Efficiency and Equity of Education Tracking
A Quantitative Analysis

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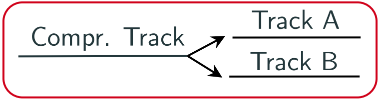
School Tracking: a Common Feature of Education Policy

- Tracking: Allocation of children into different school tracks (OECD, 2013)



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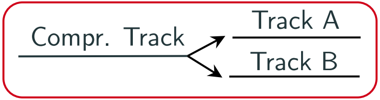

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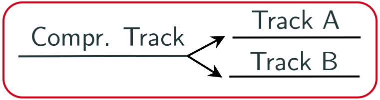

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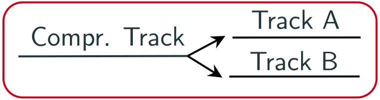

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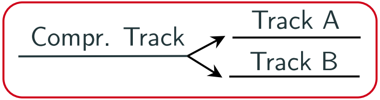

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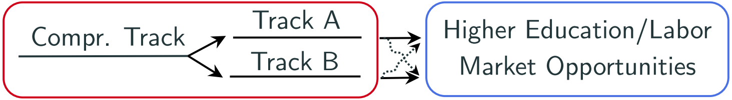

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This Paper

- Macroeconomic general equilibrium model of human capital accumulation over life-cycle and across generations (e.g.Lee and Seshadri (2019))
  - → Focus on schooling years: Child skill formation depends on *peers* and *instruction pace* in school tracks + *skill shocks*
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- Tracking Policy Counterfactuals:
  1. Postponing tracking: social mobility ↑ but agg. output ↓ (Benabou, 1996)
     - → Learning losses in comprehensive years outweigh benefits from less uncertainty
  2. Limiting parental influence in track choice: mobility ↑, agg. output ↑
     - → Tracking becomes more efficient

Related Literature

1. Quantitative Family Macro Literature on inequality and mobility
    - Higher Education (Abbott et al., 2019; Capelle, 2022)
    - Early Childhood and Parental Influence (Daruich, 2022; Lee and Seshadri, 2019; Yum, 2022)
    - School Closures (Agostinelli, Doepke, et al., 2022; Fuchs-Schündeln et al., 2022; Jang and Yum, 2022)
    - Developing Countries (Fujimoto, Lagakos, and Vanvuren, 2023)

→ We add tracking in secondary school

2. Literature on Child Skill Development
    - Theory and Estimation (Agostinelli, Saharkhiz, and Wiswall, 2019; Cunha and Heckman, 2007; Cunha, Heckman, and Schennach, 2010; Duflo, Dupas, and Kremer, 2011)

→ We focus on peer and instruction level effects across tracks

3. Reduced-form evidence on Education Tracking
    - Educational and Labor Market Outcomes (Betts, 2011; Dustmann, Puhani, and Schönberg, 2017; Hanushek and Wössmann, 2006; Matthewes, 2021) ...
    - Social Mobility (Meghir and Palme, 2005; Pekkarinen, Uusitalo, and Kerr, 2009)

→ We build structural model to gauge macro effects of tracking

Model

Environment

- Households live for 20 periods in OLG structure: Full Details

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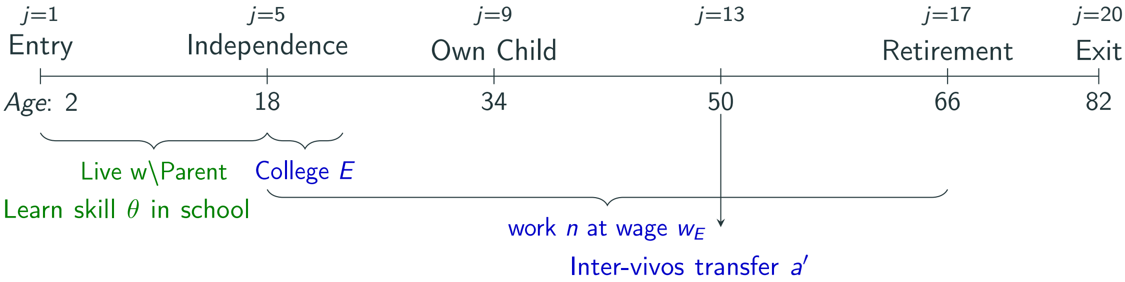

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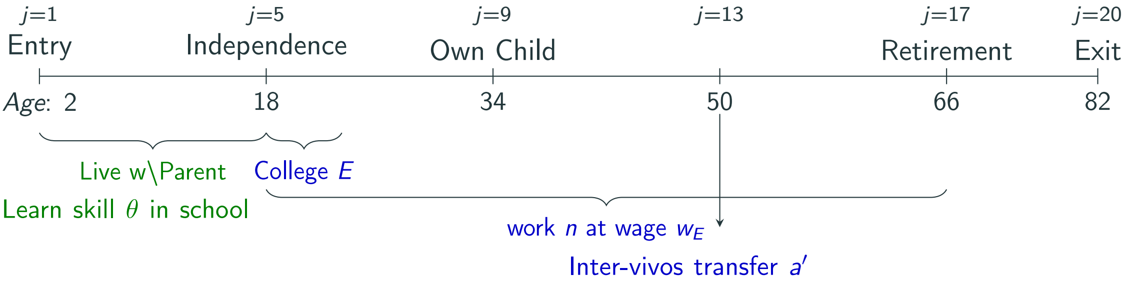

- Representative firm produces output using physical capital K + two types of human capital H_E Details

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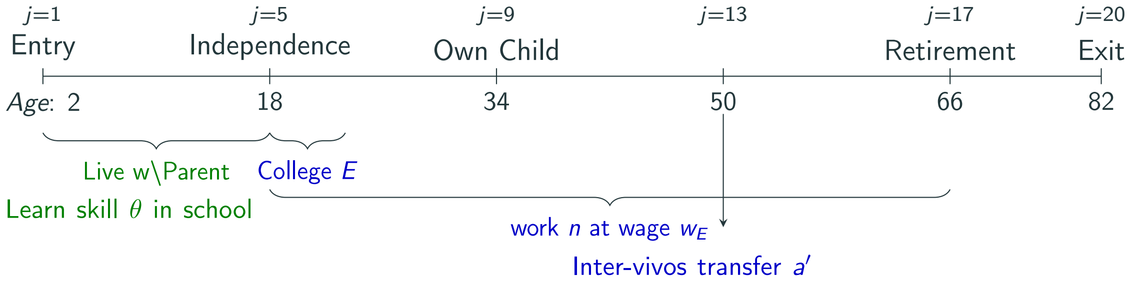
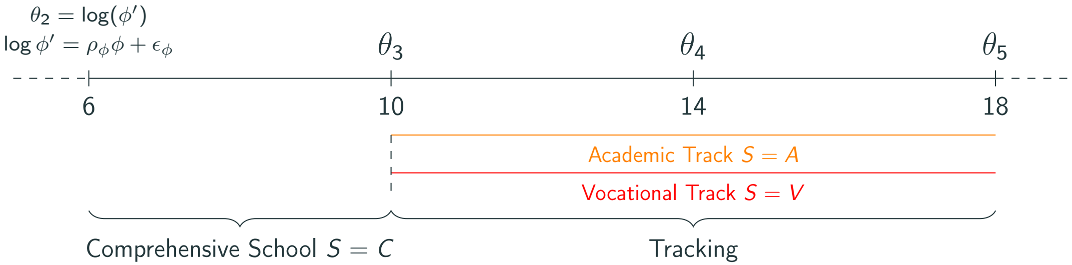


- Representative firm produces output using physical capital K + two types of human capital H_E Details
- Government collects labor + capital income taxes; finances pensions and lump-sum transfers

Skill Formation during School Years



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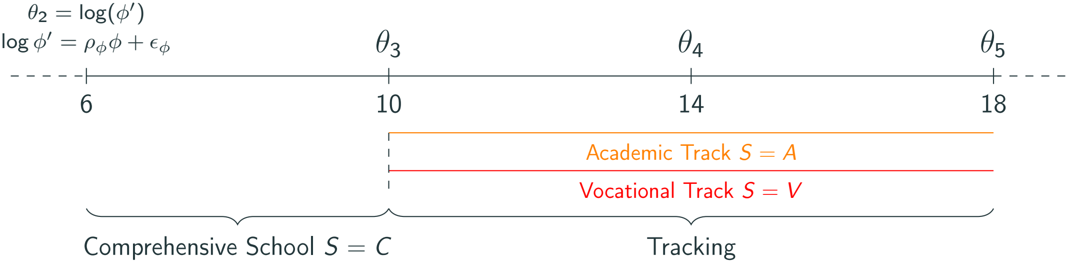

- Child skill θ_j evolution during school years $j = 2 - 4$ in school of track S:

$$\theta_{j+1} = \kappa\theta_j + \alpha\bar{\theta}_j^S + g(\theta_j, P_j^S) + \xi E + \eta_{j+1} \tag{1}$$

- $\bar{\theta}_j^S$: Average skill in track S
- P_j^S: Instruction pace in track S
- E: Parental inputs *(exogenous)*
- $\eta_{j+1} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\eta_{j+1}})$: Skill shock

Pace of Instruction

$$g(\theta_j, P_j) = \beta P_j + \gamma \theta_j P_j - \frac{\delta}{2} P_j^2 \tag{2}$$

- $\gamma > 0$: Complementarity btw. skills and pace (Aucejo et al., 2022; Duflo, Dupas, and Kremer, 2011)

$\rightarrow$ Unique $P_j^*(\theta_j)$ for a child with θ_j that maximizes θ_{j+1}, with $\frac{\partial P_j^*}{\partial \theta_j} > 0$

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Implications: Theory Illustration

- θ_{j+1} decreases in $(\theta_j - \bar{\theta}_j^S)^2$ $\forall S \implies$ strict segregation of children efficient
- Dynamically (w\o re-tracking): skill shocks incur efficiency losses

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→ Model: track allocation by parents

Parent makes the School Track Decision $(j = 11)$ Other Periods

- Parent with education E, human capital h_{11}, assets a_{11} and child with skills θ_3, ability ϕ' solves

$$V_{11}(E, h_{11}, a_{11}; \theta_3, \phi') = \max\{W_{11}(E, h_{11}, a_{11}; S = V, \theta_3, \phi'), \\ W_{11}(E, h_{11}, a_{11}; S = A, \theta_3, \phi') - \chi(E)\}$$

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$$W_{11}(E, h_{11}, a_{11}; S, \theta_3, \phi') = \max_{c_{11}, a_{12}, n_{11}} \left\{ u(\frac{c_{11}}{q}, n_{11}) + \beta \, \mathbb{E}_{\varepsilon_{12}, \eta_4} V_{12}(E, h_{12}, a_{12}; S, \theta_4, \phi') \right\}$$

$$\text{s.t. } \theta_4 = \underbrace{f(\theta_3, \bar{\theta}_3^S, E, \eta_4)}_{\text{(1) w\textbackslash optimal } P_3^S} \quad + \quad \text{BC + Time Constraint + Borrowing Constraint}$$

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- Parent (correctly) anticipates average skill level in each track $\bar{\theta}_3^S$

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- $\chi(E)$: Asymmetric utility costs from sending child to A-track by E Evidence

Calibration

Strategy and Data

1a. Child skill formation technology estimated using *German National Education Panel Study (NEPS)* data

→ Treat skills as latent variables and use (log)-linear measurement system (Agostinelli, Doepke, et al., 2023; Cunha, Heckman, and Schennach, 2010)

→ Measures: child achievement test scores Details Estimates

1b. *German Socio Economic Panel (SOEP)*: labor market moments

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1b. *German Socio Economic Panel (SOEP)*: labor market moments

2a. Set "standard" parameters exogenously Details

2b. Method of simulated moments to calibrate remaining 20 parameters to match 20 data targets Results

Calibration Highlights and Model Fit

1. $\chi(E)$: Deviations from track recommendations from primary school teachers
2. Skill shock variances $\sigma^2_{\eta_j}$, $j = 3, 4, 5$: Regression coefficients from regressing skill rank_j on skill rank_{j-1}

Calibration Highlights and Model Fit

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Model successfully reproduces:

- Children and child skills across school tracks and parental education Child Skills
- Intergenerational mobility and cross-sec. inequality Mobility and Inequality
- Small long-term labor market effects of track choice for children *at the margin* between school tracks (Dustmann, Puhani, and Schönberg, 2017) Details

Quantitative Results

The Timing of School Tracking

- Macroeconomic effects of postponing the school tracking age by 4 years?
  → Theory: Can be efficiency-enhancing if child skill uncertainty is high Details

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Tracking Age	10	14
Outcome	Baseline	Partial Equilibrium
Y	2.05	-0.8%
College Share	0.35	-6.9%
A-Track Share	0.42	-7.4%
A-Track on Income	0.5	-2.8%
IGE	0.31	-1.9%

- Output losses: longer comprehensive school harms learning efficiency, outweighs effect from more homogeneous peer groups later ($\bar{\theta}_5 \downarrow$)

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- Social mobility gains: end-of-school skills become more equal across tracks
  → track depends less on parents + college depends less on track

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A-Track on Income	0.5	-2.8%	-1.4%
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- General equilibrium: Wages and instruction paces adjust → A-track share rises again but still lower as less "strategic" incentives wrt to college chances

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- Trade-off between output losses and mobility gains becomes weaker

Determinants and Consequences of School Track Choice

- Track choice driven by child skills + parental education
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- Track choice driven by child skills + parental education
  - → Direct intput into child skills, preferences for tracks and for college Estimates
- Reducing parental track preferences improves inter-generational mobility and increases learning efficiency + aggregate output

Outcome	Baseline	$\chi = 0$
$\bar{\theta}_5$	0.04	0.1%
Y	2.05	0.1%
A-Track on Skills	1.02	1.4%
IGE	0.31	-2.2%

Conclusion

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School tracking policies important from macroeconomic perspective:

- Initial school track pre-determines sizable share of lifetime inequality Details
- Postponing tracking improves social mobility but harms learning and output (Arenas and Hindriks, 2021; Benabou, 1996)
  → "Second-chance" opportunities become more important
- Parental preferences in track choice important for social mobility and for aggregate output
  → Potential of (mentoring) programs that alleviate parental influence (Falk, Kosse, and Pinger, 2020)

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Thanks!

Any feedback to lukas.mahler@gess.uni-mannheim.de

Back-up

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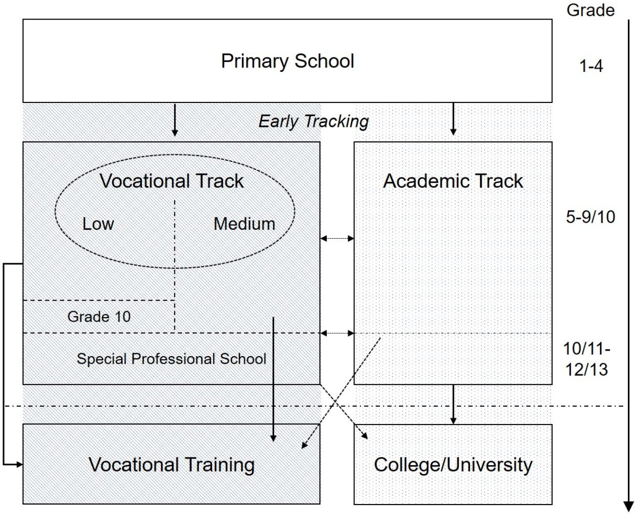
German Education System



- Main differences: Teaching level and peer exposure (Dustmann, Puhani, and Schönberg, 2017) Details Back

Institutional Setting in Germany

- See summaries in Biewen and Tapa (2017), Dustmann, Puhani, and Schönberg (2017), and Matthewes (2021)
    - Some states have a three-tier system (most of former West Germany), others have a two-tier system (former East) with integrated schools
    - Academic track (*Gymnasium*) ends in university-entry qualification, other tracks prepare for vocational career
    - Switches between tracks are possible
    - Majority of states track after 4 years of primary school (i.e. children are aged 9-10), some (Berlin, Brandenburg) track after 6 years
    - In most states, track selection is done by parents, in some states academic track is only possible with min grades in German and Math in primary school (Bavaria, Saxony, Thuringia, Brandenburg)
    - Teacher recommendations based on grades and subjective assessment or primary school teacher should "inform" track choice

Differences in School Tracks

	Vocational Track	Academic Track
Teaching Level		
Curriculum	vocational subjects, "applied" learnings goals	foreign languages, science
Intensity	32 hrs/week	35 hrs/week
Quality of Peers		
Avrg. Reading Score	-0.491	0.558
Avrg. Math Score	-0.583	0.663

- Both important for child's skill accumulation (Burke and Sass, 2013; Duflo, Dupas, and Kremer, 2011; Epple and Romano, 2011; Sacerdote, 2011)
- Average per-pupil expenditures and teacher quality do not differ much between tracks

Optimal Tracking Policy

- If objective is $\max \mathbb{E}[\theta_4]$: Track children at threshold $\tilde{\theta}_3 = \mathbb{E}[\theta_3]$
  - $\rightarrow$ Improves expected learning relative to C-track
  - $\rightarrow$ May increase inequality $Var(\theta_4)$ relative to C-track
  - $\rightarrow$ Children around θ_3 learn less relative to C-track Conditions

Explanation Pace Explanation Results

Optimal Tracking Policy

- If objective is max $\mathbb{E}[\theta_4]$: Track children at threshold $\tilde{\theta}_3 = \mathbb{E}[\theta_3]$
  - $\rightarrow$ Improves expected learning relative to C-track
  - $\rightarrow$ May increase inequality $Var(\theta_4)$ relative to C-track
  - $\rightarrow$ Children around θ_3 learn less relative to C-track Conditions

- If objective is max $\mathbb{E}[\theta_5]$ (and re-tracking not possible): Track children early (in $j = 3$) only if skill shock variance $\sigma^2_{\eta 4}$ is small, o\w track late (in $j = 4$)
  - $\rightarrow$ Late tracking foregoes learning benefits early, achieves more homogeneous groups later Condition

Explanation Pace Explanation Results

Optimal Tracking Policy

- If objective is max $\mathbb{E}[\theta_4]$: Track children at threshold $\tilde{\theta}_3 = \mathbb{E}[\theta_3]$
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  - → May increase inequality $Var(\theta_4)$ relative to C-track
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- If objective is max $\mathbb{E}[\theta_5]$ (and re-tracking not possible): Track children early (in $j = 3$) only if skill shock variance $\sigma^2_{\eta 4}$ is small, o\w track late (in $j = 4$)
  - → Late tracking foregoes learning benefits early, achieves more homogeneous groups later Condition

→ Track choice by <u>parents may not align with optimal</u> tracking threshold

Explanation Pace Explanation Results

Details on Proposition 2

- Aggregate end-of-school skills in a full tracking system are larger than in a full comprehensive system. This holds regardless of who makes the track decision, i.e. regardless of the tracking skill threshold. $\tilde{\theta}_1$. The expected gain from tracking is

$$\mathbb{E}(\theta_2|T) - \mathbb{E}(\theta_3|C) = \frac{\gamma^2}{2\delta}\left(\sigma^2_{\theta_2} - \mathbb{E}(Var[\theta_2|S])\right) \tag{3}$$

- The end-of-school skill distribution in a full tracking system has a "fatter" right tail. In case of tracking at the optimal skill threshold $\tilde{\theta}_1 = \mathbb{E}(\theta_1)$, the variance of end-of-school skills in a full tracking system is larger than the variance in a full comprehensive system iff

$$\alpha^2 + 2\alpha\left(1 + \frac{\beta\gamma}{\delta}\right) - (8-\pi)\frac{\gamma^4}{\pi\delta^2}\sigma^2_{\theta_2} > 0. \tag{4}$$

- Children with initial skills inside an non-empty interval lose from a full tracking system in terms of their end-of-school skills relative to a full comprehensive system. With $\alpha = 0$ the losses are symmetric in both tracks. With $\alpha > 0$, the losses are concentrated in the track with the lower average skill level. Back

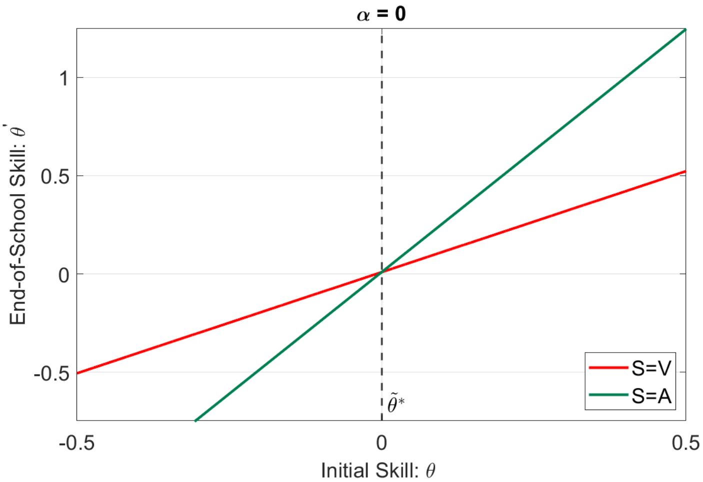
Details on Proposition 3

Proposition 3
Average end-of-school skills are larger in an optimal late tracking system than in an optimal early tracking system iff

$$\frac{\sigma_{\eta_3}^2}{\sigma_{\theta_2}^2} > \alpha + \left(\alpha + (1 + \frac{\beta\gamma}{\delta})\right)^2 - \frac{\beta\gamma}{\delta}\left(1 + \frac{\beta\gamma}{2\delta}\right) + \frac{\gamma^4}{2\delta^2\pi}\sigma_{\theta_2}^2.$$

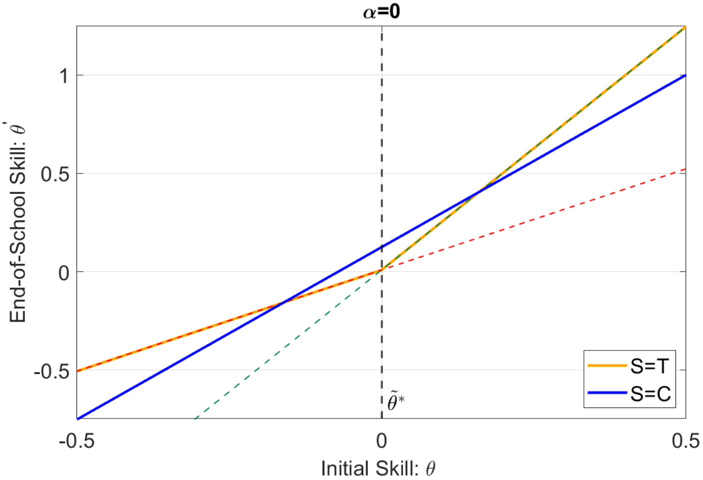
Back

Illustration: Full Tracking vs Full Comprehensive System



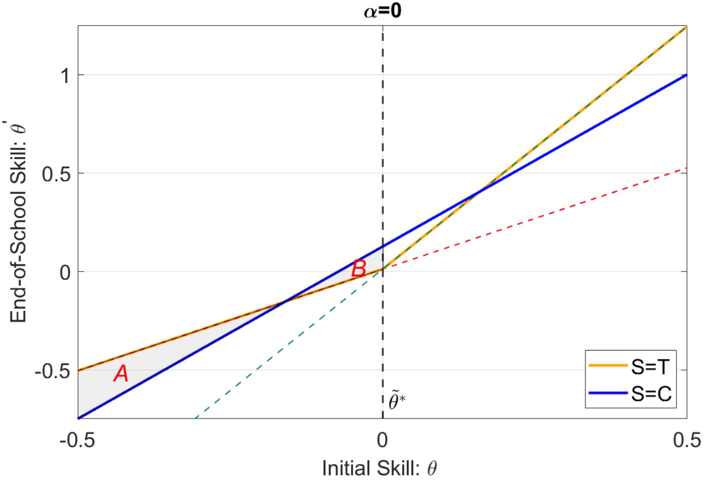
- Child with $\theta = \tilde{\theta}^* = \tilde{\theta}^p$ is indifferent between tracks

Illustration: Full Tracking vs Full Comprehensive System



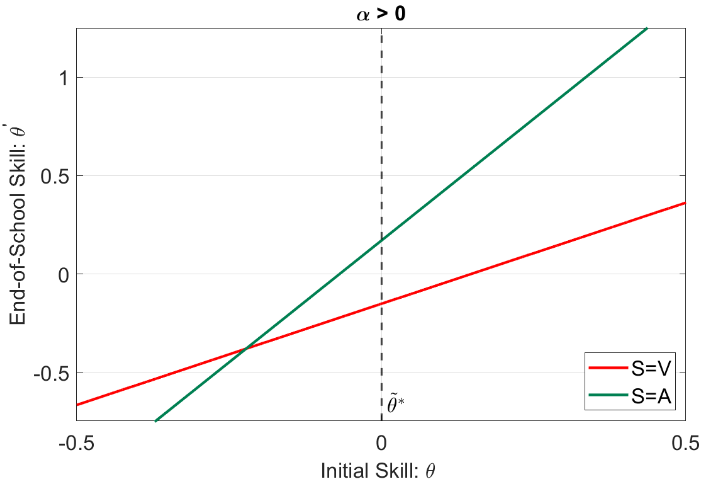
- Children around $\tilde{\theta}^*$ lose from tracking (symmetrically)

Illustration: Full Tracking vs Full Comprehensive System



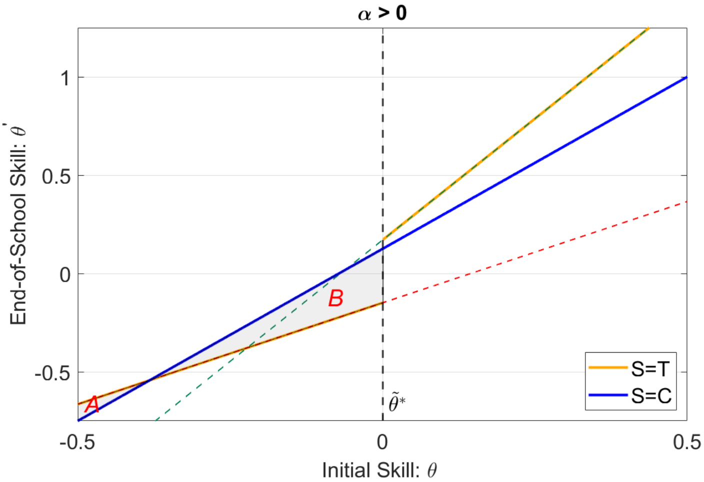
- But gains (A) from tracking outweigh losses (B) in both tracks

Illustration: Full Tracking vs Full Comprehensive System



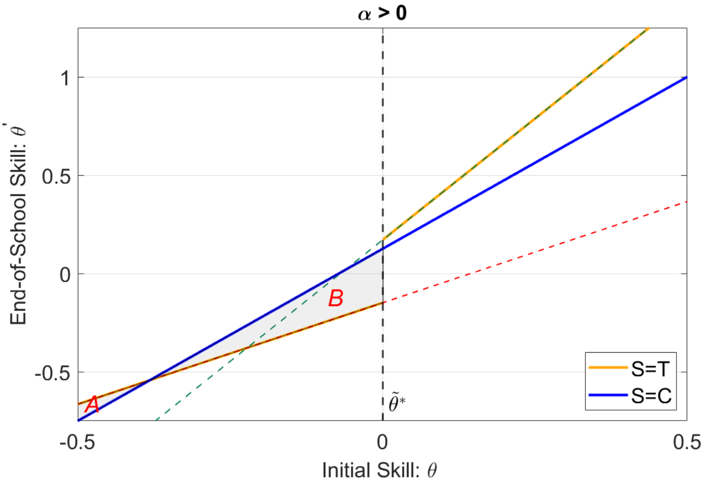
- $\alpha > 0 \implies \tilde{\theta}^*$ is not "incentive-compatible"

Illustration: Full Tracking vs Full Comprehensive System



- Children in V-Track lose on average more from tracking than children in A-Track

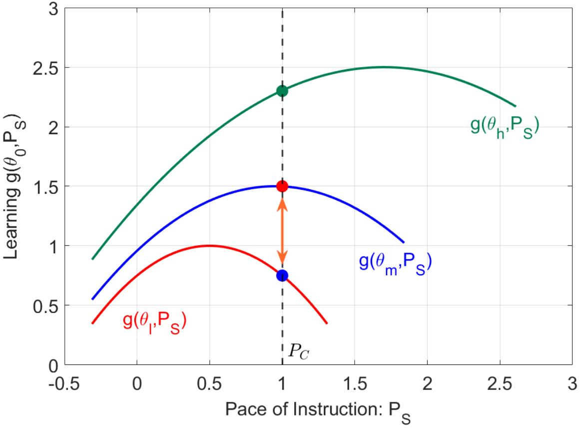
Illustration: Full Tracking vs Full Comprehensive System



- Children in V-Track may even, on average, learn less than in comprehensive system Back

Illustration: Tracking vs Comp. School after Shock Realization

Learning in Comprehensive Track when θ_l is shocked to θ_m and θ_m is shocked to θ_l

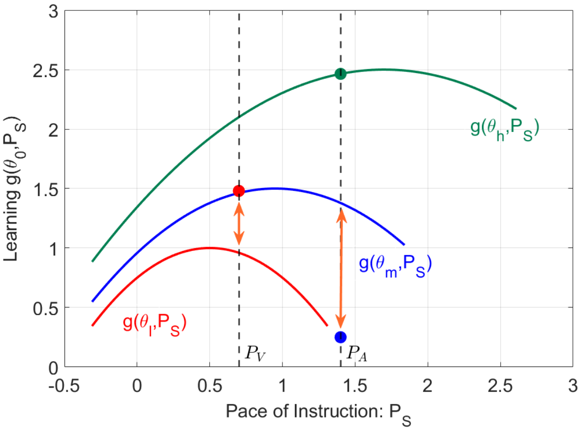


- Aggregate learning remains unaffected

Back

Illustration: Tracking vs Comp. School after Shock Realization

Learning in Tracking System when θ_l is shocked to θ_m and θ_m is shocked to θ_l



- Shocks can lead to aggregate learning losses

Back

School Track Selection by Parental Education

Back

	Academic Track	
High SES	0.35***	0.24***
	0.02	0.02
Controls:		
Age & Gender	yes	yes
Tests	no	yes
R^2	0.2	0.36
N	2,480	2,475

Deviations from Teacher Recom.	High SES	Low SES
Academic Recom.		
Follow	94%	81%
Deviate	6%	19%
Vocational Recom.		
Follow	78%	91%
Deviate	22%	9%

Notes: Data from NEPS Starting Cohort 3. High SES = 1 if at least one parent has an academic school degree and household income $\geq$ 2,000 EUR/month.

- Significant conditional SES-gap in academic track attendance in grade 5 (Falk, Kosse, and Pinger, 2020)
- Parents bias track selection towards their own educational background

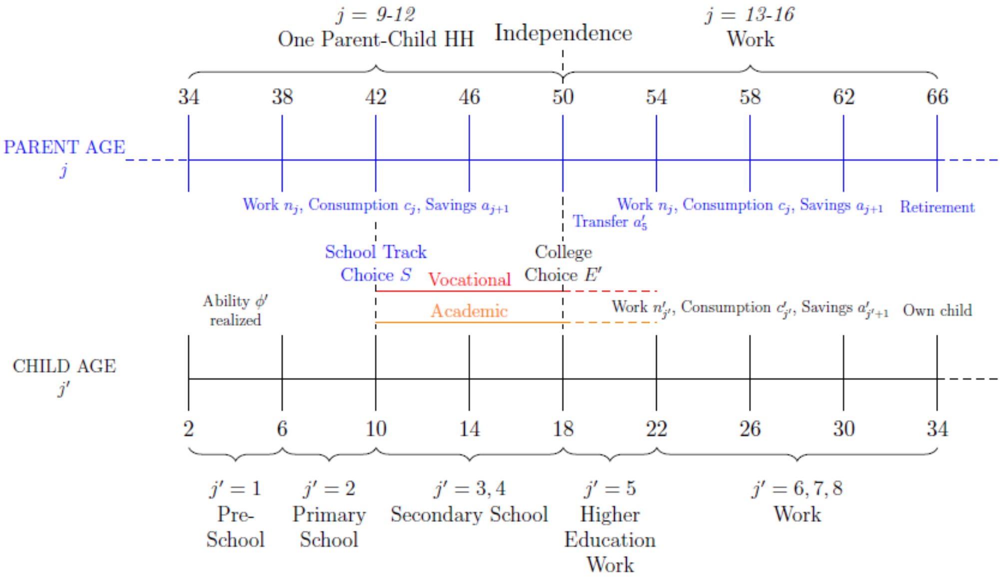
Teacher Recommendations

	Academic Track Recom.	
	Cohort 2	*Cohort 3*
High SES	0.11***	0.14***
	0.01	0.02
Controls:		
Age & Gender	Yes	Yes
Tests	Grades (4)	Test (5)
R	0.44	0.36
N	3575	2634

Back

Detailed Timeline of Life-Cycle Events

Back



Constraints affecting the Decision Problems each Period

Back

- Budget Constraint (during work years)

$$c_j + a_{j+1} = w_E h_j n_j + (1+r)a_j + T(y_j, a_j), \tag{5}$$

where $T(y_j, a_j)$ gives lump-sum transfers g net of progressive labor income taxes and linear capital income taxes; during retirement, agents receive pension benefits $\pi(h_{17})$

- Borrowing Constraint: $a_{j+1} \geq \frac{g}{1+r}$, where r is the interest rate
- Time Constraint (during working years): $n_j \in [0, 1]$
- Human Capital Growth (during working years):

$$h_{j+1} = \gamma_{j,E} h_j \varepsilon_{j+1}, \quad \log \varepsilon_j \sim \mathcal{N}(0, \sigma_\varepsilon^2), \tag{6}$$

where $\gamma_{j,E}$ are age- and education-specific deterministic growth rates and ε_{j+1} are market luck shocks

Value at Independence $(j = 5)$ Back

- Newly independent adult decides on college $(E = 1) \rightarrow$ access to college-skilled labor market in future periods

$$V_5(S, \theta_5, a_5, \phi, \nu(E^p)) = \max\{W_5(E = 0, h_5, a_5, \phi),$$
$$W_5(E = 1, h_5, a_5, \phi) - \psi(S, \theta_5, \nu(E^p))\}$$

$$W_5(E, h_5, a_5, \phi) = \max_{c_5, a_6,, n_5 \in [0, \bar{n}(E)]} \{u(c_5, n_5) + \beta\, \mathbb{E}_{\varepsilon_6}\, V_6(E, h_6, a_6, \phi)\}$$
$$\text{s.t. } h_5 = \exp(\theta_5) \quad + \quad \text{Constraints}$$

Value at Independence $(j = 5)$ Back

- Newly independent adult decides on college $(E = 1) \to$ access to college-skilled labor market in future periods

$$V_5(S, \theta_5, a_5, \phi, \nu(E^p)) = \max\{W_5(E = 0, h_5, a_5, \phi), \\ W_5(E = 1, h_5, a_5, \phi) - \psi(S, \theta_5, \nu(E^p))\}$$

$$W_5(E, h_5, a_5, \phi) = \max_{c_5, a_6,, n_5 \in [0, \bar{n}(E)]} \{u(c_5, n_5) + \beta \, \mathbb{E}_{\varepsilon_6} V_6(E, h_6, a_6, \phi)\}$$

$$\text{s.t. } h_5 = \exp(\theta_5) \quad + \quad \text{Constraints}$$

- $\psi(S, \theta_5, \nu(E^p))$: "Psychic" college costs depend on school track S, end-of-school skills θ_5, and parent-specific preference shock $\nu(E^p)$

Value at Independence $(j = 5)$ Back

- Newly independent adult decides on college $(E = 1) \rightarrow$ access to college-skilled labor market in future periods

$$V_5(S, \theta_5, a_5, \phi, \nu(E^p)) = \max\{W_5(E = 0, h_5, a_5, \phi), \\ W_5(E = 1, h_5, a_5, \phi) - \psi(S, \theta_5, \nu(E^p))\}$$

$$W_5(E, h_5, a_5, \phi) = \max_{c_5, a_6, , n_5 \in [0, \bar{n}(E)]} \{u(c_5, n_5) + \beta\, \mathbb{E}_{\varepsilon_6} V_6(E, h_6, a_6, \phi)\}$$

$$\text{s.t. } h_5 = \exp(\theta_5) \quad + \quad \text{Constraints}$$

- a_5 : Inter-vivos transfer from altruistic parent

Value of young Parent without Child $(j = 6, 7, 8)$

$$V_j(E, h_j, a_j, \phi) = \max_{c_j > 0, a_{j+1}, n_j} \left\{ u(c_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}} V_{j+1}(E, h_{j+1}, a_{j+1}, \phi) \right\}$$

$$h_{j+1} = \gamma_{j,s} h_j \varepsilon_{j+1}$$

BC + Time Constraint + Borrowing Constraint

- In $j = 8$, parents takes expectations over future child's ability ϕ'

$\rightarrow$ $\log \phi' = \rho_\phi \log \phi + \varepsilon_\phi, \quad \varepsilon_\phi \sim \mathcal{N}(0, \sigma_\phi^2)$

Value of Parent with young Child $(j = 9, 10)$

$$V_j(E, h_j, a_j; \theta_{j'}, \phi') = \max_{c_j, a_j, n_j} \left\{ u(\frac{c_j}{q}, n_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}, \eta_{j'+1}}\, V_{j+1}(E, h_{j+1}, a_{j+1}, \theta_{j'+1}, \phi') \right\}$$

$$\text{s.t. } \theta_3 = f(\theta_2, \bar{\theta}_2, E, \eta_3)$$

$$\theta_2 = \log(\phi')$$

BC + Time Constraint + Borrowing Constraint

- In $j = 9$ no expectation over child skill uncertainty

Value of Parent with Child in Secondary School $(j = 12)$

$$V_j(E, h_j, a_j; \theta_{j'}, \phi', S) = \max_{c_j, a_j, n_j} \left\{ u(\frac{c_j}{q}, n_j) + \beta\, \mathbb{E}_{\varepsilon_{j+1}, \eta_{j'+1}}\, V_{j+1}(E, h_{j+1}, a_{j+1}, \theta_{j'+1}, \phi', S) \right\}$$

$$\text{s.t. } \theta_5 = f(\theta_4, \bar{\theta}_4, E, \eta_5)$$

BC + Time Constraint + Borrowing Constraint

Value when own Child becomes independent $(j = 13)$

- Parent makes transfer decision s_5 just before child becomes independent, not knowing college taste shock $\nu'(E)$

$$V_{13}(E, h_{13}, a_{13}, \phi, \theta_5, S) = \max_{a'_5 \geq 0} \left\{ \widetilde{V}_{13}(E, h_{13}, a_{13} - a'_5) + \Lambda \, \mathbb{E}_{\nu'} \, V_{j'=5}(\theta_5, a'_5, \phi, S, E, \nu'(E)) \right\}$$

- $\widetilde{V}_{13}$ is the value for a parent with savings a_{13} after the inter-vivos transfer has been made

$$\widetilde{V}_{13}(E, h_{13}, a_{13}) = \max_{c_{13}, a_{14}, n_{13}} \left\{ u(c_{13}, n_{13}) + \beta \, \mathbb{E}_{\varepsilon_{j+1}} \, V_{14}(E, h_{14}, a_{14}) \right\}$$

BC + Time Constraint + Borrowing Constraint

Values during Work before Retirement and Retirement Back

Model period: $j = 14, 15, 16$; Age: 54-65

$$V_j(E, h_j, a_j, \phi) = \max_{c_j, a_{j+1}, n_j} \left\{ u(c_j, n_j) + \beta \, \mathbb{E}_{\varepsilon_{j+1}} V_{j+1}(E, h_{j+1}, a_{j+1}, \phi) \right\}$$

BC + Time Constraint + Borrowing Constraint

- In $j = 16$, no expectation over market luck shock

Model period: $j = 17, 18, 19, 20$; Age: 66-81

$$V_j(E, h_{17}, a_j) = \max_{c_j > 0, a_{j+1} \geq \underline{a}} \{ u(c_j, 0) + \beta V_{j+1}(E, h_{17}, a_{j+1}) \}$$

$$\text{s.t. } c_j + a_{j+1} = \pi_j(h_{17}) + (1 + r) a_j - T(0, a_j).$$

Firm's Problem and Government

- Representative Firm produces output Y using aggregate capital K and labor inputs H according to

$$Y = K^{\alpha} H^{1-\alpha}, \tag{7}$$

where H is aggregated using a CES technology:

$$H = \{\varphi H_0^{\sigma_f} + (1-\varphi) H_1^{\sigma_f}\}^{\frac{1}{\sigma_f}} \tag{8}$$

- Government taxes labor income y according to $y_{net} = \lambda y^{1-\tau_n}$, capital income according to $\tau_s r s_j$ and financed retirement benefits π_j and lump-sum social welfare g

Equilibrium Back

Equilibrium i

Let $x_j \in X_j$ be the age-specific state vector of an individual of age j, as defined by the recursive representation of the individual's problems. Let its stationary distribution be $\Theta(X)$. Then, a stationary recursive competitive equilibrium for this economy is a collection of: (i) decision rules for college graduation $\{d^S(x_5)\}$, for school track $\{d^{S^c}(x_{11})\}$, consumption, labor supply, and assets holdings $\{c_j(x_j), n_j(x_j), s_j(x_j)\}$, and parental transfers $\{s_5\,(x_j)\}$; value functions $\{V_j\,(x_j)\}$; (iii) aggregate capital and labor inputs $\{K, H_V, H_A\}$; (iv) prices $\left\{r, w^V, w^A\right\}$; and (v) average skill levels among children in school track S^c $\{\bar{\theta}_{j,S^c}\}$ such that:

1. Given prices and average skill levels among children in each school track, decision rules solve the respective household problems and $\{V_j\,(x_j)\}$ are the associated value functions.
2. Given prices, aggregate capital and labor inputs solve the representative firm's problem, i.e. it equates marginal products to prices.

Equilibrium ii

3. Given average skill levels among children in each school track, allocation of children in school track solves the parent's problem, i.e. actual average skill levels are consistent with parents' prior.

4. Labor market for each education level clears.
   For high-school level:

$$H_V = \sum_{j=5}^{J_r} \int_{X_j} n_j(x_j)\, h_j\,(x_j)\, d\Theta(X \mid S = V) + \sum_{j=5}^{5} \int_{X_j} n_j(x_j)\, h_j\,(x_j)\, d\Theta(X \mid S = A)$$

   where the first summation is the supply of high-school graduates while the second is that labor supply of college students.

Equilibrium iii

For college level:

$$H_A = \sum_{j=6}^{J_r} \int_{X_j} n_j(x_j)\; h_j\,(x_j)\, d\Theta(X \mid S = A).$$

5. Asset market clears

$$K = \sum_{j=J_e}^{J_d} \int_{X_j} s_j(x_j) d\Theta(X),$$

which implies that the goods market clears;

6. The distribution of X is stationary: $\Theta(X) = \int \Gamma(X) d\Theta(X)$.

Back

Externally set Parameters

Parameter	Value	Description	Source
Household			
σ	2.0	Inverse EIS	Lee and Seshadri (2019)
γ	0.5	Frisch Elasticity	Fuchs-Schündeln et al., (2022)
q	1.56	Equiv. Scale	Jang and Yum (2022)
$\bar{n}(E=1)$	0.40	Time Cost of College	
Firm			
σ_f	1/3	E.o.S (H_V, H_A)	Ciccone and Peri (2005)
δ_f	6%	Annual Depreciation	
Government			
τ_n	0.128	Labor Tax Progressivity	Kindermann, Mayr, and Sachs (2020)
λ	0.679	Labor Tax Scale	Kindermann, Mayr, and Sachs (2020)
τ_s	0.35	Capital Tax Rate	
g	0.06	Lump-sum Transfers	

Internally calibrated Parameters

Parameter	Value	Description	Target	Data	Model
Preferences					
β	0.935	Discount Factor	Annl. Interest Rate	0.04	0.04
b	6.8	Labor Disutility	Avrg. Labor Supply	0.53	0.53
Λ	0.475	Parental Altruism	CL Expenses/Income	0.60	0.59
χ_V	0.017	Own V-Track Bias	% Deviations from Track	0.16	0.18
χ_A	0.021	Own A-Track Bias	Recommendations by S	0.23	0.21
College Costs					
ψ	0.88	Intercept	Share A $\rightarrow$ College	0.71	0.71
ψ_V	0.25	Add. Costs for V-Track	Share V $\rightarrow$ College	0.11	0.08
ψ_θ	0.7	Coefficient on θ_5	Regression Coefficient	0.80	0.94
$\mu_{S=V}$	0.1	Mean Taste Shock if $S = V$	Share in CL from Non-CL HH	0.20	0.18
$\mu_{\nu,S=A}$	-0.1	Mean Taste Shock if $S = A$	Share in CL from CL HH	0.64	0.66
σ_ν	0.001	Std. Taste Shock	Variance of Residual	0.218	0.122

Internally calibrated Parameters

Parameter	Value	Description	Target	Data	Model
Idiosyncratic Shocks					
σ_ε	0.008	Std. Luck Shock	Std(Log Labor Income)	0.73	0.82
σ_ϕ	0.07	Std. Ability Shock	Var(Test Scores Grade 1)	0.12	0.12
ρ_ϕ	0.65	Persistence of Ability	Test Scores Diff. by S	0.47	0.52
σ_{η_3}	0.07	Std. Learning Shock $j = 3$	$\text{Rank}_{j=2}$-$\text{Rank}_{j=3}$	0.59	0.60
σ_{η_4}	0.065	Std. Learning Shock $j = 4$	$\text{Rank}_{j=3}$-$\text{Rank}_{j=4}$	0.63	0.68
σ_{η_5}	0.05	Std. Learning Shock $j = 5$	$\text{Rank}_{j=4}$-$\text{Rank}_{j=5}$	0.72	0.74
Miscellaneous					
Ω	0.14	Pension Anchor	Replacement Rate	0.40	0.39
A	2.5	TFP	Avrg. Labor Earnings	1.0	1.0
φ	0.54	Weight V. Human Capital	College Share	0.35	0.35

Back

Model Verification: Non-targeted moments I Back

Moment	Data	Model
Child Skill Moments		
Mean Differences by Parental Background (in Standard Deviations)		
Beginning Secondary School	0.58	0.65
Middle Secondary School	0.70	0.71
Mean Differences by School Track (in Standard Deviations)		
Beginning Secondary School	0.87	0.92
Middle Secondary School	1.01	0.77
School Track Choice		
Relative share A-track children from CL. HH	0.74	0.72
Relative share A-track children from Non-CL HH	0.24	0.25
Coefficient A-track on Skill Rank	0.87	1.02

Model Verification: Non-targeted moments II Back

Moment	Data	Model
Intergenerational Mobility		
Parental Income Gradient (Dodin et al., 2021)	0.52	0.50
Q5/Q1 A-track on income (Dodin et al., 2021)	2.13	2.50
Q1 A-track on income (Dodin et al., 2021)	0.34	0.26
IGE (Kyzyma and Groh-Samberg, 2018)	0.27-0.37	0.30-0.33
Inequality - Returns to College		
Gini Coefficient of Labor Income	0.29	0.26
CL/Non-CL Earnings	1.69	1.76

Model Verification: Effect of Track on Labor Market Outcomes

- Dustmann, Puhani, and Schönberg (2017): Track choice has no long-term labor market effects for children at the *margin* between school tracks
- In our model, track choice, conditional on all other states is characterized by skill threshold $\tilde{\theta}_3$

Model Verification: Effect of Track on Labor Market Outcomes

- Dustmann, Puhani, and Schönberg (2017): Track choice has no long-term labor market effects for children at the *margin* between school tracks
- In our model, track choice, conditional on all other states is characterized by skill threshold $\tilde{\theta}_3$

→ Compare children with *same states* $(E, h_{11}, a_{11}, \phi')$ and skills in a 10% interval around threshold $\tilde{\theta}_3$ who go to different initial school tracks

→ Academic track choice for these *model-marginal* children yields 4.5% higher PV of lifetime earnings, 4.6% higher PV of lifetime wealth

Back

Contributions to Inequality in Lifetime Outcomes Back

- Decompose variance of lifetime earnings (LFE) and lifetime wealth (LFW) into variance conditional on states at different life stages (Huggett, Ventura, and Yaron, 2011; Lee and Seshadri, 2019)

Contributions to Inequality in Lifetime Outcomes Back

- Decompose variance of lifetime earnings (LFE) and lifetime wealth (LFW) into variance conditional on states at different life stages (Huggett, Ventura, and Yaron, 2011; Lee and Seshadri, 2019)

Life Stage	States	Explained Variance LFE	Explained Variance LFW
Independence (age 18)	$(S, \phi, h_5, a_5, E, E^p)$	70%	65%
	(S, ϕ, a_5, E, E^p)	54%	45%
School Track Choice (age 10)	$(S, \phi', \theta_3, h_{11}, a_{11}, E)$	23%	30%
	(S)	12%	13%
Pre-Birth (parent age 30)	(E, ϕ, h_8, a_8)	10%	20%

$\rightarrow$ Initial school track alone accounts for large share of lifetime inequality

Track Choice Determinants

Back

Dependent Variable: $S = A$				
	Stand. Coefficient Estimates			
	(1)	(2)	(3)	(4)
	Baseline	$\omega_{5,j=3,4} = 0$	$\mu_{\nu,1} = \mu_{\nu,0} = 0$	$\chi_0 = \chi_1 = 0$
ϕ'	0.09	0.10	0.10	0.10
θ_3	0.53	0.54	0.58	0.56
$E = 1$	0.34	0.24	0.23	0.21
h_{11}	0.00	-0.01	0.00	0.00
a_{11}	0.02	0.01	0.02	0.02

Notes: Standardized coefficient estimates of regressions of an academic school track dummy on all states at the time of the tracking decision. Column (1) corresponds to the baseline economy. In Column (2), we shut down the channel of differential parental inputs in periods 3 and 4. Column (3) considers the case of identical college taste shock by parental education. In Column (4), we remove the parental preference bias for education.

Consequences of Parental Preferences on Track Choice

- Skill technology: (mis)-allocation of children across tracks due to parental preferences $\implies$ average learning losses as instruction pace adjusts
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Outcome	Baseline	$\chi_S = 0$	50:50 split
Y	1.11	0.0%	0.0%
College Share	0.36	-0.8%	-0.6%
A-Track Share	0.42	4.0%	18.8%
A-Track on Income	0.44	-33.1%	-45.3%
CL on Skills	0.79	0.3%	1.9%
IGE	0.28	-3.6%	-1.8%
Gini Earnings	0.22	0.5%	0.0%

$\rightarrow$ No effects on output but social mobility increases Learning Gains Back

Late Tracking Effects on Learning

Outcome	(1) Early Tracking	(2) PE + ET Pace	(3) PE + Pace adjusts	(4) GE + ET Pace	(5) GE + Pace adjusts
$\bar{\theta}_4$	-0.215	-7.9%	-3.3%	-12.6%	-12.6%
$\bar{\theta}_5$	-0.233	-4.3%	-7.3%	-8.2%	-12.4%
$Std(\theta_{4\|S^c=V})$	0.21	-11.4%	-41.0%	-10.5%	-37.6%
$Std(\theta_{4\|S^c=A})$	0.327	-12.8%	-9.8%	-14.7%	-12.8%
$Std(\theta_{5\|S^c=V})$	0.283	-8.8%	-13.4%	-8.5%	-31.8%
$Std(\theta_{5\|S^c=A})$	0.279	-5.0%	-7.2%	-6.8%	-9.3%

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Details on Estimation of Child Skill Formation

Skill Formation Technology and Measurement

- Specify formation of child i's (log) skills in period $j^c + 1$ according to

$$\begin{aligned} \theta_{i,j+1} = \omega_0 + \omega_1 \theta_i + \omega_2 \theta_i^2 + \omega_3 \bar{\theta}_{-i|C_S(i)} \\ + \omega_4 (\theta_i - \bar{\theta}_{S^c(i)})^2 + \omega_5 E_i + \eta_{i,j+1}. \end{aligned} \tag{9}$$

  - $\bar{\theta}_{-i,C_{S^c(i)}}$: Average (log) skills of child i's peers in class $C_S^c(i)$
  - $\bar{\theta}_S$: Average (log) skills in school track $S^c(i)$
  - E_i : Parental-education dummy variable

$\rightarrow$ rearranged version of (1) after substituting optimal P_j^S

- Treat child skills as latent variables (Agostinelli, Saharkhiz, and Wiswall, 2019; Cunha, Heckman, and Schennach, 2010)

Log-Linear Measurement System

- For each j, we have (at least) three measures $m = 1, 2, 3$ for latent skills, given by:

$$M_{i,j,m} = \mu_{j,m} + \lambda_{j,m}\theta_{i,j} + \epsilon_{i,j,m} \tag{10}$$

- The measures for skills constitute test scores for different domains
- $\epsilon_{i,j,m}$: Measurement errors with $\mathbb{E}[\epsilon_{j,m}] = 0 \ \forall \ m, j$

Assumptions

1. $\lambda_{j,1} = 1 \ \forall \ j$
2. $\mathbb{E}[\theta_{i,j}] = 0 \ \forall \ j$
3. Measurement errors are independent contemporaneously across measures, and from latent variables

Identification

- Under the assumptions, we can identify $\mu_{j,m}$ and $\lambda_{j,m}$ from ratios of covariances of the measures

$\rightarrow$ identify latent skills up to measurement error

$$\theta_{i,j} = \frac{M_{i,j,m} - \mu_{j,m}}{\lambda_j} - \frac{\epsilon_{i,j,m}}{\lambda_{j,m}} = \tilde{M}_{i,j,m} - \frac{\epsilon_{i,j,m}}{\lambda_{j,m}} \tag{11}$$

- Rewrite empirical analogue of (9) in terms of observed $\tilde{M}_{i,j,m}$ such that it can be estimated from data

$\rightarrow$ Residuals contain structural errors η and measurement errors ϵ

$\rightarrow$ Aggregate measure into unbiased factor using Bartlett scores (Agostinelli, Doepke, et al., 2023)

Data

- German National Educational Panel Survey (NEPS), Starting Cohorts 2,3,4: 2011-2018

→ longitudinal data on child competencies and school, classroom and home environments

→ independent ($\sim$ biannual) tests on math, reading, scientific, and other domains

- We estimate using test measures $\tilde{M}_{i,j,m}$ between four periods:

  $j^c = 2$ primary school (ages 6-10)

  $j^c = 3$ first stage of secondary school (ages 10-14)

  $j^c = 4$ second stage of secondary school (ages 14-18)

- Assume latent variables and errors are normally distributed
- For now: Use only estimates from SC 3

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Estimates of Child Skill Technology

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Dependent Variable: $\theta_{i,j+1}$ Grade 9 on Grade 5		
Coefficient	Variable	
$\hat{\omega}_1$	$\theta_{i,j}$	0.65
		(0.026)
$\hat{\omega}_2$	$\theta_{i,j}^2$	0.02
		(0.02)
$\hat{\omega}_3$	$\bar{\theta}_{-i,j,S}$	0.12
		(0.082)
$\hat{\omega}_4$	$(\theta_{i,j} - \bar{\theta}_{j,S})^2$	-0.05
		(0.025)
$\hat{\omega}_5$	$E = 1$	0.10
		(0.04)
N Children		1,675

Track Choice and College by Parental Background

	Data	Model
Share $S' = A\|$ Academic Parent	0.74	0.67
Share $S' = A\|$ Vocational Parent	0.24	0.3
Share $S = A\|$ Academic Parent	0.71	0.59
Share $S = A\|$ Vocational Parent	0.19	0.33
$\hat{\beta}_y$	0.52	0.33
$\hat{\beta}_\theta$	0.87	1.38

- $\hat{\beta}_y$: Coefficient from regression of academic track of child on parental income rank Dodin et al. (2021)
- $\hat{\beta}_\theta$: Coefficient from regression of academic track of child on parental income rank Back