rho_t)$. Use the equilibrium condition again

$$e^{\rho_t k_t^L}\sqrt{\frac{\tau_{\varepsilon,t}^{-1}}{(\tau_{s,t}+a_t^2\tau_{u,t})^{-1}+\tau_{\varepsilon,t}^{-1}}} = 1$$

and then rewrite the change rate that is the proxy for the IP effect as:

$$\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \frac{\mathscr{D}_{lce}(\boldsymbol{\tau}'_t;\rho_t)}{\mathscr{D}_{hce}(\boldsymbol{\tau}'_t;\rho_t)}-1 = \frac{\tau_{s,t}(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}(\tau_{s,t}+a_t^2\tau_{u,t})e^{2\rho_t k_t^L}}{\tau_{s,t}^2}$$

$$= n_t\left(1+n_t\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right)e^{2\rho_t k_t^L}+n_t\left(1+\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right) \tag{A.22}$$

Thus, the effect is linear in $e^{2\rho_t k_t^L}$ with a positive slope $\zeta_1(\boldsymbol{\tau}'_t;\rho_t) = n_t\left(1+n_t\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right)$ and a positive intercept $\zeta_0(\boldsymbol{\tau}'_t;\rho_t) = n_t\left(1+\frac{\tau_{u,t}\tau_{\varepsilon,t}}{\rho_t^2}\right)$.

Second, the change rate in $\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$, as the proxy for the IE effect, equals to

$$\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \frac{\mathscr{N}_{lce}(\boldsymbol{\tau}'_t;\rho_t)}{\mathscr{N}_{hce}(\boldsymbol{\tau}'_t;\rho_t)}-1 = \frac{\tau_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}(\tau_{s,t}+\tau_{\varepsilon,t})}\frac{a_t^2\tau_{u,t}+\tau_{\varepsilon,t}}{\tau_{u,t}\tau_{\varepsilon,t}}-1$$

$$= \frac{1}{1+\frac{\rho_t^2}{\tau_{\varepsilon,t}(1-\mathcal{Q}_t)}}\left[\frac{\tau_{u,t}}{1-\mathcal{Q}_t}-1\right] \tag{A.23}$$

The sign of $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ depends on the aggregation of the degree of the noise trading $\tau_{u,t}$ and the relative information quality $\mathcal{Q}_t$. Hence, its absolute size $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ follows the piecewise function:

$$|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| = \begin{cases} \frac{\tau_{s,t}\tau_{\varepsilon,t}}{\tau_{s,t}\tau_{\varepsilon,t}+\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})}\left[\frac{\tau_{u,t}}{1-\mathcal{Q}_t}-1\right] & \text{, if } \tau_{u,t}+\mathcal{Q}_t>1 \\ 0 & \text{, if } \tau_{u,t}+\mathcal{Q}_t=1 \\ \frac{\tau_{s,t}\tau_{\varepsilon,t}}{\tau_{s,t}\tau_{\varepsilon,t}+\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})}\left[1-\frac{\tau_{u,t}}{1-\mathcal{Q}_t}\right] & \text{, if } \tau_{u,t}+\mathcal{Q}_t<1 \end{cases} \tag{A.24}$$

Together with the effect delivered by the factor of $\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$, the overall effect rests on a few contingent cases.

- ▷ **Claim 1.1:** $\tau_{u,t}+\mathcal{Q}_t \geqslant 1$: there are simultaneously $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) \geqslant 0$. Hence, the overall effect on the welfare must satisfy $\widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{lce} > \widetilde{\mathscr{W}}(\boldsymbol{\tau}'_t;\rho_t)_{hce}$.
- ▷ **Claim 1.2:** $\tau_{u,t}+\mathcal{Q}_t < 1$: there are simultaneously $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < 0$. Hence, the two effects are in the battlefield.

- If $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)|$, then welfare at the LCE must be higher: $\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{lce} > \widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{hce}$. The sufficient condition holds if for some quadruples $\{\boldsymbol{\tau}_t'; \rho_t\}$ such that $[(\tau_{s,t} + \tau_{\varepsilon,t})(\tau_{\varepsilon,t} + a_t^2\tau_{u,t}) + \tau_{\varepsilon,t}\tau_{s,t}]/\tau_{s,t}^2 > \{a_t^2[\tau_{s,t} - \tau_{u,t}(\tau_{\varepsilon,t} + \tau_{s,t})]\}/[a_t^2\tau_{s,t} + \tau_{\varepsilon,t}\tau_{s,t} + \tau_{\varepsilon,t}^2]$.
- If $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) < |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)|$, then welfare at the HCE must be higher: $\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{lce} < \widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{hce}$. The sufficient condition holds if for some quadruples $\{\boldsymbol{\tau}_t'; \rho_t\}$ such that $[(\tau_{s,t} + \tau_{\varepsilon,t})(\tau_{\varepsilon,t} + a_t^2\tau_{u,t}) + \tau_{\varepsilon,t}\tau_{s,t}]/\tau_{s,t}^2 < \{a_t^2[\tau_{s,t} - \tau_{u,t}(\tau_{\varepsilon,t} + \tau_{s,t})]\}/[a_t^2\tau_{s,t} + \tau_{\varepsilon,t}\tau_{s,t} + \tau_{\varepsilon,t}^2]$.
- If $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) = |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)|$, then welfare is the same at either the LCE or the HCE: $\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{lce} = \widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{hce}$. The sufficient condition holds if some quadruples $\{\boldsymbol{\tau}_t'; \rho_t\}$ such that $[(\tau_{s,t}+\tau_{\varepsilon,t})(\tau_{\varepsilon,t}+a_t^2\tau_{u,t})+\tau_{\varepsilon,t}\tau_{s,t}]/\tau_{s,t}^2 = \{a_t^2[\tau_{s,t} - \tau_{u,t}(\tau_{\varepsilon,t} + \tau_{s,t})]\}/[a_t^2\tau_{s,t} + \tau_{\varepsilon,t}\tau_{s,t} + \tau_{\varepsilon,t}^2]$.

□

■ ***Proposition 2.*** (i) $\frac{\partial\Delta\mathscr{D}_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t';\rho_t)}{\partial k_t} = \frac{2\zeta_1(\boldsymbol{\tau}_t';\rho_t)e^{2\rho_t k_t}\rho_t}{(1-\varphi^L)^2} \geqslant 0$ for any $0 < k_t < k_t^L$, so $\frac{\partial\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t';\rho_t)_{h+\rightsquigarrow l-}}{\partial k_t} \geqslant 0$. Hence, for any $\Delta k_t < 0$, $\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{h+\rightsquigarrow l-} \leqslant 0$.

(ii) As $k_t \searrow 0$, $\lim_{k_t\searrow 0}\Delta\mathscr{D}_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t) = \zeta_1(\boldsymbol{\tau}_t'; \rho_t) + \zeta_0(\boldsymbol{\tau}_t'; \rho_t) \geqslant 0$ and therefore $\lim_{k_t\searrow 0}\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t'; \rho_t)_{h+\rightsquigarrow l-} \geqslant 0$.

(iii) $\frac{\partial\mathscr{D}_{k_t>k_t^H}(\boldsymbol{\tau}_t';\rho_t)}{\partial k_t} \equiv 0$ for any $k_t > k_t^H$ and therefore $\Delta\mathscr{D}_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t) = 0$.

(iv) The cross-partial derivatives:

$$\frac{\partial\Delta\mathscr{D}^2_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t)}{\partial k_t\partial\rho_t} = \frac{2e^{2\rho_t k_t}}{(1-\varphi^L)^2}\frac{\tau_{\varepsilon,t}}{\tau_{s,t}}\left[1 + 2k_t\rho_t + \frac{\tau_{\varepsilon,t}^2\tau_{u,t}}{\rho_t}\left(2k_t - \frac{1}{\rho_t}\right)\right] \tag{A.25}$$

$$\frac{\partial\Delta\mathscr{D}^2_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t)}{\partial k_t\partial\tau_{\varepsilon,t}} = \frac{2e^{2\rho_t k_t}\rho_t}{(1-\varphi^L)^2}\frac{\partial\zeta_1(\boldsymbol{\tau}_t'; \rho_t)}{\partial\tau_{\varepsilon,t}} \geqslant 0 \tag{A.26}$$

$$\frac{\partial\Delta\mathscr{D}^2_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t)}{\partial k_t\partial\tau_{s,t}} = \frac{2e^{2\rho_t k_t}\rho_t}{(1-\varphi^L)^2}\frac{\partial\zeta_1(\boldsymbol{\tau}_t'; \rho_t)}{\partial\tau_{s,t}} \leqslant 0 \tag{A.27}$$

$$\frac{\partial\Delta\mathscr{D}^2_{h+\rightsquigarrow l-}(\boldsymbol{\tau}_t'; \rho_t)}{\partial k_t\partial\tau_{u,t}} = \frac{2e^{2\rho_t k_t}\rho_t}{(1-\varphi^L)^2}\frac{\partial\zeta_1(\boldsymbol{\tau}_t'; \rho_t)}{\partial\tau_{u,t}} \geqslant 0 \tag{A.28}$$

The first derivative is open ended but if $2k_t > \frac{1}{\rho_t}$, the sign must be positive. It means that the risk tolerance is constrained. The signs for the rest derivatives follow Remark 2.1 in the main text. □

■ ***Proposition 3.*** The limits of two effects when the insurance opportunities are fully realized are:

$$\begin{aligned}\lim_{\tau_{s,t}\nearrow\infty}\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) &= \lim_{\tau_{s,t}\nearrow\infty}\frac{(\tau_{s,t} + \tau_{\varepsilon,t})(\tau_{\varepsilon,t} + a_t^2\tau_{u,t}) + \tau_{\varepsilon,t}\tau_{s,t}}{\tau_{s,t}^2} \\ &= \lim_{\tau_{s,t}\nearrow\infty}\left(\frac{2\tau_{\varepsilon,t}}{\tau_{s,t}} + \frac{\tau_{\varepsilon,t}^2\tau_{u,t}}{\rho_t^2\tau_{s,t}} + \frac{\tau_{\varepsilon,t}^3\tau_{u,t}}{\rho_t^2\tau_{s,t}^2} + \frac{\tau_{\varepsilon,t}^2}{\tau_{s,t}^2}\right) = 0\end{aligned} \tag{A.29}$$

$$\lim_{\tau_{s,t}\nearrow\infty}\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)=\lim_{\tau_{s,t}\nearrow\infty}\frac{a_t^2[-\tau_{s,t}+\tau_{u,t}(\tau_{\varepsilon,t}+\tau_{s,t})]}{a_t^2\tau_{s,t}+\tau_{\varepsilon,t}\tau_{s,t}+\tau_{\varepsilon,t}^2}$$

$$=\frac{\tau_{\varepsilon,t}^2}{\rho_t^2}\frac{\lim_{\tau_{s,t}\nearrow\infty}\left(-1+\frac{\tau_{u,t}(\tau_{\varepsilon,t}+\tau_{s,t})}{\tau_{s,t}}\right)}{\lim_{\tau_{s,t}\nearrow\infty}\left(\frac{\tau_{\varepsilon,t}^2}{\rho_t^2}+\tau_{\varepsilon,t}+\frac{\tau_{\varepsilon,t}^2}{\tau_{s,t}}\right)}$$

$$=\frac{\tau_{\varepsilon,t}(\tau_{u,t}-1)}{\rho_t^2+\tau_{\varepsilon,t}} \tag{A.30}$$

Hence, the IP effect $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)$ can be zero whereas the sign of the IE effect $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)$ is not determined, but rather contingent to $\tau_{u,t}$.

Suppose that the event of no insurance is $A=\{\theta_{t+1}:\tau_{s,t}\lll\infty\}$ and the event of welfare cuts is $B=\{\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t';\rho_t):\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t';\rho_t)>0\}$. The Hirshleifer effect implies $A\Rightarrow B$. If we observe that full insurance have caused welfare cuts, this directly entails $\bar{A}=\{\theta_{t+1}:\tau_{s,t}\nearrow\infty\}\Rightarrow B$. Its contrapositive is $\bar{B}=\{\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t';\rho_t):\Delta\widetilde{\mathscr{W}}(\boldsymbol{\tau}_t';\rho_t)\leqslant 0\}\Rightarrow A$ that is the negation of $\bar{B}\Rightarrow\bar{A}$ that is the contrapositive of $A\Rightarrow B$. Thus, the Hirshleifer effect is false. Furthermore, suppose that B' means no welfare cuts such that $B'=\bar{B}$, If we observe $\bar{A}\Rightarrow B'$, this will be the converse to the Hirshleifer effect's contrapositive $\bar{B}\Rightarrow\bar{A}$. Hence, the Hirshleifer effect is irrelevant to us. □

■ ***Proposition 4.*** (i) In the first case, the limits of the IP effect and the IE effect are

$$lim_{\tau_{u,t}\searrow 0}\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)=\frac{2\tau_{s,t}\tau_{\varepsilon,t}+\tau_{\varepsilon,t}^2}{\tau_{s,t}^2}\geqslant 0 \tag{A.31}$$

$$lim_{\tau_{u,t}\searrow 0}\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)=-\frac{\tau_{s,t}\tau_{\varepsilon,t}}{(\tau_{s,t}+\tau_{\varepsilon,t})\rho_t^2+\tau_{s,t}\tau_{\varepsilon,t}}\leqslant 0 \tag{A.32}$$

When $\tau_{u,t}\searrow 0$ and $\mathscr{Q}_t\leqslant 1$, the second effect must be non-positive and necessarily create welfare improvements, which backward affirms the finding in Proposition 1. The overall welfare effect $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)$ is:

$$\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)=\frac{\tau_{s,t}\tau_{\varepsilon,t}^3+\tau_{\varepsilon,t}^3\rho_t^2+2\tau_{s,t}^2\tau_{\varepsilon,t}^2+3\tau_{s,t}\tau_{\varepsilon,t}^2\rho_t^2+2\tau_{s,t}^2\tau_{\varepsilon,t}\rho_t^2-\tau_{s,t}^3\tau_{\varepsilon,t}}{\tau_{s,t}^2\left[\tau_{s,t}\tau_{\varepsilon,t}+\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})\right]} \tag{A.33}$$

The overall welfare effect is welfare cuts if $\tau_{s,t}^3<(2\tau_{s,t}+\tau_{\varepsilon,t})\left[\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})+\tau_{s,t}\tau_{\varepsilon,t}\right]$ and is welfare gains otherwise.

(ii) In the second case, market prices become $\mathscr{P}(s_t)=s_t-\frac{1}{a_t}$ due to $u_t=\mathbb{E}(u_t)=-1$ and market prices display a one-for-one relationship with the signals. As the signal has been fully revealed, which means that prices reveal the signals and people can learn the signals from prices without purchasing, the welfare must stay, so there is $lim_{\tau_{u,t}\nearrow\infty}\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)=e^{\rho_t k_t}$. Otherwise there is no closed-form solution since $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t';\rho_t)$ encounter poor approximation.

(iii) $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ are increasing in $\tau_{u,t}$:

$$\frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}} = \frac{\tau_{s,t}\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}^3}{\tau_{s,t}^2\rho_t^2} \geqslant 0 \tag{A.34}$$

$$\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}} = \left(\frac{\tau_{s,t}}{\tau_{\varepsilon,t}+\tau_{s,t}}+\frac{\rho_t^2}{\tau_{\varepsilon,t}}\right)^{-1} \geqslant 0 \tag{A.35}$$

The second order derivatives are zero, say $\frac{\partial^2\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}^2} = \frac{\partial^2\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}^2} = 0$ so both effects are linear in $\tau_{u,t}$. The absolute responsiveness of the overall welfare in $\tau_{u,t}$ is defined as $\gamma^a_{\tau_{u,t}} = \frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}} + \frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}}$ that can be shown constant and positive.

$$\gamma^a_{\tau_{u,t}} = \frac{\tau_{s,t}\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}^3}{\tau_{s,t}^2\rho_t^2} + \left(\frac{\tau_{s,t}}{\tau_{\varepsilon,t}+\tau_{s,t}}+\frac{\rho_t^2}{\tau_{\varepsilon,t}}\right)^{-1} \geqslant 0 \tag{A.36}$$

Moreover, the relative responsiveness of the overall welfare in $\tau_{u,t}$ is defined as $\gamma^r_{\tau_{u,t}}$ that is $\frac{\frac{\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{u,t}}}{\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}$. As high $\tau_{u,t}$ raises the overall welfare while the $\gamma^a_{\tau_{u,t}}$ is constant, $\gamma^r_{\tau_{u,t}}$ is falling in $\tau_{u,t}$, when the net welfare is cuts whereas it is rising when the net welfare is gains. Assume that $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)\neq 0$, the relative responsiveness can be written as:

$$\gamma^r_{\tau_{u,t}} = \frac{\frac{\tau_{s,t}\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}^3}{\tau_{s,t}^2\rho_t^2} + \frac{\tau_{\varepsilon,t}(\tau_{\varepsilon,t}+\tau_{s,t})}{\tau_{\varepsilon,t}^2+\rho_t^2(\tau_{\varepsilon,t}+\tau_{s,t})}}{\frac{(\tau_{s,t}+\tau_{\varepsilon,t})\left(\tau_{\varepsilon,t}+\tau_{\varepsilon,t}^2\rho_t^{-2}\tau_{u,t}\right)+\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{s,t}^2} + \frac{\tau_{u,t}\tau_{\varepsilon,t}^2+(\tau_{u,t}-1)\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{\varepsilon,t}\tau_{s,t}+\rho_t^2(\tau_{\varepsilon,t}+\tau_{s,t})}} \geqslant 0 \tag{A.37}$$

(iv) Suppose that when $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)\geqslant 0$, $\tau^*_{u,t}$ poises the two effects, say $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau^*_{u,t}) = \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau^*_{u,t})$. Recall that the slopes of both effects in $\tau_{u,t}$ are linear so they can only intersect once. Thus, given the limits at zero $\tau_{u,t}$, if $\tau^*_{u,t}<0$, this means that in $\tau_{u,t}\in(0,\infty)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ always dominates $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$. Otherwise if $\tau^*_{u,t}>0$, it is the fixed point under which $\tau_{u,t}\in(0,\tau^*_{u,t})$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ dominates $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$, and above which $\tau_{u,t}\in(\tau^*_{u,t},\tau^\dagger_{u,t})$ the dominance switches. This $\tau^*_{u,t}$ can be solved as:

$$\tau^*_{u,t} = \frac{\rho_t^2(\tau_{s,t}^3+2\tau_{s,t}^2\tau_{\varepsilon,t}+2\tau_{s,t}^2\rho_t^2+\tau_{s,t}\tau_{\varepsilon,t}^2+3\tau_{s,t}\tau_{\varepsilon,t}\rho_t^2+\tau_{\varepsilon,t}^2\rho_t^2)}{\tau_{s,t}^3\rho_t^2-\tau_{s,t}^2\tau_{\varepsilon,t}^2-\tau_{s,t}\tau_{\varepsilon,t}^3-2\tau_{s,t}\tau_{\varepsilon,t}^2\rho_t^2-\tau_{\varepsilon,t}^3\rho_t^2} \tag{A.38}$$

Consequently, the sign of $\tau^*_{u,t}$ depends on the sign of $\tau_{s,t}^3\rho_t^2-\tau_{s,t}^2\tau_{\varepsilon,t}^2-\tau_{s,t}\tau_{\varepsilon,t}^3-2\tau_{s,t}\tau_{\varepsilon,t}^2\rho_t^2-\tau_{\varepsilon,t}^3\rho_t^2$.

The second constraint concerns the stark difference in the limits at zero $\tau_{u,t}$. When $\tau_{u,t}=1-\mathscr{Q}_t$, there is $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)=0$. Before $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ touches down to zero, the conflict raises the concern about if there is a switch in the sign of the overall welfare. The question entails the comparison between $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$. Suppose that $\tilde{\tau}_{u,t}\in(0,1-\mathscr{Q}_t)$ such that $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tilde{\tau}_{u,t}) = |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tilde{\tau}_{u,t})|$ and this is also a fixed point:

$$\tilde{\tau}_{u,t} = \frac{\rho_t^2(\tau_{s,t}^3-2\tau_{s,t}^2\tau_{\varepsilon,t}-2\tau_{s,t}^2\rho_t^2-\tau_{s,t}\tau_{\varepsilon,t}^2-3\tau_{s,t}\tau_{\varepsilon,t}\rho_t^2-\tau_{\varepsilon,t}^2\rho_t^2)}{\tau_{s,t}^3\rho_t^2+\tau_{s,t}^2\tau_{\varepsilon,t}^2+\tau_{s,t}\tau_{\varepsilon,t}^3+2\tau_{s,t}\tau_{\varepsilon,t}^2\rho_t^2+\tau_{\varepsilon,t}^3\rho_t^2} \tag{A.39}$$

That $\lim_{\tau_{u,t}\searrow 0}|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$is larger than $\lim_{\tau_{u,t}\searrow 0}\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ entails $\tilde{\tau}_{u,t}\in(0,1-\mathscr{Q}_t)$ necessarily.

▷ **Claim 4.1:** If there are some tuples $\{\tau_{s,t}, \tau_{\varepsilon,t}, \rho_t\}$ such that $\tau_{s,t}^3\rho_t^2 > \tau_{\varepsilon,t}^2[(\tau_{s,t} + \rho_t^2)(\tau_{\varepsilon,t} + \tau_{s,t}) + \rho_t^2\tau_{s,t}]$ and $\tau_{s,t}^3\tau_{\varepsilon,t} > (2\tau_{s,t}\tau_{\varepsilon,t} + \tau_{\varepsilon,t}^2)[(\tau_{s,t} + \tau_{\varepsilon,t})\rho_t^2 + \tau_{s,t}\tau_{\varepsilon,t}]$, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) < 0$ in $\tau_{u,t} \in (0, \tilde{\tau}_u)$ and the size ordering is $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$. $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ in $\tau_{u,t} \in (\tilde{\tau}_u, 1 - \mathscr{Q}_t)$ and the size ordering is $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$. The size ordering for any $\tau_{u,t} \in (1 - \mathscr{Q}_t, \tau_{u,t}^*)$ is $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ and for any $\tau_{u,t} \in (\tau_{u,t}^*, \tau_{u,t}^\dagger)$ is $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$. For any $\tau_{u,t} \in (1 - \mathscr{Q}_t, \tau_{u,t}^\dagger)$, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$.

▷ **Claim 4.2:** If there are some tuples $\{\tau_{s,t}, \tau_{\varepsilon,t}, \rho_t\}$ such that $\tau_{s,t}^3\rho_t^2 > \tau_{\varepsilon,t}^2[(\tau_{s,t} + \rho_t^2)(\tau_{\varepsilon,t} + \tau_{s,t}) + \rho_t^2\tau_{s,t}]$ and $\tau_{s,t}^3\tau_{\varepsilon,t} < (2\tau_{s,t}\tau_{\varepsilon,t} + \tau_{\varepsilon,t}^2)[(\tau_{s,t} + \tau_{\varepsilon,t})\rho_t^2 + \tau_{s,t}\tau_{\varepsilon,t}]$, there is no such a non-negative $\tilde{\tau}_u$ such that the overall welfare switches its sign and therefore, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ for all possibly positive $\tau_{u,t}$ but there is a non-negative τ_u^* such that the importance for the overall welfare cuts switches. The size ordering: for any $\tau_{u,t} \in (0, 1 - \mathscr{Q}_t)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$, while for any $\tau_{u,t} \in (1 - \mathscr{Q}_t, \tau_{u,t}^*)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ and for any $\tau_{u,t} \in (\tau_{u,t}^*, \tau_{u,t}^\dagger)$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$.

▷ **Claim 4.3:** If there are some tuples $\{\tau_{s,t}, \tau_{\varepsilon,t}, \rho_t\}$ such that $\tau_{s,t}^3\rho_t^2 < \tau_{\varepsilon,t}^2[(\tau_{s,t} + \rho_t^2)(\tau_{\varepsilon,t} + \tau_{s,t}) + \rho_t^2\tau_{s,t}]$ and $\tau_{s,t}^3\tau_{\varepsilon,t} > (2\tau_{s,t}\tau_{\varepsilon,t} + \tau_{\varepsilon,t}^2)[(\tau_{s,t} + \tau_{\varepsilon,t})\rho_t^2 + \tau_{s,t}\tau_{\varepsilon,t}]$, there is no such a non-negative $\tau_{u,t}^*$ but there is a non-negative $\tilde{\tau}_u$ in $(0, 1 - \mathscr{Q}_t)$. Thus, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) < 0$ and the size ordering is $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ for any $\tau_{u,t} \in (0, \tilde{\tau}_u)$, and $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ for any $\tau_{u,t} \in (\tilde{\tau}_u, \tau_{u,t}^\dagger)$. For any $\tau_{u,t} \in (\tilde{\tau}_u, 1 - \mathscr{Q}_t)$, the size ordering is $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ while it is $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$, for any $\tau_{u,t} \in (1 - \mathscr{Q}_t, \tau_{u,t}^\dagger)$.

▷ **Claim 4.4:** If there are some tuples $\{\tau_{s,t}, \tau_{\varepsilon,t}, \rho_t\}$ such that $\tau_{s,t}^3\rho_t^2 < \tau_{\varepsilon,t}^2[(\tau_{s,t} + \rho_t^2)(\tau_{\varepsilon,t} + \tau_{s,t}) + \rho_t^2\tau_{s,t}]$ and $\tau_{s,t}^3\tau_{\varepsilon,t} < (2\tau_{s,t}\tau_{\varepsilon,t} + \tau_{\varepsilon,t}^2)[(\tau_{s,t} + \tau_{\varepsilon,t})\rho_t^2 + \tau_{s,t}\tau_{\varepsilon,t}]$, there is no either a non-negative $\tilde{\tau}_u$ or a non-negative $\tau_{u,t}^*$. Thus, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$ for any non-negative $\tau_{u,t}$. The size ordering: for any $\tau_{u,t} \in (0, 1 - \mathscr{Q}_t)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ while for any $\tau_{u,t} \in (1 - \mathscr{Q}_t, \tau_{u,t}^\dagger)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0$.

□

■ ***Proposition 5.*** (i) When $\tau_{\varepsilon,t} \searrow 0$, the two effects are being trivial and therefore, the net welfare change goes to zero because evidently, there are:

$$\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t|\tau_{\varepsilon,t} \searrow 0) = 0 \tag{A.40}$$

$$\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t|\tau_{\varepsilon,t} \searrow 0) = \lim_{\tau_{\varepsilon,t}\searrow 0} \frac{\tau_{u,t}\frac{\tau_{\varepsilon,t}+\tau_{s,t}}{\tau_{s,t}} - 1}{1 + \frac{\rho_t^2}{\tau_{\varepsilon,t}\frac{\tau_{s,t}}{\tau_{\varepsilon,t}+\tau_{s,t}}}} = \frac{\tau_{u,t} - 1}{\infty} = 0 \tag{A.41}$$

Thus, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t|\tau_{\varepsilon,t} \searrow 0) \approx \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t|\tau_{\varepsilon,t} \searrow 0) + \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t|\tau_{\varepsilon,t} \searrow 0) = 0$, which means that the net welfare effect is almost non-existent. However, $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) = 0$ does not necessarily imply $\tau_{\varepsilon,t} = 0$.

(ii) When $\tau_{\varepsilon,t} \nearrow \infty$, the limit of the first effect is evidently positive infinity:

$$\lim_{\tau_{\varepsilon,t}\nearrow\infty} \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \infty \tag{A.42}$$

The limit of the second effect is

$$\lim_{\tau_{\varepsilon,t}\nearrow\infty} \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \frac{\lim_{\tau_{\varepsilon,t}\nearrow\infty}[\tau_{s,t}(\tau_{u,t}-1)+\tau_{u,t}\tau_{\varepsilon,t}]}{\lim_{\tau_{\varepsilon,t}\nearrow\infty}\left[\tau_{s,t}+\rho_t^2\left(\frac{\tau_{s,t}}{\tau_{\varepsilon,t}}+1\right)\right]} = \frac{\infty}{\tau_{s,t}+\rho_t^2} = \infty \tag{A.43}$$

and therefore, $\lim_{\tau_{\varepsilon,t}\nearrow\infty} \Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = \infty$.

(iii) The absolute responsiveness of the overall welfare to informational advantages is:

$$\gamma^a_{\tau_{\varepsilon,t}} = \frac{\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} = \frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} + \frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} \tag{A.44}$$

where

$$\frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} = \left\{\frac{3\tau_{\varepsilon,t}^2\tau_{u,t}+2\tau_{\varepsilon,t}\rho_t^2+2\tau_{s,t}\tau_{\varepsilon,t}\tau_{u,t}+2\tau_{s,t}\rho_t^2}{\tau_{s,t}^2\rho_t^2}\right\} \tag{A.45}$$

$$\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} = \left\{\frac{\tau_{s,t}\tau_{\varepsilon,t}^4\tau_{u,t}+\tau_{\varepsilon,t}^4\rho_t^2\tau_{u,t}+2\tau_{s,t}\tau_{\varepsilon,t}^3\rho_t^2\tau_{u,t}+\tau_{s,t}^2\tau_{\varepsilon,t}^2\rho_t^2(\tau_{u,t}-1)}{\left[\tau_{s,t}\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})\right]^2}\right\} \tag{A.46}$$

$\frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} \geqslant 0$ is evident while the sign of $\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}}$ is not unambiguous. The second order derivatives are $\frac{\partial^2\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}^2} = \frac{6\tau_{u,t}\tau_{\varepsilon,t}+2\tau_{u,t}\tau_{s,t}+2\rho_t^2}{\rho_t^2\tau_{s,t}^2} \geqslant 0$ and $\frac{\partial^2\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}^2} = \frac{2\tau_{s,t}^2\rho_t^2\tau_{\varepsilon,t}^3[\tau_{s,t}(1-\tau_{u,t})+\rho_t^2]}{\left[\tau_{s,t}\tau_{\varepsilon,t}^2+\rho_t^2\tau_{\varepsilon,t}(\tau_{\varepsilon,t}+\tau_{s,t})\right]^3}$. Thus, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ is convex and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ is convex if $\tau_{s,t}(1-\tau_{u,t})+\rho_t^2 > 0$ and is concave if $\tau_{s,t}(1-\tau_{u,t})+\rho_t^2 < 0$. But this is not too surprising because the IE works differently on information of different quality. Define a function $\mathcal{O}(\tau_{\varepsilon,t};\rho_t,\tau_{s,t},\tau_{u,t}) = \tau_{s,t}\tau_{\varepsilon,t}^4\tau_{u,t}+\tau_{\varepsilon,t}^4\rho_t^2\tau_{u,t}+2\tau_{s,t}\tau_{\varepsilon,t}^3\rho_t^2\tau_{u,t}+\tau_{s,t}^2\tau_{\varepsilon,t}^2\rho_t^2(\tau_{u,t}-1)$ that pins down the indeterminacy of the sign. $\mathcal{O}(\tau_{\varepsilon,t};\rho_t,\tau_{s,t},\tau_{u,t}) = 0$ delivers three real roots and wherein the first one is weakly negative, and the second is zero, and the last one, however, owns an ambiguous sign. In fact, the indeterminacy can be affirmed by pinning down this ambiguous sign. This root can be shown equal to $\frac{\tau_{s,t}\rho_t\left[\sqrt{\tau_{u,t}(\tau_{s,t}-\tau_{s,t}\tau_{u,t}+\rho_t^2)}-\tau_{u,t}\rho_t\right]}{\tau_{u,t}(\tau_{s,t}+\rho_t^2)}$, denoted as $\tilde{\tau}_{\varepsilon,t}$. Since $\mathcal{O}(\tau_{\varepsilon,t};\rho_t,\tau_{s,t},\tau_{u,t})$ has a critical point at $\tau_{\varepsilon,t}=0$, there are only two cases where this critical point is a local maxima once $\tilde{\tau}_{\varepsilon,t} > 0$ or a local minima once $\tilde{\tau}_{\varepsilon,t} \leqslant 0$. Given that $\mathcal{O}(\tau_{\varepsilon,t};\rho_t,\tau_{s,t},\tau_{u,t}|\tau_{\varepsilon,t}=0)=0$, the first case implies that $\mathcal{O}(\tau_{\varepsilon,t};\rho_t,\tau_{s,t},\tau_{u,t})$ will becomes negative in $\tau_{\varepsilon,t}\in\mathbb{R}_+$ whereas the second case implies that it turns positive in $\tau_{\varepsilon,t}\in\mathbb{R}_+$.

- ▷ If $\tilde{\tau}_{\varepsilon,t} \leqslant 0$, there is $\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} > 0$ for any $\tau_{\varepsilon,t} > 0$. The condition $\frac{\tau_{s,t}\rho_t\left[\sqrt{\tau_{u,t}(\tau_{s,t}-\tau_{s,t}\tau_{u,t}+\rho_t^2)}-\tau_{u,t}\rho_t\right]}{\tau_{u,t}(\tau_{s,t}+\rho_t^2)} \leqslant 0$ implies $(\tau_{s,t}+\rho_t^2)(1-\tau_{u,t}) \leqslant 0$ and further entails $\tau_{u,t} \geqslant 1$ that is, however, not sufficient to pin down the curvature.
- ▷ If $\tilde{\tau}_{\varepsilon,t} > 0$, there is $\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} < 0$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < 0$ in $0 < \tau_{\varepsilon,t} < \tilde{\tau}_{\varepsilon,t}$, given $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{\varepsilon,t}\searrow 0) = 0$ while it will be $\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} > 0$ for any

$\tau_{\varepsilon,t} > \tilde{\tau}_{\varepsilon,t}$. The condition $\frac{\tau_{s,t}\rho_t\left[\sqrt{\tau_{u,t}(\tau_{s,t}-\tau_{s,t}\tau_{u,t}+\rho_t^2)}-\tau_{u,t}\rho_t\right]}{\tau_{u,t}(\tau_{s,t}+\rho_t^2)} > 0$ implies $(\tau_{s,t}+\rho_t^2)(1-\tau_{u,t}) > 0$ and further entails $\tau_{u,t} < 1$ that enforces strict convexity due to $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{u,t} < 1) > 0$.

Likewise, the relative responsiveness can be written as $\gamma^r_{\tau_{\varepsilon,t}} = \frac{\frac{\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}}}{\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}$, if assume $\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) \neq 0$.

$$\gamma^r_{\tau_{\varepsilon,t}} = \frac{\frac{3\tau_{\varepsilon,t}^2\tau_{u,t}+2\tau_{\varepsilon,t}\rho_t^2+2\tau_{s,t}\tau_{\varepsilon,t}\tau_{u,t}+2\tau_{s,t}\rho_t^2}{\tau_{s,t}^2\rho_t^2}+\frac{\tau_{s,t}\tau_{\varepsilon,t}^4\tau_{u,t}+\tau_{\varepsilon,t}^4\rho_t^2\tau_{u,t}+2\tau_{s,t}\tau_{\varepsilon,t}^3\rho_t^2\tau_{u,t}+\tau_{s,t}^2\tau_{\varepsilon,t}^2\rho_t^2(\tau_{u,t}-1)}{\left[\tau_{s,t}\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}\rho_t^2(\tau_{s,t}+\tau_{\varepsilon,t})\right]^2}}{\frac{(\tau_{s,t}+\tau_\varepsilon)\left(\tau_{\varepsilon,t}+\tau_{\varepsilon,t}^2\rho_t^{-2}\tau_{u,t}\right)+\tau_\varepsilon\tau_{s,t}}{\tau_{s,t}^2}+\frac{\tau_{u,t}\tau_{\varepsilon,t}^2+(\tau_{u,t}-1)\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{\varepsilon,t}\tau_{s,t}+\rho_t^2(\tau_{\varepsilon,t}+\tau_{s,t})}} \tag{A.47}$$

(iv) The overall welfare patterns are subject to two constraints. The first one is if $\tilde{\tau}_{\varepsilon,t} > 0$ such that for some small $\tau_{\varepsilon,t}$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ is negative and convex, and the second one is if two effects can cross such that the dominance in increasing overall welfare cuts might switch, when both effects cause welfare cuts. In the first place, define $\tau^*_{\varepsilon,t} > 0$ at which the two effects are poised, say $\Delta\mathscr{D} = \Delta\mathscr{N}$:

$$\tau^*_{\varepsilon,t} = \frac{\rho_t\chi_t - \tau_{s,t}\tau_{u,t}\rho_t^2 - \tau_{s,t}\rho_t^2 - \rho_t^4}{2[(\tau_{s,t}+\rho_t^2)\tau_{u,t}]} \tag{A.48}$$

where $\chi_t = (4\tau_{s,t}^3\tau_{u,t}^2 - 4\tau_{s,t}^3\tau_{u,t} + 5\tau_{s,t}^2\tau_{u,t}^2\rho_t^2 - 10\tau_{s,t}^2\tau_{u,t}\rho_t^2 + \tau_{s,t}^2\rho_t^2 - 6\tau_{s,t}\tau_{u,t}\rho_t^4 + 2\tau_{s,t}\rho_t^4 + \rho_t^6)^{\frac{1}{2}}$[11]. The further scrutinizing enforces $\tau^*_{\varepsilon,t} < 0$ and it implicitly requires $\tau_{u,t} \in \left(0, \frac{2\rho_t^2+\tau_{s,t}}{\tau_{s,t}}\right)$. Therefore, when $\tau_{u,t} < 1$, $\tau^*_{\varepsilon,t}$ must be negative. This means that when $\Delta\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$ is convex, there will be no positive intersection between $\Delta\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$. Moreover, when $\tau_{u,t} \in \left(1, \frac{2\rho_t^2+\tau_{s,t}}{\tau_{s,t}}\right)$, there is still $\tau^*_{\varepsilon,t} < 0$ whereas once $\tau_{u,t} > \frac{2\rho_t^2+\tau_{s,t}}{\tau_{s,t}}$, $\tau^*_{\varepsilon,t} > 0$ and therefore, the intersection can be seen.

On the one hand, let us focus on the case of $\tau^*_{\varepsilon,t} > 0$ and the straight-up question is in what patterns the two effects move to the point $\tau^*_{\varepsilon,t}$. Intuitively, the variety depends on the curvature of $\Delta\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$, given that $\Delta\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ is known convex at all times. Using the derived second-order derivative, the convexity is obtained if $\tau_{u,t} \in \left(1, \frac{\tau_{s,t}+\rho_t^2}{\tau_{s,t}}\right)$ and otherwise if $\tau_{u,t} > \frac{\tau_{s,t}+\rho_t^2}{\tau_{s,t}}$, it becomes concave. Irrespective of the curvature, once $\tau^*_{\varepsilon,t} > 0$, there are the following size orderings. When $\tau_{\varepsilon,t} < \tau^*_{\varepsilon,t}$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$, and when $\tau_{\varepsilon,t} > \tau^*_{\varepsilon,t}$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$. On the other hand, if $\tau^*_{\varepsilon,t} \leqslant 0$, there will be no intersection in $\mathbb{R}_+$, which means that one effect always dominates the other one. Given the root distribution of $\tau^*_{\varepsilon,t} = 0$, it implies that $\Delta\mathscr{D} - \Delta\mathscr{N}$ must be positive for $\tau_{\varepsilon,t} > 0$ and therefore, it is $\Delta\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ that dominates. Denote $\hat{\tau}_{\varepsilon,t} = \frac{(1-\tau_{u,t})\tau_{s,t}}{\tau_{u,t}}$ such that $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = 0$.

▷ **Claim 5.1:** If $\tau_{u,t} < 1$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) < 0$ in $\tau_{\varepsilon,t} \in (0, \hat{\tau}_{\varepsilon,t})$ and it drops to the minimum at $\tau_{\varepsilon,t} = \tilde{\tau}_{\varepsilon,t}$ before bouncing back. In this case, $\tau^*_{\varepsilon,t} < 0$. Define $\tau'_{\varepsilon,t} > 0$ at which $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| = \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ ($\tau'_{\varepsilon,t}$ not shown but solv-

[11] Denote the four roots as $\{\tau^{[1]}_{\varepsilon,t}, \tau^{[2]}_{\varepsilon,t}, \tau^{[3]}_{\varepsilon,t}, \tau^{[4]}_{\varepsilon,t}\}$ and the solutions are $\tau^{[1]}_{\varepsilon,t} = 0$, $\tau^{[2]}_{\varepsilon,t} = -\tau_{s,t}$, $\tau^{[3]}_{\varepsilon,t} = \tau^*_{\varepsilon,t}$, $\tau^{[4]}_{\varepsilon,t} = -\frac{\rho_t\chi_t+\tau_{s,t}\tau_{u,t}\rho_t^2+\tau_{s,t}\rho_t^2+\rho_t^4}{2[(\tau_{s,t}+\rho_t^2)\tau_{u,t}]}$. Among them, $\tau^{[2]}_{\varepsilon,t}$ and $\tau^{[4]}_{\varepsilon,t}$ are weakly negative for sure, so they are not useful to pin down the question. $\tau^{[1]}_{\varepsilon,t}$ is zero and affirms the derivation earlier that zero is a fixed point, irrespective of all else. $\tau^*_{\varepsilon,t}$ is used here for pinning down the question once it is positive.

able because it is also a fixed point). $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ and $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ can cross if $\lim_{\tau_{\varepsilon,t}\searrow 0}\frac{\partial|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|}{\partial\tau_{\varepsilon,t}} > \lim_{\tau_{\varepsilon,t}\searrow 0}\frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}}$, say $\tau_{u,t} > 1+\frac{2\rho_t^2}{\tau_{s,t}}$, and this is the same restriction for $\Delta\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ crossing $\Delta\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$. Thus, the size ordering is that when $\tau_{\varepsilon,t}\in(0,\tau'_{\varepsilon,t})$, there are net welfare improvements and $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$. When $\tau_{\varepsilon,t}\in(\tau'_{\varepsilon,t},\hat{\tau}_\varepsilon)$, the welfare cuts dominate the welfare gains and $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$, and when $\tau_{\varepsilon,t}\in(\hat{\tau}_\varepsilon,\infty)$, , the welfare cuts caused by $\Delta\mathscr{D}(\boldsymbol{\tau}'_t;\rho_t)$ dominate the welfare cuts caused by $\Delta\mathscr{N}(\boldsymbol{\tau}'_t;\rho_t)$ and $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$.

- ▷ **Claim 5.2:** The only difference from Claim 5.1 is that $\tau_{u,t}\leqslant 1+\frac{2\rho_t^2}{\tau_{s,t}}$ and $\tau'_{\varepsilon,t}>0$ cannot be found. Thus, the size ordering is that when $\tau_{\varepsilon,t}\in(0,\hat{\tau}_\varepsilon)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$, and when $\tau_{\varepsilon,t}\in(\hat{\tau}_\varepsilon,\infty)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$.
- ▷ **Claim 5.3:** If $\tau_{u,t}\geqslant 1$ and $\tau^*_{\varepsilon,t}>0$, $\frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} > 0$ and $\frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{\varepsilon,t}} > 0$ for any $\tau_{\varepsilon,t}>0$, but there is a switch in dominance at $\tau^*_{\varepsilon,t}$. Moving to this point, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ is concave if $\tau_{u,t} > \frac{\tau_{s,t}+\rho_t^2}{\tau_{s,t}}$ and is convex if $\tau_{u,t}\in\left(1,\frac{\tau_{s,t}+\rho_t^2}{\tau_{s,t}}\right)$. Hence, the size ordering is that when $\tau_{\varepsilon,t}\in(0,\tau^*_{\varepsilon,t})$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$, and when $\tau_{\varepsilon,t}\in(\tau^*_{\varepsilon,t},\infty)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$.
- ▷ **Claim 5.4:** If $\tau_{u,t}\geqslant 1$ and $\tau^*_{\varepsilon,t}\leqslant 0$, the only difference from Claim 5.3 is the losses of the switch in dominance at $\tau^*_{\varepsilon,t}$. Hence, the size ordering is that for any $\tau_{\varepsilon,t}>0$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ is always the case.

□

■ ***Proposition 6.*** (i) All else constant, when $\tau_{s,t}\searrow 0$, the limits of each effect are:

$$\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{s,t}\searrow 0) = \lim_{\tau_{s,t}\searrow 0}\frac{2\tau_{\varepsilon,t}+\frac{\tau_{\varepsilon,t}^2}{\rho_t^2}\tau_{u,t}}{2\tau_{s,t}} = \infty \tag{A.49}$$

$$\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{s,t}\searrow 0) = \frac{\tau_{\varepsilon,t}\tau_{u,t}}{\rho_t^2} \geqslant 0 \tag{A.50}$$

Given that the tuples $(\tau_{u,t},\tau_{\varepsilon,t},\rho_t)$ are limited, $\lim_{\tau_{s,t}\searrow 0}\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)=\infty$.

(ii) When $\tau_{s,t}\nearrow\infty$, the limits of each effect are solved in Proposition 3. When $\tau_{s,t}\nearrow\infty$, the overall welfare is $\lim_{\tau_{s,t}\nearrow\infty}\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)=\frac{\tau_{\varepsilon,t}(\tau_{u,t}-1)}{\tau_{\varepsilon,t}+\rho_t^2}$ that is positive if $\tau_{u,t}>1$ and that is, however, negative if $\tau_{u,t}<1$.

(iii) The absolute responsiveness of the overall welfare to signal informativeness is defined as:

$$\begin{aligned}\gamma^a_{\tau_{s,t}} &= \frac{\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{s,t}} = \frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{s,t}} + \frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\tau_{s,t}} \\ &= \frac{-\tau_{\varepsilon,t}^2\tau_{u,t}(\tau_{s,t}+2\tau_{\varepsilon,t})-2\tau_{\varepsilon,t}\rho_t^2\tau_{s,t}-2\tau_{\varepsilon,t}^2\rho_t^2}{\rho_t^2\tau_{s,t}^3} + \frac{\tau_{\varepsilon,t}^2(-\tau_{\varepsilon,t}^3\tau_{u,t}-\tau_{\varepsilon,t}^2\rho_t^2)}{(\tau_{\varepsilon,t}^2\tau_{s,t}+\tau_{\varepsilon,t}\rho_t^2\tau_{s,t}+\tau_{\varepsilon,t}^2\rho_t^2)^2}\end{aligned} \tag{A.51}$$

It is evident that $\frac{\partial \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial \tau_{\varepsilon,t}} \leqslant 0$ and $\frac{\partial \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial \tau_{\varepsilon,t}} \leqslant 0$. The relative responsiveness is:

$$\gamma^r_{\tau_{s,t}} = \frac{\frac{-\tau_{\varepsilon,t}^2\tau_{u,t}(\tau_{s,t}+2\tau_{\varepsilon,t})-2\tau_{\varepsilon,t}\rho_t^2\tau_{s,t}-2\tau_{\varepsilon,t}^2\rho_t^2}{\rho_t^2\tau_{s,t}^3} + \frac{\tau_{\varepsilon,t}^2(-\tau_{\varepsilon,t}^3\tau_{u,t}-\tau_{\varepsilon,t}^2\rho_t^2)}{(\tau_{\varepsilon,t}^2\tau_{s,t}+\tau_{\varepsilon,t}\rho_t^2\tau_{s,t}+\tau_{\varepsilon,t}^2\rho_t^2)^2}}{\frac{(\tau_{s,t}+\tau_{\varepsilon,t})\left(\tau_{\varepsilon,t}+\tau_{\varepsilon,t}^2\rho_t^{-2}\tau_{u,t}\right)+\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{s,t}^2} + \frac{\tau_{u,t}\tau_{\varepsilon,t}^2+(\tau_{u,t}-1)\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{\varepsilon,t}\tau_{s,t}+\rho_t^2(\tau_{\varepsilon,t}+\tau_{s,t})}} \tag{A.52}$$

The second derivatives are

$$\frac{\partial^2 \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial \tau_{s,t}^2} = \frac{2\tau_{\varepsilon,t}(\tau_{\varepsilon,t}\tau_{u,t}\tau_{s,t}+2\rho_t^2\tau_{s,t}+3\tau_{\varepsilon,t}^2\tau_{u,t}+3\tau_{\varepsilon,t}\rho_t^2)}{\rho_t^2\tau_{s,t}^4} \geqslant 0 \tag{A.53}$$

$$\frac{\partial^2 \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial \tau_{s,t}^2} = -\frac{2\tau_{\varepsilon,t}^2(\tau_{\varepsilon,t}^3\tau_{u,t}+\tau_{\varepsilon,t}^2\rho_t^2)(\tau_{\varepsilon,t}^2+\tau_{\varepsilon,t}\rho_t^2)}{(\tau_{\varepsilon,t}^2\tau_{s,t}+\tau_{\varepsilon,t}\rho_t^2\tau_{s,t}+\tau_{\varepsilon,t}^2\rho_t^2)^3} \geqslant 0 \tag{A.54}$$

so both effects are convex.

(iv) The first constraint is from the limits in (ii). On the one hand, if the overall welfare converges to a positive number, say $\tau_{u,t} > 1$, then the effect of $\mathscr{N}$ is positive and declining in $\tau_{s,t}$. To poise the two individual effects, there are three real roots[12] and $\tau_{u,t} > 1$ can determine only one positive root that is $\tau_{s,t}^{[2]}$, denoted as $\tilde{\tau}_{s,t}$. On the other hand, if it converges to a negative number, say $\tau_{u,t} < 1$, $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ does not cross $\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ even once in all possible $\tau_{s,t} \in \mathbb{R}_+$. But, the fixed point theorem enforces $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ to cross $|\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)|$ once because they are continuous and $\lim_{\tau_{s,t}\searrow 0} \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) - \lim_{\tau_{s,t}\searrow 0} |\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > 0$ and $\lim_{\tau_{s,t}\nearrow\infty} \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) - \lim_{\tau_{s,t}\nearrow\infty} |\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| < 0$, there must exist a $\tau_{s,t}^*$ such that $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{s,t} = \tau_{s,t}^*) = |\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t|\tau_{s,t} = \tau_{s,t}^*)|$. Furthermore, when $\tau_{s,t} = \frac{\tau_{\varepsilon,t}\tau_{u,t}}{1-\tau_{u,t}}$, $\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) = 0$, as denoted as $\hat{\tau}_{s,t}$. Hence, there are only two distinct cases:

- ▷ **Claim 6.1:** If $\tau_{u,t} > 1$, the welfare cuts are reinforced in general by either IP or IE. But when $\tau_{s,t} < \tilde{\tau}_{s,t}$, the IP dominates, say $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$ whereas the dominance switches to the IE as long as $\tau_{s,t} > \tilde{\tau}_{s,t}$, say $\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$.
- ▷ **Claim 6.2:** If $\tau_{u,t} < 1$, the welfare cuts caused by both effects but the IP happen to be the dominant force when $\tau_{s,t} < \hat{\tau}_{s,t}$, say $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0$. Even though the IE starts to create welfare gains, it cannot offset the welfare cuts caused by the IP when $\hat{\tau}_{s,t} < \tau_{s,t} < \tau_{s,t}^*$, say $\Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > |\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > 0 > \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$. It, however, turns out to be net welfare improvements produced by the IE, say $|\Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)| > \Delta \mathscr{D}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t) > 0 > \Delta \mathscr{N}_{h \rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)$ as

[12] Denote these roots as $\tau_{s,t}^{[1]}, \tau_{s,t}^{[2]}, \tau_{s,t}^{[3]}$, which equal to:

$$\tau_{s,t}^{[1]} = -\tau_{\varepsilon,t}$$

$$\tau_{s,t}^{[2]} = \frac{-\sqrt{\mathcal{B}^2 - 4(\rho_t^2 - \tau_{u,t}\rho_t^2)(\tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^2 + \tau_{\varepsilon,t}\rho_t^4)} - \mathcal{B}}{2\left[\rho_t^2\left(1-\tau_{u,t}\right)\right]}$$

$$\tau_{s,t}^{[3]} = \frac{\sqrt{\mathcal{B}^2 - 4(\rho_t^2 - \tau_{u,t}\rho_t^2)(\tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^2 + \tau_{\varepsilon,t}\rho_t^4)} - \mathcal{B}}{2\left[\rho_t^2\left(1-\tau_{u,t}\right)\right]}$$

where $\mathcal{B} = \tau_{\varepsilon,t}^2\tau_{u,t} + \tau_{\varepsilon,t}\tau_{u,t}\rho_t^2 + \tau_{\varepsilon,t}\rho_t^2 + 2\rho_t^4$. Evidently, $\tau_{u,t} > 1$ implies $[\mathcal{B}^2 - 4(\rho_t^2 - \tau_{u,t}\rho_t^2)(\tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^2 + \tau_{\varepsilon,t}\rho_t^4)]^{\frac{1}{2}} > \mathcal{B}$ and therefore, $\tau_{s,t}^{[2]} > 0$ and $\tau_{s,t}^{[3]} < 0$. Moreover, $\tau_{u,t} < 1$ implies $[\mathcal{B}^2 - 4(\rho_t^2 - \tau_{u,t}\rho_t^2)(\tau_{\varepsilon,t}^2\tau_{u,t}\rho_t^2 + \tau_{\varepsilon,t}\rho_t^4)]^{\frac{1}{2}} < \mathcal{B}$ and therefore, $\tau_{s,t}^{[2]} < 0$ and $\tau_{s,t}^{[3]} < 0$. $\tau_{s,t}^{[1]} \leqslant 0$ for any cases.

long as $\tau_{s,t} > \tau^*_{s,t}$.

$\square$

■ ***Proposition 7.*** (i) The limits of the two effects as $\rho_t \searrow 0$ are:

$$\lim_{\rho_t \searrow 0} \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \infty \tag{A.55}$$

$$\lim_{\rho_t \searrow 0} \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \frac{\tau_{u,t}\tau_{\varepsilon,t} + \tau_{s,t}(\tau_{u,t} - 1)}{\tau_{s,t}} \begin{cases} > 0, & \text{if } \tau_{u,t} + \mathcal{Q}_t > 1 \\ \leqslant 0, & \text{if } \tau_{u,t} + \mathcal{Q}_t \leqslant 1 \end{cases} \tag{A.56}$$

Therefore, as $\rho_t \searrow 0$, the overall welfare effect goes to $\lim_{\rho_t \searrow 0} \Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \infty$.

(ii) The limits of the two effects as $\rho_t \nearrow \infty$ are:

$$\lim_{\rho_t \nearrow \infty} \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \frac{\tau^2_{\varepsilon,t} + 2\tau_{s,t}\tau_{\varepsilon,t}}{\tau^2_{s,t}} \geqslant 0 \tag{A.57}$$

$$\lim_{\rho_t \nearrow \infty} \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = 0 \tag{A.58}$$

Therefore, as $\rho_t \nearrow \infty$, the overall welfare effect goes to $\lim_{\rho_t \nearrow \infty} \Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = \frac{\tau^2_{\varepsilon,t} + 2\tau_{s,t}\tau_{\varepsilon,t}}{\tau^2_{s,t}}$.

(iii) The absolute responsiveness is

$$\begin{aligned} \gamma^a_{\rho_t} &= \frac{\partial\Delta\widetilde{\mathscr{W}}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)}{\partial\rho_t} = \frac{\partial\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)}{\partial\rho_t} + \frac{\partial\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)}{\partial\rho_t} \\ &= -\frac{2\tau^2_{\varepsilon,t}\tau_{u,t}(\tau_{s,t} + \tau_{\varepsilon,t})}{\tau^2_{s,t}\rho^3_t} + \frac{2\tau^3_{\varepsilon,t}\rho_t(\tau_{\varepsilon,t} + \tau_{s,t})[(1 - \tau_{u,t})\tau_{s,t} - \tau_{u,t}\tau_{\varepsilon,t}]}{(\tau_{s,t}\tau^2_{\varepsilon,t} + \tau_{s,t}\tau_{\varepsilon,t}\rho^2_t + \tau^2_{\varepsilon,t}\rho^2_t)^2} \end{aligned} \tag{A.59}$$

and the relative responsiveness:

$$\gamma^r_{\rho_t} = \frac{-\frac{2\tau^2_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{\tau^2_{s,t}\rho^3_t} + \frac{2\tau^3_{\varepsilon,t}\rho_t(\tau_{\varepsilon,t}+\tau_{s,t})[(1-\tau_{u,t})\tau_{s,t}-\tau_{u,t}\tau_{\varepsilon,t}]}{(\tau_{s,t}\tau^2_{\varepsilon,t}+\tau_{s,t}\tau_{\varepsilon,t}\rho^2_t+\tau^2_{\varepsilon,t}\rho^2_t)^2}}{\frac{(\tau_{s,t}+\tau_{\varepsilon,t})\left(\tau_{\varepsilon,t}+\tau^2_{\varepsilon,t}\rho_t^{-2}\tau_{u,t}\right)+\tau_{\varepsilon,t}\tau_{s,t}}{\tau^2_{s,t}} + \frac{\tau_{u,t}\tau^2_{\varepsilon,t}+(\tau_{u,t}-1)\tau_{\varepsilon,t}\tau_{s,t}}{\tau_{\varepsilon,t}\tau_{s,t}+\rho^2_t(\tau_{\varepsilon,t}+\tau_{s,t})}} \tag{A.60}$$

When $\tau_{u,t} + \mathcal{Q}_t > 1$ holds, $\gamma^a_{\rho_t} < 0$ and $\gamma^r_{\rho_t} < 0$. The change rate in the overall welfare is not monotonic if $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$. The second derivatives are $\frac{\partial^2\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\rho^2_t} = \frac{6\tau^2_{\varepsilon,t}\tau_{u,t}(\tau_{s,t}+\tau_{\varepsilon,t})}{\tau^2_{s,t}\rho^4_t} \geqslant 0$ and $\frac{\partial^2\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)}{\partial\rho^2_t} = \frac{2\tau^4_{\varepsilon,t}(\tau_{s,t}\tau_{\varepsilon,t}+3\tau_{\varepsilon,t}\rho^2_t+3\tau_{s,t}\rho^2_t)(\tau_{s,t}+\tau_{\varepsilon,t})[(\tau_{u,t}-1)\tau_{s,t}+\tau_{s,t}\tau_{\varepsilon,t}]}{v(\tau_{s,t}\tau^2_{\varepsilon,t}+\tau_{s,t}\tau_{\varepsilon,t}\rho^2_t+\tau^2_{\varepsilon,t}\rho^2_t)^3} \geqslant 0$ if $\tau_{u,t} + \mathcal{Q}_t > 1$ holds and the two effects are convex. If $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)$ is concave in ρ_t, which offers the other possible welfare pattern.

(iv) If $\tau_{u,t} + \mathcal{Q}_t \leqslant 1$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)$ is negative and concave, and converges to zero when $\rho_t \nearrow \infty$. $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)$ is positive and convex, and also converges to zero when $\rho_t \nearrow \infty$. The overall welfare pattern concerns if the absolute size of $\mathscr{N}$ can be larger than $\mathscr{D}$. So let $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t) = |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t; \rho_t)|$ and denote $u_t = \rho^2_t$ and $u^2_t = \rho^4_t$ and simplify it:

$$\lambda_1 u_t^2 + \lambda_2 u_t + \lambda_0 = 0 \tag{A.61}$$

$$\text{where} \quad \lambda_0 = \tau_{\varepsilon,t}^3 \tau_{u,t} \tau_{s,t}^2 + \tau_{\varepsilon,t}^4 \tau_{u,t} \tau_{s,t}$$

$$\lambda_1 = \tau_{\varepsilon,t}^3 + 2\tau_{\varepsilon,t}\tau_{s,t}^2 + 3\tau_{\varepsilon,t}^2\tau_{s,t}$$

$$\lambda_2 = \tau_{\varepsilon,t}^4 \tau_{u,t} + \tau_{\varepsilon,t}\tau_{u,t}\tau_{s,t}^3 - \tau_{\varepsilon,t}\tau_{s,t}^3 + 2\tau_{\varepsilon,t}^2\tau_{u,t}\tau_{s,t}^2 + 2\tau_{\varepsilon,t}^2\tau_{s,t}^2 + 2\tau_{\varepsilon,t}^3\tau_{u,t}\tau_{s,t} + \tau_{\varepsilon,t}^3\tau_{s,t}$$

This becomes a simple one-dimension quadratic equation. If the discriminant $\Delta^\dagger = \lambda_2^2 - 4\lambda_1\lambda_0 > 0$, there will be four distinct real roots in which two positive and two negative are the solutions[13]. The two positive roots are of interest and denote the smaller one as ρ_L^* and the larger one as ρ_H^*. The net welfare gains only occur between the two cutoffs whereas out of the range, the net welfare effect is welfare loss. Otherwise if $\Delta^\dagger = \lambda_2^2 - 4\lambda_1\lambda_0 \leqslant 0$, the net welfare cuts are for all possible ρ_t, which are explained mainly by the IP.

If $\tau_{u,t} + \mathscr{Q}_t > 1$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ and $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ are positive and convex, and converge to zero when $\rho_t \nearrow \infty$. The overall welfare pattern concerns the exact number of $\mathscr{N}$ and $\mathscr{D}$. So in this case, let $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) = \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$ and denote $u_t = \rho_t^2$ and $u_t^2 = \rho_t^4$ and simplify it:

$$\nu_1 u_t^2 + \nu_2 u_t + \nu_0 = 0 \tag{A.62}$$

$$\text{where} \quad \nu_0 = \lambda_0$$

$$\nu_1 = \lambda_1$$

$$\nu_2 = \tau_{\varepsilon,t}^4 \tau_{u,t} - \tau_{\varepsilon,t}\tau_{u,t}\tau_{s,t}^3 + \tau_{\varepsilon,t}\tau_{s,t}^3 + 2\tau_{\varepsilon,t}^2\tau_{s,t}^2 + \tau_{\varepsilon,t}^3\tau_{s,t} + 2\tau_{\varepsilon,t}^3\tau_{u,t}\tau_{s,t}$$

If the discriminant $\Delta^\ddagger = \lambda_2^2 - 4\lambda_1\lambda_0 > 0$, again there will be four distinct real roots two of which are positive and the other two of which are negative. The two positive roots are of interest and denote the smaller one as $\rho_L^{'}$ and the larger one as $\rho_H^{'}$. There are no welfare gains but rather, the IE causes more welfare cuts only between the two cutoffs and out of this range, the other effect explains the main welfare cuts. Otherwise if $\Delta^\dagger = \lambda_2^2 - 4\lambda_1\lambda_0 \leqslant 0$, the net welfare cuts are for all possible ρ_t. Also, it is worth noting that there are some special cases. For example, when $(\tau_{u,t}, \tau_{s,t}, \tau_{\varepsilon,t}) = (0.5, 0.5, 0.5)$, there is $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) = 0$ for any $\rho_t > 0$. In addition, when $\Delta^\dagger = \Delta^\ddagger = 0$, there is only one intersection between the two effects but in either case, it does not affect the overall dominance of one effect over the other effect. Thus, these special cases are trivial.

- ▷ **Claim 7.1:** If $\tau_{u,t} + \mathscr{Q}_t \leqslant 1$ and $\Delta^\dagger > 0$, welfare cuts can be expected when $\rho_t < \rho_L^*$ or $\rho_t > \rho_H^*$ whereas welfare gains can be expected when $\rho_L^* < \rho_t < \rho_H^*$. The size ordering: when $\rho_t \in (0, \rho_L^*) \cup (\rho_H^*, \infty)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$. When $\rho_t \in (\rho_L^*, \rho_H^*)$, $|\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > \Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$.
- ▷ **Claim 7.2:** If $\tau_{u,t} + \mathscr{Q}_t \leqslant 1$ and $\Delta^\dagger \leqslant 0$, welfare cuts are always expected. The size ordering is $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t) > |\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)| > 0 > \Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}_t'; \rho_t)$.

[13] To save space, they are not shown. It can be seen that u_t and u_t^2 must be non-negative. For instance, each $u_t > 0$ can generate two roots - $\rho_t^{[1]} = \sqrt{u_t}$ and $\rho_t^{[2]} = -\sqrt{u_t}$. There are at most four distinct real roots.

▷ **Claim 7.3:** If $\tau_{u,t}+\mathcal{Q}_t>1$ and $\Delta^\ddagger>0$, welfare cuts are always expected but the dominance switches at two cutoffs. When $\rho_t\in(0,\rho'_L)\cup(\rho^*_H,\infty)$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>0$ whereas as $\rho_t\in(\rho^*_L,\rho^*_H)$, $\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>0$.

▷ **Claim 7.4:** If $\tau_{u,t}+\mathcal{Q}_t>1$ and $\Delta^\ddagger\leqslant 0$, $\Delta\mathscr{D}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>\Delta\mathscr{N}_{h\rightsquigarrow l}(\boldsymbol{\tau}'_t;\rho_t)>0$ entails welfare cuts at all possible ρ_t.

□

■ ***Lemma 5.*** Given that $\omega_t=s_t+\frac{u_t}{\mu^*_t(k_t)a_t}$ contains the same information as price, plug the informed and uninformed demand into the market clearing condition and then apply Bayes' Rule.

$$\mu^*_t(k_t)\cdot\underbrace{a_t(s_t-\mathscr{P}(s_t,u_t))}_{X_I(s_t,p_t)}+(1-\mu^*_t(k_t))\cdot\underbrace{\frac{\mathbb{E}_t(\dot\theta_{t+1}|\omega_t)-\mathscr{P}(s_t,u_t)}{\rho_t\mathbb{V}_t(\dot\theta_{t+1}|\omega_t)}}_{X_U(p_t)}+u_t=0$$

$$\begin{aligned}\mathscr{P}(s_t,u_t)&=\frac{\mu^*_t(k_t)a_ts_t+u_t+\frac{(1-\mu^*_t(k_t))\mathbb{E}_t(\dot\theta_{t+1}|\omega_t)}{\rho_t\mathbb{V}_t(\dot\theta_{t+1}|\omega_t)}}{\mu^*_t(k_t)a_t+\frac{1-\mu^*_t(k_t)}{\rho_t\mathbb{V}_t(\dot\theta_{t+1}|\omega_t)}}\\&=\underbrace{\frac{\frac{(1-\mu^*_t(k_t))(\tau_{s,t}\bar\theta_{t+1}+\mu^*_t(k_t)a_t\tau_{u,t})}{\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}}}{\rho_t\mu^*_t(k_t)a_t\left\{[\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}]^{-1}+\sigma_{\varepsilon t}^{2}\right\}+1-\mu^*_t(k_t)}}_{\alpha_1}\\&+\underbrace{\frac{\left[\frac{(1-\mu^*_t(k_t))(\mu^*_t(k_t)a_t)^2\tau_{u,t}}{\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}}+\rho_t\mu^*_t(k_t)a_t\left\{[\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}]^{-1}+\sigma_{\varepsilon t}^{2}\right\}\right]}{\rho_t\mu^*_t(k_t)a_t\left\{[\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}]^{-1}+\sigma_{\varepsilon t}^{2}\right\}+1-\mu^*_t(k_t)}}_{\alpha_2}\cdot\omega_t\end{aligned}\quad\text{(A.63)}$$

where α_1 and α_2 are constants and can be further simplified:

$$\alpha_1=\frac{\rho_t(\tau_{\varepsilon,t}\tau_{u,t}\mu^*_t(k_t)+\bar\theta_{t+1}\tau_{s,t}\rho_t)(1-\mu^*_t(k_t))}{\tau_{\varepsilon,t}^2\tau_{u,t}\mu^*_t(k_t)^2+\tau_{\varepsilon,t}\rho_t^2\mu^*_t(k_t)+\tau_{s,t}\rho_t^2}\quad\text{(A.64)}$$

$$\alpha_2=\frac{\mu^*_t(k_t)(\tau_{\varepsilon,t}^2\tau_{u,t}\mu^*_t(k_t)+\tau_{\varepsilon,t}\rho_t^2+\tau_{s,t}\rho_t^2)}{\tau_{\varepsilon,t}^2\tau_{u,t}\mu^*_t(k_t)^2+\tau_{\varepsilon,t}\rho_t^2\mu^*_t(k_t)+\tau_{s,t}\rho_t^2}\quad\text{(A.65)}$$

□

■ ***Proposition 8.*** Provided that the only randomness in the random variable $\mathcal{X}_t$ is from $\omega_t=s_t+\frac{u_t}{\mu^*_t(k_t)a_t}\sim N\left(\bar\theta_{t+1}-\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}},\tau_{s,t}^{-1}+\frac{1}{\mathcal{B}-\tau_{s,t}}\right)$, $\mathcal{X}_t$ also follows a normal distribution. The mean of $\mathcal{X}_t$ is

$$\begin{aligned}\mu_{\mathcal{X}}&=\frac{\frac{\tau_{s,t}\bar\theta_{t+1}+\sqrt{\tau_{u,t}(\mathcal{B}-\tau_{s,t})}}{\mathcal{B}}+\left[\frac{\mathcal{B}-\tau_{s,t}}{\mathcal{B}}-\alpha_2\right]\left(\bar\theta_{t+1}-\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}}\right)-\alpha_1}{\sqrt{2\left\{[\tau_{s,t}+(\mu^*_t(k_t)a_t)^2\tau_{u,t}]^{-1}+\sigma_{\varepsilon t}^{2}\right\}}}\\&=\frac{1}{\sqrt2}\cdot\frac{\bar\theta_{t+1}(1-\alpha_2)+\alpha_2\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}}-\alpha_1}{\sqrt{\mathcal{B}^{-1}+\tau_{\varepsilon,t}^{-1}}}\end{aligned}\quad\text{(A.66)}$$

and the variance of $\mathcal{X}_t$ is

$$\sigma^2_{\mathcal{X}} = \frac{\left(\frac{\mathcal{B}-\tau_{s,t}}{\mathcal{B}} - \alpha_2\right)^2 \left(\tau_{s,t}^{-1} + \frac{1}{\mathcal{B}-\tau_{s,t}}\right)}{2\left(\mathcal{B}^{-1} + \tau_{\varepsilon,t}^{-1}\right)}$$

$$= \frac{1}{2}\frac{\tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B} - \tau_{s,t}\right]^2}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})\left(\mathcal{B}+\tau_{\varepsilon,t}\right)} \tag{A.67}$$

Hence, the welfare of an interior equilibrium can be written as

$$\mathscr{W}_{int}(\boldsymbol{\tau}'_t;\rho_t) = \mathbb{E}_t\left(-e^{-\mathcal{X}_t^2}\right) = -\frac{exp\left\{-\frac{\mu^2_{\mathcal{X}}}{1+\sigma^2_{\mathcal{X}}}\right\}}{\sqrt{1+\sigma^2_{\mathcal{X}}}}$$

$$= -\frac{exp\left\{-\frac{1}{2}\frac{\left[\bar{\theta}_{t+1}(1-\alpha_2)+\alpha_2\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}}-\alpha_1\right]^2\mathcal{B}\tau_{\varepsilon,t}\tau_{s,t}(\mathcal{B}-\tau_{s,t})}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})(\mathcal{B}+\tau_{\varepsilon,t})+\tau_{\varepsilon,t}[(1-\alpha_2)\mathcal{B}-\tau_{s,t}]^2}\right\}}{\sqrt{1+\frac{\tau_{\varepsilon,t}[(1-\alpha_2)\mathcal{B}-\tau_{s,t}]^2}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})(\mathcal{B}+\tau_{\varepsilon,t})}}} \tag{A.68}$$

The logarithm linearized welfare function $\widetilde{\mathscr{W}}(\cdot)_{int}$ is derived à la the same decomposition as the discrete case. Define the terms $\mathscr{D}_{int}$ and $\mathscr{N}_{int}$ on behave of the behavior in IP and the behavior in IE:

$$\widetilde{\mathscr{W}}_{int}(\cdot) = -\ln\left(-\mathscr{W}_{int}(\boldsymbol{\tau}'_t;\rho_t)\right) = \frac{1}{2}\mathscr{D}_{int}(\mu^*_t(k_t)) + \frac{1}{2}\ln\left(1+\mathscr{N}_{int}(\mu^*_t(k_t))\right)$$

$$\approx \mathscr{D}_{int}(\mu^*_t(k_t)) + \mathscr{N}_{int}(\mu^*_t(k_t)) \tag{A.69}$$

where

$$\mathscr{D}_{int}(\mu^*_t(k_t)) = \frac{\tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B}-\tau_{s,t}\right]^2}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})\left(\mathcal{B}+\tau_{\varepsilon,t}\right)} \tag{A.70}$$

$$\mathscr{N}_{int}(\mu^*_t(k_t)) = \frac{\left[\bar{\theta}_{t+1}(1-\alpha_2)+\alpha_2\sqrt{\frac{\tau_{u,t}}{\mathcal{B}-\tau_{s,t}}}-\alpha_1\right]^2\mathcal{B}\tau_{\varepsilon,t}\tau_{s,t}(\mathcal{B}-\tau_{s,t})}{\tau_{s,t}(\mathcal{B}-\tau_{s,t})\left(\mathcal{B}+\tau_{\varepsilon,t}\right)+\tau_{\varepsilon,t}\left[(1-\alpha_2)\mathcal{B}-\tau_{s,t}\right]^2} \tag{A.71}$$

□