Persistent Winners and Reserve Prices in Repeated Auctions

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- Application: *Auctions for online display advertising* ($143 billion US market in 2022)
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    - First- and Second-Price sealed-bid Auctions (FPA/SPA)

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    - First- and Second-Price sealed-bid Auctions (FPA/SPA)
  - Advertisers bid through *Demand Side Platforms* that allow
    - Management of advertising campaign with a fixed budget
    - Automatic real-time bidding in multiple auctions
  - 90% transactions use automatic technology

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    - Reserve in SPA $<$ FPA for high incumbents
- *What are the implications in dynamic FPA and SPA?*
  - Increasing reserve reduces seller's information
  - **Revenue** in SPA $>$ FPA iff incumbent's capacity is low
  - **Trade** is non-monotonic in capacity

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- Markovian structure with state equal to last winner — the **incumbent**:
  1. No winner ($\emptyset$) in $t \Rightarrow n$ new bidders in $t + 1$
  2. Winner θ in $t \Rightarrow$ in $t + 1$ $\begin{cases} n \text{ new bidders} & \text{prob. } \eta \\ \text{incumbent } \theta \text{ and } n - 1 \text{ new bidders} & \text{prob. } 1 - \eta \end{cases}$

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- Sufficient weaker form of unsophistication: **one-shot myopia**
  - Bidders are myopic only the first time they bid in the repeated auctions
    - Forward myopia only matters in first period
    - Backward myopia is irrelevant for the incumbent

Outline

1. **Static auctions** with exogenous incumbent
   - Optimal reserve price
   - Seller's revenue: FPA vs. SPA

2. **Dynamic auctions**
   - Transition dynamics
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$$\pi^F\left(\theta, R\right) = \eta \pi_n\left(R\right) + \left(1 - \eta\right) \pi^F_{n-1,\theta}\left(R\right)$$

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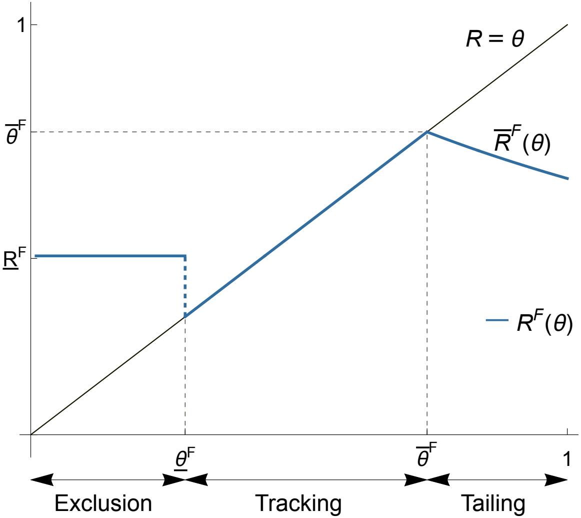
Exclusion	$(R^F > \theta)$	if	$\theta < \underline{\theta}^F$
Tracking	$(R^F = \theta)$	if	$\underline{\theta}^F \leq \theta \leq \overline{\theta}^F$
Tailing	$(R^F < \theta)$	if	$\theta > \overline{\theta}^F$



$\theta \sim \mathcal{U}(0,1) \quad \eta = 0.5 \quad n = 4$

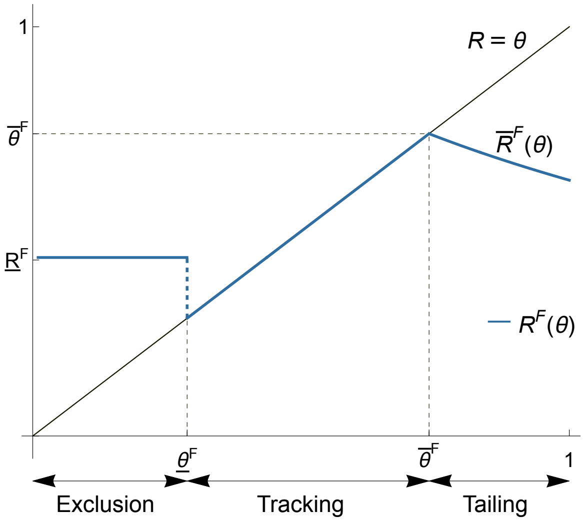
▸ Static Revenues

Tailing in FPA

- Tracking high θ is too costly: excessive reserve if θ leaves
- Tailing reserve solves

$$\psi(\overline{R}^F) = \frac{(1-\eta)(1-(n-1)\log(F(\theta)))}{n\eta f(\overline{R}^F)}$$

$\Rightarrow$ **decreasing** in θ



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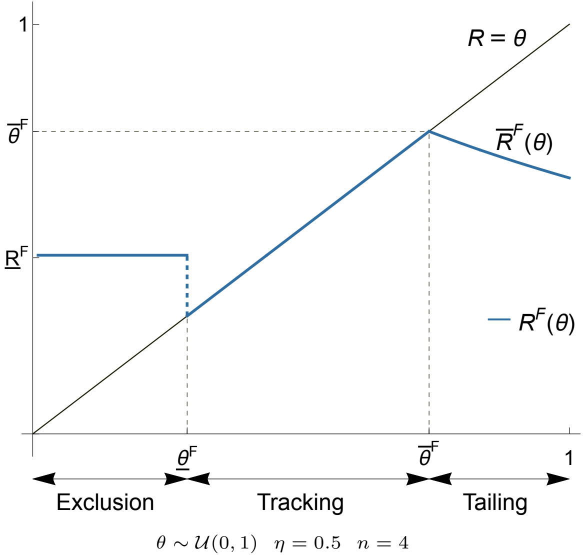

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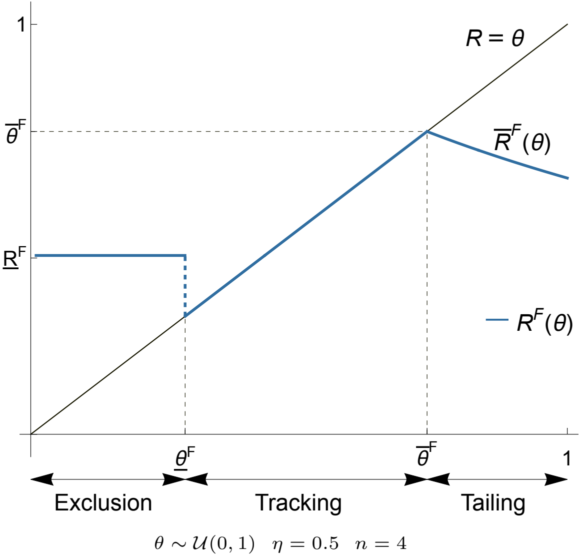
- Cost of increasing R (risk of no trade if θ leaves) independent of θ
- Benefit of increasing R (higher winning bid if θ stays) *decreases* in θ
  - Bidders with higher values are less sensitive to R:

$$\frac{\partial^2}{\partial R \partial \theta} b^F(\cdot) < 0$$



Tailing in SPA

- In SPA: exclusion/tracking/tailing but ...
- Tailing reserve solves

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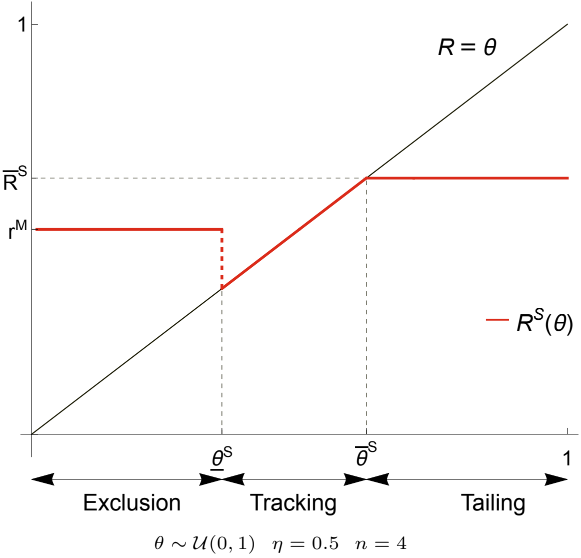

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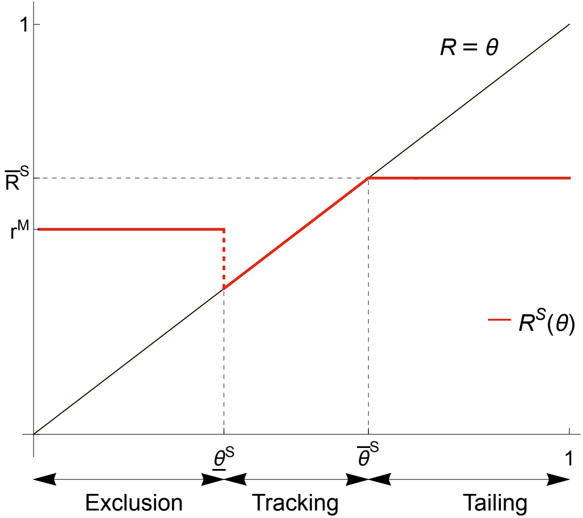

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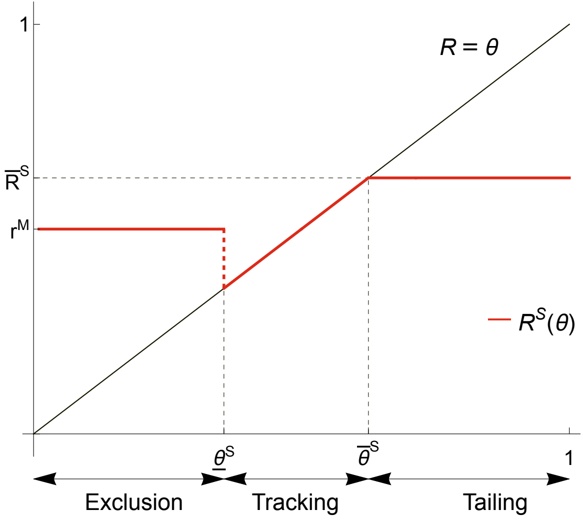
- Cost *and* benefit of increasing R independent of θ
  - Bids are independent of R
- Lower benefit of R than in FPA
  - Losing incumbent substitutes R for high new bidders



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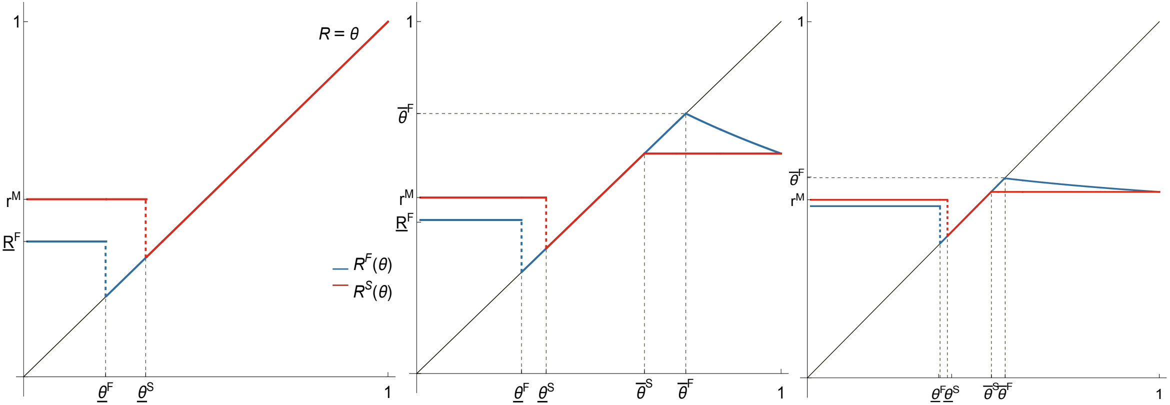
Effect of Persistence on Tracking



Low η: no tailing

Medium η

High η: $R^i \to r^M$

- Increasing η reduces tracking (since incumbent is less likely to stay)
- More tracking in FPA than SPA (and same reserve in FPA/SPA at $\theta = 1$)

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- If the incumbent stays:
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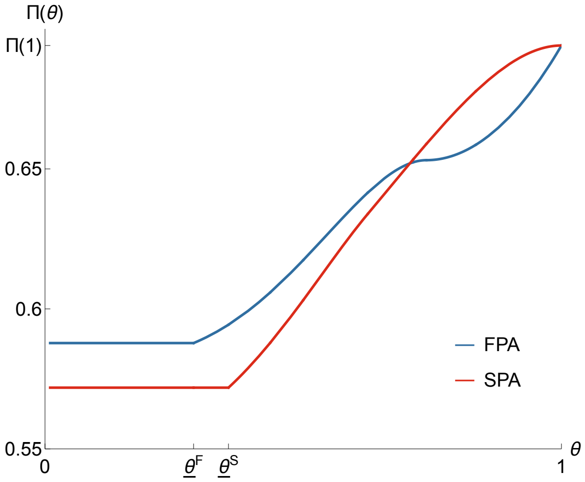
  - In FPA, more aggressive bidding (myopia and higher reserve) *but*
  - In SPA, **incumbent acts as "reserve"** $\Rightarrow$ high revenue when he loses (regardless of seller's R)
    - Reserve works even if unannounced in SPA, but not in FPA

  $\Rightarrow$ Seller can tailor R to new bidders

Seller's Revenue: FPA vs. SPA

Higher revenue in SPA for high θ (if $\overline{\theta}^S < 1$)



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- At $\theta = 1$, same R and revenue in SPA/FPA
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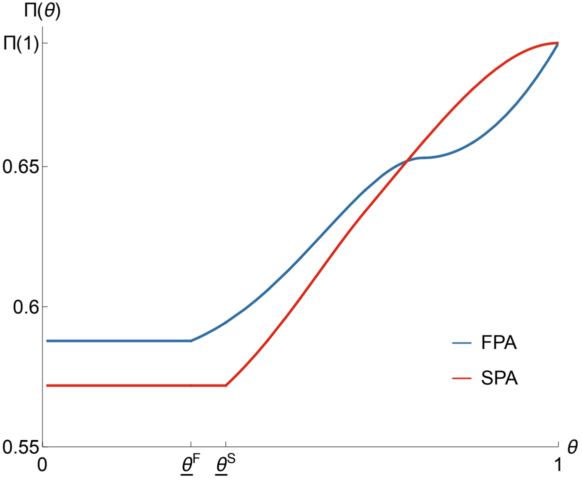

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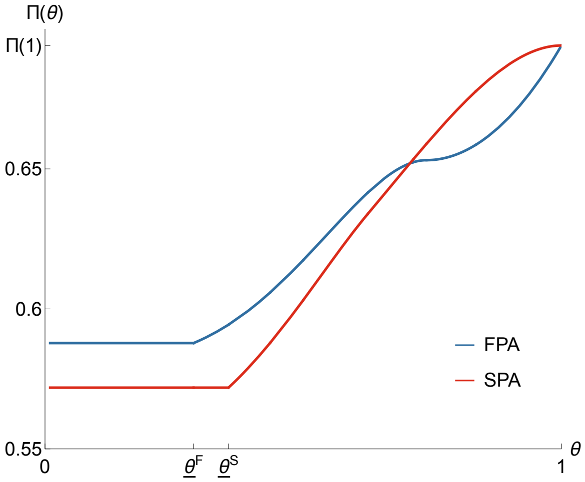
- At $\theta = 1$, same R and revenue in SPA/FPA
  - Highest incumbent never loses (when he stays)
- Marginally reducing θ has first-order effect on FPA (since θ pays his bid)
  - ... but not on SPA (since incumbent's payment is independent of θ)



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Dynamic Auctions

- Recursive representation with **state = incumbent** and value function

$$V(\theta) = \max_R \underbrace{\pi(\theta, R)}_{\substack{\text{Static} \\ \text{Revenue}}} + \beta \overbrace{\mathbb{E}_{\theta,R}}^{\substack{\text{Transition} \\ \text{Dynamics}}} [V(\theta')]$$

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  – Same transition dynamics in FPA and SPA (because same winner given R) ▸ Value Function

Transition Dynamics with Incumbent θ

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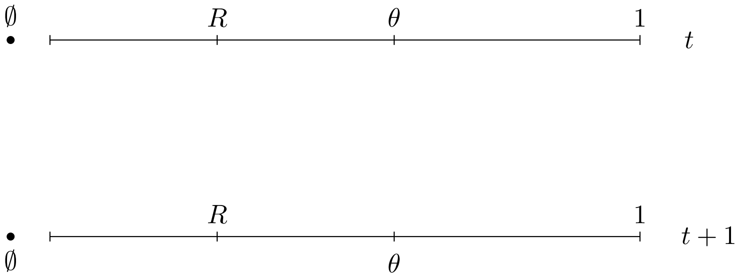

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Incumbent
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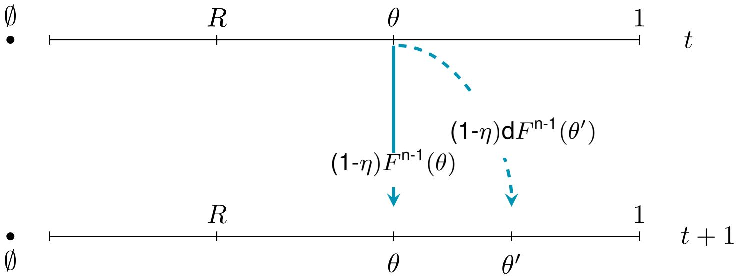

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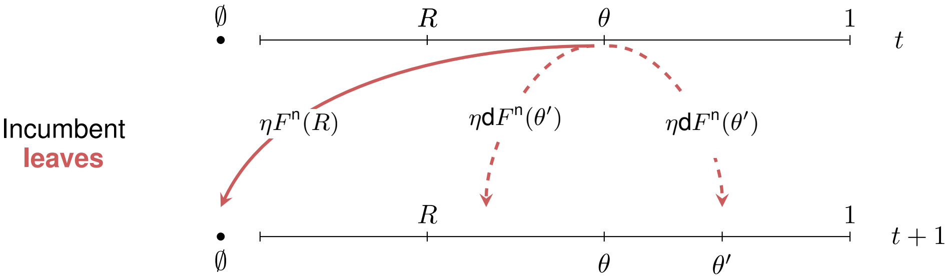

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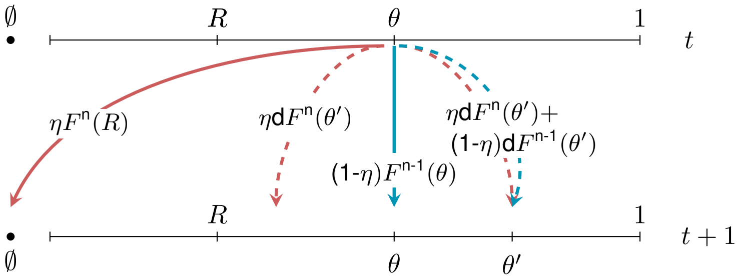

Dynamic Effect of R

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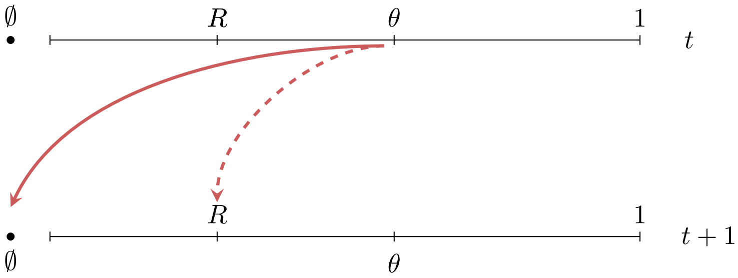

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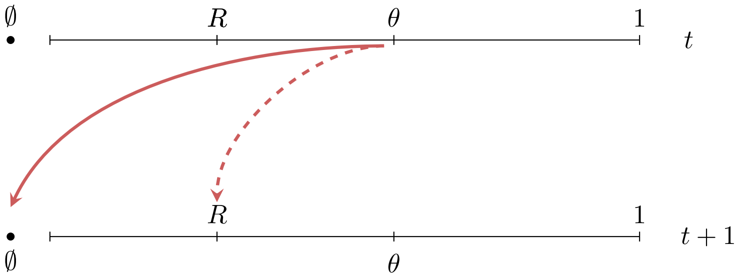
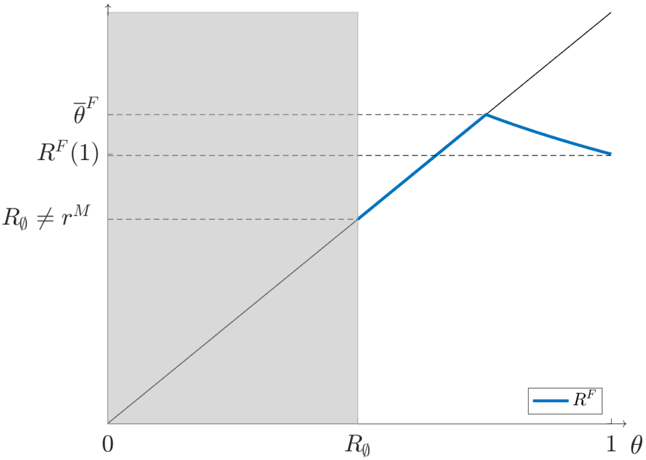


- R only matters when incumbent leaves and highest new bidder is R
  - $R \uparrow$ **reduces seller's information** (from to R to $\emptyset$)
  - Dynamic cost of excluding R is $\beta\left(V(R) - V(\emptyset)\right)$, independent of θ

Dynamic Optimal Reserve Price in FPA

Tracking and Tailing as in static case but

1. Lowest reserve $R_\emptyset$ (with no incumbent) is lowest possible winner



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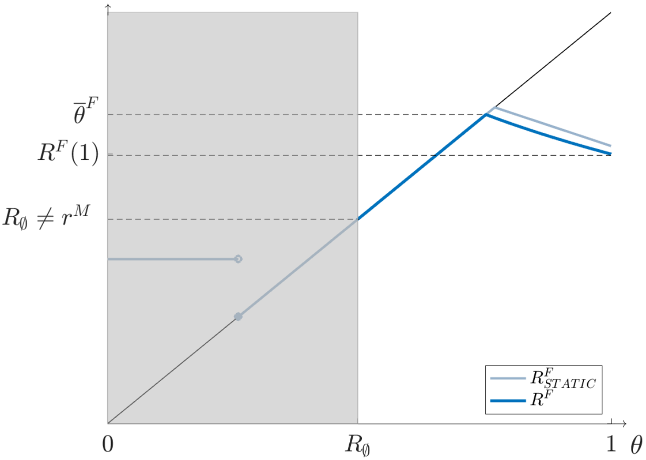
▸ $R_\emptyset$ Comparative Statics



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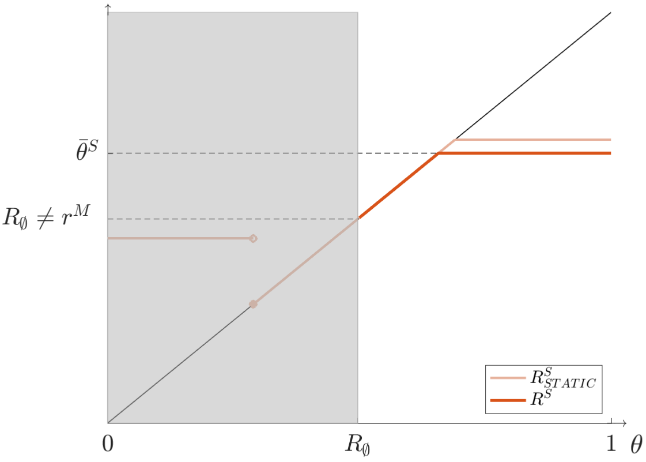

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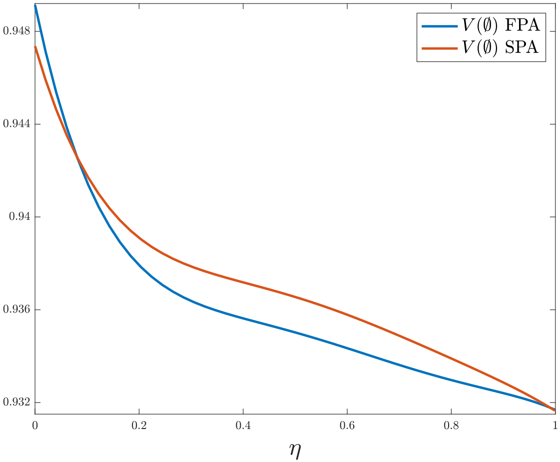
Seller's Value: FPA vs. SPA

- Seller's value $V(\emptyset)$ depends on stationary distribution of θ
  - Decreasing in η because less persistence $\Rightarrow$ lower incumbents



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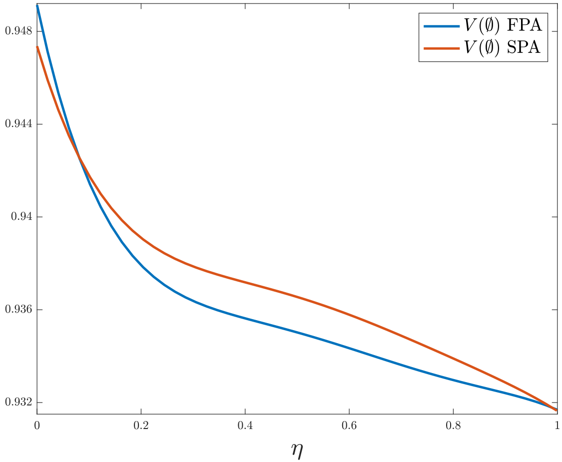

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- **SPA $\succ$ FPA if η is not too low**:
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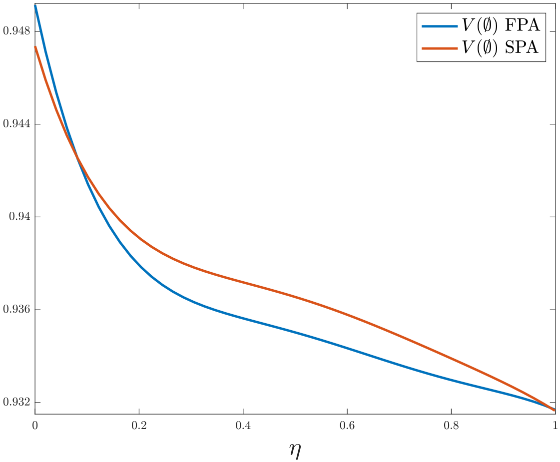
  - Winners are likely to have high θ



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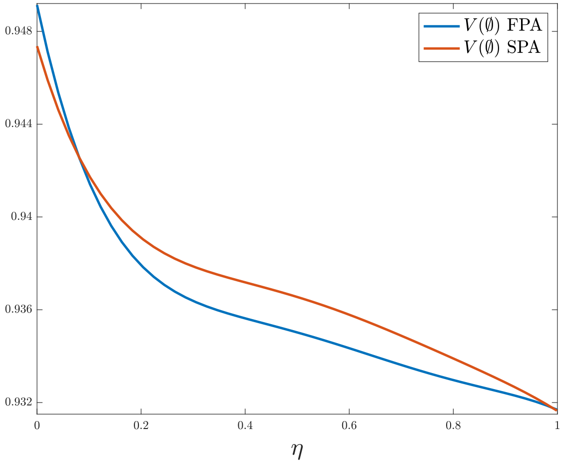
- $\eta = 1$: symmetric bidders, reserve r^M and revenue equivalence



$\beta = 0.15 \quad n = 10$

Trade

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- Given incumbent θ, **trade fails** with probability

$$\overbrace{\eta}^{\theta \text{ leaves}} \times \underbrace{F(R(\theta))^n}_{\text{all entrants} < R(\theta)}$$

Trade: Effect of η

- As η increases

$$\underset{\uparrow}{\eta} \times F(\underset{\downarrow}{R\,(\theta)})^n$$

  – Direct effect: θ leaves more often
  – Seller reduces $R\,(\theta)$

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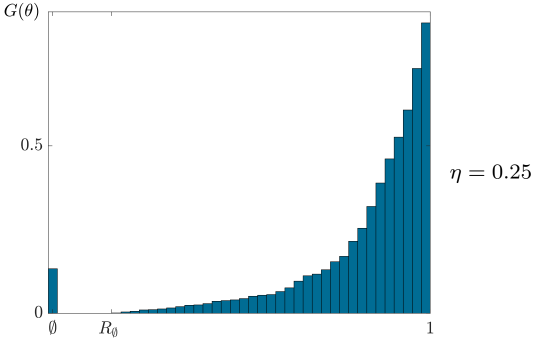
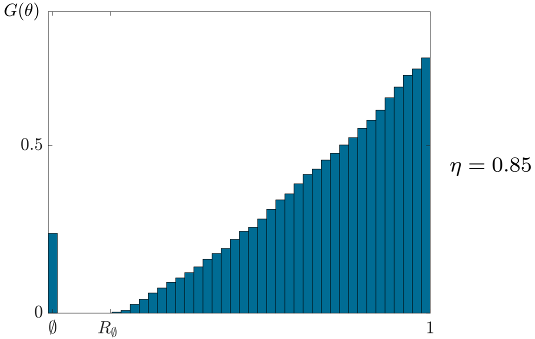




Trade: Effect of η

- As η increases

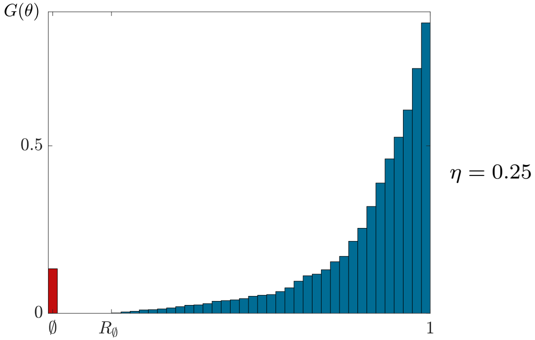
$$\underset{\uparrow}{\eta} \times \underset{\downarrow}{F(R(\theta))^n}$$

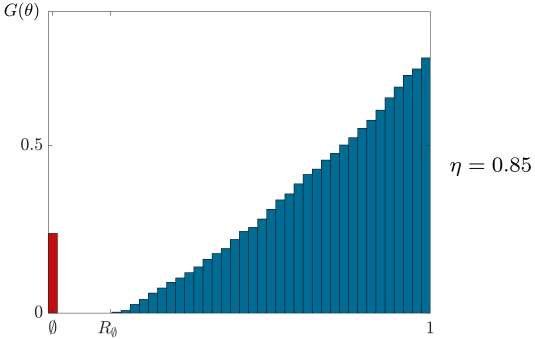
  - Direct effect: θ leaves more often
  - Seller reduces $R(\theta)$

- **Stationary distribution** $G(\theta)$ also depends on η

- Long-run trade is

$$T = 1 - \underbrace{\frac{\int_{R_\emptyset}^{1} \eta F(R(\theta'))^n \mathrm{d}G(\theta')}{1 - F(R_\emptyset)^n}}_{G(\emptyset)}$$





Trade

- Long-run trade is one minus the stationary distribution of state $\emptyset$

$$T = 1 - G(\emptyset)$$

  - $\eta = 0$: $T = 1$
  - $\eta = 1$: static auction,

$$T = 1 - F\left(r^M\right)^n$$

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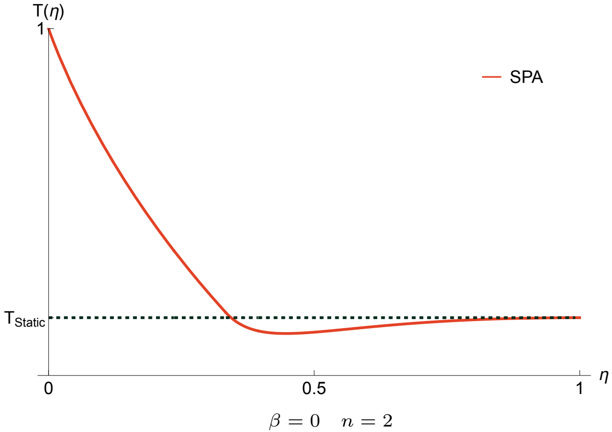
  - T can be non-monotonic in η



Conclusions

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Conclusions

- **Repeated auctions** with **myopic bidders** that buy up to capacity
- Optimal **reserve price** solves
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  - Dynamic information acquisition (additional cost of reserve)

  $\Rightarrow$ *Tail* high-value winners, with decreasing reserve in FPA
- Low winners' persistence reduces reserve prices (less tracking and lower tailing)
  - May increase trade
  - Higher revenue in SPA than FPA
    - More aggressive bidding in FPA with tracking (myopia) but
    - Lower reserve in SPA with tailing (incumbent substitutes reserve)

Static Seller's Revenue

- Let $b(\theta, R, n)$ be the expected payment of type θ conditional on winning a standard auction with n (symmetric) bidders and reserve R
- Static revenue in FPA is

$$\pi^F(\theta, R) = \eta \pi_n(R) + (1-\eta)\,\pi^F_{n-1,\theta}(R)$$

$$= \eta \int_R^1 b(x, R, n)\,\mathrm{d}F(x)^n \qquad \text{Incumbent leaves}$$

$$+ (1-\eta)\,\mathbb{I}[R \le \theta]\left(F(\theta)^{n-1} b(\theta, R, n) + \int_\theta^1 b(x, R, n)\,\mathrm{d}F(x)^{n-1}\right) \qquad \text{Track or tail incumbent}$$

$$+ (1-\eta)\,\mathbb{I}[R > \theta] \int_R^1 b(x, R, n)\,\mathrm{d}F(x)^{n-1} \qquad \text{Exclude incumbent}$$

- Static revenue in SPA is

$$\pi^S(\theta, R) = \eta \int_R^1 b(x, R, n)\,\mathrm{d}F(x)^n \qquad \text{Incumbent leaves}$$

$$+ (1-\eta)\,\mathbb{I}[R \le \theta]\left(F(\theta)^{n-1} b(\theta, R, n) + \int_\theta^1 b(x, \theta, n-1)\,\mathrm{d}F(x)^{n-1}\right) \qquad \text{Track or tail incumbent}$$

$$+ (1-\eta)\,\mathbb{I}[R > \theta] \int_R^1 b(x, R, n-1)\,\mathrm{d}F(x)^{n-1} \qquad \text{Exclude incumbent}$$

  - Aggressive myopic bidding in FPA (n vs $n-1$ bidders)
  - Losing incumbent substitutes reserve in SPA

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Value Function

- Let $b(\theta, R, n)$ be the expected payment of type θ conditional on winning a standard auction with n symmetric bidders and reserve R
- Value functions in auction $i = S, F$ are

$$V_{\emptyset}^{i} = \max_{R} \int_{R}^{1} b(x, R, n)\,\mathrm{d}F(x)^{n} + \beta\left(F(R)^{n} V_{\emptyset}^{i} + \int_{R}^{1} V^{i}(\theta')\,\mathrm{d}F(\theta')^{n}\right)$$

$V^{i}(\theta) = \max_{R}\pi^{i}(\theta, R)$	Static Revenue
$+\beta\Big[\eta\left(F(R)^{n} V_{\emptyset}^{i} + \int_{R}^{1} V^{i}(\theta')\,\mathrm{d}F(\theta')^{n}\right)$	Incumbent leaves
$+(1-\eta)\,\mathbb{I}[R \leq \theta]\left(F(\theta)^{n-1} V^{i}(\theta) + \int_{\theta}^{1} V^{i}(\theta')\,\mathrm{d}F(\theta')^{n-1}\right)$	Track or tail incumbent
$+(1-\eta)\,\mathbb{I}[R > \theta]\left(F(R)^{n-1} V_{\emptyset}^{i} + \int_{R}^{1} V^{i}(\theta')\,\mathrm{d}F(\theta')^{n-1}\right)\Big]$	Exclude incumbent

  – Auction formats affects static revenue but not transition dynamics

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