## **Consumer Bankruptcy:** the Role of Financial Frictions

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Importance of consumer credit markets

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- Through what channels and to what extent do FFs shape the welfare implication of a consumer bankruptcy law?

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### Key Findings

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- **Banks**: Agency problem with depositors (i.e., HH savers)

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Timing Value function with h = 0 Value function with h = 1

#### **Firms**



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Capital investment is financed by undefaultable bank loansGross rates of return on physical capital and labor:

$$1 + r_k = F_K(K, E) + (1 - \delta)$$
$$w = F_E(K, E)$$



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Price schedule of bank loans

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Exogenous calibration

Internal calibration

Untargeted Moments

#### **Consumer Credit with Financial Frictions**

■ Incentive constraint:

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where  $W(N) = \xi \cdot N$ 

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**Divestment channel**:

Reduced loans to firms  $\rightarrow$  lower capital, production, wages

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• Solving transition path ) • Transition path of banking leverage ratio:  $\eta$  (e.g.)

Variable	Benchmark	Longer Exclusion
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Welfare (CEV in %)	Benchmark	Longer Exclusion
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 Lower borrowing cost + Mitigated adverse effects of FFs > Costly filing
 Why so bad for HHs with bad credit history? Loss of borrowing ability in the short run ≫ Benefits in the long run

# Conclusion

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- Study the role of financial frictions in consumer credit markets
- Incentive and divestment channels
- Interaction between bankruptcy strictness & FFs
- Stricter rules are favored by most HHs, but not all

#### Thank you and find me on Twitter **9 @Li\_Econ**

# Appendix

# **Bankruptcy Regimes in US**

- Chapter 7
  - Most unsecured debts are discharged in exchange for non-exempt assets
  - Filers do not have to use future income to repay debts
  - Filers must pay filing and legal fees
  - Such record stays on credit report for 10 years
  - In 2017, the non-business bankruptcy filings under Ch.  $7 \approx 60\%$
- Chapter 13
  - It involves reorganization
  - Filers have to make a plan to repay debtors over 3 to 5 years
  - Filers can keep property
  - Such record stays on credit report for 7 years

#### **Related Literature / Contributions**

- Consumer default: Chatterjee et al. (2007), Livshits et al. (2007)
   Financial frictions: Gertler and Kiyotaki (2010), G. and Karadi (2011)
   First to model endogenous consumer default and financial frictions
- Consumer bankruptcy debate: Athreya (2002), Li and Sarte (2006), Livshits et al. (2007), Nakajima (2017), Exler et al. (2020)
   First to analyze the role of financial frictions consumer credit markets and its welfare implications

◀ Back

# Timing

- Households begin each period with state (a, e, v, h)
- Given borrowing prices q(a', e), households with good credit history h = 0 choose to either repay debt d = 0 or file for bankruptcy d = 1
  - If d = 0, they also choose a' and consume  $c = w \cdot \exp(e) + a q(a', e) \cdot a'$
  - If d = 1, they consume the leftover earnings  $c = (1 \eta) \cdot w \cdot \exp(e)$  and their credit history turns bad h' = 1
- Households may die at a rate of  $(1 \rho)$ 
  - Among households who survive, e' and  $\nu'$  are drawn from  $Q^e(e'|e)$  and  $Q^{\nu}(\nu')$ . Bad credit history could be removed with probability  $\mathbb{P}_h$
  - Newborn households begin with no assets a' = 0, labor productivity e' drawn from G<sup>e</sup>, no present bias v' = 1, and good credit history h' = 0

#### Vale Function with h = 0

The value function of households with good credit history is thus given by:

$$V(\epsilon, a, e, \nu, h = 0) = \max_{d} \left[ V^{d=0}(a, e, \nu, h = 0) + \epsilon^{d=0}, V^{d=1}(q, e, \nu, h = 0) + \epsilon^{d=1} \right],$$

where  $\epsilon^d$  is drawn from the following extreme value distribution  $EV(\epsilon^d)$ :

$$EV(\epsilon^d) = \exp\left\{-\exp\left(-\frac{\epsilon^d - \mu_{\epsilon}}{\zeta}\right)\right\},\,$$

where  $\zeta > 0$  determines the variance of the shock and  $\mu_{\epsilon} = -\zeta \cdot \gamma_E$  makes the shock mean zero and  $\gamma_E$  is the Euler's constant

◀ Back

#### Vale Function with h = 0 (cont.)

The conditional value function of repayment is given by:

$$\begin{aligned} V^{d=0}(a,e,\nu,h=0) &= \max_{a'} \left[ u \left( w \cdot \exp(e) + a - q(a',e) \cdot a' \right) \right. \\ &+ \nu \cdot \beta \cdot \rho \cdot \sum_{(e',\nu')} Q^e(e'|e) \cdot Q^\nu(\nu') \cdot V(a',e',\nu',h'=0) \right], \end{aligned}$$

The conditional value function of defaulting is then given by:

$$V^{d=1}(a, e, \nu, h = 0) = u \left( (1 - \eta) \cdot w \cdot \exp(e) \right) + \nu \cdot \beta \cdot \rho \cdot \sum_{(e', \nu')} Q^e(e'|e) \cdot Q^{\nu}(\nu') \cdot V(a' = 0, e', \nu', h' = 1),$$

Assume that filing for bankruptcy is feasible only if  $a < -\eta \cdot \exp(e)$ 

#### Vale Function with h = 0 (cont.)

Under the distributional assumption on the utility shocks, the default choice probability  $g_d$  takes the following form:

$$g_d(a, e, \nu, h = 0) = \begin{cases} \frac{\exp\{V^{d=1}(a, e, \nu, h = 0)/\zeta\}}{\exp\{V^{d=0}(a, e, \nu, h = 0)/\zeta\} + \exp\{V^{d=1}(a, e, \nu, h = 0)/\zeta\}} & \text{if } a < -\eta \cdot \exp(e);\\ 0 & \text{otherwise.} \end{cases}$$

The unconditional value function of households with good credit history is then given by:

$$V(a, e, \nu, h = 0) = \mathbb{E}_{\epsilon} V(\epsilon, a, e, \nu, h = 0)$$
  
=  $\zeta \cdot \ln\left(\exp\left\{\frac{V^{d=0}(a, e, \nu, h = 0)}{\zeta}\right\} + \exp\left\{\frac{V^{d=1}(a, e, \nu, h = 0)}{\zeta}\right\}\right)$ 


### Vale Function with h = 1

The value function of households with bad credit history h = 1 is given by:

$$\begin{split} V(a,e,\nu,h=1) &= \max_{a' \ge 0} \bigg[ u \left( w \cdot \exp(e) + a - \bar{q} \cdot a' \right) + \nu \cdot \beta \cdot \rho \cdot \sum_{(e',z',h')} Q^e(e'|e) \cdot Q^\nu(\nu') \\ & \cdot \Big( \mathbb{P}_h \cdot V(a',e',\nu',h'=0) + (1 - \mathbb{P}_h) \cdot V(a',e',\nu',h'=1) \Big) \bigg], \end{split}$$

where  $\bar{q} \equiv \rho/(1 + r_f)$  denotes the discount risk-free rate and bad credit record could be removed with probability  $\mathbb{P}_h$ . I use  $\mu(a, e, \nu, h)$  to denote the cross-sectional distribution of households

$$W(N) = \max_{K', \mathcal{A}'} \left[ \beta_f (1 - \psi) \pi' + \beta_f W(N') \right]$$
(lifetime dividends)  
s.t.  $N' = \psi \pi'$  (retained earnings)  
 $\pi' = (1 + r'_k - \delta)K' + (1 + r'_l)L' - (1 + r_f)D'$  (profit)  
 $K' + L' = D' + N$  (balance sheet)  
 $W(N) \ge \theta(K' + L')$  (incentive constraint)

#### **Return on Unsecured Loans**

■ It is defined as:

$$1 + r'_{l} = \frac{-\sum_{a' < 0, e} \left[ \int_{e'} R(a', e') \, dF(e'|e) \right] \mathcal{A}'(a', e)}{L'}$$

Numerator consists of full repayment and wage garnishment

$$R(a', e') = (1 - d'(a', e'))a' + d'(a', e')\eta w' \exp(e')$$

Denominator denotes aggregate discount loans

$$L' = -\sum_{a'<0,e} \left[q(a',e)a'\right] \mathcal{A}'(a',e)$$



## **Agency Problem b/w Banks and Depositors**

Incentive constraint:

$$W(N) \ge \theta(K' + L') \to \xi N \ge \theta(K' + L') \to \frac{\xi}{\theta} \ge \left(\frac{K' + L'}{N}\right) \equiv LR'$$

where  $W(N) = \xi N$  has been widely shown in the literature This translates to an endogenous leverage constraint

• Necessary and sufficient conditions are:

$$\Lambda' \left[ r'_k - (\delta + r_f) \right] = \lambda \theta$$
  
$$\Lambda' \left[ \int_{e'} R(a', e') \, dF(e'|e) \right] = \left[ \Lambda'(1 + \tau + r_f) + \lambda \theta \right] q(a', e')$$
  
$$\lambda \left[ \xi N - \theta \left( K' + L' \right) \right] = 0$$

where  $\Lambda' = \beta_f (1 - \psi + \psi \xi')$  is the adjusted discount factor and  $\lambda$  denotes the multiplier on the incentive constraint

## **No-Arbitrage Conditions**

• Excess returns are equal:

$$r'_k - (\delta + r_f) = r'_l - (\tau + r_f) = \iota \equiv \frac{\lambda \theta}{\Lambda'} \ge 0$$

*i*: Leverage premium,  $\lambda$ : IC multiplier,  $\Lambda'$ : Adjusted discount factor

- $\iota$  is determined by whether and how much IC is binding
  - $\iota = 0$  when IC is slack
  - $\iota > 0$  when IC is binding  $\longrightarrow \iota \gg 0$  if IC becomes more binding

#### **Price Schedule of Bank Loans**

• For each loan contract  $\mathcal{A}'(a' < 0, e)$ ,

$$q(a',e) = \frac{\rho \int_{e'} \left[ (1 - d'(a',e')) + d'(a',e') \left(\frac{\eta w' \exp(e')}{a'}\right) \right] dF(e'|e)}{1 + r_f + \iota}$$

1 - individual-level default premium

opportunity cost + aggregate-level incentive premium

- Recall that:  $e = (e_1, e_2, e_3) \rightarrow q(a', e_1, e_2)$
- $\theta = 0$  resembles the frictionless case (only default premium)

• Note that: 
$$q(a' > 0, e) = (1 + r_f)^{-1}$$

#### Back

Parameter		Value	Source / Target
<b>Households</b> CRRA coefficient Household survival rate Household discount factor	γ ρ β	2 0.98 0.9592	Standard Avg. working lifespan of 50 years Effective discount factor of 0.94
<b>Production</b> Capital share Depreciation rate	α δ	0.36 0.08	Standard Standard

# **Exogenous Calibration (cont.)**

Parameter		Value	Source / Target
Financial market			
Risk-free rate	rf	0.04	McGrattan and Prescott (2000)
Wage garnishment rate	ή	0.25	25% of disposable income
Probability of flag removal	$\mathbb{P}_h$	0.10	Avg. exclusion of 10 years
Bank survival rate	ψ	0.8926	Avg. planning period of 10 years
Diverting fraction	$\dot{\theta}$	0.2918	25% lower than the targeted ratio
Transfer to newly entering banks	ω	0.0101	1% of total assets intermediated
Exogenous processes			
S.D. of permanent labor productivity	$\sigma_1$	0.448	Storesletten et al. (2004)
AR(1) of persistent labor productivity	$\rho_2$	0.957	Storesletten et al. (2004)
S.D. of persistent labor productivity	$\sigma_2$	0.129	Storesletten et al. (2004)
S.D. of transitory labor productivity	$\sigma_3$	0.351	Storesletten et al. (2004)
Support of household preferences	$(\nu_1, \nu_2)$	(0,1)	Hand-to-mouth households

- Dispersion of extreme value shock  $(\zeta_d)$
- Probability of preference shocks  $(\mathbb{P}_{\nu})$

Parameter	Value	Target	Data	Model
$\mathbb{P}_{\nu}$ $\zeta$	0.01057	Banking leverage ratio	4.57	4.57
	0.02150	Chapter 7 default rate (%)	0.61	0.61



### **Untargeted Moments Aligned with Data**

Source: SCF (2004), Exler and Tertilt (2020)

### **Untargeted Moments Aligned with Data**

Moment (in %)	Data	Model
Fraction of households in debt	7.05	8.63
Debt-to-earnings ratio	2.56	1.87
Average borrowing interest rate	10.93 - 12.84	12.18

◀ Back

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## **Benchmark vs. Frictionless Economy in %**

Variable	Benchmark	Frictionless
Incentive premium	-	-100.0000
Avg. borrowing interest rate Conditional default rate	-	-12.5789 -14.5683
Fraction of HHs in debt Debt-to-earnings ratio	-	5.1374 4.2824
GDP Wage	- -	2.9035 2.9035



## Effects of Varying Diverting Fraction $\theta$

# Effects of Varying Diverting Fraction $\theta$

Variable	$\theta = 0.2888$	$\theta = 0.2918$	$\theta = 0.2947$
Consumer credit markets			
Avg. borrowing interest rate (%)	12.1411	12.1829	12.2221
Conditional default rate (%)	0.6073	0.6082	0.6090
Fraction of HHs in debt (%)	8.6511	8.6335	8.6175
Debt-to-earnings ratio (%)	1.8796	1.8748	1.8705
Incentive & divestment channels			
Incentive premium (%)	0.5935	0.6264	0.6570
GDP	1.8055	1.8028	1.8004
Wage	1.1555	1.1538	1.1522

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GDP	1.8055	1.8028	1.8004
Wage	1.1555	1.1538	1.1522

# Effects of Varying $\psi$

Variable	$\psi = 0.9091$	$\psi = 0.9000$	$\psi = 0.8889$
Consumer credit markets			
Avg. borrowing interest rate (%)	11.8810	12.0933	12.2303
Fraction of HHs in debt (%)	8.8426	8.6720	8.6143
Debt-to-earnings ratio (%)	1.9602	1.8855	1.8697
Conditional default rate (%)	0.5969	0.6064	0.6091
Incentive & divestment channels			
Incentive premium (%)	0.4670	0.5562	0.6635
GDP	1.8157	1.8085	1.7998
Wage	1.1621	1.1574	1.1519

# **Solving Transition Path**

- Solve old and new equilibria and set the number of transition periods
- Guess the transition path of banking leverage ratio and the implied aggregate prices over time
- First solve household problem backward to get policy functions and then use them to simulate the economy forward
- Compute aggregate variables and the updated banking leverage ratio
- Compare between the old and new ratios; if not close enough, update it and do the above procedures again

## **Transition Path of Banking Leverage Ratio**

(a)  $\eta = 0.25 \rightarrow 0.20$  (b)  $\eta = 0.25 \rightarrow 0.30$ 

