Replicating Business Cycles and Asset Returns with Sentiment and Low Risk Aversion[1]

Kevin J. Lansing
Federal Reserve Bank of San Francisco

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[1] Any opinions expressed here do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

What is the most important business cycle shock?

Productivity and labor supply shocks: Chari, Kehoe, McGrattan (2007, JPE).

Productivity and wage markup shocks: Smets, Wouters (2007, AER).

Investment shocks: Justiniano, Primiceri, Tambalotti (2010, JME).

Risk shocks: Christiano, Motto, Rostagno (2014, AER).

Confidence shocks: Angeletos, Collard, Dellas (2018, Econometrica).

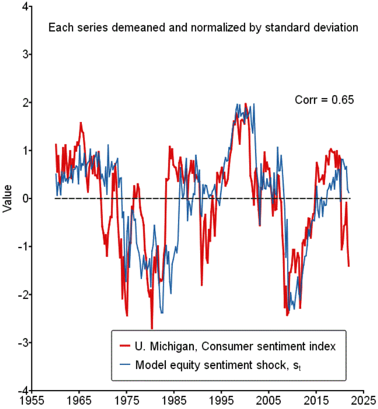
! None of these studies employ data on U.S. asset returns showing the equity risk premium or the bond term premium.

! None of these studies allow for fluctuations in U.S. capital share of income (models are missing a key technology shock).

Summary of paper

- Solve for 9 RBC model shock sequences to exactly replicate:
  - 8 U.S. macro times series: y_t, c_t, i_t, h_t, k_t, $r_t k_t / y_t$, $p_{s,t}$, d_t.
  - 3 types of U.S. asset returns: $r_{s,t}$, $r_{b,t}$, $r_{c,t}$.
- Capital law of motion shocks and equity sentiment shocks are important drivers of movements in most U.S. variables and asset returns.
- But other shocks (e.g., capital share of income shocks) also play a significant role, particularly for low frequency movements.
- There is no "most important shock." Rather, U.S. outcomes have been shaped by a complex and time-varying mixture of fundamental and non-fundamental disturbances.

Preview of results: Sentiment and risk aversion shocks

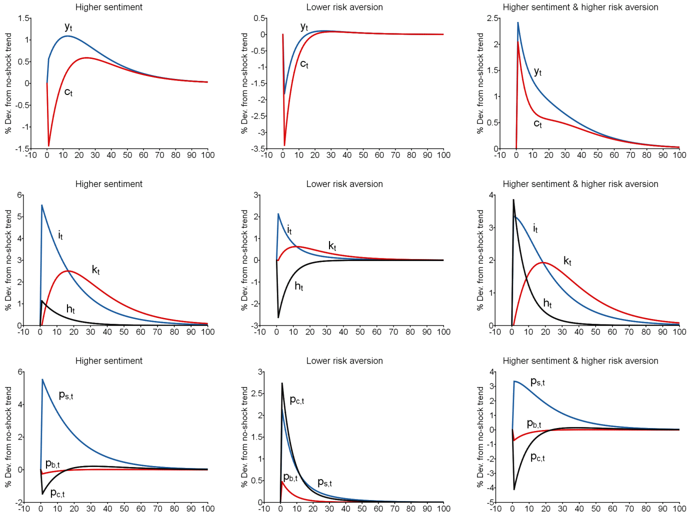


Empirical evidence of pro-cyclical risk aversion

Each study employs data on equity options for S&P 500 stock index.

- Barone-Adesi, Mancini, and Shefrin (2017, WP): Estimated risk aversion is higher after market gains and lower after market losses.
- Bliss and Panigirtzoglou (2004, *Journal of Finance*): Estimated risk aversion is lower during sample periods with high stock market volatility, such as crises.
- Kosolapova, Hanke and Weissensteiner (2023, *Quantitative Finance*): Estimated risk aversion is pro-cyclical.
- Hypothesis: Representative investor changes as market changes. Higher risk-averse investors tend to enter the stock market during booms but leave the market during busts.

Procyclical risk aversion solves comovement problem



RBC model with fundamental shocks in red

$$\max_{\substack{c_t,\, k_{t+1},\\ h_{1,t},\, h_{2,t}}} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t - \kappa_t C_t) - D \exp(u_t) \frac{(h_{1,t} + h_{2,t})^{1+\gamma}}{1+\gamma} \right]$$

$$c_t \;+\; i_t \;=\; w_t h_{1,t} + r_t k_t = y_t = A\, k_t^{\alpha_t} \left[\exp(z_t)\, h_{1,t}\right]^{1-\alpha_t}$$

$$k_{t+1} = B \exp(v_t)\, k_t^{1-\delta_t-\varphi_t}\, i_t^{\delta_t} \left[\exp(z_t)\, h_{2,t}\right]^{\varphi_t}$$

$\eta_t \equiv (1-\kappa_t)^{-1}$ = risk aversion shock, I(0)

u_t = labor disutility shock, I(0)

z_t = labor augmenting productivity shock, I(1)

α_t = factor distribution shock, I(0)

v_t = capital law of motion multiplier shock, I(0)

δ_t = capital law of motion investment shock, I(0)

φ_t = capital law of motion investor effort shock, I(0)

First-order conditions: Equity value is linked to investment

$$D\exp(u_t)(h_{1,t}+h_{2,t})^{\gamma} = \frac{\eta_t}{c_t}\frac{(1-\alpha_t)y_t}{h_{1,t}}$$

$$D\exp(u_t)(h_{1,t}+h_{2,t})^{\gamma} = \frac{\eta_t}{c_t}\frac{\varphi_t i_t}{\delta_t h_{2,t}}$$

$$\underbrace{\frac{i_t}{\delta_t}}_{p_{s,t}} = \widehat{E}_t \underbrace{\beta\frac{c_t}{c_{t+1}}\frac{\eta_{t+1}}{\eta_t}}_{M_{t+1}}[\underbrace{\alpha_{t+1}y_{t+1}-(1+\frac{\varphi_{t+1}}{\delta_{t+1}})i_{t+1}}_{d_{t+1}}+\underbrace{\frac{i_{t+1}}{\delta_{t+1}}}_{p_{s,t+1}}]$$

<u>Equilibrium bond prices</u>:

$$p_{b,t} = E_t M_{t+1} \qquad (E_t \equiv \text{RE})$$

$$p_{c,t} = E_t M_{t+1}\left[1+\bar{\delta}_c\exp(\omega_{t+1})p_{c,t+1}\right] \qquad (E_t \equiv \text{RE})$$

$$\omega_{t+1} = \text{bond coupon decay shock, I(0)}$$

Introducing equity sentiment

Transformed FOC: $$q_t = \eta_t \alpha_t + \beta \left[1 - \delta_t (1 - \alpha_t) - \varphi_t\right] \widehat{E}_t q_{t+1}$$

Fundamental solution:

$$q_t^{\mathrm{f}} \simeq \overline{q}^{\mathrm{f}} \left[\eta_t / \overline{\eta}\right]^{\gamma_\eta} \left[\alpha_t / \overline{\alpha}\right]^{\gamma_\alpha} \left[\delta_t / \overline{\delta}\right]^{\gamma_\delta} \left[\varphi_t / \overline{\varphi}\right]^{\gamma_\varphi}$$

Agent's Perceived Law of Motion (PLM):

$$q_t = q_t^{\mathrm{f}} \exp(s_t)$$

$$s_t = \overline{s} + \rho_s (s_{t-1} - \overline{s}) + \varepsilon_{s,t}, \quad \text{equity sentiment shock, I(0)}$$

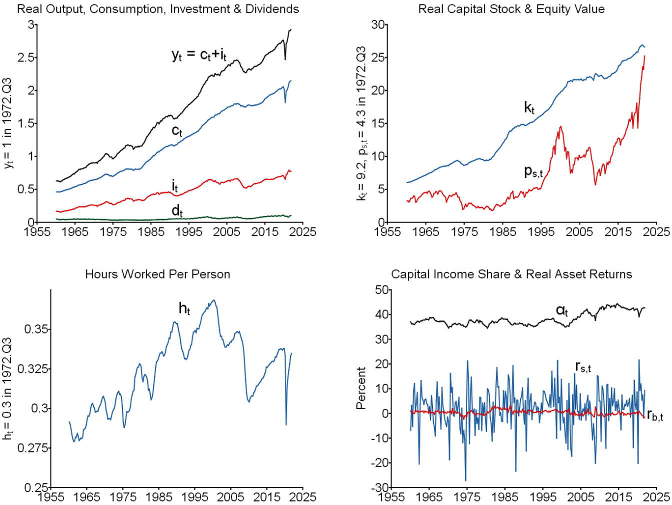
$$\widehat{E}_t q_{t+1} = E_t q_{t+1}^{\mathrm{f}} \times \exp[\overline{s} + \rho_s (s_t - \overline{s}) + \tfrac{1}{2} \sigma_{\varepsilon,s}^2]$$

Equilibrium Solution = Actual Law of Motion (ALM):

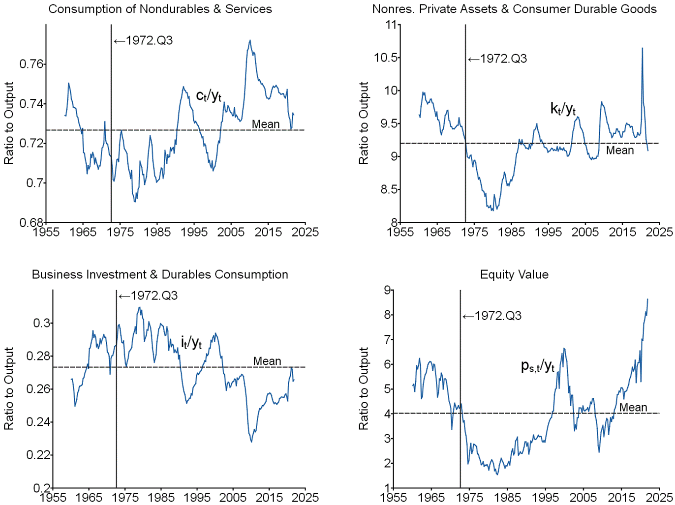
$$q_t = \mathrm{g}\left(\eta_t, \alpha_t, \delta_t, \varphi_t, s_t\right)$$

$\Rightarrow$ Equity value now depends on 4 fundamental shocks and s_t

U.S. data versions of model variables: 1960.Q1 to 2021.Q4



U.S. macro ratios close to long-run means in 1972.Q3



Model parameter values

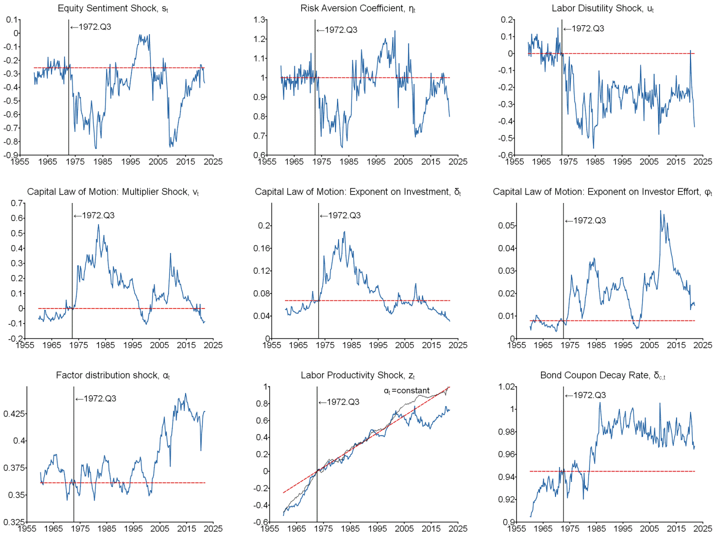
Choose parameters so that steady state, trend, or ergodic mean values of the model variables exactly match U.S. data in 1972.Q3.

Parameter	Value	Description/Target
$\overline{\eta}$	1	Risk aversion coefficient $= 1$ in 1972.Q3
γ	1	Frisch labor supply elasticity $= 1/\gamma = 1$.
$\overline{\alpha}$	0.361	Capital income share $= 0.361$ in 1972.Q3.
A	0.999	$k_t / y_t = 9.242$ with $z_t = 0$ in 1972.Q3.
$\overline{\delta}$	0.067	$i_t / y_t = 0.287$ in 1972.Q3.
$\overline{\varphi}$	0.008	$d_t / y_t = 0.041$ in 1972.Q3.
B	1.336	$k_{t+1} / k_t = \exp(\mu)$ in 1972.Q3.
D	10.476	$h_{1,t} + h_{2,t} = 0.3$ in 1972.Q3.
$\overline{s}$	$\boxed{-0.255}$	$p_{s,t} / y_t = 4.259$ in 1972.Q3.
β	1.002	$r_{b,t} = 0.245\%$ in 1972.Q3.
$\overline{\delta}_c$	0.945	$r_{c,t} = 0.869\%$ in 1972.Q3.

Solve for 9 shocks to replicate 9 observed U.S. variables

Shock	Replicates	Persistence
s_t = Equity sentiment	$p_{s,t}$	$\rho_s = 0.923$
η_t = Risk aversion	$r_{b,t}$	$\rho_\eta = 0.859$
u_t = Labor disutility	h_t	$\rho_u = 0.858$
v_t = Capital law multiplier	k_{t+1}	$\rho_v = 0.971$
δ_t = Capital law investment	i_t	$\rho_\delta = 0.980$
φ_t = Capital law investor effort	d_t	$\rho_\varphi = 0.978$
α_t = Factor distribution	$r_t k_t / y_t$	$\rho_\alpha = 0.979$
z_t = Labor productivity	y_t	$\rho_z = 1$
ω_t = Bond coupon decay rate	$r_{c,t}$	$\rho_\omega = 0.963$

Model-identified (reverse-engineered) shocks



Effect of steady state sentiment on mean equity premium

Negative sentiment in steady state is similar to models with disaster risk.

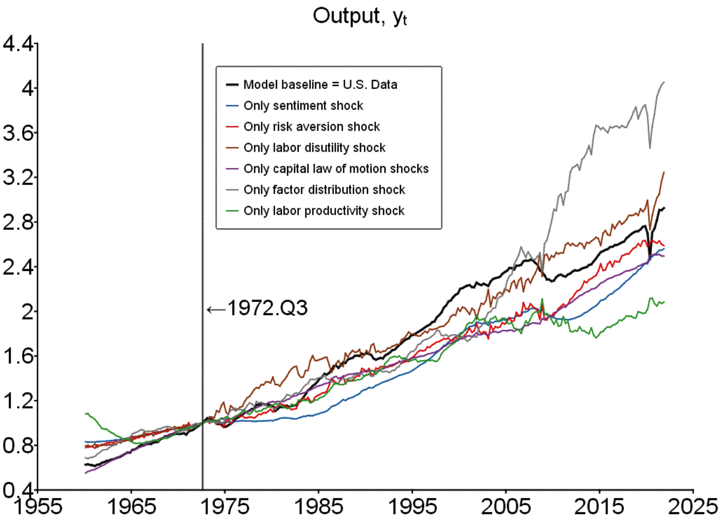
Steady state sentiment	$r_{s,t}$	$r_{b,t}$	$r_{s,t} - r_{b,t}$
$\bar{s} = -0.255$, Baseline	2.15	0.38	1.77
$\bar{s}' = -0.1$	1.30	0.85	0.44
$\bar{s}' = 0$	0.83	1.16	-0.34
$\bar{s}' = 0.1$	0.41	1.48	-1.07
$\bar{s}' = 0.2$	0.05	1.79	-1.75

Notes: Each number is the mean quarterly return in percent.

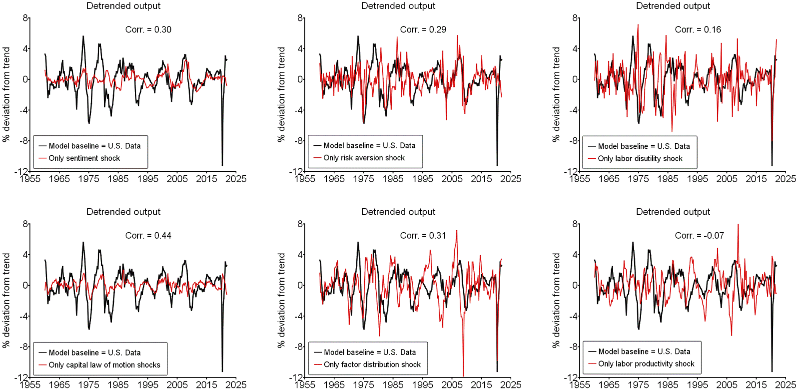
Top row matches mean U.S. asset returns with shock realizations s_t.

Other rows use the shock realizations $s_t' = s_t + (\bar{s}' - \bar{s})$.

Turn on one or more shocks, with other shocks turned off



Impacts of individual shocks on detrended output



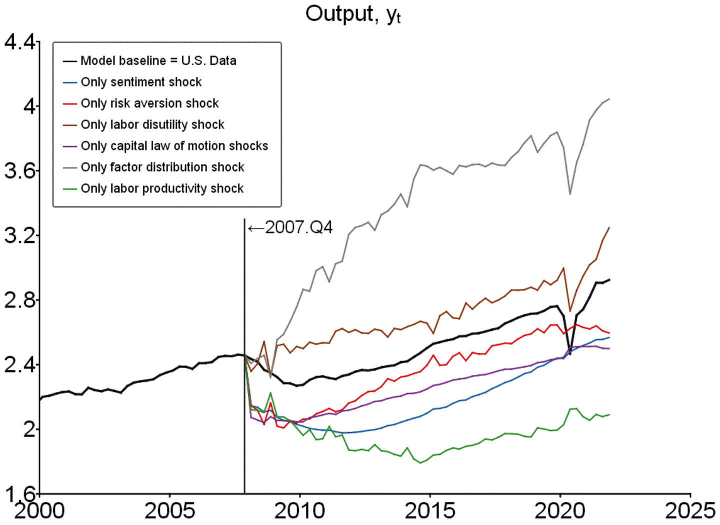
Quantifying the importance of each shock

Model vs. data correlation coefficients: Detrended variables

Shock scenario	y_t	c_t	i_t	h_t
Baseline model = U.S. data	1.00	1.00	1.00	1.00
Only equity sentiment shock	0.30	−0.21	**0.43**	0.25
Only risk aversion shock	0.29	0.23	−0.32	0.22
Only labor disutility shock	0.16	0.12	0.15	**0.37**
Only capital law of motion shocks	**0.44**	0.21	0.40	−0.27
Only factor distribution shock	0.31	**0.35**	0.21	−0.10
Only labor productivity shock	−0.07	0.01	−0.14	0.10

Boldface indicates the largest correlation coefficient for each variable.

Great Recession versus Covid Recession



Cumulative growth impacts of individual shocks

Cumulative growth impacts, 1972.Q3 to 2021.Q4

Shock scenario	c_t	i_t	k_t	$p_{s,t}$
Baseline model = U.S. data	201.1	170.9	187.7	492.8
Only equity sentiment shock	163.4	137.5	137.0	137.5
Only risk aversion shock	145.4	192.0	179.4	192.0
Only labor disutility shock	224.5	224.5	205.1	224.5
Only capital law of motion shocks	172.7	91.82	155.6	319.8
Only factor distribution shock	**284.0**	**357.7**	**326.2**	**357.7**
Only labor productivity shock	108.3	108.3	111.7	108.3

Notes: Each number is the cumulative growth rate in percent starting in 1972.Q3.
Boldface indicates largest cumulative growth for each variable.

Conclusion

- Model-identified sentiment shock is highly correlated with survey-based measures of U.S. consumer sentiment.
- Model-identified risk aversion coefficient is higher in good times. This pattern is consistent with empirical studies that estimate risk aversion using stock option prices.
- Angeletos, Collard, and Dellas (AER 2020, p. 3054): "In principle, any of the reduced-form objects contained in our anatomy [that identifies a main business cycle shock] may map into a uninterpretable combination of multiple theoretical shocks..."
- I find that multiple theoretical shocks are indeed necessary to fully explain the historical patterns of U.S. business cycles and asset returns. There is no "most important shock."

Fundamental equity value

Rewrite FOC:

$$\frac{\eta_t p_{s,t}}{c_t} = \beta \widehat{E}_t \{ \underbrace{\eta_{t+1}\alpha_{t+1} + [1 - \delta_{t+1}(1-\alpha_{t+1}) - \varphi_{t+1}] \frac{\eta_{t+1} p_{s,t+1}}{c_{t+1}}}_{\equiv q_{t+1} \text{ (object to be forecasted)}} \}$$

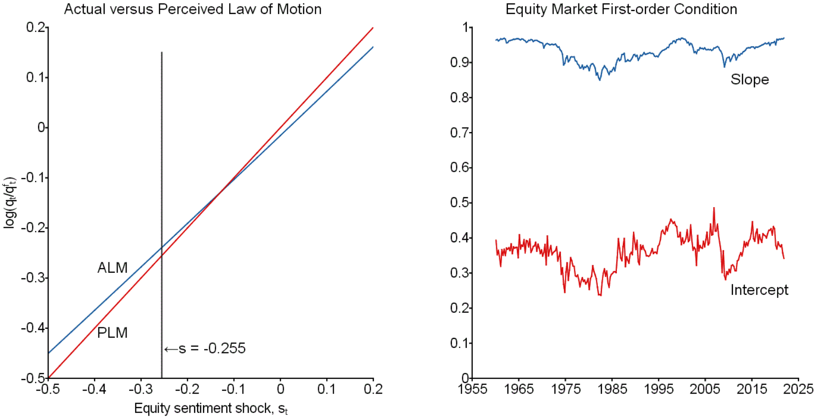
Transformed FOC: $$q_t = \eta_t \alpha_t + \beta [1 - \delta_t (1-\alpha_t) - \varphi_t] \widehat{E}_t q_{t+1}$$

Fundamental solution when $\widehat{E}_t q_{t+1} = E_t q_{t+1}$ (RE):

$$q_t^{\mathrm{f}} \simeq \overline{q}^{\mathrm{f}} [\eta_t / \overline{\eta}]^{\gamma_\eta} [\alpha_t / \overline{\alpha}]^{\gamma_\alpha} \left[\delta_t / \overline{\delta}\right]^{\gamma_\delta} [\varphi_t / \overline{\varphi}]^{\gamma_\varphi}$$

$\Rightarrow$ Equity value depends on 4 fundamental shocks.

Agent's perceived law of motion is close to self-fulfilling



Properties of in-sample forecast errors

Statistic	Subjective Forecast	Model-consistent Forecast
$Mean\,(err_t)$	0.07	−0.06
$\sqrt{Mean(err_t^2)}$	0.63	0.61
$Corr(err_t,\ err_{t-1})$	−0.24	−0.22
$Corr(err_t,\ err_{t-2})$	0.08	0.08
$Corr(err_t,\ err_{t-3})$	0.07	0.08

- $err_t = q_t - \widehat{E}_{t-1}\, q_t$, where in-sample mean of $q_t = 5.39$.
- Agent's forecast errors are close to white noise with near-zero mean.
- Little incentive for an individual agent to adjust forecast rule.

Cross-correlation of shock innovations

	$\varepsilon_{s,t}$	$\varepsilon_{\eta,t}$	$\varepsilon_{u,t}$	$\varepsilon_{v,t}$	$\varepsilon_{\delta,t}$	$\varepsilon_{\varphi,t}$	$\varepsilon_{\alpha,t}$	$\varepsilon_{z,t}$
$\varepsilon_{s,t}$	1.00	0.95	0.90	−0.69	−0.47	−0.42	−0.14	0.25
$\varepsilon_{\eta,t}$		1.00	0.92	−0.52	−0.26	−0.30	−0.11	0.23
$\varepsilon_{u,t}$			1.00	−0.51	−0.28	−0.28	−0.17	0.30
$\varepsilon_{v,t}$				1.00	0.88	0.65	0.11	−0.13
$\varepsilon_{\delta,t}$					1.00	0.49	−0.05	0.07
$\varepsilon_{\varphi,t}$						1.00	0.45	−0.37
$\varepsilon_{\alpha,t}$							1.00	−0.85
$\varepsilon_{z,t}$								1.00

Note: $\varepsilon_{\omega,t}$ is weakly correlated with most other shock innovations

- Strong correlations imply that model-identified shocks should be viewed as *residuals*, not as deep structural elements of the economy (Andrle 2014, IMF WP).

Asset returns from stochastic simulations

Simulated shocks are orthogonal to each other.

	U.S. data		Model simulations	
Return	Mean	Median	Mean	Median
$r_{s,t}$	2.15	3.11	1.77	1.27
$r_{b,t}$	0.38	0.37	0.27	0.19
$r_{c,t}$	1.11	0.50	0.90	0.02

Notes: Quarterly returns in percent. Model statistics are average values from 100,000 simulations, each 247 quarters in length.

- The unique set of shock realizations that account for the U.S. data sample have produced higher real equity returns than should be expected going forward, based on the theoretical model.