Democratic Policy Decisions with Decentralized Promises Contingent on Vote Outcome*

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Abstract

We study pre-vote interactions in a committee that enacts a welfare-improving reform through voting. Committee members use decentralized promises contingent on the reform enactment to influence the vote outcome. Equilibrium promises prevent beneficial coalitional deviations and minimize total promises. We show that multiple equilibria exist, involving promises from high- to low-intensity members to enact the reform. Promises dissuade reform opponents from enticing the least enthusiastic reform supporters to vote against the reform. We explore whether some recipients of the promises can be supporters of the reform and discuss the impact of polarization on the total promises.

Keywords: promises contingent on vote outcome, political failure, majority coercion, direct democracy, stability, principle of least action

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1 Introduction

The act of voting in political elections is often regarded as a moral duty, and trading votes for money or favors can be seen as abhorrent. Similarly, voting rights in corporations are an important means for stakeholders to voice their opinions on decisions that affect their economic ownership. Decoupling voting rights from economic ownership may therefore undermine the ideal of allocative efficiency in capital markets. Vote trading is thus a controversial practice that can be viewed as undermining the principles of democracy and efficient capital markets. Despite these negative connotations, vote trading is a widespread practice that can take various accepted forms within legal and societal norms. For example, legislative logrolling is a common practice where politicians exchange their votes on particular issues in return for votes on other issues.[1] In the case of corporate votes, activist investors may borrow shares for a nominal fee and use them to vote in favor of their own private agendas.[2]

Although vote trading is a commonly used practice, its normative properties are not well understood. In a recent survey, Casella and Macé [2021] noticed that, "Given the prominence of vote trading in all groups' decision-making, it is very surprising how little we know and understand about it." On the one hand, vote trading enables voters to adjust the intensity of their preferences, an action which is not possible

[1]Early evidence of logrolling was identified in the British parliament during the 1840s "railway mania". Railway companies had to petition Parliament for a Private Act allowing them to begin construction of their lines. During the railway mania, an early case of a technology bubble, substantial funds were drawn from optimistic investors, including Members of Parliament (MPs). Parliamentary rules prevent MPs from directly voting on private acts concerning companies at arm's length, aiming to safeguard against personal interests influencing the approval of projects. Even so, evidence suggests that vote trading occurred between MPs, prioritizing individual interests (see Esteves and Geisler Mesevage [2021]). While logrolling in the British parliament historical example appears to be motivated by greed, it is more typically driven by the interest of the constituents in political institutions such as the US congress.

[2]Hu and Black [2005] offer an overview of the "decoupling techniques" which are used in practice to unbundle the common shares' economic interest from voting rights.

with a binary vote. Early literature (Buchanan and Tullock [1962] and Coleman [1966]) highlighted that this feature should, in principle, have a positive impact on the efficiency of the vote outcome. On the other hand, when a coalition of voters trade votes, externalities are induced on non-trading voters (Downs [1957]) which can harm efficiency by creating a conflict between private and collective interests.

In this paper, we evaluate one form of "vote trading", where prior to casting their votes, voters make decentralized promises contingent on the voting outcome. We examine a model in which a committee makes a vote-based collective decision, e.g., majority vote, on whether to enact a reform or retain the status quo. Committee members disagree on which alternative they prefer because they have heterogeneous intensities of preferences. We assume that the reform is socially optimal in that the sum of preference intensities for the reform relative to the status quo is positive. Before voting, however, members can freely make credible and enforceable promises contingent on the committee's decision. The promises are unconstrained and involve coalitions of any size ranging from a pair, to the entire committee. The promises alter the incentives to vote, but voters retain control of their voting rights and sincerely cast their votes to maximize self-interest.

The promises from our model capture the process of liquidation voting in corporate bankruptcy proceedings where creditors and sometimes shareholders are asked to vote on a proposed plan of reorganization or liquidation, against the alternative—resolving the issues in court. The wedge between the expected court rulings' terms and the proposal's terms represents promises of transfers between stakeholders, as we envision in our model. Our model also captures a common practice in legislative bodies, including the US Congress, where bills are often amended before they are voted on. The final bill bundles the initial bill and the amendments, which we view as promises between voters. The process of amending bills is meant to give voters who initially oppose the bill reasons to vote for it. For example, the Patient Protection and Affordable Care Act (commonly known as Obamacare) was passed by the US

Congress in 2010. During the legislative process, the bill was amended in order to gain enough support from both Democrats and Republicans. Several concessions to moderate Democrats and Republicans were made, including removing a public option and scaling back the scope of the bill. More generally, the promises can represent favors (e.g. logrolling), legislative amendments, terms of liquidation, monetary payments, or any commitment to certain future actions that increase the advantage to the recipients of the promises.

In the absence of promises and when the committee decides by majority rule, the median voter theorem holds because heterogeneity is unidimensional and the decision is binary (Black [1958]). In this case, the political equilibrium aligns with the preferred policy of the median voter. The introduction of promises creates a multidimensional set of alternatives, which renders the median voter theorem invalid.

We study the equilibrium promises that prohibit the formation of *blocking coalitions*. When promises are in place, the members of a blocking coalition create incremental promises among themselves, overturn the committee decision, and achieve a better individual outcome compared to the pre-deviation outcome. The absence of blocking coalitions ensures the stability of the equilibrium. The resulting promises are in the *core*, that is, the set of promises that cannot be overturned once they are in place.

Our equilibrium also requires that the total transfer promised by all parties is kept as low as possible. By minimizing the transfer promises made, we reduce the various transaction costs associated with promises that are not explicitly modeled. For example, in a political context when a bill is being amended, it can become more complex and potentially deviate from its original purpose. In such cases, politicians may aim to minimize the scope and number of amendments made if they believe that their constituents or donors would not approve of too many changes. In addition, if promises take the form of future transfers, as in logrolling or in corporate liquidation, these promises may generate uncertain costs and benefits for both parties involved. Thus, minimizing promises can help reduce uncertainty and its associated costs. Finally, transfer promises may also be illegal in some settings such as the British parliament during the "railways mania" of the 19th century, and minimizing their magnitude can minimize the risk of detection.[3] More broadly, minimizing the total promises can reduce the costs of building the institutions that guarantee the enforceability, the commitment, and the elicitation of intensities that are necessary to implement the promises.

Based on the assumptions of stability and minimality, we explore the equilibrium-voting outcome when promises are allowed before voting. The first insight from the model is that equilibria exist and achieve the social optimum, meaning that the reform is enacted. The second insight is that there are multiple equilibria: while reducing the set of stable promises profiles, the minimal total transfer assumption does not imply the uniqueness of the equilibrium. The indeterminacy arises from the multiple ways to divide the total transfers among the promisers and to distribute the promises among the recipients. However, the assumption of minimality implies that all equilibria share common characteristics. Specifically, in all equilibria, the total promise transfers remain the same, and these promises flow from committee members with higher intensities to those with lower intensities. In fact, for all voting rules and all distributions of *ex ante* intensities, there is a unique critical committee member such that, in all equilibria, the promisers have larger *ex ante* intensities than that member while the promises recipients have lower intensities that that member.

To provide more detailed predictions on the equilibrium flow of transfer and identity of the promisers and promisees, more specific assumptions on the distribution of

[3]Our assumption of total transfer minimization parallels Maupertuis's *principle of least action* in Physics (Maupertuis [1748]). According to the principle of least action, the behavior of a physical system can be thought of as an attempt to minimize a quantity called action that measures the work to be done on the system, e.g., Ekeland [2006]. To the extent that promises represent costly actions for the promisers, minimizing the total promises transfer is tantamount to minimizing the actions deployed by the promisers to improve their self-interest from the vote.

ex ante intensities are necessary.

When the coalition of reform supporters does not have enough voting power to enact the reform, the voting outcome without promises is inefficient. This is the case of *frustrated minorities* where the reform is defeated despite the presence of a minority of members who would experience a large increase in utility when the reform is enacted. In that case, the promisers are reform supporters and the recipients of the promises are reform opponents. Thus, all equilibrium promises share the common characteristic of being in "the reaching across the aisle" type where the promisers support the reform while the recipients of the promises oppose it.

In the alternative scenario where reform supporters possess sufficient voting power to enact the reform, the attainment of stability and minimality may still necessitate the use of promises. Blocking coalitions can exist where opponents of the reform entice reform supporters with the weakest intensities, with promises contingent on defeating the reform. These promises can persuade weakly motivated supporters to become reform opponents. Once these promises are made, a new coalition is established by combining the opponents of the reform and the supporters who have been persuaded to switch stances. The new coalition becomes a blocking coalition if it satisfies two conditions. First, it must have enough voting power to influence the committee to reject the reform. Second, all of its members should strictly increase their utilities compared to the voting outcome in the absence of promises. When a blocking coalition exist, promises become crucial as they serve the purpose of providing compensation to its members.

When considering the equilibrium implications for the identity of promisers and the recipients of the promises, we find that as a general rule, there always exist equilibria where the recipients of the promise are reform opponents. However, under some conditions for the distribution of *ex ante* intensities, equilibria where some promises recipients are reform supporters with weak intensities may also exist. We also discuss the impact of polarisation on the equilibrium promises and propose a selection mechanism based on a sequential procedure of promises that implements a unique equilibrium in a finite number of steps for all distributions of *ex ante* intensities and all the voting rules we consider.

Related literature. The main contribution of this paper is to demonstrate the feasibility of evaluating promise-related practices in decentralized settings. Our results intersect with three streams of literature.

Our finding, that the reform is enacted in equilibrium, provides compelling evidence that the practice of contingent promises based on vote outcomes can effectively address the problem of majority coercion. This result contributes to the broader literature on political failures, which aims to identify situations where the political equilibrium outcome can be Pareto-improved, with a focus on restoring efficiency. Becker [1958] and Wittman [1989] compared political failures to market failures and discussed institutional responses to political failures. By demonstrating the efficacy of promises in mitigating majority coercion, our study contributes to this literature by reporting a formal mechanism to improve the overall efficiency in collective decisions based on direct democracy. Our result on the effectiveness of promises is reminiscent of the Coase theorem operating in the political market (Coase [1960]). The corollary of our result is that efforts to design institutions that promote the practice of promises and decrease the transaction costs associated with them push decentralized decisions mediated by voting toward an efficient outcome.

Second, we contribute to the literature on vote trading. Despite the significant strides made in the 1960s and 1970s, the subject of vote trading has somewhat faded from scholarly focus in recent decades. A pivotal argument in the development of this literature is that the voting externality could make trading votes undesirable (Riker and Brams [1973]). Despite the declining interest, recent work by Casella and Palfrey [2019] has offered a more optimistic perspective on trading votes, in committee settings where members vote on multiple issues and sequentially trade votes on one issue against votes for other issues.[4] In our research, we extend this literature by examining a decentralized framework, similar to that of Casella and Palfrey [2019], but with a focus on trading through *simultaneous* promises. We also offer a more positive perspective on vote trading within a different context and highlight the potential benefits of permitting promises contingent on the vote outcome. We view this addition as important because many pre-vote interactions in corporate votes, referenda, or political elections can be thought of as promises contingent on the vote outcome, particularly in cases where the final proposal bundles the initial proposal with additional transfers. Such bundled proposals align well with our model, where the enforceability and credibility of promises are assumed to be given. In this context, the assumption of enforceability and credibility of promises can be seen as an assumption about the credibility of the institutions that implement the promises. This assumption naturally holds in democratic systems that prioritize the rule of law and encourage competition for political office, since reneging on promises would entail significant costs.

Finally, we contribute to the literature on promises, e.g., Myerson [1993], Groseclose and Snyder [1996], Dal Bo [2007], Dekel et al. [2008], Dekel et al. [2009], and more recently Chen and Zápal [2022]. This literature has largely concentrated on models with political representatives and revolves around leaders vying for votes by making pledges or campaign promises. Within this framework, much attention has been devoted to leaders' capacity to utilize budgetary resources to manipulate vote results and gain profits while in office. Through varying assumptions about the game structure, these studies have made significant advancements with an emphasis on the sequencing of promises. Our study offers a different perspective by investigating the potential advantages of outcome-contingent promises in a decentralized setting. In our

[4] Philipson and Snyder [1996] also offer a more positive result on vote trading in the context of an organized centralized vote market where the vote buyers could be party leaders or committee chairs. See also Xefteris and Ziros [2017] for a similar message under different assumptions on the vote trading market.

context, promises happen simultaneously within the committee (or the electorate), and without a designated leader. Rather than examining this practice solely within the political agency framework, we demonstrate how pre-vote interactions through promises can facilitate agreement and drive efficient outcomes in direct democracies. By exploring the potential benefits of vote-contingent promises in this distinct context, our research underscores the need for a broader examination of this practice.

Our approach also differs from previous studies on outcome-contingent promises by incorporating the principle of least action, which assumes minimal total transfer promises. This principle aligns with the standard economic theory of cost minimization, since the utilitarian cost of reaching an efficient agreement is kept at its minimum level. It also shares similarities with the principle of stability. The principle of stability operates on the assumption that voters can effectively coordinate and form blocking coalitions. Likewise, the principle of minimum total promises operates on the assumption that promisers can coordinate their actions to achieve reform enactment at a lower overall cost.[5] The principle of least action is bundled with the principle of stability in our analysis, allowing us to make general predictions about the political equilibrium. Despite the inherent indeterminacy that can arise in decentralized decision-making settings where the median voter theorem is not applicable, our approach provides a framework for understanding and predicting the outcomes of such settings.

The paper proceeds as follow. Section 2 sets forth the general setting, Section 3 defines and characterizes stable promises, Section 4 defines and establishes the existence and indeterminacy of the equilibrium and provides its general implications. On a case-by-case basis, we describe the equilibrium implications in Section 5. Section 6 explores additional implications of the equilibria. We conclude in the paper section

[5]Both principles can be enforced in a model of political competition where a skilled political entrepreneur creates a platform that includes the transfers required by the equilibrium and aims to achieve electoral success.

7. Appendix A provides the proofs of the paper's main results, and Appendix B presents the supplementary results related to the discussions in Section 6.

2 The model

Consider a committee $\mathbb{I} = \{1, \cdots, I\}$ of I members faced with a vote on a reform. Based on the vote, the committee's decision is to adopt the reform "R" or to turn it down and support the status quo "S." Each voter i is characterized by a parameter u_i representing member i's *intensity of preferences* for the reform relative to the status quo (or simply intensity). Normalizing the (cardinal) utility derived from the status quo for each voter to 0, the parameter u_i represents the utility experienced by voter i when the reform is adopted. A committee member with intensity $u_i \geq 0$ supports the reform while a member with intensity $u_i < 0$ favors the status quo.[6] We order the committee members so that the intensities u_i are non-decreasing in i, that is,

$$u_1 \leq \cdots \leq u_I, \tag{1}$$

and denote by $\boldsymbol{u}$ the vector $\boldsymbol{u} = (u_1, u_2, .., u_I)$. Relative to the status quo, adopting the reform is socially optimal,

$$\sum_{i \in \mathbb{I}} u_i > 0. \tag{2}$$

The aggregate intensity $\sum_{i \in \mathbb{I}} u_i > 0$ represents the utilitarian welfare gain that the reform generates relative to the status quo. Condition (2) is not restrictive, because the alternative assumption $\sum_{i \in \mathbb{I}} u_i < 0$ would imply that the status quo is the efficient outcome and by symmetry all the results of the model would hold for the status quo alternative. We denote the coalition of reform opponents (or status quo supporters) by $\mathcal{C}^S$ and the coalition of reform supporters by $\mathcal{C}^R$:

$$\mathcal{C}^S := \{i : u_i < 0\} \equiv \{1, \cdots, n\}, \qquad \mathcal{C}^R := \{i : u_i \geq 0\} \equiv \{n+1, \cdots, I\}, \tag{3}$$

[6]Breaking the tie $u_i = 0$ by favoring the status quo instead of the reform or by randomizing the committee's choice between the two policies does not change the main conclusions of our analysis.

where $n \in \mathbb{I}$ denotes the number of reform opponents. The aggregate intensity of preferences of reform and status quo supporters, respectively, are denoted by

$$U^R := \sum_{\mathcal{C}^R} |u_i| \equiv \sum_{\mathcal{C}^R} u_i, \quad U^S := \sum_{\mathcal{C}^S} |u_i| \equiv \sum_{\mathcal{C}^S} (-u_i). \tag{4}$$

Notice that both U^R and U^S are non negative quantities and that $U^R - U^S = \sum_{\mathbb{I}} u_i > 0$.

Prior to voting, we consider a transferable utility framework where any committee member can make a utility transfer promise contingent on the committee decision to any other committee member(s). The transfer promises can take the form of money payments or favors as long as these transfers translate into utilities. The transferable utility assumption holds if utility is quasilinear in money and transfers take the form of money payments.[7]

Contingent transfer promises are binding commitments. That is, any transfer promises contingent on a specific voting outcome *must* be honored by the promisers in favor of the recipients ("promisees") if that outcome is realized. Transfer promises alter the incentives to vote for the reform but voters retain their voting rights. As is standard in the literature, we assume that the transfer promises are credible and enforceable.[8]

Transfer promises are unconstrained in terms of both their cardinal impact on the utility and in terms of the identity of the promisers and the promisees. A promises profile contingent on adopting the reform is denoted by $\boldsymbol{r} = (r_1, ..., r_I) \in \mathbb{R}^I$. When $r_i \geq 0$, committee member i is a net promisee and she gets a net utility increase of r_i when the reform is adopted by the committee. When $r_i \leq 0$, committee member i is a net promisor and her utility decreases by $|r_i|$ when the reform is adopted by the committee. Notice that the net promise r_i is the aggregation of all promises that have

[7] See Bergstrom and Varian [1985] and Chiappori and Gugl [2020] for more general conditions on preferences and market environments allowing transferable utility representations.

[8] The assumption would be natural in a repeated setting where group members have incentives to develop a reputation for not reneging on promises.

been made to member i by other committee members, net of the promises that have been made by member i to other committee members. For each committee member, the incentive to vote in favor of the reform is solely based on the net aggregate flow of promises. Hence we can assume without loss of generality that the net aggregate flow of promises, represented by the profile $\boldsymbol{r}$, is the sole factor determining the voting decisions of every committee member. Because the utilities are transferred from the promisers to the promisees, any promises profile must satisfy the zero sum condition:

$$\boldsymbol{r} \in \mathcal{P} := \{\boldsymbol{x} = (x_1, ..., x_I) \in \mathbb{R}^I \mid \sum_{\mathbb{I}} x_i = 0\}. \tag{5}$$

Similarly, a promises profile contingent on the reform being defeated is denoted by $\boldsymbol{s} = (s_1, .., s_I) \in \mathcal{P}$ where s_i is the net utility increase of committee member i when the reform is defeated.

When facing a vote on the reform with the outstanding promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$, each committee member casts her vote to maximize self-interest. When $u_i + r_i \geq s_i$, member i votes for the reform; otherwise, she votes for the status quo. We assume thus all committee members vote *sincerely*. Sincere voting is a natural assumption in our framework since all voters have the same information and voting is costless.[9]

The committee is ruled by a κ-majority rule for some $\kappa \in \mathbb{I}$; the reform is adopted if and only if at least κ votes are in favor of the reform. Special cases of the κ-majority rule range from simple majority ($\kappa = \kappa_I$ where κ_I is the smallest integer larger than $I/2$), to unanimity where $\kappa = I$. When $\kappa_I < \kappa < I$, the κ-voting rule is a supermajority rule. The voting share threshold to adopt the status quo policy is given by $\widehat{\kappa}$ where

$$\widehat{\kappa} := I - \kappa + 1. \tag{6}$$

When $\widehat{\kappa}$ committee members vote for the status quo, then the committee adopts the status quo. Any κ-voting rule can alternatively be defined by two sets of *decisive*

[9]In our model, all model parameters are common knowledge, and therefore there is no reason for voters to be strategic. Sincere voting is an undominated strategy in our context.

coalitions, $\mathcal{D}^R$ and $\mathcal{D}^S$, that are sets of subsets of $\mathbb{I}$ — that is, elements of $2^{\mathbb{I}}$. A coalition is decisive for adopting (resp. defeating) the reform if the group adopts (resp. defeats) the reform when all members of that coalition vote for (resp. against) the reform. Therefore,

$$\mathcal{D}^R := \{\mathcal{C} \subseteq \mathbb{I} \ : \ |\mathcal{C}| \geq \kappa\}, \quad \mathcal{D}^S := \{\mathcal{C} \subseteq \mathbb{I} \ : \ |\mathcal{C}| \geq \widehat{\kappa}\}. \tag{7}$$

We denote by $D(\boldsymbol{r}, \boldsymbol{s}) \in \{R, S\}$ the committee decision or voting outcome when facing the promises profile $(\boldsymbol{r}, \boldsymbol{s})$.

The utility vector derived by committee members from the voting outcome is denoted by $\boldsymbol{v}^{\boldsymbol{r},\boldsymbol{s}} = (v_1^{\boldsymbol{r},\boldsymbol{s}}, v_2^{\boldsymbol{r},\boldsymbol{s}}, ..., v_I^{\boldsymbol{r},\boldsymbol{s}})$ and is given by

$$\boldsymbol{v}^{\boldsymbol{r},\boldsymbol{s}} := \begin{cases} \boldsymbol{u} + \boldsymbol{r}, & \text{if} \quad D(\boldsymbol{r}, \boldsymbol{s}) = R; \\ \boldsymbol{s}, & \text{otherwise} \end{cases} \tag{8}$$

or more succinctly, $\boldsymbol{v}^{\boldsymbol{r},\boldsymbol{s}} = (\boldsymbol{u} + \boldsymbol{r})1_{\{D(\boldsymbol{r},\boldsymbol{s})=R\}} + \boldsymbol{s}1_{\{D(\boldsymbol{r},\boldsymbol{s})=S\}}$. In particular, the utility derived from the voting outcome in the absence of the promises—meaning $\boldsymbol{r} = \boldsymbol{s} = \boldsymbol{0}$ where $\boldsymbol{0}$ is the zero vector of size I—is $\boldsymbol{v}^{\boldsymbol{0},\boldsymbol{0}} = \boldsymbol{u}1_{\{D(\boldsymbol{0},\boldsymbol{0})=R\}}$.

3 Stable promises: Definition and characterization

We focus our attention on promises profiles that are stable in the sense that once in place, no coalition can arrange additional promises among its members that would reverse the committee decision and yield a better outcome for all its members. If such coalitions exist, we qualify them as *blocking coalitions* and define them as follows.

Definition 1. *[**Blocking coalitions**] Let the promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ be in place. When $D(\boldsymbol{r}, \boldsymbol{s}) = R$, a coalition $\mathcal{C}$ blocks the promises profile $(\boldsymbol{r}, \boldsymbol{s})$ if there exists an alternative promises profile contingent on adopting the status quo $\boldsymbol{s'} \in \mathcal{P}$ and satisfying $\mathcal{C} = \{i : s'_i \neq s_i\}$, such that a deviation from $(\boldsymbol{r}, \boldsymbol{s})$ to $(\boldsymbol{r}, \boldsymbol{s'})$ implies*

1. *a reversal of the committee decision: $D(\boldsymbol{r}, \boldsymbol{s'}) = S$ and,*

2. *individual benefits for all members of that coalition:* $v_i^{\boldsymbol{r},\boldsymbol{s'}} > v_i^{\boldsymbol{r},\boldsymbol{s}}$ *for all* $i \in \mathcal{C}$.

When $D(\boldsymbol{r}, \boldsymbol{s}) = S$*, a coalition* $\mathcal{C}$ *blocks the promises profile* $(\boldsymbol{r}, \boldsymbol{s})$ *if there exists an alternative promises profile contingent on adopting the reform* $\boldsymbol{r'} \in \mathcal{P}$ *and satisfying* $\mathcal{C} = \{i : r'_i \neq r_i\}$*, such that a deviation from* $(\boldsymbol{r}, \boldsymbol{s})$ *to* $(\boldsymbol{r'}, \boldsymbol{s})$ *implies*

1. *a reversal of the committee decision:* $D(\boldsymbol{r'}, \boldsymbol{s}) = R$ *and,*

2. *individual benefits for all members of that coalition:* $v_i^{\boldsymbol{r'},\boldsymbol{s}} > v_i^{\boldsymbol{r},\boldsymbol{s}}$ *for all* $i \in \mathcal{C}$.

To illustrate Definition 1, assume the contingent transfers $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ are promised and that the committee decision is $D(\boldsymbol{r}, \boldsymbol{s}) = R$. When a blocking coalition $\mathcal{C}$ exists, some of its members deviate from $(\boldsymbol{r}, \boldsymbol{s})$ by launching an additional round of promises contingent on rejecting the reform, targeting the remaining members of that coalition. These promises are designed to entice the promisees to vote against the reform and thereby reverse the committee decision. Denote these incremental promises by $\tilde{\boldsymbol{s}} \in \mathcal{P}$ and notice that $\mathcal{C} = \{i : \tilde{s}_i \neq 0\}$. Since these promises are internal to the coalition $\mathcal{C}$, we have $\sum_{i \in \mathcal{C}} \tilde{s}_i = 0$. The resulting *cumulative* promises profile contingent on the status quo is given by $\boldsymbol{s'} = \boldsymbol{s} + \tilde{\boldsymbol{s}}$ and we have, $s'_i \neq s_i$ if and only if $i \in \mathcal{C}$ and, $D(\boldsymbol{r}, \boldsymbol{s}') = S$. The condition $v_i^{\boldsymbol{r},\boldsymbol{s'}} > v_i^{\boldsymbol{r},\boldsymbol{s}}$ holds for all $i \in \mathcal{C}$ and says that all members of the blocking coalition $\mathcal{C}$ benefit from the deviation, that is, $s'_i \equiv v_i^{\boldsymbol{r},\boldsymbol{s'}} > v_i^{\boldsymbol{r},\boldsymbol{s}} \equiv u_i + r_i$.

Similarly, when $D(\boldsymbol{r}, \boldsymbol{s}) = S$, a blocking coalition can deviate with incremental promises contingent on reform enactment. We now define stable promises.

Definition 2. *[**Stable promises**] The promises profile* $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ *is stable if and only if there exists no blocking coalition for* $(\boldsymbol{r}, \boldsymbol{s})$*.*

We denote the set of stable promises profiles by

$$\mathcal{S} := \{(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2 \mid (\boldsymbol{r}, \boldsymbol{s}) \text{ is stable}\} \tag{9}$$

and provide a characterization of this set in the next proposition.

Proposition 1. *[**Characterization of stable promises**] The promises profile* $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ *is stable if and only if*

$$\sum_{\mathcal{C}} (u_i + r_i) \geq \sum_{\mathcal{C}} s_i \text{ for all coalitions } \mathcal{C} \in \mathcal{D}^S. \tag{10}$$

Moreover, any stable promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{S}$ *leads the committee to enact the reform, that is,* $D(\boldsymbol{r}, \boldsymbol{s}) = R$.

Proposition 1 shows that stable promises enact the reform. Promises contingent on the committee decision are effective in removing any inefficiency caused by voting externalities. To see this, assume that more than $\widehat{\kappa}$ members vote against the reform in the absence of promises. The committee then rejects the reform and creates a frustrated losing coalition: the coalition supporting the reform has a larger aggregate intensity of preferences than the aggregate intensity for the status quo of the coalition of reform opponents and yet the committee rejects the reform because reform supporters do not have enough voting power. The inefficiency arises because binary voting does not allow the expression of intensity of preferences. Allowing promises with arbitrary transfers permits reform supporters to modulate their actions. They can express their intensity of preferences not only by voting, but also by launching promises contingent on the reform in order to convert reform opponents to reform supporters.

Proposition 1 also shows that promises profiles enabling the reform are stable if and only if, when compared to the status quo alternative, the reform is socially optimal for any decisive coalitions that can induce the group to overturn the reform. As a result, stability is characterized by the system of linear inequalities (10), which define a convex polyhedron, and this allows us to carry out a fairly detailed analysis of stable promises.

To illustrate the usefulness of the characterization (10), assume that the zero (empty) promises profile ($\boldsymbol{r} = \mathbf{0}, \boldsymbol{s} = \mathbf{0}$) leads the committee to enact the reform: $D(\mathbf{0}, \mathbf{0}) = R$. Given the ordering of *ex ante* intensities (1), inequalities (10) hold if and only if $\sum_1^{\widehat{\kappa}} u_i \geq 0$. The characterization in Proposition 1 offers, therefore, a practical tool to verify whether a promises profile is stable. We now discuss an example to illustrate further the result in Proposition 1.

Example 1. *Consider a committee with* 3 *members whose intensities are given by*

$$\boldsymbol{u} = (u_1, u_2, u_3) = (-4,\ 1,\ 5).$$

Assume the committee is ruled by the simple majority rule $\kappa = \widehat{\kappa} = 2$ *so that both* $\mathcal{D}^R$ *and* $\mathcal{D}^S$ *consist of coalitions that have a cardinality* 2 *or higher. The reform is socially optimal since* $\sum_i u_i = 2 > 0$. *In the absence of promises, the reform is adopted since both members* 2 *and* 3 *vote for it, and thus* $\boldsymbol{v}^{\mathbf{0},\mathbf{0}} = (-4,\ 1,\ 5)$. *By Proposition 1, the empty promises profile* $(\mathbf{0}, \mathbf{0})$ *is not stable because* $u_1 + u_2 = -3 < 0$. *This happens because member* 1 *can promise to transfer to member* 2 *an amount* $s_2 = 2$ *if the reform is defeated. The resulting group decision after the promises* $(\boldsymbol{r} = \mathbf{0},\ \boldsymbol{s} = (-2, 2, 0))$ *are in place is thus to defeat the reform because both members* 1 *and* 2 *vote for* S. *The voting outcome is then* $D(\mathbf{0}, \boldsymbol{s}) = S$ *and the resulting intensity is* $\boldsymbol{v}^{\mathbf{0},\boldsymbol{s}} = (-2,\ 2,\ 0)$. *Because* $v_1^{\mathbf{0},\boldsymbol{s}} = \ -2 \ > \ -4 = v_1^{\mathbf{0},\mathbf{0}}$ *and,* $v_2^{\mathbf{0},\boldsymbol{s}} = \ 2 \ > \ 1 = v_1^{\mathbf{0},\mathbf{0}}$, *the coalition* $\{1, 2\}$ *is a blocking coalition for the promises profile* $(\mathbf{0}, \mathbf{0})$.

Consider now the promises profile $(\boldsymbol{r}, \boldsymbol{s}) := ((3,\ 0,\ -3),\ \mathbf{0})$ *that enacts the reform and imply the ex post intensities* $\boldsymbol{v}^{\boldsymbol{r},\mathbf{0}} = (-1,\ 1,\ 2)$. *Since inequality (10) holds for the promises profile* $(\boldsymbol{r}, \mathbf{0})$ *for all coalitions of size* 2 *and above, the promises profile* $(\boldsymbol{r} = (3, 0, -3), \mathbf{0})$ *is stable. Following the same logic, one can check that there are multiple other stable promises profiles.*

Example 1 illustrates that stable promises enact the reform. The example also points to an indeterminacy of stable promises. Precluding the formation of blocking coalitions still leaves multiple ways to structure the promises of transfers across committee members to achieve stability. Moreover, the example illustrates another cause of multiplicity related to the linearity of intensities in promises. For example, we can associate the stable promises profile $((3, 0, -3), \mathbf{0})$ with an equivalence class of stable promises of the form $((3,0-3)+\boldsymbol{s},\boldsymbol{s})$ for an arbitrary $\boldsymbol{s}\in\mathcal{P}$. This is because the inequalities (10) hold for any pair of promises $(\boldsymbol{r},\boldsymbol{s})$ if and only if it holds for the pair of promises $(\boldsymbol{r}-\boldsymbol{s},\boldsymbol{0})$, an implication of our assumption that utility is linear in promises.

In the following proposition, we establish the existence, multiplicity, and efficiency of stable promises. Before stating the proposition, denote by $\mathcal{S}_0$ the set of stable promises that are contingent only on the reform being enacted:

$$\mathcal{S}_0 := \{\boldsymbol{r}\in\mathcal{P} \mid (\boldsymbol{r},\boldsymbol{0})\in\mathcal{S}\}.$$

Proposition 2. *[**Existence and multiplicity of stable promises**]*

(i) There exist multiple stable promises: the set $\mathcal{S}_0$ is non-empty and contains multiple elements.

(ii) The promises profile $(\boldsymbol{r},\boldsymbol{s})\in\mathcal{S}$ if and only if $(\boldsymbol{r}-\boldsymbol{s},\boldsymbol{0})\in\mathcal{S}_0$, and hence

$$\mathcal{S}=\{(\boldsymbol{r}+\boldsymbol{s},\boldsymbol{s}) \mid \boldsymbol{r}\in\mathcal{S}_0 \text{ and } \boldsymbol{s}\in\mathcal{P}\}. \tag{11}$$

The existence result in Proposition 2 sharply contrasts with the findings of the "chaos theorems" literature, highlighting the core's non-existence in voting games with multidimensional alternatives[10] and the presence of voting cycles[11]. The non-applicability of chaos theorems in our framework arises from several technical reasons. First, unlike in the literature where the set of alternatives is typically assumed to be compact, our set of policy alternatives, denoted as $\mathcal{P}$, is unbounded and lacks a bliss point in individual preferences. Consequently, our stable promises do not exhibit the Condorcet winner property, as it is always possible to win a binary vote against any promises profile by forcing one member to promise large transfers to all other members. Second, individual preferences in our framework are locally non-differentiable in promises, and the gradient of utility is a step function of promises.

[10] See, e.g., Plott [1967], McKelvey and Wendell [1976], and Schofield [1983].

[11] See, e.g., McKelvey [1976], Schofield [1984], Gibbard [1973], Satterthwaite [1975], and Duggan and Schwartz [2000].

This lack of differentiability renders the gradient method used in Plott's approach inapplicable, as it relies on first-order conditions to identify the equilibrium. As a result, the conclusions from the chaos theorems literature do not directly apply to our model. Instead, our analysis demonstrates that stability requires the satisfaction of the inequalities defined by (10), which in turn define a convex polyhedron in the set of promises. Importantly, in the proof of Proposition 2, we identify specific elements from this polyhedron and proved that it has a dimension of $I-1$ and non-empty interior. This ensure that our stable promises are robust in the sense that the set of stable promises remains non-empty even if there are slight changes in individual preferences. The result also implies that there are multiple equilibria even when we restrict the promises to be in the set $\mathcal{S}_0$ where promises only occur when the reform is enacted. The multiplicity of stable promises in the set $\mathcal{S}$ is even more fundamental: the restriction of stability allows *any* promises contingent on the reform being adopted to be part of a stable promises profile from the set $\mathcal{S}$ when the promises contingent on the status quo $\boldsymbol{s}$ are chosen appropriately.[12]

4 Equilibrium promises: existence and properties

The political equilibrium is defined as the set of promises profiles that maintain stability while minimizing total transfer promises.

We define the total promises transfer for any promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ by

$$\mathcal{T}_{\boldsymbol{r},\boldsymbol{s}} := \frac{1}{2}\sum_{\mathbb{I}} |r_i| + \frac{1}{2}\sum_{\mathbb{I}} |s_i|. \tag{12}$$

For any stable promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{E}$, we have $|r_i - s_i| \leq |r_i| + |s_i|$ and hence, $\mathcal{T}_{\boldsymbol{r}-\boldsymbol{s},\boldsymbol{0}} \leq \mathcal{T}_{\boldsymbol{r},\boldsymbol{s}}$. In order to minimize the total promises transfer (12) we can restrict our attention to the set $\mathcal{S}_0$ of equilibrium promises which only require transfers if the

[12] To see this, consider a stable profile $(\boldsymbol{r}, \boldsymbol{0})$ and observe that the promises profile $(\boldsymbol{0}, -\boldsymbol{r})$ is also stable. Therefore, for any promises profile contingent on adopting the reform $\boldsymbol{r'} \in \mathcal{P}$, the promises profile $(\boldsymbol{r'}, \boldsymbol{r'} - \boldsymbol{r})$ is also stable.

reform is adopted. To enhance clarity, we introduce the following changes in notation:

$$\mathcal{T}_{\boldsymbol{r}} := \mathcal{T}_{\boldsymbol{r},\boldsymbol{0}} \equiv \frac{1}{2}\sum_{i\in\mathbb{I}} |r_i|, \qquad \boldsymbol{v}^{\boldsymbol{r}} := \boldsymbol{v}^{\boldsymbol{r},\boldsymbol{0}}, \tag{13}$$

and define equilibrium promises profiles as follows.

Definition 3 (**Equilibrium promises profiles**). *A promises profile $\boldsymbol{r} \in \mathcal{P}$ contingent on the reform is an equilibrium if (i) it is stable and, (ii) it minimizes the total promises transfer among all stable promises. The set of equilibrium promises is, therefore, given by*

$$\mathcal{E} := \{\boldsymbol{r} \in \mathcal{S}_0 \mid \mathcal{T}_{\boldsymbol{r}} = \inf_{\boldsymbol{r'}\in\mathcal{S}_0} \mathcal{T}_{\boldsymbol{r'}}\}.$$

We have the following existence result.

Proposition 3 (**Existence**). *The set of equilibrium promises $\mathcal{E}$ is not empty.*

Relative to stability, the equilibrium adds the restriction that stability is achieved in the "cheapest possible way". The set of equilibrium promises $\mathcal{E}$ is smaller than the set of stable promises $\mathcal{S}_0$, but indeterminacy is not eliminated by the minimum total transfer restriction as the following example illustrates.

Example 2. *Reconsider the committee consisting of* 3 *members, with intensities* $\boldsymbol{u} = (-4, 1, 5)$ *discussed in Example 1. The committee operates under a majority rule:* $\kappa = \widehat{\kappa} = 2$*. Inequality* (10) *shows that* $\boldsymbol{r} \in \mathcal{P}$ *is stable if any only if:*

$$r_1 + r_2 \geq 3, \quad r_1 + r_3 \geq -1, \quad r_2 + r_3 \geq -6, \quad r_1 + r_2 + r_3 = 0.$$

Direct calculations show that $\mathcal{S}_0 = \{\boldsymbol{r} \in \mathcal{P} | r_1 \leq 6,\ r_3 \leq -3,\ -r_2 = r_1 + r_3 \geq -1\}$*. Therefore, there are multiple stable promises and the set* $\mathcal{S}_0$ *is a triangle in the plane* (r_1, r_3) *with vertices* $(2, -3)$*,* $(6, -3)$ *and* $(6, -7)$*. For any* $r \in \mathcal{S}_0$*, we have* $\mathcal{T}_{\boldsymbol{r}} = \frac{1}{2}(|r_1| + |r_1 + r_3| + |r_3|) \geq |r_3| \geq 3$*. On the other hand, any* $\boldsymbol{r} \in \mathcal{S}_0$ *satisfies* $\mathcal{T}_{\boldsymbol{r}} = 3$ *if and only if* $r_3 = -3$ *and* $2 \leq r_1 \leq 3$*. That is, in the plane* (r_1, r_3)*, the set of equilibrium promises is a subset of* $\mathcal{S}_0$ *represented by the horizontal side of* *the triangle bounded by the points* $(2,-3)$ *and (3,-3). Therefore, multiple equilibrium promises generate the minimum total transfer* $\mathcal{T}_{\boldsymbol{r}} = 3$*. To summarize, the set of stable promises is a triangle and the set of equilibria is a subset of one of the edges of that triangle.*

Example 2 illustrates that multiplicity of the stable promises persists even when we focus on the promises that minimize the total promises transfer (12). However, by focusing on these minimizing equilibria, we identify in the next proposition some general properties of the structure of promises that will hold for all equilibria.

Proposition 4. *[**Push toward equality**] For any equilibrium promises profile* $\boldsymbol{r} \in \mathcal{E}\backslash\{\boldsymbol{0}\}$*, there exists a committee member* $k_* \in \mathcal{C}^R$ *such that*

$$-u_j \leq r_j \leq 0 \leq r_i, \quad \textit{and} \quad v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}} \quad \textit{for all} \quad i < k_* \leq j, \tag{14}$$

and the total promises transfer of the promises profile $\boldsymbol{r}$ *satisfies:*

$$\mathcal{T}_{\boldsymbol{r}} = \sum_{i<k_*} r_i = \sum_{j \geq k_*} (-r_j). \tag{15}$$

Proposition 4 shows that, in any equilibrium, the flow of promises should be initiated by reform supporters with an *ex ante* intensity that is larger than that of a critical reform supporter k_*. Moreover, the promisers are subject to an individual rationality constraint precluding them from promising more than the utility gained when the reform is adopted. Promise recipients have an *ex ante* intensity that is lower than that of the critical member k_* and can be reform opponents or reform supporters. Thus, members with large *ex ante* intensities promise transfers to members with lower intensities. The specific location of the critical committee member k_* depends on the voting rule and the *ex ante* distribution of the intensity of preferences.[13] Despite the transfers, the *ex post* intensities of the promisers are always larger than those of the promisees as inequalities (14) show.

[13] In the proof of Proposition 4, we allow the critical member k_* to depend on $\boldsymbol{r}$. However, as we will see later in Remark 1 below, we can find a common critical member k_* for all $\boldsymbol{r} \in \mathcal{E}\backslash\{\boldsymbol{0}\}$.

Proposition 4 is, however, agnostic about the amount of transfer promises as well as the identity of promisees. In particular the promises recipients can be reform supporters or reform opponents. The amount of transfer promises depends on the *ex ante* distribution of the intensity of preferences and the majority threshold κ. In the next section, we make specific assumptions on the distribution of *ex ante* intensities and derive a more detailed description of the equilibrium transfer promises.

5 Equilibrium promises implications

In this section, we characterize the set $\mathcal{E}$ of equilibrium promises. We identify the magnitude and directions of the flow of equilibrium promises. In the first subsection, we study the case where reform supporters lack the voting power to enact the reform in the absence of promises. This situation is commonly referred to as "frustrated minorities" or equivalently as "coercive majority". In the second subsection, we study the case where reform supporters possess sufficient voting power to enact the reform in the absence of promises.

5.1 Committee with a decisive coalition of reform opponents

We assume in this subsection that the reform does not have enough voting support in the absence of promises, $\mid \mathcal{C}^R \mid < \kappa$ with $\kappa \geq 2$.[14] Under this assumption, $D(\mathbf{0}, \mathbf{0}) = S$. When the committee decision is taken by simple majority rule, the assumption $\mid \mathcal{C}^R \mid < \kappa$ on the distribution of intensity of preferences creates a "majority coercion" problem leading the committee to an inefficient outcome. The next proposition provides some general properties of the transfers that need to be promised in any equilibrium when $\mid \mathcal{C}^R \mid < \kappa$.

[14]When $\kappa = 1$, the reform will always be enacted by voting as long as it has support from at least one member. The assumption (2) implies that at least one member supports the reform. We, therefore, excluded the case $\kappa = 1$ in this section.

Proposition 5. ***[Equilibrium transfer promises with majority coercion]*** *Consider a committee with weak support for the reform,* $\mid \mathcal{C}^R \mid < \kappa$. *A promises profile* $\boldsymbol{r} \in \mathcal{P}$ *is an equilibrium if and only if*

1. *All promisers are reform supporters and the promises* $(r_i)_{i \in \mathcal{C}^R}$ *satisfy:*

   (a) *The individual rationality and non-positivity constraint* $-u_i \leq r_i \leq 0$ *for all* $i \in \mathcal{C}^R$ *and,*

   (b) *the aggregate promises transfer of the coalition* $\mathcal{C}^R$ *is* $\sum_{\mathcal{C}^R} r_i = -U^S$.

2. *All promises recipients are reform opponents and the promises* $(r_i)_{i \in \mathcal{C}^S}$ *satisfy:*

   (a) *When* $|\mathcal{C}^R| < \kappa - 1$, *each member* $i \in \mathcal{C}^S$ *is promised the transfer* $r_i = -u_i > 0$ *just to make her indifferent between the reform and the status quo,* $v_i^{\boldsymbol{r}} = 0$.

   (b) *When* $|\mathcal{C}^R| = \kappa - 1$, *the promises to the members of* $\mathcal{C}^S$ *are non-negative,* $r_i \geq 0$ *for all* $i \in \mathcal{C}^S$ *and satisfy:*

      i. $\sum_{i \in \mathcal{C}^S} r_i = U^S$.

      ii. *The ex post intensities of members in* $\mathcal{C}^S$ *cannot exceed those of members in* $\mathcal{C}^R$: $v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}}$ *for any* $i \in \mathcal{C}^S$ *and* $j \in \mathcal{C}^R$.

Moreover, the equilibrium promises profiles $\boldsymbol{r} \in \mathcal{E}$ *are indeterminate but all have identical total transfer promises* $\mathcal{T}_{\boldsymbol{r}} = U^S$.

Proposition 5 shows that equilibrium promises profiles satisfy $-\sum_{\mathcal{C}^R} r_i = U^S = \sum_{\mathcal{C}^S} r_i$. All equilibria consist of promises profiles of the *reaching across the aisle* type, that is, reform supporters compensate reform opponents. Moreover, in all equilibria, the total transfer compensates reform opponents for the aggregate disutility that they experience when the reform is enacted. The critical member k_* from Proposition 4 is given by $k_* = n+1$, so that the reform supporters are promisers and reform opponents are promisees. The equilibrium leaves multiple ways for the members of the coalition

$\mathcal{C}^R$ to divide among themselves the total transfer directed towards reform opponents. Whether there is an additional indeterminacy on the side of the promises' recipients depends on the size of the coalition $\mathcal{C}^R$.

If the coalition $\mathcal{C}^R$ is short at least 2 members of being decisive ($|\mathcal{C}^R| < \kappa - 1$), then the distribution of equilibrium promises among promisees is unique. After receiving the promises, all members of $\mathcal{C}^S$ are indifferent between voting for or against the reform. In that case, the promises to the members of the coalition $\mathcal{C}^S$ are identical for all equilibria and are determined by the *ex ante* intensity of preferences, $r_i = -u_i$ for all $i \in \mathcal{C}^S$. Recalling our tie breaking assumption the committee members vote for the reform when indifferent between the alternatives R and S, notice that when $|\mathcal{C}^R| < \kappa - 1$, there is a unanimous vote for the reform after the equilibrium promises have been made.

When the coalition $\mathcal{C}^R$ is just one member short of being decisive ($|\mathcal{C}^R| = \kappa - 1$), the promises to the members of $\mathcal{C}^S$ are indeterminate. In that case, multiple distributions of promises across the members of the receiving coalition $\mathcal{C}^S$ can form an equilibrium provided that they satisfy the requirement $2.(b).ii$ of Proposition 5. The requirement restricts the promises directed to each member of the coalition $\mathcal{C}^S$ to be non-negative and to produce *ex post* intensities of preferences that maintain the ordering across the coalitions of promisers and promisees of the *ex ante* intensity of preferences. Changing the ordering of interim intensities across the coalition $\mathcal{C}^R$ and $\mathcal{C}^S$ would create the incentives to engage in additional rounds of promises and contradict the stability requirement of the equilibrium. When $|\mathcal{C}^R| = \kappa - 1$, the committee adopts the reform after the promises are made but the vote for the reform may not be unanimous.

We now discuss an example to illustrate the results of Proposition 5.

Example 3. *Consider a committee with* 5 *members ruled by a simple majority rule,* $\kappa = 3$. *The intensities are given by*

$$\boldsymbol{u} = (u_1, u_2, u_3, u_4, u_5) = (-2, \ -1, \ -1, \ 8, \ 10).$$

The coalition supporting the reform is $\mathcal{C}^R = \{4, 5\}$ *and has an aggregate intensity*

$U^R = 18$. *The coalition supporting the status quo is* $\mathcal{C}^S = \{1, 2, 3\}$ *and has an aggregate intensity* $U^S = 4$. *The minority coalition* $\mathcal{C}^R = \{4, 5\}$ *is one member short of being decisive. Without promises, the reform is defeated,* $D(\mathbf{0}, \mathbf{0}) = S$ *and each member of the coalition* $\mathcal{C}^R$ *is dissatisfied with the committee choice. By Proposition 5, in any equilibrium, members of the coalition* $\mathcal{C}^R$ *promise the aggregate transfer* $U^S = 4$ *contingent on reform adoption to the members of coalition* $\mathcal{C}^S$. *Promises* r_4 *and* r_5 *satisfy the individual rationality and non-positivity constraints* $-8 \leq r_4 \leq 0$, $-10 \leq r_5 \leq 0$ *and the promises satisfy the no dissipation constraint* $r_4 + r_5 = -4$. *Following condition* $2.(b).ii$ *from Proposition 5 the promises* r_1, r_2 *and* r_3 *must satisfy the non-negativity constraint* $0 \leq \min\{r_1, r_2, r_3\}$ *and the ordering of ex post intensities constraint* $\max\{-2 + r_1, \ -1 + r_2, \ -1 + r_3\} \leq \min\{8 + r_4, \ 10 + r_5\}$. *For example the equilibrium promises profiles* $\boldsymbol{r_1} = (2, \ 1, \ 1, \ 0, \ -4)$ *and* $\boldsymbol{r_2} = (0, \ 0, \ 4, \ -2, \ -2)$ *produce the respective ex post intensities* $\boldsymbol{v}^{\boldsymbol{r_1}} = (0, \ 0, \ 0, \ 8, \ 6)$, $\boldsymbol{v}^{\boldsymbol{r_2}} = (-2, \ -1, \ 3, \ 6, \ 8)$. *Under the promises profile* $\boldsymbol{r_1}$, *there is a unanimous vote for the reform. Under the equilibrium promises profile* $\boldsymbol{r_2}$, *there is majority support for the reform but members* 1 *and* 2 *vote against it.*

The promises profile $\boldsymbol{r_3} = (0, \ 0, \ 4, \ -4, \ 0)$ *produces the ex post intensities* $\boldsymbol{v}^{\boldsymbol{r_3}} = (-2, \ -1, \ 3, \ 2, \ 10)$ *and leads the committee to adopt the reform. The promises profile* $\boldsymbol{r_3}$ *is not an equilibrium, however, because the utility of member* 3, $v_3^{\boldsymbol{r_3}} = 3$ *is larger than that of member* 4, $v_4^{\boldsymbol{r_3}} = 2$. *In fact, the promises profile* $\boldsymbol{r_3}$ *is not stable because the coalition* $\{1, \ 2, \ 4\}$ *has an aggregate intensity of* $-2 + (-1) + 2 = -1 < 0$ *and can thus form a blocking coalition to profile* $\boldsymbol{r_3}$. *This is because member* 4 *promises a transfer of* 4 *and ended up with an ex post intensity of* 2. *Due to her low level of intensity, member* 4 *becomes an attractive target for enticement from members* 1 *and* 2. *They form a blocking coalition by promising member* 4 *a transfer of up to* $s_4 = 3$ *in the event that the reform is rejected.*

In the next subsection, we study the equilibrium when the committee would adopt the reform in the absence of promises.

5.2 Committee with a decisive coalition of reform supporters

In this subsection, we assume that the reform has enough support, $\mid \mathcal{C}^R \mid \geq \kappa$ so that the group adopts the reform, $D(\mathbf{0},\mathbf{0}) = R$. Recalling definitions (3), notice that $n \leq I - \kappa \equiv \widehat{\kappa} - 1$ and that $u_i < 0 \leq u_j$, for $i \leq n < j$. In the absence of promises, members of the coalition $\mathcal{C}^R$ are numerous enough to lead the group to their preferred policy R. Members of the coalition $\mathcal{C}^S$ can, however, promise transfers contingent on defeating the reform to some members of the coalition $\mathcal{C}^R$ to increase the support for their preferred policy S and lead the committee to defeat the reform. The members of the coalition $\mathcal{C}^R$ who are more susceptible to being enticed to vote for the status quo are those with the lowest *ex ante* intensity of preferences for the reform. When the enticements are persuasive, some reform supporters are converted to reform opponents and these conversions represent a pivotal event if a blocking coalition exists. We denote the coalition of these members by

$$\underline{\mathcal{C}}^R := \{n+1, .., \widehat{\kappa}\}.$$

Notice that the members of $\underline{\mathcal{C}}^R$ are the cheapest to entice into voting for S and, since $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$, members of $\underline{\mathcal{C}}^R$ are numerous enough to form a decisive coalition for the status quo with $\mathcal{C}^S$. Recall that $u_i \geq 0$ for all $i \in \underline{\mathcal{C}}^R$ and denote by

$$\underline{U}^R := \sum_{\underline{\mathcal{C}}^R} u_i \equiv \sum_{i=n+1}^{\widehat{\kappa}} u_i$$

the aggregate intensity of preferences for the reform of the coalition $\underline{\mathcal{C}}^R$.

For the members of the coalition $\mathcal{C}^S$, the incentive to entice members of the coalition $\underline{\mathcal{C}}^R$ to vote for the alternative S arises only when there are gains from trade. Specifically, by switching from the decision R to the decision S, members of the coalition $\mathcal{C}^S$ gain the aggregate amount U^S. To entice members of the coalition $\underline{\mathcal{C}}^R$ to vote for S, members of the coalition $\mathcal{C}^S$ incur the aggregate cost $\underline{U}^R$. Therefore, we define the aggregate gains from trade of the members of the coalition $\mathcal{C}^S$ as

$$G^S := U^S - \underline{U}^R = -\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i \equiv -\sum_{i=1}^{\widehat{\kappa}} u_i. \tag{16}$$

5.2.1 Equilibria when there are no gains from trade: $G^S \leq 0$

The following results identify the unique equilibrium promises profile when there are no gains from trade.

Proposition 6. *[**No promises equilibrium in the absence of gains from trade**] Consider a committee with more reform supporters than the κ-majority requirement, $\mid \mathcal{C}^R \mid \geq \kappa$. Assume the gain from trade defined in equation (16) is non-positive, $G^S \leq 0$. Then $\boldsymbol{r} = \mathbf{0}$ is the unique equilibrium, that is, $\mathcal{E} = \{\mathbf{0}\}$.*

When the gain from trade G^S is negative, reform opponents have a smaller aggregate intensity of preferences than the required cost, $\underline{U}^R$, to change the group decision from R to S. When the gain from trade G^S is non-positive, the coalition $\mathcal{C}^S$ has no incentive to entice committee members supporting the reform to vote against it. Therefore, no blocking coalition for the zero promises profile exists. Proposition 6 shows that, in that case, there will be no need for the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ to preempt attempts to form blocking coalitions and the resulting equilibrium is a no promises equilibrium.

5.2.2 Equilibria with first-order preemption

We now consider the case where the gain from trade G^S is positive with $\mid \mathcal{C}^R \mid \geq \kappa$. We also assume without loss of generality that $\kappa \geq 2$.[15]

When the gains from trade are positive, blocking coalitions for the zero promises profile exist. The members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ then have to promise transfers to

[15] If $\kappa = 1$, then $\widehat{\kappa} = I$ so $\underline{\mathcal{C}}^R = \mathcal{C}^R$ and, given condition (2), we have $G^S = -\sum_{i \in \mathbb{I}} u_i < 0$ which contradicts the assumption that $G^S > 0$.

members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ to preempt attempts to form blocking coalitions. We first discuss an example to illustrate the equilibrium promises intuitively, before giving a proposition that covers more general situations.

Example 4. *Consider a committee with $I = 4$ members, $\kappa = 3$, and $\boldsymbol{u} = (-5,\ 1,\ 3,\ 10)$. In this case $\widehat{\kappa} = 2$, $\mathcal{C}^S = \{1\}$, $\mathcal{C}^R = \{2,\ 3,\ 4\}$, $\underline{\mathcal{C}}^R = \{2\}$, $\mathcal{C}^R/\underline{\mathcal{C}}^R = \{3,\ 4\}$, and $G^S = 4$.*

The intuition suggests that, in equilibrium, the coalition $\mathcal{C}^R/\underline{\mathcal{C}}^R = \{3, 4\}$ should promise a total of $G^S = 4$ to the coalition $\{1, 2\}$. The aggregate intensity of the coalition $\{3, 4\}$ is $3 + 10 = 13$, and therefore it can afford to promise a transfer of 4 without violating the ordering condition for ex post intensities and creating incentives for new rounds of promises.

Consistent with the case of an intense minority covered in Section 5.1, the equilibria are not unique. For example, all the following promises are equilibria: $\boldsymbol{r_1} = (2,\ 2,\ 0,\ -4)$, $\boldsymbol{r_2} = (3,\ 1,\ -1,\ -3)$. They produce the respective ex post intensities $\boldsymbol{v}^{\boldsymbol{r_1}} = (-3,\ 3,\ 3,\ 6)$, $\boldsymbol{v}^{\boldsymbol{r_2}} = (-2,\ 2,\ 2,\ 7)$. However, the promises profile $\boldsymbol{r_3} = (2,\ 2,\ -1,\ -3)$ achieves the minimum total transfer promises $\mathcal{T}_{\boldsymbol{r_3}} = 4$ but is not an equilibrium. This is because the resulting ex post instensities $\boldsymbol{v}^{\boldsymbol{r_3}} = (-3,\ 3,\ 2,\ 7)$ violate the ordering condition $v_2^{\boldsymbol{r_3}} > v_3^{\boldsymbol{r_3}}$. Because $v_1^{\boldsymbol{r_3}} + v_3^{\boldsymbol{r_3}} = -1 < 0$, Proposition 1 shows that the promises profile $\boldsymbol{r_3}$ is not stable. This happens because, after $\boldsymbol{r_3}$ is in place, member 3 becomes a new target for enticement by member 1. The promises profile $\boldsymbol{r_3}$ is unstable since the coalition $\{1, 3\}$ is blocking it.

In the following proposition, we show that the intuition in Example 4 holds more generally and provide conditions under which the coalition of promisers can afford to preempt new rounds of promises and achieve stability. Before stating the results in a proposition, we define the aggregate surplus of intensity of preferences of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ relative to each member $k \in \underline{\mathcal{C}}^R$ by

$$\Delta U_k := \sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} [u_j - u_k] \equiv \sum_{j=\widehat{\kappa}+1}^{I} [u_j - u_k]. \tag{17}$$

The variable ΔU_k represents the maximum aggregate transfer that members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ can promise while keeping their *ex post* intensities above that of member k. When $\Delta U_{\widehat{\kappa}}$ is larger than G^S, the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ can afford to promise a total transfer of G^S while maintaining their *ex post* intensities above that of all the members of the coalition of promisees $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. In this case, members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ can be compensated for the forgone gain realized by forming a blocking coalition, i.e. G^S, without creating new targets for enticement against the reform.

The following proposition establishes that when $G^S \leq \Delta U_{\widehat{\kappa}}$, all equilibria involve a total transfer promises of G^S from the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ to the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. Importantly, in all equilibria, the *ex post* individual intensities remain larger for all members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ than that of any member of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. Before stating the proposition, we recall that $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$ and $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R = \{\widehat{\kappa}+1, .., I\}$.

Proposition 7. *[**Equilibrium promises with first order preemption.**] Consider a committee in which the support for the reform is larger than the κ-majority requirement, $\mid \mathcal{C}^R \mid \geq \kappa$ with $\kappa \geq 2$. Assume the gain from trade defined in equation (16) satisfies $0 < G^S \leq \Delta U_{\widehat{\kappa}}$. Equilibrium promises profiles $\boldsymbol{r} \in \mathcal{E}$ are indeterminate and any promises profile $\boldsymbol{r} \in \mathcal{P}$ is an equilibrium if and only if*

1. *Members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ are promisors subject to individual rationality constraints while members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ are promisees:*

$$-u_j \leq r_j \leq 0 \leq r_i, \quad \forall i \leq \widehat{\kappa} < j. \tag{18}$$

2. *The ex post intensities of members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ cannot exceed those of members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$*

$$v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}}, \quad \forall i \leq \widehat{\kappa} < j. \tag{19}$$

3. *The total promises transfer induced by $\boldsymbol{r}$ is given by*

$$\mathcal{T}_{\boldsymbol{r}} = G^S = \sum_{i=1}^{\widehat{\kappa}} r_i = -\sum_{j=\widehat{\kappa}+1}^{I} r_j. \tag{20}$$

Proposition 7 shows that, under the assumption $|\mathcal{C}^R| \geq \kappa$ and $0 < G^S \leq \Delta U_{\widehat{\kappa}}$, all equilibria require members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ to promise a total transfer of G^S to the members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. In particular, the critical member k_* from Proposition 4 is given by $k_* = \widehat{\kappa} + 1$. The objective of this transfer is to preempt members of the coalition $\mathcal{C}^S$ from enticing members of the coalition $\underline{\mathcal{C}}^R$ to cast their votes against the reform. Proposition 7 also shows that multiplicity occurs for two reasons. First, the members of the promisers coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ can divide the cost of preemption in different ways among themselves. Multiplicity also occurs because the transfer promises to the coalition of promisees $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ can be distributed in different ways. However, the distributions of promises are constrained by inequality (19) to generate *ex post* intensities of preferences that are lower for the members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ than for the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$.

The constraint (19) represents a binding political constraint. To build intuition on the constraint, consider a slight variation of Example 4, where a committee with $I = 4$ members has simple majority rule, $\kappa = 3$, and where the *ex ante* intensities are now given by $\boldsymbol{u} = (-5,\ 1,\ 3,\ 3)$. Proposition 7 applies to this example with $\Delta U_{\widehat{\kappa}} \equiv \Delta U_2 = G^S = 4$ and in any equilibrium, the coalition $\{3, 4\}$ must promise a transfer of 4 to the coalition $\{1, 2\}$. In equilibrium however, the coalition of promisers $\{3, 4\}$ cannot transfer anything to the weak reform supporter member 2 without reversing the order of *ex post* intensities. This means that all equilibrium promises need to be directed to member 1. In this example, there is a single equilibrium given by $\boldsymbol{r} = (4,\ 0,\ -2,\ -2)$ and the equilibrium is of reaching across the aisle type. By contrast, Example 4 shows that, when $\Delta U_{\widehat{\kappa}} = 11 > G^S = 4$, there are equilibria of the "circle the wagon" type where some of the promises recipients are reform supporters. We will discuss in Section 6 below the conditions under which all equilibria are of reaching across the aisle type in a general setting.

5.2.3 Equilibria with higher-order preemption

We now consider the case where $\mid \mathcal{C}^R \mid \geq \kappa$ with $\kappa \geq 2$ and with $0 \leq \Delta U_{\widehat{\kappa}} < G^S$. When $\Delta U_{\widehat{\kappa}} < G^S$, members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ cannot promise the aggregate transfer G^S while maintaining *ex post* intensities above the intensities of all members in $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. In that case, when the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ collectively promise G^S, the *ex post* intensities of some of them are reduced enough that they become targets for new rounds of enticements.

A *higher-order* blocking coalition may then emerge after the first round of promises: members of the coalition $\mathcal{C}^S$ together with some reform supporters from the coalition $\mathcal{C}^R$ with the lowest post-promises intensities may form a decisive coalition defeating the reform. To preempt such attempts, other members of the coalition $\mathcal{C}^R$ make new round of promises. These new promises may create opportunities for new blocking coalitions to form which will require another round of promises. The additional rounds of promises to prevent the formation of higher order blocking coalitions imply that the total transfer is larger than G^S. The precise characterization of the equilibrium will be reported in Proposition 8 below, and we first provide some insight on the equilibrium promises structure through a numerical example.

Example 5. *Consider a committee with $I = 7$ members, and a $\kappa-$majority requirement with $\kappa = 3$ and $\widehat{\kappa} = 5$. Assume that the ex ante intensities are $\boldsymbol{u} = (u_1 = -14,\ u_2 = -8,\ u_3 = 2,\ u_4 = 4,\ u_5 = 6,\ u_6 = 8,\ u_7 = 8)$. In this case* $\mathcal{C}^S = \{1,2\}, \quad \underline{\mathcal{C}}^R = \{3,4,5\}, \quad \mathcal{C}^R/\underline{\mathcal{C}}^R = \{6,7\}, \quad$ *and* $\quad G^S = 10, \quad \Delta U_5 = 4.$

Let us assume that members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R = \{6,7\}$ promise a transfer of $G^S = 10$ to the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1,2,3,4,5\}$. After the promises are issued, either member 6 or 7 will have an ex post intensity that is lower than $u_5 = 6$. Indeed, if for example we set

$$\boldsymbol{r} = (6,4,0,0,0,-5,-5), \qquad \textit{then} \quad \boldsymbol{v^r} = (-8,-4,2,4,6,3,3). \tag{21}$$

Then $v_6^{\boldsymbol{r}} < v_5^{\boldsymbol{r}}$ and the coalition $\{1,2,3,6,7\}$ forms a blocking coalition for the promises *profile* $\boldsymbol{r}$*. Therefore, the promises profile* $\boldsymbol{r}$ *defined in* (21) *is not an equilibrium and additional promises need to be made to reach an equilibrium.*

Who will promise these additional transfers? We see from the ex post intensities given in (21) *that, member* 5 *becomes the member with the largest intensity* $v_5^{\boldsymbol{r}} = 6$ *even though she does not belong to the coalition* $\mathcal{C}^R/\underline{\mathcal{C}}^R$*. Thus, member* 5 *will initiate the second round of promises. We thus see that each round of promises creates the incentives for a new round of promises that adds to the existing ones.*

To build intuition on the structure of equilibrium promises, we now show how an equilibrium can be constructed for this example through a three steps procedure:

Step 1. *The coalition* $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ *promises a total of* $\Delta U_5 = 4$ *to the coalition* $\mathcal{C}^S$*, aligning their ex post intensities with member* 5*,* $u_5 = 6$*. This uniquely implies* $r_6 = r_7 = -2$*. We also need to insure that ex post intensities of* $\mathcal{C}^S = \{1, 2\}$ *do not exceed* $u_5 = 6$*. This allows multiple choices for* r_1 *and* r_2 *but for illustration we choose*

$$\boldsymbol{r} = (3, 1, 0, 0, 0, -2, -2), \quad \boldsymbol{v}^{\boldsymbol{r}} = (-11, -7, 2, 4, 6, 6, 6). \tag{22}$$

Notice also that $\mathcal{T}_{\boldsymbol{r}} = 4 < G^S$ *and the transfer promises are not sufficient to preclude the formation of blocking coalitions. Indeed, an inspection of the ex post intensities given in* (22) *shows that the promises profile* $\boldsymbol{r}$ *is not an equilibrium because the coalition* $\{1,\ 2,\ 3,\ 4,\ 5\}$ *is a blocking coalition for* $\boldsymbol{r}$*.*

Step 2. *Members of the coalition* $\{5, 6, 7\}$ *promise the members of* $\mathcal{C}^S$ *an amount that reduces the formers' ex post intensities to* $\boldsymbol{v}_4^{\boldsymbol{r}} = 4$*. Thus,* $\tilde{r}_5 = \tilde{r}_6 = \tilde{r}_7 = -2$*. The algorithm also caps the ex post intensities of* $\mathcal{C}^S = \{1, 2\}$ *at* $u_4 = 4$*. Again there are multiple possible choices and for illustration, we choose*

$$\tilde{\boldsymbol{r}} = (6, 0, 0, 0, -2, -2, -2), \quad \textit{which means} \quad \boldsymbol{r} + \tilde{\boldsymbol{r}} = (9, 1, 0, 0, -2, -4, -4),$$
$$\textit{and thus} \quad \boldsymbol{v}^{\boldsymbol{r}+\tilde{\boldsymbol{r}}} = (\boldsymbol{v}^{\boldsymbol{r}})^{\tilde{\boldsymbol{r}}} = (-5, -7, 2, 4, 4, 4, 4).$$

Although the total transfer $\mathcal{T}_{\boldsymbol{r}+\tilde{\boldsymbol{r}}} = 10 = G^S$*, the "effective" transfer from* $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ *to* $\mathcal{C}^S$ *is only* $\Delta U_4 = 4 + 4 = 8$*, which is still less than* $G^S = 10$*. Indeed, the promises profile* $\boldsymbol{r} + \tilde{\boldsymbol{r}}$ *is not an equilibrium because it can be blocked by the coalition* $\{1,\ 2,\ 3,\ 4,\ 5\}$*.*

Step 3. *If we continue to reduce the intensities of the members of the coalition* $\{4,5,6,7\}$ *to* $\boldsymbol{v}_3^{\boldsymbol{r}+\tilde{\boldsymbol{r}}} = 2$*, each member of the coalition* $\{4,5,6,7\}$ *needs to transfer* 2 *more units of utility. The "effective" transfer from* $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ *to* $\mathcal{C}^S$ *will be* $\Delta U_3 = 6+6 = 12 > G^S = 10$*. That transfer will be more than necessary to meet the minimum total transfer. This means we need to reduce the common transfer so that the effective transfer originating from the coalition* $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ *is just equal to* 10*. Let's assume that, after the promises profile* $\boldsymbol{r}+\tilde{\boldsymbol{r}}$ *is in place, each member of the coalition* $\{4,5,6,7\}$ *makes a promise of* $x > 0$ *to the members of the coalition* $\mathcal{C}^S$*. Then the aggregate cumulative transfer from the coalition* $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ *is* $8+2x$*. Equalizing that effective transfer to* 10 *gives* $x = 1$*. Note that the resulting ex post intensities of the promisers* $\{4,5,6,7\}$ *are all equal to* 3 *and that they are larger than* $\boldsymbol{v}_3^{\boldsymbol{r}+\tilde{\boldsymbol{r}}} = 2$*. The distribution of these promises among the members of coalition* $\mathcal{C}^S$ *is indeterminate except for the fact that the ex post intensities must not exceed the ex post intensity* $\boldsymbol{v}_4^{\boldsymbol{r}+\tilde{\boldsymbol{r}}} = 3$*. We can choose, for example,*

$$\begin{gathered}\bar{\boldsymbol{r}} = (2,2,0,-1,-1,-1,-1), \textit{ or say } \hat{\boldsymbol{r}} := \boldsymbol{r}+\tilde{\boldsymbol{r}}+\bar{\boldsymbol{r}} = (11,3,0,-1,-3,-5,-5), \\ \textit{and thus} \quad \boldsymbol{v}^{\hat{\boldsymbol{r}}} = (\boldsymbol{v}^{\boldsymbol{r}+\tilde{\boldsymbol{r}}})^{\bar{\boldsymbol{r}}} = (-3,-5,2,3,3,3,3).\end{gathered} \tag{23}$$

This is an equilibrium, with total transfer $\mathcal{T}_{\hat{\boldsymbol{r}}} = 14 > G^S = 10$*. By Proposition 8 below we see that* 14 *is indeed the minimum total transfer. Of the* 14 *units of utility transferred,* 10 *units is a compensation to preclude the formation of first order blocking coalitions and* 4 *units sway members to forswear higher order blocking coalitions. So, this algorithm achieves the intended result of determining an equilibrium. The algorithm leaves a degree of freedom on how to allocate the promises among the coalition of promisees which reflects the indeterminacy of the equilibrium set. However, the algorithm determines uniquely the allocation of promises among promisers. To see this let us make some additional observations.*

First, note that in Step 3 *of the algorithm, members of the coalition* $\{4,5,6,7\}$ *issue equal incremental promises. If they do not promise an equal amount, then some members will become more vulnerable to enticement, and consequently further* *promises will be required in order to reach an equilibrium. While at the end of this process a stable promises profile is reached, that profile will not realize the minimum total transfer. To see this, assume, instead of* (23)*, that the incremental promises in Step* 3 *are not equalized among the members of the coalition* $\{4,5,6,7\}$*,*

$$\bar{\boldsymbol{r}}' = (2,2,0,0,-2,-1,-1), \quad \text{or say} \quad \hat{\boldsymbol{r}}' := \boldsymbol{r} + \tilde{\boldsymbol{r}} + \bar{\boldsymbol{r}}' = (11,3,0,0,-4,-5,-5),$$
$$\text{and thus} \quad \boldsymbol{v}^{\hat{\boldsymbol{r}}'} = (\boldsymbol{v}^{\boldsymbol{r}+\tilde{\boldsymbol{r}}})^{\bar{\boldsymbol{r}}'} = (-3,-5,2,4,2,3,3).$$

This is not a stable profile because $\{1,2,3,5,6\}$ *is a blocking coalition. However,* $\mathcal{T}_{\hat{\boldsymbol{r}}'} = 14$ *and the new round of promises to preclude the formation of this new blocking coalition will inflate the total promises and contradict the minimum total promises constraint. The constraint of equal transfer among promisers uniquely determines the promisers' individual (net) transfer. This result is proven in Proposition 8.*

Second, the allocation of total transfers among promisees is not unique. In our numerical example, members of the coalition $\{4,5,6,7\}$ *promise a total transfer of* 14 *to the member of the coalition* $\{1,2,3\}$*. The coalition* $\{1,2,3\}$ *can distribute the total transfer of* 14 *in an arbitrary way among the members of the coalition* $\{1,2,3\}$*, as long as their ex post intensities do not exceed* 3*, which is the ex post intensity of the members of the coalition* $\{4,5,6,7\}$*. For example, the promises profile* $\boldsymbol{r}_1 = (10,4,0,-1,-3,-5,-5)$ *is an equilibrium and produces the ex post intensities* $v^{\boldsymbol{r}_1} = (-4,-4,2,3,3,3,3)$*. Similarly, the promises profile* $\boldsymbol{r}_2 = (9,4,1,-1,-3,-5,-5)$ *is an equilibrium and produces the ex post intensities* $\boldsymbol{v}^{\boldsymbol{r}_2} = (-5,-4,3,3,3,3,3)$*. In particular, profile* $\boldsymbol{r}_2$ *exhibits a circle the wagon transfer since member* 3 *is a promises recipient even though she is a reform supporter* ($u_3 = 2 > 0$)*.* □

We now present the general characterization of equilibrium promises profiles for the case $\Delta U_{\widehat{\kappa}} < G^S$, and assume again that $\kappa \geq 2$ without loss of generality. Before stating the proposition, define the intensity

$$u_* := \frac{1}{\kappa - 1} \sum_{j \in \mathbb{I}} u_j > 0 \tag{24}$$

and the reform supporter $k_* \in \underline{\mathcal{C}}^R$ by[16]

$$u_{k_*-1} \leq u_* < u_{k_*}. \tag{25}$$

Proposition 8. *[* ***Equilibria with higher-order preemption*** *] Consider a committee in which support for the reform is in excess of κ-majority requirement, $\mid \mathcal{C}^R \mid \geq \kappa$ with $\kappa \geq 2$. Assume the gain from trade defined in equation (16) is not affordable to promise, i.e., $\Delta U_{\widehat{\kappa}} < G^S$. A promises profile $\boldsymbol{r} \in \mathcal{P}$ is an equilibrium if and only if*

1. *Members of the coalition $\{k_*, \cdots, I\}$ are promisers subject to individual rationality constraints*

$$-u_j \leq r_j = -u_j + u_* < 0 \quad \textit{for all} \quad j \geq k_*\,, \tag{26}$$

and experience equal ex post intensities

$$v_j^{\boldsymbol{r}} = u_* > 0 \quad \textit{for all} \quad j \geq k_*. \tag{27}$$

2. *Members of the coalition $\{1, \cdots, k_* - 1\}$ are promisees, and their ex post intensities cannot exceed those of promisers*

$$r_i \geq 0 \quad \textit{and} \quad v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}} \equiv u_*, \quad \textit{for all} \quad i < k_* \leq j. \tag{28}$$

Moreover, equilibrium promises profiles are indeterminate and they all generate the common total promises transfer

$$\mathcal{T}_{\boldsymbol{r}} = \mathcal{T}_* \textit{ where } \mathcal{T}_* := \sum_{j \geq k_*} \left[u_j - u_*\right] > G^S. \tag{29}$$

Proposition 8 shows that in all equilibria, members of the promisers coalition promise a transfer that is affine in their *ex ante* intensities as described in equation (26). This implies that members with larger intensities promise a larger transfer

[16] Proposition 8 shows that the member k_* defined in (25) meet the criteria of the critical member defined in Proposition 4. That is why we choose the same notation k_*.

and this results in an equalized distribution of *ex post* intensities among the coalition of promisers. Therefore, in all equilibria the distribution of promised transfers among promisers is unique. This uniqueness is novel and contrasts with the cases covered in Proposition 5 and Proposition 7 where the distribution of transfer promises among promisers was indeterminate. The intuition of this uniqueness is that if the promisers have unequal *intensities*, some of them will become subject to enticement to cast their vote against the reform and this in turn contradicts the stability requirement as additional promises are required to preclude the enticement. On the other hand, the multiplicity associated with the transfer distribution among promisees remains valid as in the cases covered in Proposition 7.

Proposition 8 also shows that the total transfer is larger than the gain from trade G^S as we illustrated in the example. More specifically, a direct calculation shows that the aggregate promises from the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$

$$\sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} r_j = \sum_{j=\widehat{\kappa}+1}^{I} \left[u_j - u_* \right] = G^S.$$

This shows that members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ will promise an aggregate transfer of G^S. An aggregate transfer of G^S will be sufficient to achieve an equilibrium when the gain from trade is affordable. However, this is not the case since $\Delta U_{\widehat{\kappa}} < G^S$ and as a result, some members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ will have lower *ex post* intensities than some members of $\underline{\mathcal{C}}^R$ and hence become targets for enticement. To preempt this to happen, additional transfer promises are needed from the members of the coalition $\{k_*, .., \widehat{\kappa}\} \subseteq \underline{\mathcal{C}}^R$ with *interim* intensities that are larger than those of some members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$. The aggregation of these promises are given by $\sum_{j=k_*}^{\widehat{\kappa}} \left[u_j - u_* \right] > 0$, which results in a total transfer $\mathcal{T}_* > G^S$.

Remark 1. *We remark that among all equilibria, the coalition of promisers* $\{k_*, \cdots, I\}$ *and the coalition of promises recipients* $\{1, \cdots, k_* - 1\}$ *are unique. This is because the critical member* k_* *in Proposition 4 can be chosen to be the same for all* $\boldsymbol{r} \in \mathcal{E} \backslash \{\mathbf{0}\}$ *in all subcases. In the case of Proposition 5 we have* $k_* = n + 1$*; in the case of*

Proposition 7 we have $k_* = \widehat{\kappa} + 1$*; in the case of Proposition 8, the common* k_* *is determined by* (25)*; while in the case of Proposition 6 the* k_* *is irrelevant since in this case* $\mathcal{E} = \{\mathbf{0}\}$*. We also recall that the total payment transfer* $\mathcal{T}_{\boldsymbol{r}}$ *is also the same for all* $\boldsymbol{r} \in \mathcal{E}$*. So the multiplicity of equilibria is only due to how to divide the total transfer among the promisers* $k_*, \cdots, I$ *and/or how to distribute the total transfer among the promisees* $1, \cdots, k_* - 1$*.*

6 Discussion

In this section, we discuss the predictions of our equilibria. In the first subsection, we discuss whether the promises are of the "reaching across the aisle" type. In the second subsection, we discuss the impact of an increase in polarization on our equilibria. In the third subsection, we propose a sequential procedure of transfers that allows the selection of a unique equilibrium.

6.1 Reaching across the aisle promises

We say that a promises profile $\boldsymbol{r} \in \mathcal{P}$ is of the *reaching across the aisle* type if all the promises recipients are reform opponents and all the promisers are reform supporters:

$$r_i \geq 0 \text{ for all } i \in \mathcal{C}^S, \text{ and } r_i \leq 0 \text{ for all } i \in \mathcal{C}^R.$$

Alternatively, when some promises flow from reform supporters to other reform supporters with weakest intensities, we say that the promises profile $\boldsymbol{r}$ has some *circle the wagon* transfers.

When the reform lacks sufficient voting support to be enacted without the use of promises, Proposition 5 indicates that all equilibrium promises will be of the reaching across the aisle type. Therefore, according to our model, in cases of majority coercion, promises involving "circle the wagon" transfers are not expected to occur. When the reform has enough voting support, Proposition 9 from Appendix B provides necessary and sufficient conditions under which all equilibrium promises are of the across the aisle type. The proposition states that, in order to eliminate transfers of circle the wagon type, we generally require a uniform distribution of intensities among a specific subset of reform supporters with the weakest intensities. In other words, the proposition shows that when the weakest reform supporters derive uniform utility from the reform, then all equilibrium promises are expected to be of the reaching across aisle type. If the condition does not hold, some equilibria may feature promises directed to reform supporters with the weakest intensities.

6.2 Impact of polarization

Intuitively, when the polarization of the *ex ante* intensities distribution increases, persuading reform opponents to change their stance becomes more challenging. This intuition suggests that as the distribution of intensities becomes more polarized, the magnitude of equilibrium promise transfers is likely to increase in order to overcome the resistance of reform opponents. To formalize this intuition within our model, consider an increase in polarization where all *ex ante* intensities are multiplied by a scalar $\lambda > 1$. In the context of our model, it can be demonstrated that there exists a one-to-one correspondence between the equilibrium promises when $\lambda = 1$ and those when $\lambda > 1$. This implies that the increase in the polarization of the intensity distribution, represented by λ, has an amplifying effect on the total promise transfers, scaling it by a factor of λ. Recently, earmarking, which refers to the practice of legislators allocating funds for specific projects in their district or state, has been revived in the US Congress.[17] Given that earmarks provide lawmakers with incentives to vote for legislation they may not support otherwise, they can be broadly interpreted as promises within the framework of our model. The observed increase in polarization within the US Congress, coupled with the restoration of earmarking, aligns with the

[17] https://www.science.org/content/article/congress-restores-spending-earmarks-rules-remove-odor

predictions of our model on the impact of polarization.

6.3 Equilibrium selection

In this section, we present an algorithm (i.e. a procedure) that implements a sequence of promise transfers, leading to an equilibrium state within a finite number of steps. A fully informed planner or an institution mandating the algorithm will reach a unique equilibrium after a finite number of steps. Moreover, the equilibrium attained through the sequence of promises associated with the algorithm achieve the distribution of *ex post* intensities that has the lowest dispersion among all equilibria. The algorithm operates based on a guiding principle where, at each step, committee member(s) with the highest intensities promise a transfer to committee member(s) with the lowest intensities. In each step, a fixed amount is transferred to ensure that the post-promise intensities of either the promisers or the recipients of the promises align with the next lowest or highest intensity committee member, respectively. This iterative process continues until the gains from trade (U^S or G^S) of the resulting post-promise intensities are effectively nullified. The algorithm serves as an equilibrium selection mechanism, accompanied by a dynamic implementation involving transfers between members in the upper and lower tails of the distribution of intensities. It is worth noting that alternative algorithms based on different principles could select different types of equilibria. In the next numerical example, we illustrate the algorithm. Proposition 10 from Appendix B shows that the algorithm is valid in all the cases covered in this paper and that it achieves the distribution of *ex post* intensities that have the lowest dispersion (or equivalently range) after a finite number of iterations.

Example 6. *Recall the committee of Example 5 where $I = 7$, $\kappa = 3$, $\widehat{\kappa} = 5$, and*

$$\boldsymbol{u} = (-14, \ -8, \ 2, \ 4, \ 6, \ 8, \ 8).$$

Note that in this case $\sum_{i=1}^{5} u_i < 0$, so $\boldsymbol{r} = 0$ is unstable (see Proposition 1). For the first step, the promises profile $\boldsymbol{r}^1 = (4, \ 0, \ 0, \ 0, \ 0, \ 2, \ 2)$ is implemented. This is *done to align the next step intensities on member 6 and 7 to that of member 5. The resulting interim intensities profile is then* $\boldsymbol{v}^1 = (-10, \ -8, \ 2, \ 4, \ 6, \ 6, \ 6)$.

We notice that $\sum_{i=1}^{5} v_i^1 < 0$*, and thus the promises profile* $\boldsymbol{r}^1$ *is unstable. In the second step, the incremental promises profile* $\boldsymbol{r}^2 = (2, \ 0, \ 0, \ 0, \ -2/3, \ -2/3, \ -2/3)$ *is implemented which results in the intensities* $\boldsymbol{v}^2 = (-8, \ -8, \ 2, \ 4, \ 16/3, \ 16/3, \ 16/3)$.

We notice again that $\sum_{i=1}^{5} v_i^2 < 0$*, and hence the promises profile* $\boldsymbol{r}^1 + \boldsymbol{r}^2$ *is also unstable. In the third step, the incremental promises profile* $\boldsymbol{r}^3 = (2, \ 2, \ 0, \ 0, \ -4/3, -4/3, \ -4/3)$ *is implemented which results in the intensities* $\boldsymbol{v}^3 = (-6, \ -6, \ 2, \ 4, \ 4, \ 4, \ 4)$.

Since $\sum_{i=1}^{5} v_i^3 < 0$*, the promises profile* $\boldsymbol{r}^1 + \boldsymbol{r}^2 + \boldsymbol{r}^3$ *is unstable. If in the fourth step, we set the incremental promises profile* $\boldsymbol{r}^4 = (4, \ 4, \ 0, \ -2, \ -2, -2, \ -2)$*, the resulting intensities would be* $v^4 = (-2, \ -2, \ 2, \ 2, \ 2, \ 2, \ 2)$*. We see that* $\sum_{i=1}^{5} v_i^4 > 0$*, so the promises profile* $\sum_{i=1}^{4} \boldsymbol{r}^4$ *is stable. However, we can achieve stability with a lower total transfer. Assume that instead of promising a total transfer of* 8*, the total incremental transfer is* $X^* < 8$ *so that the ex post intensities* $\boldsymbol{v}^*$ *satisfy*

$$\boldsymbol{v}^* = (-6 + \frac{X^*}{2}, \ -6 + \frac{X^*}{2}, \ 2, \ 4 - \frac{X^*}{4}, \ 4 - \frac{X^*}{4}, \ 4 - \frac{X^*}{4}, \ 4 - \frac{X^*}{4}).$$

We then set X^* *just large enough to attain stability,* $0 = \sum_{i=1}^{5} v_i^* = -2 + \frac{X^*}{2}$*, that is,* $X^* = 4$ *and hence* $\boldsymbol{v}^* = (-4, \ -4, \ 2, \ 3, \ 3, \ 3, \ 3)$*. This is one of the equilibria constructed in Example 5.* □

7 Conclusion

Our study focuses on a binary voting model, where committee members can make outcome-contingent promises before casting their votes. We define the political equilibrium using the classical notion of stability, combined with the assumption of total transfer promises minimization. Our findings show that multiple equilibria exist and always lead to reform enactment. The intensity of promises flows from supporters with the strongest intensities, to those with weaker intensities. In committees with a frustrated coalition of reform supporters, promises flow across the aisle from reform supporters to reform opponents. In cases where reform supporters hold enough voting power to enact the reform, promises may be necessary to prevent weaker supporters from voting against it. However, these promises need not necessarily cross the aisle unless the weakest reform supporters have homogeneous preferences.

It is important to acknowledge that the theoretical framework presented in this paper focuses on a specific type of political failure, namely the problem of majority coercion. Other forms of political failures, such as the tragedy of the commons (Olson [1965]) voter ignorance (Downs [1957]) and rent-seeking (Tullock [1967]) are also significant, and warrant further investigation. Therefore, there is still much work to be done in developing a comprehensive evaluation of the practice of promises, particularly in environments where informational issues play a crucial role. It is also worth noting that we do not explicitly model the voting game leading to an equilibrium. What we can say is that once an equilibrium promises profile is reached, it cannot be overturned, whereas other profiles do not exhibit the same level of stability. In practice, promises often take the form of amendments to legislative bills, involving multiple rounds of negotiation. Thus, it would be valuable to model a dynamic game of sequential decentralized promises, aiming to generate equilibria based on the farsighted core and minimal total transfers. If possible, this extension would align with the existing literature on promises with political representation, providing novel understanding of the sequencing of promises in decentralized settings.

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Online Appendix

Appendix A contains the proofs of all propositions from the paper. Appendix B contains results that support the discussions in Section 6. Our proofs often rely on a contradiction approach, where we construct modified promises profiles that contradict our assumptions. Notably, we do not rely on external tools and instead, we utilize basic logical arguments and work directly with the mathematical objects. To assist the reader, we provide a summarized table of notation for easy reference.

Table 1: Summary of the notations

Notations	**Definition**
I	Number of committee members
$\mathbb{I} = \{1, \cdots, I\}$	Set of committee members
$n \leq I$	Number of reform opponents
$\kappa \leq I$	Voting share threshold to enact the reform
$\widehat{\kappa} = I - \kappa + 1$	Voting share threshold to defeat the reform
$\mathcal{C}^R = \{i : u_i \geq 0\} \equiv \{n+1, \cdots, I\}$	Coalition of reform supporters
$\mathcal{C}^S = \{i : u_i < 0\} \equiv \{1, \cdots, n\}$	Coalition of reform opponents
$\mathcal{D}^R$ resp. $\mathcal{D}^S$	The set of decisive coalitions enacting (resp. defeating) the reform
$U^R = \sum_{i=n+1}^{I} u_i$	Aggregate intensity for the reform
$U^S = \sum_{i=1}^{n}(-u_i)$	Aggregate intensity for the status quo
$\boldsymbol{u} = (u_1, \cdots, u_I)$	Vector of *ex ante* intensities
$\boldsymbol{r} = (r_1, \cdots, r_I)$	Promises profile contingent on enacting the reform
$\boldsymbol{s} = (s_1, \cdots, s_I)$	Promises profile contingent on defeating the reform
$D(\boldsymbol{r}, \boldsymbol{s}) \in \{R, S\}$	Committee decision once the promises $(\boldsymbol{r}, \boldsymbol{s})$ are in place
$\boldsymbol{v}^{\boldsymbol{r},\boldsymbol{s}} = (\boldsymbol{u} + \boldsymbol{r})1_{\{D(\boldsymbol{r},\boldsymbol{s})=R\}} + \boldsymbol{s}1_{\{D(\boldsymbol{r},\boldsymbol{s})=S\}}$	Profile of *ex post* intensities implied by the promises profile $(\boldsymbol{r}, \boldsymbol{s})$
$\boldsymbol{v}^{\boldsymbol{r}} = (\boldsymbol{u} + \boldsymbol{r})1_{\{D(\boldsymbol{r},\boldsymbol{s})=R\}}$	Profile of *ex post* intensities implied by the promises profile $(\boldsymbol{r}, \boldsymbol{0})$
$\mathcal{S}_0$ resp. $\mathcal{E}$	Set of stable promises resp. set of equilibrium promises with $\boldsymbol{s} = 0$
$\mathcal{T}_{\boldsymbol{r}} = \frac{1}{2}\sum_{i=1}^{I} \lvert r_i \rvert$	Total transfer promises associated with the promises profile $(\boldsymbol{r}, \boldsymbol{0})$
$\underline{\mathcal{C}}^R = \{n+1, \cdots . \widehat{\kappa}\}$	Coalition of weakest reform supporters
$\underline{U}^R = \sum_{\underline{\mathcal{C}}^R} u_i$	Aggregate *ex ante* intensity of the coalition $\underline{\mathcal{C}}^R$
$G^S = U^S - \underline{U}^R$	Aggregate gain from trade of the members of the coalition $\mathcal{C}^S$
$\Delta U_k = \sum_{j \in \mathcal{C}^R / \underline{\mathcal{C}}^R} [u_j - u_k]$	Aggregate surplus of intensity of the coalition $\mathcal{C}^R / \underline{\mathcal{C}}^R$ relative to member $k \in \underline{\mathcal{C}}^R$
$u_* = \frac{1}{\kappa - 1} \sum_{j \in \mathbb{I}} u_j$	Scaled average *ex ante* intensity
$u_{k_*-1} \leq u_* < u_{k_*}$	Definition of κ^*, the critical promiser

Appendix A: Proofs

Proof of Proposition 1:

We proceed in two steps.

Step 1. In this step we verify the "only if" direction of the proposition ($\Rightarrow$) by proving that if $(\boldsymbol{r},\boldsymbol{s}) \in \mathcal{S}$, then inequalities (10) hold. Assume that $(\boldsymbol{r},\boldsymbol{s}) \in \mathcal{S}$.

We first prove that $D(\boldsymbol{r},\boldsymbol{s}) = R$ by contradiction. Suppose that $D(\boldsymbol{r},\boldsymbol{s}) = S$. In that case, the grand coalition $\mathbb{I}$ itself is a blocking coalition for the promises profile $(\boldsymbol{r},\boldsymbol{s})$. The committee members can initiate the alternative promises contingent on adopting the reform

$$r'_i := s_i - u_i + \frac{1}{I}\sum_{j\in\mathbb{I}} u_j.$$

Since $\boldsymbol{s} \in \mathcal{P}$, we have $\sum_{\mathbb{I}} r'_i = 0$, and thus $\boldsymbol{r'} \in \mathcal{P}$. When voting under the alternative promises profile $(\boldsymbol{r'},\boldsymbol{s})$, member i compares the utility if the reform is adopted, $u_i + r'_i = s_i + \frac{1}{I}\sum_{\mathbb{I}} u_j$ with the utility when the reform is rejected s_i. By assumption, $\sum_{\mathbb{I}} u_j > 0$, and hence committee members unanimously vote for the reform, leading the committee to adopt the reform, $D(\boldsymbol{r'},\boldsymbol{s}) = R$. Committee member i then gets the utility $v_i^{\boldsymbol{r'},\boldsymbol{s}} = s_i + \frac{1}{I}\sum_{\mathbb{I}} u_j$. Because we assumed $D(\boldsymbol{r},\boldsymbol{s}) = S$, we have $v^{\boldsymbol{r},\boldsymbol{s}} = s_i$. Thus, $v_i^{\boldsymbol{r},\boldsymbol{s}} < v_i^{\boldsymbol{r'},\boldsymbol{s}}$ for any committee member $i \in \mathbb{I}$. We conclude that if $D(\boldsymbol{r},\boldsymbol{s}) = S$, then the promises profile $(\boldsymbol{r},\boldsymbol{s})$ is not stable because the grand coalition $\mathbb{I}$ can block it. Hence, $D(\boldsymbol{r},\boldsymbol{s}) = R$.

Second, we prove the inequalities (10). We proceed by contradiction and assume that there exists a coalition $\mathcal{C} \in \mathcal{D}^S$ such that

$$\sum_{\mathcal{C}} (u_i + r_i) < \sum_{\mathcal{C}} s_i \tag{30}$$

Define $\tilde{u}_i := u_i + r_i - s_i$, and consider the partition of $\mathcal{C} = \underline{\mathcal{C}} \cup \overline{\mathcal{C}}$ where $\overline{\mathcal{C}} := \{i \in \mathcal{C} : \tilde{u}_i \geq 0\}$ and, $\underline{\mathcal{C}} := \{i \in \mathcal{C} : \tilde{u}_i < 0\}$. Inequality (30) implies $\underline{\mathcal{C}} \neq \emptyset$. Moreover, if $\overline{\mathcal{C}} = \emptyset$ then $\underline{\mathcal{C}} = \mathcal{C} \in \mathcal{D}^S$, and therefore $D(\boldsymbol{r},\boldsymbol{s}) = S$ since all members of the decisive coalition $\underline{\mathcal{C}}$ vote for the status quo. This is a contradiction since we know from the first step of this proof that $D(\boldsymbol{r}, \boldsymbol{s}) = R$ and therefore, $\overline{\mathcal{C}} \neq \emptyset$.

Denote, for any $\varepsilon \geq 0$,

$$\alpha_\varepsilon := \frac{\sum_{i\in\overline{\mathcal{C}}}(\tilde{u}_i + \varepsilon)}{\sum_{i\in\underline{\mathcal{C}}} |\tilde{u}_i|} > 0.$$

Note that (30) implies $\sum_{i\in\overline{\mathcal{C}}} \tilde{u}_i < \sum_{i\in\underline{\mathcal{C}}} |\tilde{u}_i|$, so that $\alpha_0 < 1$, and thus we may choose $\varepsilon > 0$ small enough such that $0 < \alpha_\varepsilon < 1$.

If the promises profile $(\boldsymbol{r}, \boldsymbol{s})$ is in place, members of the coalition $\mathcal{C}$ can propose additional internal promises $\tilde{\boldsymbol{s}}$ contingent on the status quo and defined by

$$\tilde{s}_i := \begin{cases} u_i + r_i + \varepsilon - s_i, & \text{if} \quad i \in \overline{\mathcal{C}}; \\ -\alpha_\varepsilon |\tilde{u}_i|, & \text{if} \quad i \in \underline{\mathcal{C}}; \\ 0, & \text{if} \quad i \notin \mathcal{C}; \end{cases}$$

which in turn induces the cumulative promises profile $\boldsymbol{s'}$ contingent on the status quo:

$$s'_i := \begin{cases} u_i + r_i + \varepsilon, & \text{if} \quad i \in \overline{\mathcal{C}}; \\ s_i - \alpha_\varepsilon |\tilde{u}_i|, & \text{if} \quad i \in \underline{\mathcal{C}}; \\ s_i, & \text{if} \quad i \notin \mathcal{C}. \end{cases} \tag{31}$$

Notice that $\boldsymbol{s'} \in \mathcal{P}$ because,

$$\begin{aligned} \sum_{i\in\mathbb{I}} s'_i &= \sum_{i\in\overline{\mathcal{C}}}[u_i + r_i + \varepsilon] + \sum_{i\in\underline{\mathcal{C}}} s_i - \alpha_\varepsilon \sum_{i\in\underline{\mathcal{C}}} |\tilde{u}_i| + \sum_{i\notin\mathcal{C}} s_i \\ &= \sum_{i\in\overline{\mathcal{C}}}[s_i + \tilde{u}_i + \varepsilon] + \sum_{i\in\underline{\mathcal{C}}} s_i - \sum_{i\in\overline{\mathcal{C}}}(\tilde{u}_i + \varepsilon) + \sum_{i\notin\mathcal{C}} s_i = \sum_{i\in\mathbb{I}} s_i = 0. \end{aligned}$$

When facing the promises profile $(\boldsymbol{r}, \boldsymbol{s'})$, the voting decision of the members of the coalition $\mathcal{C}$ is determined by the inequalities

$$\begin{aligned} &\text{for } i \in \overline{\mathcal{C}}: \quad s'_i = u_i + r_i + \varepsilon > u_i + r_i; \\ &\text{for } i \in \underline{\mathcal{C}}: \quad s'_i = s_i - \alpha_\varepsilon |\tilde{u}_i| > s_i - |\tilde{u}_i| = s_i + \tilde{u}_i = u_i + r_i; \end{aligned}$$

where we use $\tilde{u}_i < 0$ for all $i \in \underline{\mathcal{C}}$ and $\alpha_\varepsilon < 1$ in the second inequality. Hence, all members of $\mathcal{C}$ vote for the status quo. Because the coalition $\mathcal{C}$ is decisive for the status quo alternative, the collected support for the alternative S from all members of $\mathcal{C}$ sways the committee decision in favor of the status quo, $D(\boldsymbol{r}, \boldsymbol{s'}) = S$. Moreover, all members of the coalition $\mathcal{C}$ strictly improve their utilities with the new promises profile $(\boldsymbol{r}, \boldsymbol{s'})$ relative to the promises profile $(\boldsymbol{r}, \boldsymbol{s})$: Any member i of the coalition $\mathcal{C}$ derives the utility $u_i + r_i$ when facing the promises profile $(\boldsymbol{r}, \boldsymbol{s})$, and the utility $s'_i > u_i + r_i$ when they face the promises profile $(\boldsymbol{r}, \boldsymbol{s'})$. To sum up, members of the coalition $\mathcal{C}$ propose the alternative promises profile $(\boldsymbol{r}, \boldsymbol{s'})$ and each one of them achieves a strictly higher utility with the new promises profile $(\boldsymbol{r}, \boldsymbol{s'})$ relative to $(\boldsymbol{r}, \boldsymbol{s})$. From Definition 2, we deduce that the promises profile $(\boldsymbol{r}, \boldsymbol{s})$ is not stable. We conclude from the contradiction that inequalities (10) hold.

Step 2. Now we proceed in the "if' direction ($\Leftarrow$). Given a promises profile $(\boldsymbol{r}, \boldsymbol{s}) \in \mathcal{P}^2$ satisfying inequality (10) for any coalition $\mathcal{C} \in \mathcal{D}^S$, we want to show that it is stable.

First, observe that if inequalities (10) hold, then $D(\boldsymbol{r}, \boldsymbol{s}) = R$. This is because if $D(\boldsymbol{r}, \boldsymbol{s}) = S$, then there must exist a coalition $\mathcal{C} \in \mathcal{D}^S$ whose members vote unanimously for the status quo. This cannot be true because when $\sum_{\mathcal{C}}(u_i + r_i) \geq \sum_{\mathcal{C}} s_i$, we cannot have $u_i + r_i < s_i$ for all $i \in \mathcal{C}$.

To prove that $(\boldsymbol{r}, \boldsymbol{s})$ is stable, proceed again by contradiction and assume that $(\boldsymbol{r}, \boldsymbol{s})$ is not stable. By Definition 2 and the fact that $D(\boldsymbol{r}, \boldsymbol{s}) = R$, there must exist a coalition $\mathcal{C}$ and an alternative promises profile $\boldsymbol{s'} \in \mathcal{P}$ such that $\mathcal{C} = \{i : s'_i \neq s_i\}$, $D(\boldsymbol{r}, \boldsymbol{s'}) = S$ and, $v_i^{\boldsymbol{r},\boldsymbol{s'}} = s'_i > v_i^{\boldsymbol{r},\boldsymbol{s}} = u_i + r_i$ for all $i \in \mathcal{C}$. Since $\boldsymbol{s}, \boldsymbol{s'} \in \mathcal{P}$, by the zero sum property we have

$$\sum_{i \in \mathcal{C}} s_i = -\sum_{i \notin \mathcal{C}} s_i = -\sum_{i \notin \mathcal{C}} s'_i = \sum_{i \in \mathcal{C}} s'_i. \tag{32}$$

Consider now the coalition $\mathcal{B} := \{i : s'_i > u_i + r_i\} \supset \mathcal{C}$ and observe that it is a decisive coalition for the status quo alternative, that is, $\mathcal{B} \in \mathcal{D}^S$. This is because only the members of $\mathcal{B}$ vote for the status quo and the status quo is adopted by the committee, $D(\boldsymbol{r}, \boldsymbol{s'}) = S$. Thus, it must be that the coalition $\mathcal{B}$ has enough voting power to defeat the reform: $|\mathcal{B}| \geq \hat{\kappa}$. We now establish that the coalition $\mathcal{B}$ violates the inequality (10). Using equality (32) and $\mathcal{C} = \{i : s_i' \neq s_i\}$ we observe that

$$\sum_{i\in\mathcal{B}} s_i = \sum_{i\in\mathcal{C}} s_i + \sum_{i\in\mathcal{B}\setminus\mathcal{C}} s_i = \sum_{i\in\mathcal{C}} s_i' + \sum_{i\in\mathcal{B}\setminus\mathcal{C}} s_i' = \sum_{i\in\mathcal{B}} s_i' > \sum_{i\in\mathcal{B}} (u_i + r_i).$$

To sum up we have $\mathcal{B} \in \mathcal{D}^S$ and $\sum_{i\in\mathcal{B}} s_i > \sum_{i\in\mathcal{B}}(u_i + r_i)$, and therefore inequality (10) is contradicted. Consequently, the promises profile $(\boldsymbol{r}, \boldsymbol{s})$ is stable. □

Proof of Proposition 2:

(i) Since any promises profile $\boldsymbol{r} \in \mathcal{S}_0$ satisfies $D(\boldsymbol{r}, \boldsymbol{0}) = R$, the set of utility outcomes associated with stable promises from the set $\mathcal{S}_0$ is given by

$$\mathcal{Z} := \{\boldsymbol{v}^{r,\boldsymbol{0}} | r \in \mathcal{S}_0\} = \mathcal{S}_0 + \boldsymbol{u} \equiv \{\boldsymbol{z} = \boldsymbol{u} + \boldsymbol{r} | \boldsymbol{r} \in \mathcal{S}_0\}.$$

Since $\boldsymbol{r} \in \mathcal{P}$, by Proposition 1 we see that

$$\mathcal{Z} = \left\{\boldsymbol{z} \in \mathbb{R}^I : \sum_{i\in\mathbb{I}} z_i = \sum_{i\in\mathbb{I}} u_i \quad \text{and} \quad \sum_{i\in\mathcal{C}} z_i \geq 0 \text{ for all coalitions } \mathcal{C} \in \mathcal{D}^S\right\}.$$

The above set $\mathcal{Z}$ is non empty; it includes, for example, the vector $(\alpha_1 \sum_{\mathbb{I}} u_i, .., \alpha_I \sum_{\mathbb{I}} u_i)$, where $\alpha_i \geq 0$ with $\sum_{i\in\mathbb{I}} \alpha_i = 1$. We note that the set $\mathcal{Z}$ depends only on the aggregate intensity $\sum_{\mathbb{I}} u_i$ and not on the distribution of intensities. The set of stable promises $\mathcal{S}_0$ can be defined as a translation of the set $\mathcal{Z}$

$$\mathcal{S}_0 = \mathcal{Z} - \boldsymbol{u} = \{\boldsymbol{r} \in \mathcal{P} \quad | \quad \exists \boldsymbol{z} \in \mathcal{Z} \text{ such that } \boldsymbol{r} = \boldsymbol{z} - \boldsymbol{u}\}.$$

(ii) If the inequalities (10) hold for the promises profile $(\boldsymbol{r}, \boldsymbol{s})$, then they also hold for the payment promises profile $(\boldsymbol{r} - \boldsymbol{s}, \boldsymbol{0})$. Therefore, all equilibria can be categorized by equivalence classes where each element of an equivalence class is congruent to the same element of the set of equilibria with transfers that are only promised conditional on the reform being adopted. This is formally described in equation (11). □

Proof of Proposition 3:

Note that $\mathcal{S}_0$ is nonempty. Let $\boldsymbol{r}_n \in \mathcal{S}_0$ be a minimizing sequence: $\lim_{n\to\infty} \mathcal{T}_{\boldsymbol{r}_n} = \inf_{\boldsymbol{r}\in\mathcal{S}_0} \mathcal{T}_{\boldsymbol{r}} \geq 0$. Since $\mathcal{T}_{\boldsymbol{r}_n} \leq \mathcal{T}_{\boldsymbol{r}_1} < \infty$ for all $n \geq 1$, then $\{\boldsymbol{r}_n\}_{n\geq 1} \subset \mathbb{R}^I$ is bounded. By the local compactness of $\mathbb{R}^I$, there exists a subsequence $\{\boldsymbol{r}_{n_k}\}_{k\geq 1}$ such that $\lim_{k\to\infty} \boldsymbol{r}_{n_k} = \boldsymbol{r}^*$ for some $\boldsymbol{r}^* \in \mathbb{R}^I$. Since $\sum_{i\in\mathbb{I}} r_{n_k,i} = 0$ for all k, then clearly $\sum_{i\in\mathbb{I}} r_i^* = 0$, that is, $\boldsymbol{r}^* \in \mathcal{P}$. Moreover, by Proposition 1 (with $\boldsymbol{s} = 0$) we see that $\boldsymbol{r}_{n_k}$ satisfies (10) for each k. Then by sending $k \to \infty$ we see that $\boldsymbol{r}^*$ also satisfies (10), and thus $\boldsymbol{r}^* \in \mathcal{S}_0$. Finally, observe that $\mathcal{T} : \mathcal{S}_0 \to [0, \infty)$ is continuous and hence $\mathcal{T}_{\boldsymbol{r}^*} = \lim_{k\to\infty} \mathcal{T}_{\boldsymbol{r}_{n_k}} = \inf_{\boldsymbol{r}\in\mathcal{S}_0} \mathcal{T}_{\boldsymbol{r}}$. This implies that $\boldsymbol{r}^* \in \mathcal{E}$. $\square$

Proof of Proposition 4:

The proof relies on the following three lemmas.

Lemma 1. *For any equilibrium promises profile $\boldsymbol{r} \in \mathcal{E}\backslash\{\boldsymbol{0}\}$, there exists no pair of members (i, j) such that*

$$r_i < 0 < r_j \quad \textit{and} \quad v_i^{\boldsymbol{r}} < v_j^{\boldsymbol{r}}. \tag{33}$$

Proof. Assume by contradiction that (33) holds true. For some $\varepsilon > 0$ small, set

$$\tilde{r}_i = r_i + \varepsilon \leq 0, \quad \tilde{r}_j = r_j - \varepsilon \geq 0, \quad \text{and} \quad \tilde{r}_k = r_k \text{ for all } k \neq i, j. \tag{34}$$

Notice that $\tilde{\boldsymbol{r}} \in \mathcal{P}$ because the promises of member i are reduced by ε and the promises recipient j also sees a reduction in his transfer by the same amount ε. Next, for any $\mathcal{C} \in \mathcal{D}^S$, since $\boldsymbol{r} \in \mathcal{E} \subset \mathcal{S}_0$, using inequality (10) with $\boldsymbol{s} = 0$ and inequalities (33) give

$$\sum_{k\in\mathcal{C}} (u_k + \tilde{r}_k) = \begin{cases} \displaystyle\sum_{k\in\mathcal{C}} v_k^{\boldsymbol{r}} \geq 0, & \text{if } i, j \in \mathcal{C} \text{ or } i, j \notin \mathcal{C}; \\ \displaystyle\sum_{k\in\mathcal{C}} v_k^{\boldsymbol{r}} + \varepsilon \geq \sum_{k\in\mathcal{C}} v_k^{\boldsymbol{r}} \geq 0, & \text{if } i \in \mathcal{C}, j \notin \mathcal{C}. \end{cases} \tag{35}$$

For the last case that $j \in \mathcal{C}, i \notin \mathcal{C}$, by setting $\varepsilon < v_j^{\boldsymbol{r}} - v_i^{\boldsymbol{r}}$, we have $v_i^{\boldsymbol{r}} < v_j^{\boldsymbol{r}} - \varepsilon = u_j + \tilde{r}_j$. Then, since $(\mathcal{C}\backslash\{j\}) \cup \{i\}$ is also in $\mathcal{D}^S$, we have

$$\sum_{k\in\mathcal{C}}(u_k + \tilde{r}_k) \geq \sum_{k\in(\mathcal{C}\backslash\{j\})\cup\{i\}} v_k^{\boldsymbol{r}} \geq 0, \quad \text{if } j \in \mathcal{C}, i \notin \mathcal{C}. \tag{36}$$

Combining (35) and (36) and recalling that $\mathcal{C}$ is an arbitrary coalition from the set $\mathcal{D}^S$, by Proposition 1 again (with $\boldsymbol{s} = 0$) we see that $\tilde{\boldsymbol{r}} \in \mathcal{S}_0$.

Note further that

$$|\tilde{r}_i| = -\tilde{r}_i = -r_i - \varepsilon = |r_i| - \varepsilon, \quad |\tilde{r}_j| = \tilde{r}_j = r_j - \varepsilon = |r_j| - \varepsilon, \quad |\tilde{r}_k| = |r_k|,\ k \neq i, j.$$

Then

$$\mathcal{T}_{\tilde{\boldsymbol{r}}} = \frac{1}{2}\sum_{k=1}^{I}|\tilde{r}_k| = \frac{1}{2}\Big[\sum_{k\neq i,j}|r_k| + |r_i| - \varepsilon + |r_j| - \varepsilon\Big] = \mathcal{T}_{\boldsymbol{r}} - \varepsilon < \mathcal{T}_{\boldsymbol{r}}.$$

This contradicts the minimum total promises transfer property of $\boldsymbol{r} \in \mathcal{E}$. □

Lemma 2. *For any equilibrium promises profile $\boldsymbol{r} \in \mathcal{E}\backslash\{\mathbf{0}\}$, there exists no committee member i such that*

$$r_i < 0 \quad \textit{and} \quad v_i^{\boldsymbol{r}} < 0. \tag{37}$$

Proof. Assume by contradiction that (37) holds true. Since $\boldsymbol{r} \in \mathcal{P}$, there exists $j \neq i$ such that $r_j > 0$. Then by Lemma 1 we have $v_j^{\boldsymbol{r}} \leq v_i^{\boldsymbol{r}} < 0$. Define the promises profile $\tilde{\boldsymbol{r}}$ by (34) again. Note that $u_i + \tilde{r}_i = v_i^{\boldsymbol{r}} + \varepsilon$, $u_j + \tilde{r}_j = v_j^{\boldsymbol{r}} - \varepsilon$. Then for $\varepsilon > 0$ small enough, we have $u_j + \tilde{r}_j < u_i + \tilde{r}_i \leq 0$.

Following the same reasoning as in Lemma 1, we prove now that the inequality $\sum_{k\in\mathcal{C}}(u_k + \tilde{r}_k) \geq 0$ must hold for every coalition $\mathcal{C} \in \mathcal{D}^S$. First, observe that by Proposition 1 we have $\sum_{k\in\mathcal{C}} v_k^{\boldsymbol{r}} \geq 0$ and hence, there exists $m \in \mathcal{C}$ such that $v_m^{\boldsymbol{r}} \geq 0$. Notice that, since $v_j^{\boldsymbol{r}} \leq v_i^{\boldsymbol{r}} < 0$, we have $m \neq i, j$.

When $i, j \in \mathcal{C}$, or $i, j \notin \mathcal{C}$, or $i \in \mathcal{C}, j \notin \mathcal{C}$, (35) remains true and hence $\sum_{k\in\mathcal{C}}(u_k + \tilde{r}_k) \geq 0$. In the last case where $j \in \mathcal{C}, i \notin \mathcal{C}$, since $u_m + \tilde{r}_m = u_m + r_m \geq 0 \geq u_i + \tilde{r}_i$

and $(\mathcal{C}\backslash\{m\})\cup\{i\}\in\mathcal{D}^S$, we have

$$\sum_{k\in\mathcal{C}}(u_k+\tilde{r}_k)\geq\sum_{k\in(\mathcal{C}\backslash\{m\})\cup\{i\}}(u_k+\tilde{r}_k)=\sum_{k\in(\mathcal{C}\backslash\{m\})\cup\{i\}}(u_k+r_k)\geq 0.$$

This, together with (35) and Proposition 1, implies $\tilde{\boldsymbol{r}}\in\mathcal{S}_0$. Then, following the same argument on the total transfer used in the last step of the proof of Lemma 1, we derive the desired contradiction. □

Lemma 3. *For any equilibrium promises profile* $\boldsymbol{r}\in\mathcal{E}\backslash\{\mathbf{0}\}$, *introduce*

$$\underline{k}_*:=\max\{i|r_i>0\},\qquad \overline{k}_*:=\min\{i|r_i<0\}. \tag{38}$$

Then

$$\underline{k}_*<\overline{k}_*,\quad \textit{and}\quad r_k=0\quad \textit{for all } \underline{k}^*<k<\overline{k}_*. \tag{39}$$

Committee member $\overline{k}_*$ *is a reform supporter—*$\overline{k}_*\in\mathcal{C}^R$, *all committee members* $j\geq\overline{k}_*$ *are promisors subject to an individual rationality constraint, and all committee members* $i\leq\underline{k}_*$ *are promisees*

$$r_i\geq 0,\ \textit{for all } i\leq\underline{k}_*;\quad \textit{and}\quad -u_j\leq r_j\leq 0,\ \textit{for all } j\geq\overline{k}_*. \tag{40}$$

The ex post intensities $\boldsymbol{v}^{\boldsymbol{r}}\equiv\boldsymbol{u}+\boldsymbol{r}$ *are ranked across the coalition of promisers and the coalition of promisees, that is,*

$$v_i^{\boldsymbol{r}}\leq v_j^{\boldsymbol{r}}\quad \textit{for all}\quad i\leq\underline{k}_*<\overline{k}_*\leq j. \tag{41}$$

Proof. First, since $r_{\overline{k}_*}<0$, by Lemma 2 we have $u_{\overline{k}_*}+r_{\overline{k}_*}\geq 0$, which implies $u_{\overline{k}_*}>0$. That is, $\overline{k}_*\in\mathcal{C}^R$.

Next, since $r_{\overline{k}_*}<0<r_{\underline{k}_*}$, by Lemma 1 we have $u_{\underline{k}_*}+r_{\underline{k}_*}\leq u_{\overline{k}_*}+r_{\overline{k}_*}$, and thus $u_{\underline{k}_*}<u_{\overline{k}_*}$. Then by (1) we obtain $\underline{k}_*<\overline{k}_*$. The inequality $\underline{k}_*<\overline{k}_*$ and the definitions (38) imply that $r_k=0$ for all $\underline{k}^*<k<\overline{k}_*$, and this proves the statement in (39).

We now prove the statements in (40). First, to prove the left inequality (40), assume by contradiction that $r_i<0$ for some $i\leq\underline{k}_*$. Since $r_{\underline{k}_*}>0$, we must have

$i < \underline{k}_*$. By (1), we have $v_i^{\boldsymbol{r}} < u_i \leq u_{\underline{k}_*} < v_{\underline{k}_*}^{\boldsymbol{r}}$. This contradicts Lemma 1, and thus $r_i \geq 0$ for all $i \leq \underline{k}_*$. Similarly, to prove the right inequality in (40), assume there exists $j \geq \overline{k}_*$ such that $r_j > 0$. Then we would have $r_{\overline{k}_*} < 0 < r_j$ and $v_{\overline{k}_*}^{\boldsymbol{r}} < u_{\overline{k}_*} \leq u_j < v_j^{\boldsymbol{r}}$. This contradicts Lemma 1, and thus $r_j \leq 0$ for all $j \geq \overline{k}_*$. Moreover, if $r_j < -u_j$ for some $j \geq \overline{k}_*$, then $v_j^{\boldsymbol{r}} = u_j + r_j < 0$. Since $\overline{k}_* \in \mathcal{C}^R$, then $j \in \mathcal{C}^R$, and thus $r_j < -u_j \leq 0$. To sum up, $r_j < 0$ and $v_j^{\boldsymbol{r}} < 0$. This contradicts Lemma 2, so $r_j \geq -u_j$ for all $j \geq \overline{k}_*$, and thus (40) holds true.

We finally prove inequalities (41). Assume by contradiction that $v_i^{\boldsymbol{r}} > v_j^{\boldsymbol{r}}$ for some $i \leq \underline{k}_* < \overline{k}_* \leq j$. By (40) we have $r_j \leq 0 \leq r_i$. If $r_j < 0 < r_i$, we obtain the contradiction with Lemma 1. If $r_j = 0 < r_i$, we have $r_{\overline{k}_*} < 0 < r_i$, and by (1) we have $v_{\overline{k}_*}^{\boldsymbol{r}} < u_{\overline{k}_*} \leq u_j = v_j^{\boldsymbol{r}} < v_i^{\boldsymbol{r}}$, contradicting Lemma 1. Similarly, if $r_j < 0 = r_i$, we have $r_j < 0 < r_{\underline{k}_*}$, and by (1) we have $v_{\underline{k}_*}^{\boldsymbol{r}} > u_{\underline{k}_*} \geq u_i = v_i^{\boldsymbol{r}} > v_j^{\boldsymbol{r}}$, also contradicting Lemma 1. In the last case that $r_i = 0 = r_j$, we have $r_{\overline{k}_*} < 0 < r_{\underline{k}_*}$, and by (1), $v_{\overline{k}_*}^{\boldsymbol{r}} < u_{\overline{k}_*} \leq u_j = v_j^{\boldsymbol{r}} < v_i^{\boldsymbol{r}} = u_i \leq u_{\underline{k}_*} < v_{\underline{k}_*}^{\boldsymbol{r}}$. This again contradicts Lemma 1. In summary, we obtain a contradiction with Lemma 1 in all the sub-cases, and thus (41) holds true. □

We now prove Proposition 4. We note that, from the proof we see that in all the cases $\underline{k}_* < k_* \leq \overline{k}_*$.

Proof of Proposition 4: We first prove the statements in (14). Define $a := \min_{\overline{k}_* \leq j \leq I} v_j^{\boldsymbol{r}}$. By (39) we have $v_k^{\boldsymbol{r}} = u_k$ for $\underline{k}_* < k < \overline{k}_*$. If $a \geq v_{\overline{k}_*-1}^{\boldsymbol{r}}$, then set $k_* = \overline{k}_*$. By (1) we have $a \geq v_{\overline{k}_*-1}^{\boldsymbol{r}} \geq v_k^{\boldsymbol{r}}$ for all $\underline{k}_* < k < \overline{k}_*$, and by (41) we have $a \geq v_i^{\boldsymbol{r}}$ for all $i \leq \underline{k}_*$. Moreover, by (39) and (41) we see that $r_i \geq 0 \geq r_j$ for all $i < k_* \leq j$. So $k_* = \overline{k}_*$ satisfies all the requirements in inequalities (14). We next assume $a < v_{\overline{k}_*-1}^{\boldsymbol{r}}$. Since $a \geq v_{\underline{k}_*}^{\boldsymbol{r}}$ by (41), we may set $k_* = \inf\{k > \underline{k}_* : v_k^{\boldsymbol{r}} > a\}$. Then $\underline{k}_* < k_* \leq \overline{k}_*$, and by (1) we see that $v_k^{\boldsymbol{r}} \leq a$ for all $\underline{k}_* < k < k_*$, and $v_k^{\boldsymbol{r}} \geq v_{k_*}^{\boldsymbol{r}} > a$ for all $k_* \leq k < \overline{k}_*$. Recall again (41), then $v_i^{\boldsymbol{r}} \leq a \leq v_j^{\boldsymbol{r}}$ for all $i < k_* \leq j$, and by (39) and (40), we have $r_i \geq 0 \geq r_j$ for all $i < k_* \leq j$. This completes the proof of (14). Moreover, since $\boldsymbol{r} \in \mathcal{P}$, then $\sum_{i<k_*} r_i = \sum_{j \geq k_*}(-r_j)$, and thus obtains (15). □

Proof of Proposition 5:

We start by giving a lemma that will be useful in subsequent proofs.

Lemma 4. *For any promises profile* $\boldsymbol{r} \in \mathcal{P}$*, the following holds:*

(i) For any coalition $\mathcal{C} \subset \mathbb{I}$*, we have* $\mathcal{T}_{\boldsymbol{r}} \geq \big| \sum_{i \in \mathcal{C}} r_i \big|$.

(ii) Consider a coalition $\mathcal{C} \subset \mathbb{I}$ *such that* $|\mathcal{C}| = \widehat{\kappa}$*,* $u_i + r_i \leq u_j + r_j$ *for all* $i \in \mathcal{C}$ *and* $j \notin \mathcal{C}$*. Then* $\boldsymbol{r} \in \mathcal{S}_0$ *if and only if* $\sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0$.

Proof. (i) Since $\boldsymbol{r} \in \mathcal{P}$, we have

$$\begin{aligned} \mathcal{T}_{\boldsymbol{r}} &= \frac{1}{2}\Big[\sum_{i \in \mathcal{C}} |r_i| + \sum_{i \notin \mathcal{C}} |r_i|\Big] \geq \frac{1}{2}\Big[\big|\sum_{i \in \mathcal{C}} r_i\big| + \big|\sum_{i \notin \mathcal{C}} r_i\big|\Big] \\ &= \frac{1}{2}\Big[\big|\sum_{i \in \mathcal{C}} r_i\big| + \big| -\sum_{i \in \mathcal{C}} r_i\big|\Big] = \big|\sum_{i \in \mathcal{C}} r_i\big|. \end{aligned}$$

(ii) Note that $\mathcal{C} \in \mathcal{D}^S$ because the reform is defeated if all members of $\mathcal{C}$ vote against it. If $\boldsymbol{r} \in \mathcal{S}_0$, by Proposition 1 (and recalling $\boldsymbol{s} = \mathbf{0}$), since $\mathcal{C} \in \mathcal{D}^S$, we have $\sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0$.

We now assume $\sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0$ for $\mathcal{C}$ satisfying the conditions in part (ii) of Lemma 4 and prove that $\boldsymbol{r} \in \mathcal{S}_0$. This in particular implies that

$$\min_{j \notin \mathcal{C}} (u_j + r_j) \geq \max_{i \in \mathcal{C}} (u_i + r_i) \geq 0. \tag{42}$$

For any decisive coalition $\tilde{\mathcal{C}} \in \mathcal{D}^S$, consider the partition of $\tilde{\mathcal{C}}$ defined by $\tilde{\mathcal{C}} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3$. Members of the coalition $\mathcal{C}_1$ belong both to $\mathcal{C}$ and $\tilde{\mathcal{C}}$, that is, $\mathcal{C}_1 := \mathcal{C} \cap \tilde{\mathcal{C}}$. The coalition $\mathcal{C}_2$ is a subset of $\tilde{\mathcal{C}} \backslash \mathcal{C}$ such that when merged with the coalition $\mathcal{C}_1$, it forms a coalition with cardinality $\widehat{\kappa}$, that is, $|\mathcal{C}_1| + |\mathcal{C}_2| = \widehat{\kappa}$. Finally, the coalition $\mathcal{C}_3$ formed by the residual members of $\tilde{\mathcal{C}}$ who do not belong to $\mathcal{C}_1$ or $\mathcal{C}_2$, that is, $\mathcal{C}_3 := \tilde{\mathcal{C}} \backslash (\mathcal{C}_1 \cup \mathcal{C}_2)$. Note that $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C} \backslash \mathcal{C}_1$ and $j \in \mathcal{C}_2$ and that, $|\mathcal{C} \backslash \mathcal{C}_1| = |\mathcal{C}_2|$. Thus, the inequality (42) implies

$$\sum_{j \in \mathcal{C}_2} (u_j + r_j) \geq \sum_{i \in \mathcal{C} \backslash \mathcal{C}_1} (u_i + r_i). \tag{43}$$

Since $\mathcal{C}_3 \cap \mathcal{C} = \emptyset$, we have $u_k + r_k \geq \max_{i \in \mathcal{C}}(u_i + r_i) \geq 0$ for all $k \in \mathcal{C}_3$, and hence $\sum_{k \in \mathcal{C}_3}(u_k + r_k) \geq 0$. Using this last inequality and (43) yields

$$\begin{aligned} \sum_{i \in \tilde{\mathcal{C}}}(u_i + r_i) &= \sum_{i \in \mathcal{C}_1}(u_i + r_i) + \sum_{j \in \mathcal{C}_2}(u_j + r_j) + \sum_{k \in \mathcal{C}_3}(u_k + r_k) \\ &\geq \sum_{i \in \mathcal{C}_1}(u_i + r_i) + \sum_{i \in \mathcal{C} \setminus \mathcal{C}_1}(u_i + r_i) + 0 = \sum_{i \in \mathcal{C}}(u_i + r_i) \geq 0. \end{aligned}$$

Now it follows from Proposition 1 again that $\boldsymbol{r} \in \mathcal{S}_0$. □

We now prove Proposition 5. We first show that $\mathcal{T}_{\boldsymbol{r}} \geq U^S$ for any stable promises profile $\boldsymbol{r} \in \mathcal{S}_0$. Indeed, since $|\mathcal{C}^R| < \kappa$, we have $\mathcal{C}^S \in \mathcal{D}^S$ so Proposition 1 implies then that $\sum_{\mathcal{C}^S}(u_i + r_i) \geq 0$. Therefore,

$$\sum_{i \in \mathcal{C}^S} |r_i| \geq \sum_{i \in \mathcal{C}^S} r_i \geq \sum_{i \in \mathcal{C}^S}(-u_i) = U^S. \tag{44}$$

Then it follows from Lemma 4 (i) that

$$\mathcal{T}_{\boldsymbol{r}} \geq |\sum_{i \in \mathcal{C}^S} r_i| \geq U^S, \text{ for all } \boldsymbol{r} \in \mathcal{S}_0. \tag{45}$$

Step 1: We start by considering the case where $|\mathcal{C}^R| < \kappa - 1$, and cover first the "if" direction ($\Leftarrow$), then the "only if" direction ($\Rightarrow$).

If a promises profile $\boldsymbol{r}$ satisfies conditions 1 and 2.a from Proposition 5, then it can be directly checked that $\boldsymbol{r} \in \mathcal{P}$ and $\mathcal{T}_{\boldsymbol{r}} = U^S$. Moreover, by construction we see that $u_i + r_i = 0$ for $i \in \mathcal{C}^S$ and $u_j + r_j \geq 0$ for $j \in \mathcal{C}^R$. Therefore, $\sum_{i \in \mathcal{C}}(u_i + r_i) \geq 0$ for all coalitions $\mathcal{C} \in \mathcal{D}^S$ and hence, by Proposition 1 the promises profile $\boldsymbol{r}$ is stable, that is $\boldsymbol{r} \in \mathcal{S}_0$. Since $\mathcal{T}_{\boldsymbol{r}} = U^S$, inequality (45) shows that the promises profile $\boldsymbol{r}$ achieves the minimum total promises transfer, and thus $\boldsymbol{r} \in \mathcal{E}$.[18]

[18] In particular, we note that the following promises profile $\boldsymbol{r}$ satisfies 1 and 2.a from Proposition 5, and hence is an equilibrium with minimum total promises transfer:

$$r_i := -u_i, \ i \in \mathcal{C}^S; \qquad r_j := -\frac{U^S}{U^R} u_j, \ j \in \mathcal{C}^R. \tag{46}$$

We now prove the only if part. That is, we assume $\boldsymbol{r} \in \mathcal{E}$ and prove that conditions 1 and 2.a from Proposition 5 hold. Note that

$$\sum_{i\in\mathcal{C}^R} |r_i| \geq \sum_{i\in\mathcal{C}^R} (-r_i) = \sum_{i\in\mathcal{C}^S} r_i \geq U^S, \tag{47}$$

and combining (44) with (47) gives

$$\mathcal{T}_{\boldsymbol{r}} = \frac{1}{2}\sum_{i\in\mathcal{C}^R} |r_i| + \frac{1}{2}\sum_{i\in\mathcal{C}^S} |r_i| \geq \frac{1}{2}\sum_{i\in\mathcal{C}^R}(-r_i) + \frac{1}{2}\sum_{i\in\mathcal{C}^S}(r_i) \geq U^S. \tag{48}$$

Since $\boldsymbol{r} \in \mathcal{E}$ has minimum total promises transfer, we must have $\mathcal{T}_{\boldsymbol{r}} = U^S$. Therefore, inequalities (48) become equalities, that is,

$$U^S = \mathcal{T}_{\boldsymbol{r}} = \frac{1}{2}\sum_{i\in\mathcal{C}^R} |r_i| + \frac{1}{2}\sum_{i\in\mathcal{C}^S} |r_i| = \frac{1}{2}\sum_{i\in\mathcal{C}^R}(-r_i) + \frac{1}{2}\sum_{i\in\mathcal{C}^S}(r_i) = U^S.$$

This in turn implies that inequalities (44) and (47) are also equalities:

$$\sum_{i\in\mathcal{C}^S} |r_i| = \sum_{i\in\mathcal{C}^S} r_i = U^S, \quad \sum_{i\in\mathcal{C}^R} |r_i| = \sum_{i\in\mathcal{C}^R}(-r_i) = U^S.$$

Then

$$|r_i| = r_i,\ i \in \mathcal{C}^S; \quad |r_i| = -r_i,\ i \in \mathcal{C}^R; \quad \text{and} \quad \sum_{i\in\mathcal{C}^S} r_i = \sum_{i\in\mathcal{C}^R} |r_i| = U^S,$$

and thus

$$r_i \geq 0 \geq r_j, \quad \forall i \in \mathcal{C}^S, j \in \mathcal{C}^R, \quad \text{and} \quad \sum_{i\in\mathcal{C}^S} r_i = U^S = \sum_{j\in\mathcal{C}^R}(-r_j). \tag{49}$$

Moreover, the equality $\sum_{i\in\mathcal{C}^S} r_i = U^S$ implies that $\sum_{i\in\mathcal{C}^S}(u_i + r_i) = 0$.

Since $|\mathcal{C}^S| > \widehat{\kappa}$, then for any $i_0 \in \mathcal{C}^S$, $\mathcal{C}^S \backslash \{i_0\} \in \mathcal{D}^S$, thus by Proposition 1 we have $\sum_{i\in\mathcal{C}^S\backslash\{i_0\}}(u_i + r_i) \geq 0$. This, together with $\sum_{i\in\mathcal{C}^S}(u_i + r_i) = 0$, implies that $u_{i_0} + r_{i_0} \leq 0$, for all $i_0 \in \mathcal{C}^S$. Combining this with the equality $\sum_{i\in\mathcal{C}^S}(u_i + r_i) = 0$, we must have $u_i + r_i = 0$ or $r_i = -u_i$ for all $i \in \mathcal{C}^S$.

Finally, for each $j \in \mathcal{C}^R$, since $\mathcal{C}^S \cup \{j\} \in \mathcal{D}^S$, then by Proposition 1 we have $0 \leq \sum_{i\in\mathcal{C}^S\cup\{j\}}(u_i + r_i) = u_j + r_j$. That is, $r_j \geq -u_j$ for all $j \in \mathcal{C}^R$. To summarize, we have proven that: in addition to the relations (49), $r_i = -u_i$ for all $i \in \mathcal{C}^S$ and $-u_j \leq r_j \leq 0$ for all $j \in \mathcal{C}^R$. Thus, conditions 1 and 2.a from Proposition 5 hold. This concludes the proof for the case $|\mathcal{C}^R| < \kappa - 1$.

Step 2**:** We now consider the case $|\mathcal{C}^R| = \kappa - 1$. Notice that in this case, $|\mathcal{C}^S| = \widehat{\kappa}$. First, if $\boldsymbol{r}$ satisfies conditions 1 and 2.b of Proposition 5, then it can be checked as in Step 1 that $\boldsymbol{r} \in \mathcal{P}$, $\mathcal{T}_{\boldsymbol{r}} = U^S$, $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, and $r_j \geq -u_j$ for all $j \in \mathcal{C}^R$. Moroever, by condition 2.b of Proposition 5 and property (ii) of Lemma 4, we see that $r \in \mathcal{S}_0$. Since $\mathcal{T}_{\boldsymbol{r}} = U^S$, the promises profile $\boldsymbol{r}$ minimizes the total promises transfer and hence $\boldsymbol{r} \in \mathcal{E}$.[19]

We now prove the only if part. We assume $\boldsymbol{r} \in \mathcal{E}$ and show that conditions 1 and 2.b of Proposition 5 hold true. By the same arguments in Step 1 of this proof, we see that (49) still holds true. Moreover, for any $i_0 \in \mathcal{C}^S$ and $j_0 \in \mathcal{C}^R$, note that $(\mathcal{C}^S \backslash \{i_0\}) \cup \{j_0\} \in \mathcal{D}^S$, then by Proposition 1 we have

$$\begin{aligned} 0 \leq \sum_{i \in (\mathcal{C}^S \backslash \{i_0\}) \cup \{j_0\}} (u_i + r_i) &= \sum_{i \in \mathcal{C}^S} (u_i + r_i) - (u_{i_0} + r_{i_0}) + (u_{j_0} + r_{j_0}) \\ &= (u_{j_0} + r_{j_0}) - (u_{i_0} + r_{i_0}). \end{aligned}$$

Thus, $u_{i_0} + r_{i_0} \leq u_{j_0} + r_{j_0}$ for all $i_0 \in \mathcal{C}^S$ and $j_0 \in \mathcal{C}^R$ and hence, condition 2.$(b).ii$ of Proposition 5 is satisfied. The remaining properties of condition 2.b from Proposition 5 are implied by (49). Similarly with the exception of the inequality $-u_j \leq r_j$ for $j \in \mathcal{C}^R$, all other properties in Condition 1 from Proposition 5 are implied by (49). To prove this last property, note that if $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, then there exists $i_0 \in \mathcal{C}^S$ such that $u_{i_0} + r_{i_0} \geq 0$. Condition 2.$(b).ii$ from Proposition 5 implies that $u_j + r_j \geq u_{i_0} + r_{i_0} \geq 0$ for any $j \in \mathcal{C}^R$, and thus $r_j \geq -u_j$, for all $j \in \mathcal{C}^R$. □

[19] In particular, we note that the promises profile $\boldsymbol{r}$ constructed in (46) satisfies conditions 1 and 2.b of of Proposition 5 and hence is in $\mathcal{E}$. In contrast to the case where $|\mathcal{C}^S| < \kappa - 1$ where all members of the coalition $\mathcal{C}^S$ were *ex post* indifferent between R and S after receiving the equilibrium promises, we note that in the case $|\mathcal{C}^R| = \kappa - 1$ it is possible that $u_i + r_i > 0$ for some $i \in \mathcal{C}^S$.

Proof of Proposition 6:

Note that in this case $\sum_{i=1}^{\widehat{\kappa}} u_i = -G^S \geq 0$. It can be verified that the conditions in Lemma 4 (ii) for $\mathcal{C} = \{1, \cdots, \widehat{\kappa}\}$ and $\boldsymbol{r} = \boldsymbol{0}$ hold. Then $\boldsymbol{0} \in \mathcal{S}_0$, and this stable promises profile is both unique and has zero total transfers. Hence $\mathcal{E} = \{\boldsymbol{0}\}$. □

Proof of Proposition 7:

The proof proceeds in four steps. In the first step, we show that $\mathcal{T}_{\boldsymbol{r}} \geq G^S$. In the second step, we prove the "if" part of Proposition 7. In the third step, we give an example of across the aisle equilibrium promise that satisfies $\mathcal{T}_{\boldsymbol{r}} = G^S$. The example is used in the fourth and last step where we prove the "only if" part of Proposition 7.

Step 1. We first show that $\mathcal{T}_{\boldsymbol{r}} \geq G^S$ for any $\boldsymbol{r} \in \mathcal{S}_0$. Since $\mathcal{C}^S \cup \underline{\mathcal{C}}^R \in \mathcal{D}^S$, by Proposition 1 and (16) we have

$$0 \leq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} [u_i + r_i] = -G^S + \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i. \tag{50}$$

Then by Lemma 4 (i) we have $\mathcal{T}_{\boldsymbol{r}} \geq |\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i| \geq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S$.

Step 2. We next prove the if part: assume conditions (18), (19) and (20) hold and prove that $\boldsymbol{r} \in \mathcal{E}$. Let $\boldsymbol{r} \in \mathcal{P}$ satisfy (18), note that $r_k \geq -u_k$ or $u_k + r_k \geq 0$ for all $k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R$, and by the calculation in (50) we see that $\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} v_i^{\boldsymbol{r}} \geq 0$. These observations together with condition (19) show that all conditions required in Lemma 4 (ii) are fulfilled, and therefore we have $\boldsymbol{r} \in \mathcal{S}_0$. Condition (20) and Step 1 of this proof show that in addition to being stable, the promises profile $\boldsymbol{r}$ achieves the minimal total promises transfer $\mathcal{T}_{\boldsymbol{r}} = G^S$, and thus $\boldsymbol{r} \in \mathcal{E}$.

Step 3. In this step we construct an equilibrium $\boldsymbol{r} \in \mathcal{E}$. Consider the following across the aisle promises profile $\boldsymbol{r}$:

$$r_i := -\frac{G^S}{U^S} u_i, \ i \in \mathcal{C}^S; \quad r_j := 0, \ j \in \underline{\mathcal{C}}^R; \quad r_k := -\frac{G^S}{\Delta U_{\widehat{\kappa}}} [u_k - u_{\widehat{\kappa}}], \ k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R. \tag{51}$$

We can show directly that it satisfies conditions (18), (19) and, (20).[20] Then, using the result in Step 2 of this proof shows that the promises profile $\boldsymbol{r}$ is an equilibrium.

Step 4. We now prove the only if part. Let $\boldsymbol{r} \in \mathcal{E}$ and prove that $\boldsymbol{r}$ satisfies conditions (18), (19) and, (20). Applying Step 1 of this proof shows that, since $\boldsymbol{r} \in \mathcal{S}_0$, we have $\mathcal{T}_{\boldsymbol{r}} \geq G^S$. Recall (50) and note that

$$\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| \geq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S; \tag{52}$$

$$\sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} |r_k| \geq \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} (-r_k) = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S. \tag{53}$$

Combining (52) and (53) gives

$$\mathcal{T}_{\boldsymbol{r}} = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| + \frac{1}{2} \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} |r_k| \geq \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i + \frac{1}{2} \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} (-r_k) \geq G^S. \tag{54}$$

Since $\mathcal{T}_{\boldsymbol{r}} \geq G^S$ for any stable promises profile, and since the equilibrium promises profile defined in (51) achieves the total promises transfer G^S, it must be that $\mathcal{T}_{\boldsymbol{r}} = G^S$ for any $\boldsymbol{r} \in \mathcal{E}$. When $\mathcal{T}_{\boldsymbol{r}} = G^S$, inequalities (54) become equalities:

$$G^S = \mathcal{T}_{\boldsymbol{r}} = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| + \frac{1}{2} \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} |r_k| = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i + \frac{1}{2} \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} (-r_k) = G^S.$$

[20] To see this, first observe that $\boldsymbol{r} \in \mathcal{P}$ since, using (4) and (17), it can be checked that $\sum_{\mathbb{I}} r_i = 0$. Noticing that $u_{\widehat{\kappa}} \geq 0$, $G^S \leq U^S$, and $G^S \leq \Delta U_{\widehat{\kappa}}$, we have for any $i \in \mathcal{C}^S$, $j \in \underline{\mathcal{C}}^R$, and $k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R$,

$$r_i \geq 0, \quad r_j = 0, \quad r_k \leq 0; \qquad r_k \geq -\frac{G_s}{U_{\widehat{\kappa}}} u_k \geq -u_k \,,$$

and therefore $\boldsymbol{r}$ satisfies condition (18). Moreover, we have,

$$v_i^{\boldsymbol{r}} = [1 - \tfrac{G^S}{U_s}] u_i < 0 \leq u_{\widehat{\kappa}}, \quad v_j^{\boldsymbol{r}} = u_j \leq u_{\widehat{\kappa}},$$
$$v_k^{\boldsymbol{r}} = u_k - \tfrac{G_s}{\Delta U_{\widehat{\kappa}}} [u_k - u_{\widehat{\kappa}}] \geq u_k - [u_k - u_{\widehat{\kappa}}] = u_{\widehat{\kappa}};$$

and thus condition (19) is satisfied. Finally,

$$\textstyle\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i = \frac{G^S}{U^S} \sum_{i \in \mathcal{C}^S} [-u_i] = G^S,$$
$$\textstyle\sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_k = -\frac{G_s}{\Delta U_{\widehat{\kappa}}} \sum_{k \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [u_k - u_{\widehat{\kappa}}] = -\frac{G_s}{\Delta U_{\widehat{\kappa}}} \Delta U_{\widehat{\kappa}} = -G^S;$$

and hence condition (20) is satisfied. To sum up the promises profile $\boldsymbol{r}$ satisfy conditions (18), (19) and, (20), and therefore it is an equilibrium.

This in turn implies inequalities (52) and (53) are also equalities:

$$\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}|r_i| = \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} r_i = G^S, \qquad \sum_{j\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R}|r_j| = \sum_{j\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R}(-r_j) = G^S.$$

Then similarly to the approach used to prove (49), we deduce that $|r_i| = r_i$ for all $i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ and $|r_j| = -r_j$ for all $j \in \mathcal{C}^R\backslash\underline{\mathcal{C}}^R$. Hence

$$r_j \leq 0 \leq r_i, \quad \forall i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R, j \in \mathcal{C}^R\backslash\underline{\mathcal{C}}^R.$$

Recalling that $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$ and $\mathcal{C}^R\backslash\underline{\mathcal{C}}^R = \{\widehat{\kappa}+1, .., I\}$, we see that the last equation is equivalent to

$$r_j \leq 0 \leq r_i, \quad \forall i \leq \hat{\kappa} < j.$$

Moreover,

$$\mathcal{T}_{\boldsymbol{r}} = \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} r_i \equiv \sum_{i=1}^{\widehat{\kappa}} r_i = \sum_{j\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R}(-r_j) \equiv \sum_{j=\widehat{\kappa}+1}^{I}(-r_j) = G^S.$$

Using (16), observe further that

$$\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i + r_i) = \sum_{i\in\mathcal{C}^S} u_i + \sum_{i\in\underline{\mathcal{C}}^R} u_i + \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} r_i = -U^S + \underline{U}^R + G^S = 0.$$

To verify the individual rationality constraint $-u_j \leq r_j$ for any $j \in \mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, observe that $(\mathcal{C}^S \cup \underline{\mathcal{C}}^R) \cup \{j\} \in \mathcal{D}^S$, then by Proposition 1 we have

$$0 \leq \sum_{i\in(\mathcal{C}^S\cup\underline{\mathcal{C}}^R)\cup\{j\}}(u_i + r_i) = \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i + r_i) + (u_j + r_j) = u_j + r_j,$$

which implies $r_j \geq -u_j$. So far, we have shown that the payment promises profile $\boldsymbol{r}$ satisfies (18) and (20).

All that's left to prove is that (19) also holds. For any $j \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ and $k \in \mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, noting that $\mathcal{C}^S \cup \underline{\mathcal{C}}^R \cup \{k\}\backslash\{j\} \in \mathcal{D}^S$ because $|(\mathcal{C}^S \cup \underline{\mathcal{C}}^R \cup \{k\})\backslash\{j\}| = \widehat{\kappa}$, then by (10) from Proposition (1) we have

$$0 \leq \sum_{i\in(\mathcal{C}^S\cup\underline{\mathcal{C}}^R\cup\{k\})\backslash\{j\}}(u_i + r_i) = \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i + r_i) + (u_k + r_k) - (u_j + r_j) = v_k^{\boldsymbol{r}} - v_j^{\boldsymbol{r}}.$$

Thus, $v_j^{\boldsymbol{r}} \leq v_k^{\boldsymbol{r}}$ for all $j \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ and $k \in \mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, and the promises profile $\boldsymbol{r}$ satisfies (19). □

Proof of Proposition 8:

We start the proof with two preliminary lemmas.

Lemma 5. *Consider a committee operating under the κ-majority rule with $|\kappa| \geq 2$ and such that $G^S > \Delta U_{\widehat{\kappa}}$. The following statements hold:*

(i) Recalling the intensity u_* defined in (24), we have

$$u_* < u_{\widehat{\kappa}}. \tag{55}$$

(ii) There exists a unique group member $k_* \in \underline{\mathcal{C}}^R$, i.e. $n < \kappa_* \leq \widehat{\kappa}$, such that inequalities (25) hold, that is, $u_{k_*-1} \leq u_* < u_{k_*}$.

(iii) The constant $\mathcal{T}_*$ defined in (29) satisfies the inequality

$$\mathcal{T}_* > G^S. \tag{56}$$

Proof. To prove that (i) holds, observe that

$$\begin{aligned} G^S &= U^S - \underline{U}^R = -\sum_{i=1}^{\widehat{\kappa}} u_i = -\sum_{i=1}^{I} u_i + \sum_{i=\widehat{\kappa}+1}^{I} u_i \\ &= -\sum_{i=1}^{I} u_i + \sum_{i=\widehat{\kappa}+1}^{I} [u_i - u_{\widehat{\kappa}}] + (I - \widehat{\kappa})u_{\widehat{\kappa}} \\ &= -\sum_{i=1}^{I} u_i + \Delta U_{\widehat{\kappa}} + (\kappa - 1)u_{\widehat{\kappa}}, \end{aligned}$$

where we have used the definition of $\Delta U_{\widehat{\kappa}}$ in (17) and the relation $\widehat{\kappa} = I - \kappa + 1$. Using the definition of u_* given in (24) yields

$$G^S - \Delta U_{\widehat{\kappa}} = (\kappa - 1)u_{\widehat{\kappa}} - \sum_{i=1}^{I} u_i = (\kappa - 1)[u_{\widehat{\kappa}} - u_*]. \tag{57}$$

Since $G^S > \Delta U_{\hat{\kappa}}$, this establishes inequality (55) in part (i) of Lemma 5.

By the relations (3), (24), and (55), we have $u_n < 0 < u_* < u_{\widehat{\kappa}}$. Then by the ordering condition (1) there exists a unique $k_* \in \underline{\mathcal{C}}^R$, i.e. $n < k_* \leq \widehat{\kappa}$, such that the inequalities (25) hold[21]. This proves part (ii) of Lemma 5.

[21] For example, pick $k_* = \min k$ such that $u_k > u_*$.

To prove part (iii) of Lemma 5, observe that the total promises transfer $\mathcal{T}_*$ defined in (29) satisfies

$$\begin{aligned}\mathcal{T}_* &\equiv \sum_{j=k_*}^{\widehat{\kappa}}[u_j - u_*] + \sum_{j=\widehat{\kappa}+1}^{I}[u_j - u_*] \\ &= \sum_{j=k_*}^{\widehat{\kappa}}[u_j - u_*] + \sum_{j=\widehat{\kappa}+1}^{I}[u_j - u_{\widehat{\kappa}}] + \sum_{j=\widehat{\kappa}+1}^{I}[u_{\widehat{\kappa}} - u_*] \\ &= \sum_{j=k_*}^{\widehat{\kappa}}[u_j - u_*] + \Delta U_{\widehat{\kappa}} + (\kappa - 1)[u_{\widehat{\kappa}} - u_*] \\ &= \sum_{j=k_*}^{\widehat{\kappa}}[u_j - u_*] + G^S > G^S, \end{aligned} \tag{58}$$

where we used equation (57) in the fourth equality of the above sequence of equalities. This establishes inequality (56) in part (iii) of Lemma 5. □

Lemma 6. *Consider the across the aisle promises profile* $\boldsymbol{r}$ *defined by*

$$r_i := -\frac{\mathcal{T}_*}{U^S}u_i,\ \forall i \leq n; \qquad r_k := 0,\ \forall n < k < k_*; \quad r_j := -[u_j - u_*],\ \forall j \geq k_*, \tag{59}$$

where we recall that k_* *is defined in part (ii) of Lemma 5. Then the promises profile* $\boldsymbol{r}$ *is zero sum,* $\boldsymbol{r} \in \mathcal{P}$ *and satisfies the conditions* (26)-(29) *of Proposition 8. Moreover, the promises profile* $\boldsymbol{r}$ *defined in* (59) *is stable.*

Proof. First, $\boldsymbol{r} \in \mathcal{P}$ since

$$\sum_{i \in \mathbb{I}} r_i = \frac{\mathcal{T}_*}{U^S}\sum_{i=1}^{n}(-u_i) - \sum_{j=k_*}^{I}[u_j - u_*] = \mathcal{T}_* - \mathcal{T}_* = 0.$$

Second, the statements in (26), (27) and (29) can be directly checked from the definition of the promises profile $\boldsymbol{r}$ given in (59).

Next, to prove (28), we need first to prove the preliminary result that $0 < \mathcal{T}_* < U^S$. Using (58), observe that

$$U^S - \mathcal{T}_* = U^S - \sum_{j=k_*}^{\widehat{\kappa}}[u_j - u_*] - G^S.$$

Using the definition of G^S given in (16) yields

$$U^S - \mathcal{T}_* = \underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} [u_j - u_*] = \underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} u_j + (\widehat{\kappa} - k_* + 1)u_*. \tag{60}$$

Since $k_* \in \underline{\mathcal{C}}^R$, we have $n < k_* \leq \widehat{\kappa}$ and hence $\underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} u_j \geq 0$. Moreover, $n < k_* \leq \widehat{\kappa}$ also implies that $(\widehat{\kappa} - k_* + 1)u_* > 0$. Thus, using (60) gives $0 < \mathcal{T}_* < U^S$.

Now we are prepared to prove (28). For $i \leq n$, $n < k < k_*$, and $j \geq k_*$, we have

$$v_i^{\boldsymbol{r}} = u_i + r_i = [1 - \frac{\mathcal{T}_*}{U^S}]u_i \leq 0; \quad v_k^{\boldsymbol{r}} = u_k + r_k = u_k \geq 0; \quad v_j^{\boldsymbol{r}} = u_j + r_j = u_* > 0$$

where the left inequality is implied by $0 < \mathcal{T}_* < U^S$. Thus, we see that the condition (28) is satisfied for the promises profile $\boldsymbol{r}$ defined in (59).

Finally, to show that $\boldsymbol{r}$ is stable, set $\mathcal{C} := \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ in Lemma 4 (ii) so that $|\mathcal{C}| = \widehat{\kappa}$. Using condition (28) and observing that $u_j + r_j = u_*$ for all $j \geq k_*$ shows that we have $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C}$ and $j \notin \mathcal{C}$.

Moreover, since $\boldsymbol{r} \in \mathcal{P}$, by (55) we have

$$\begin{aligned}\sum_{i \in \mathcal{C}} (u_i + r_i) &= \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [-u_j + u_*] \\ &= \sum_{i \in \mathbb{I}} u_i - (\kappa - 1)u_* = 0.\end{aligned}$$

Then by applying Lemma 4 (ii) we see that $\boldsymbol{r} \in \mathcal{S}_0$. □

Proof of Proposition 8: We proceed in two steps.

Step 1. In this step, we prove the only if part: we fix an equilibrium promises profile $\boldsymbol{r} \in \mathcal{E}$, and show that $\boldsymbol{r}$ satisfies conditions (26)-(29) of Proposition 8. Notice that Proposition 3 shows that the set $\mathcal{E}$ is not empty, and thus it is possible to select a promises profile from $\mathcal{E}$.

Recall Lemma 3 and let $\tilde{k}_* \in \mathcal{C}^R$ satisfy

$$\underline{k}_* \equiv \max\{i : r_i > 0\} < \tilde{k}_* \leq \min\{i : r_i < 0\} \equiv \overline{k}_*, \tag{61}$$

and the requirements (14) (In this proof, we reserve the notation k_* for the requirement (25) instead of the requirements (14)), that is, $-u_j \leq r_j \leq 0 \leq r_i$ and $v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}}$ for all $i < \tilde{k}_* \leq j$. Moreover, we assume $\tilde{k}_*$ is the largest one satisfying the requirements, and thus

$$\text{either } r_{\tilde{k}_*} < 0, \quad \text{or } v^{\boldsymbol{r}}_{\tilde{k}_*} > \min_{j > \tilde{k}_*} v^{\boldsymbol{r}}_j, \tag{62}$$

because otherwise $\tilde{k}_* + 1$ would also satisfies the desired requirements[22].

Step 1.1. We first show that $\tilde{k}_* \leq k_*$. Since $k_* \leq \widehat{\kappa}$, this implies $\tilde{k}_* \leq \widehat{\kappa}$. Assume by contradiction that $\tilde{k}_* > k_*$. Then by (14) we have

$$u_j + r_j \geq u_{k_*} + r_{k_*} \geq u_{k_*}, \quad \text{for all} \quad j \geq \tilde{k}_*.$$

Moreover, in the case that $\tilde{k}_* > \widehat{\kappa}$, by (1) we also have,

$$u_j + r_j \geq u_j \geq u_{k_*}, \quad \text{for all} \quad \widehat{\kappa} \leq j < \tilde{k}_*.$$

So in all the cases we have $u_j + r_j \geq u_{k_*}$ for all $j \geq \widehat{\kappa}$. This in turn implies,

$$\sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j \geq \sum_{j > \widehat{\kappa}} [u_{k_*} - u_j] = -\Delta U_{k_*} > -G^S.$$

Thus, since $\boldsymbol{r} \in \mathcal{P}$,

$$\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} [u_i + r_i] = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j < -G^S + G^S = 0.$$

This contradicts the inequality (10) which is required here because $\boldsymbol{r} \in \mathcal{S}_0$ and $\mathcal{C}^S \cup \underline{\mathcal{C}}^R \in \mathcal{D}^S$.

Step 1.2. In this step, we prove that the *ex post* intensities of the members $j = \tilde{k}_*, \cdots, I$ are equal:

$$v^{\boldsymbol{r}}_{\tilde{k}_*} = \cdots = v^{\boldsymbol{r}}_I. \tag{63}$$

[22]To see this, assume the opposite of statement (62) holds, that is, $r_{\tilde{k}_*} \geq 0$ and $v^{\boldsymbol{r}}_{\tilde{k}_*} \leq \min_{j > \tilde{k}_*} v^{\boldsymbol{r}}_j$. Condition (14) implies that $r_{\tilde{k}_*} \leq 0$ and hence the assumption $r_{\tilde{k}_*} \geq 0$ implies that $r_{\tilde{k}_*} = 0$. Thus, we have $\underline{k}_* < \tilde{k}_* + 1 \leq \overline{k}_*$ and, $-u_j \leq r_j \leq 0 \leq r_i$ for all $i < \tilde{k}_* + 1 \leq j$. Finally, the assumption $v^{\boldsymbol{r}}_{\tilde{k}_*} \leq \min_{j > \tilde{k}_*} v^{\boldsymbol{r}}_j$ implies $v^{\boldsymbol{r}}_i \leq v^{\boldsymbol{r}}_j$ for all $i < \tilde{k}_* + 1 \leq j$. Hence $\tilde{k}_* + 1$ also satisfies the requirements (61) and (14).

Consider for notational convenience, the order statistics of $\{v_j^{\boldsymbol{r}}\}_{\tilde{k}_* \le j \le I}$:

$$v_{l_{\tilde{k}_*}}^{\boldsymbol{r}} \le \cdots \le v_{l_I}^{\boldsymbol{r}},$$

where $\{l_{\tilde{k}_*}, \cdots, l_I\}$ is a permutation of $\{\tilde{k}_*, \cdots, I\}$.

To simplify the exposition, we proceed in two substeps. First, we show that $v_{l_{\tilde{k}_*}}^{\boldsymbol{r}} = \cdots = v_{l_{\widehat{\kappa}+1}}^{\boldsymbol{r}}$. Second, we show that the equality $v_{l_{\tilde{k}_*}}^{\boldsymbol{r}} = \cdots = v_I^{\boldsymbol{r}}$ is also true.

Step 1.2.1. In this sub-step we show that

$$v_{l_{\tilde{k}_*}}^{r} = v_{l_{\widehat{\kappa}+1}}^{r} \quad \text{and hence} \quad v_{l_{\tilde{k}_*}}^{r} = \cdots = v_{l_{\widehat{\kappa}+1}}^{r}. \tag{64}$$

We emphasize that we are considering the term $l_{\widehat{\kappa}+1}$, rather than $l_{\widehat{\kappa}}$.

Assume by contradiction that

$$v_{l_{\tilde{k}_*}}^{\boldsymbol{r}} < v_{l_{\widehat{\kappa}+1}}^{\boldsymbol{r}}. \tag{65}$$

We claim that

$$r_{l_{\tilde{k}_*}} < 0. \tag{66}$$

Indeed, recall (62). In the case $r_{\tilde{k}_*} < 0$, by the definition of order statistics and the ordering (1) we have

$$u_{l_{\tilde{k}_*}} + r_{l_{\tilde{k}_*}} \le u_{\tilde{k}_*} + r_{\tilde{k}_*} < u_{\tilde{k}_*} \le u_{l_{\tilde{k}_*}},$$

which implies (66). In the case $r_{\tilde{k}_*} = 0$ and $v_{\tilde{k}_*}^{\boldsymbol{r}} > \min_{j > \tilde{k}_*} v_j^{\boldsymbol{r}}$, by (1) again we have

$$u_{\tilde{k}_*} = v_{\tilde{k}_*}^{\boldsymbol{r}} > \min_{j \ge \tilde{k}_*} v_j^{\boldsymbol{r}} = v_{l_{\tilde{k}_*}}^{\boldsymbol{r}} = u_{l_{\tilde{k}_*}} + r_{l_{\tilde{k}_*}} \ge u_{\tilde{k}_*} + r_{l_{\tilde{k}_*}},$$

implying (66) again.

Clearly $r_i > 0$ for some $i < \tilde{k}_*$, and assume without loss of generality that $r_1 > 0$. We now modify $\boldsymbol{r}$ as follows: for some $\varepsilon > 0$ small,

$$\tilde{r}_1 = r_1 - \varepsilon > 0, \quad \tilde{r}_{l_{\tilde{k}_*}} = r_{l_{\tilde{k}_*}} + \varepsilon < 0, \quad \text{and} \quad \tilde{r}_i = r_i \quad \text{for all } i \ne 1, l_{\tilde{k}_*}.$$

Set $\tilde{\mathcal{C}}=\{1,\cdots,\tilde{k}_*-1\}\cup\{l_{\tilde{k}_*},\cdots,l_{\widehat{\kappa}}\}$ with $|\tilde{\mathcal{C}}|=\widehat{\kappa}$. Note that, for $\varepsilon>0$ small enough,

$$u_1+\tilde{r}_1<u_1+r_1\le v^{\boldsymbol{r}}_{l_{\widehat{\kappa}+1}};\qquad u_{l_{\tilde{k}_*}}+\tilde{r}_{l_{\tilde{k}_*}}=v^{\boldsymbol{r}}_{l_{\tilde{k}_*}}+\varepsilon<v^{\boldsymbol{r}}_{l_{\widehat{\kappa}+1}};$$
$$u_i+\tilde{r}_i=u_i+r_i\le v^{\boldsymbol{r}}_{l_{\widehat{\kappa}+1}},\quad i\in\tilde{\mathcal{C}}\backslash\{1,l_{\tilde{k}_*}\};\quad u_j+\tilde{r}_j=u_j+r_j\ge v^{\boldsymbol{r}}_{l_{\widehat{\kappa}+1}},\quad j\notin\tilde{\mathcal{C}}.$$

So $u_i+\tilde{r}_i\le v^{\boldsymbol{r}}_{l_{\widehat{\kappa}+1}}\le u_j+\tilde{r}_j$ for all $i\in\tilde{\mathcal{C}}$ and $j\notin\tilde{\mathcal{C}}$. Note further that, since $\boldsymbol{r}\in\mathcal{E}\subset\mathcal{S}_0$,

$$\sum_{i\in\tilde{\mathcal{C}}}[u_i+\tilde{r}_i]=\sum_{i\in\tilde{\mathcal{C}}}[u_i+r_i]+\Big[(\tilde{r}_1-r_1)+(\tilde{r}_{l_{\tilde{k}_*}}-r_{l_{\tilde{k}_*}})\Big]=\sum_{i\in\tilde{\mathcal{C}}}[u_i+r_i]\ge 0.$$

Then by Lemma 4 (ii) we have $\tilde{\boldsymbol{r}}\in\mathcal{S}_0$. However, as in the last part of the proof for Lemma 1, we have $\mathcal{T}_{\tilde{\boldsymbol{r}}}=\mathcal{T}_{\boldsymbol{r}}-\varepsilon<\mathcal{T}_{\boldsymbol{r}}$. This contradicts the assumption that $\boldsymbol{r}\in\mathcal{E}$ has the minimum total promises transfer. Therefore, (64) holds true.

Step 1.2.2. Now we proceed to prove (63). Assume by contradiction that, for the order statistics in the previous step,

$$v^{\boldsymbol{r}}_{l_{\tilde{k}_*}}=\cdots=v^{\boldsymbol{r}}_{l_{k_2}}<v^{\boldsymbol{r}}_{l_{k_2+1}}\le\cdots\le v^{\boldsymbol{r}}_{l_I},\quad\text{for some } \widehat{\kappa}+1\le k_2<I.$$

First, by (66) we also have

$$r_{l_j}<0,\quad\text{for all}\quad j=\tilde{k}_*,\cdots,k_2. \tag{67}$$

Again assume $r_1>0$. We then modify $\boldsymbol{r}$ as follows: for $\varepsilon>0$ small,

$$\hat{r}_1=r_1-[\widehat{\kappa}-\tilde{k}_*+1]\varepsilon>0,\quad \hat{r}_{l_j}=r_{l_j}+\varepsilon<0,\ j=\tilde{k}_*,\cdots,k_2;$$
$$\hat{r}_{l_{k_2+1}}=r_{l_{k_2+1}}-[k_2-\widehat{\kappa}]\varepsilon<0;\quad \hat{r}_i=r_i\quad\text{for all other } i.$$

One can check that $\hat{\boldsymbol{r}}\in\mathcal{P}$:

$$\sum_{i\in\mathbb{I}}\hat{r}_i=\sum_{i\in\mathbb{I}}r_i-[\widehat{\kappa}-\tilde{k}_*+1]\varepsilon+\sum_{j=\tilde{k}_*}^{k_2}\varepsilon-[k_2-\widehat{\kappa}]\varepsilon=0$$

Similarly to Step 1.2.1, we see that, for all $i<\tilde{k}_*$ and $j>k_2+1$,

$$u_i+\hat{r}_i\le u_{l_{\tilde{k}_*}}+\hat{r}_{l_{\tilde{k}_*}}=\cdots=u_{l_{k_2}}+\hat{r}_{l_{k_2}}<u_{l_{k_2+1}}+\hat{r}_{l_{k_2+1}}<u_{l_j}+\hat{r}_{l_j},$$

where the second inequality holds for $\varepsilon > 0$ small enough. Now for the same $\tilde{\mathcal{C}} = \{1, \cdots, \tilde{k}_* - 1\} \cup \{l_{\tilde{k}_*}, \cdots, l_{\widehat{\kappa}}\}$ with $|\tilde{\mathcal{C}}| = \widehat{\kappa}$ as in Step 1.2.1, we have $u_i + \hat{r}_i \leq u_{l_{\tilde{k}_*}} + \hat{r}_{l_{\tilde{k}_*}} \leq u_j + \hat{r}_j$ for all $i \in \tilde{\mathcal{C}}$ and $j \notin \tilde{\mathcal{C}}$, and

$$\sum_{i \in \tilde{\mathcal{C}}} [u_i + \hat{r}_i] = \sum_{i \in \tilde{\mathcal{C}}} [u_i + r_i] - [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon + \sum_{j=\tilde{k}_*}^{\widehat{\kappa}} \varepsilon = \sum_{i \in \tilde{\mathcal{C}}} [u_i + r_i] \geq 0.$$

Then by Lemma 4 (ii) we see that $\hat{\boldsymbol{r}} \in \mathcal{S}_0$. Moreover, note that

$$\begin{aligned} \mathcal{T}_{\hat{\boldsymbol{r}}} - \mathcal{T}_{\boldsymbol{r}} &= \frac{1}{2}\Big[|\hat{r}_1| - |r_1| + \sum_{j=\tilde{k}_*}^{k_2} [|\hat{r}_{l_j}| - |r_{l_j}|] + |\hat{r}_{l_{k_2+1}}| - |r_{l_{k_2+1}}|\Big] \\ &= \frac{1}{2}\Big[- [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon + \sum_{j=\tilde{k}_*}^{k_2} (-\varepsilon) + [k_2 - \widehat{\kappa}]\varepsilon\Big] = -[\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon < 0, \end{aligned}$$

where the last inequality is due to $\tilde{k}_* \leq \widehat{\kappa}$ from Step 1.1. This contradicts the assumption that $\boldsymbol{r} \in \mathcal{E}$ has the minimum total promises transfer, so (63) holds true.

Step 1.3. We now collect all the results from the intermediate steps to show that $\boldsymbol{r}$ satisfies conditions (26)-(29) of Proposition 8.

Let y_* denote the common value in (63). Then $r_j = y_* - u_j \leq 0$ for all $j \geq \tilde{k}_*$. On one hand, since $\tilde{k}_* \leq k_* \leq \widehat{\kappa}$ by Step 1.1,

$$\begin{aligned} 0 &\leq \sum_{i \leq \widehat{\kappa}} [u_i + r_i] = \sum_{i \leq \widehat{\kappa}} u_i - \sum_{j > \widehat{\kappa}} r_j = \sum_{i \leq \widehat{\kappa}} u_i - \sum_{j > \widehat{\kappa}} [y_* - u_j] \\ &= \sum_{i \in \mathbb{I}} u_i - (\kappa - 1) y_* = (\kappa - 1)(u_* - y_*). \end{aligned}$$

Therefore, $y_* \leq u_*$. On the other hand, by (15),

$$\mathcal{T}_{\boldsymbol{r}} = \sum_{j \geq \tilde{k}_*} (-r_j) \geq \sum_{j \geq k_*} (-r_j) = \sum_{j \geq k_*} [u_j - y_*] \geq \sum_{j \geq k_*} [u_j - u_*] = \mathcal{T}_*. \tag{68}$$

Since $\boldsymbol{r}$ minimizes the total promises transfer and we already constructed a stable promises profile in Lemma 6 with total promises transfer $\mathcal{T}_*$, then we must have $\mathcal{T}_{\boldsymbol{r}} = \mathcal{T}_*$, and thus all the inequalities in (68) are equalities. In particular, the second inequality in (68) implies that $u_* = y_*$. Moreover, by (62) and (63) we have $r_{\tilde{k}_*} < 0$, so that the first inequality in (68) implies that $\tilde{k}_* = k_*$. Now it can be directly checked that the conditions (26)-(29) of Proposition 8 hold. This concludes the proof of the only if part of Proposition 8.

Step 2. In this step we show the if part: we fix $\boldsymbol{r} \in \mathcal{P}$ that satisfies conditions (26)-(28) of Proposition 8 and show that $\boldsymbol{r} \in \mathcal{E}$ and that (29) holds.

To show that $\boldsymbol{r} \in \mathcal{S}_0$, set $\mathcal{C} := \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ in Lemma 4 (ii) so that $|\mathcal{C}| = \widehat{\kappa}$. Using condition (28) and observing that $u_j + r_j = u_*$ for all $j \geq k_*$ (condition (27)) shows that we have $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C}$ and $j \notin \mathcal{C}$. Moreover, since $\boldsymbol{r} \in \mathcal{P}$ and $r_j = -u_j + u_*$ for $j \geq \kappa_*$, we have

$$\begin{aligned}\sum_{i\in\mathcal{C}}(u_i + r_i) &= \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} u_i - \sum_{j\in\mathcal{C}^R\setminus\underline{\mathcal{C}}^R} r_j = \sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} u_i - \sum_{j\in\mathcal{C}^R\setminus\underline{\mathcal{C}}^R} [-u_j + u_*] \\ &= \sum_{i\in\mathbb{I}} u_i - (\kappa - 1)u_* = 0.\end{aligned}$$

Then by applying Lemma 4 (ii) we see that $\boldsymbol{r} \in \mathcal{S}_0$. Moreover,

$$\mathcal{T}_{\boldsymbol{r}} = \sum_{j\geq k_*}(-r_j) = \sum_{j\geq k_*}[u_j - u_*] = \mathcal{T}_*.$$

By Proposition 3 there exists $\boldsymbol{r}^* \in \mathcal{E}$. By Step 1 (the only if direction), the promises profile $\boldsymbol{r}^*$ satisfies conditions (26)-(29), in particular, $\mathcal{T}_{\boldsymbol{r}^*} = \mathcal{T}_*$, thus $\mathcal{T}_*$ is the minimum total promises transfer for all $\boldsymbol{r} \in \mathcal{S}_0$. Finally, if $\boldsymbol{r} \in \mathcal{S}_0$ satisfies (26)-(28), we have $\mathcal{T}_{\boldsymbol{r}} = \mathcal{T}_*$, so $\boldsymbol{r}$ matches the minimum total promises transfer, and thus $\boldsymbol{r} \in \mathcal{E}$. □

Appendix B

In this appendix, we first provide necessary and sufficient conditions under which the equilibrium promises are of the reaching across the aisle type. Second, we propose an equilibrium selection based on an a finite sequence of transfers from the strongest supporters of the reform to the weakest ones.

Reaching across the aisle equilibria

The following proposition generalizes the discussions on whether the equilibrium rules out circling the wagon transfers where some promises recipients are reform supporters.

Proposition 9. *[**Reaching across the aisle equilibria.**] Consider a committee with a κ-majority voting rule. When $\kappa \geq 2$, all the equilibrium promises are of the reaching across the aisle type if and only if one of the following conditions hold:*

1. *Reform supporter lack voting power to enact the reform, $|\mathcal{C}^R| < \kappa$, as in Proposition 5.*
2. *Reform supporters have enough voting power, $|\mathcal{C}^R| \geq \kappa$, and $G^S \leq \Delta U_{\widehat{\kappa}}$ as in Proposition 7, and the two following conditions are satisfied*
   - *(a) The weakest reform supporters $\underline{\mathcal{C}}^R = \{n+1, \cdots, \widehat{\kappa}\}$ have equal ex ante intensities:*

$$u_{n+1} = \cdots = u_{\hat{\kappa}}.$$

   - *(b) Either $G^S = \Delta U_{\hat{\kappa}}$, or $G^S < \Delta U_{\hat{\kappa}}$ and the reform supporter $\widehat{\kappa}+1$ has an ex ante intensity that is equal to that of the members $\{n+1, \cdots, \widehat{\kappa}\}$:*

$$G^S = \Delta U_{\hat{\kappa}}, \quad \text{or} \quad G^S < \Delta U_{\hat{\kappa}} \text{ and } u_{n+1} = \cdots = u_{\hat{\kappa}} = u_{\hat{\kappa}+1}.$$

3. *Reform supporters have enough voting power, $|\mathcal{C}^R| \geq \kappa$, and $\Delta U_{\widehat{\kappa}} < G^S$ as in Proposition 8, and*

$$u_{n+1} = \cdots = u_{k_*-1} = u_*.$$

Proposition 9 shows that in the case of a frustrated $\kappa-$minority covered in Proposition 5, promises recipients are reform opponents in all equilibria. Despite their multiplicity, equilibrium promises share the common feature of being of the reaching across the aisle type. By contrast, in the cases covered in Proposition 7 and Proposition 8 where there are enough supporters to enact the reform to begin with ($|\mathcal{C}^R| \geq \kappa$), we show in Proposition 9 that for equilibrium promises to always be of the reaching across the aisle type additional restrictions are required. To rule out circle the wagon type transfers, we broadly need a stale distribution of intensities among a specific subset of reform supporters with weakest intensities. In words, the proposition shows that when the weakest reform supporters derive uniform utility from the reform, then equilibrium requires that all promises recipients are reform opponents. We now give the proof of Proposition 9.

Proof of Proposition 9:

When $|\mathcal{C}^R| < \kappa$, it is clear from Proposition 5 that all promises recipients are reform supporters, and thus statement 1 in Proposition 9 holds.

We now prove statement 2. We first show that properties $2.a.$ and $2.b$ collectively imply that any $\boldsymbol{r} \in \mathcal{E}$ belongs to the reaching across the aisle type.

Consider now the first subcase where $u_{n+1} = \cdots = u_{\widehat{k}}$ and $G^S = \Delta U_{\widehat{k}}$. Note that

$$u_j + r_j = v_j^{\boldsymbol{r}} \geq v_{\hat{\kappa}}^{\boldsymbol{r}} \geq u_{\hat{\kappa}}, \quad \forall j > \hat{\kappa}.$$

Then

$$\mathcal{T}_{\boldsymbol{r}} = \sum_{j>\hat{\kappa}} (-r_j) \leq \sum_{j>\hat{\kappa}} (u_j - u_{\hat{\kappa}}) = \Delta U_{\hat{\kappa}} = G^S.$$

Since $\boldsymbol{r} \in \mathcal{E}$, then $\mathcal{T}_{\boldsymbol{r}} = G^S$ (Proposition 7, condition 3), and thus equality holds above. This implies that $-r_j = u_j - u_{\hat{\kappa}}$, and thus $v_j^{\boldsymbol{r}} = u_{\hat{\kappa}}$ for all $j > \hat{\kappa}$. Note further that $u_i \leq v_i^{\boldsymbol{r}} \leq v_j^{\boldsymbol{r}} = u_{\hat{\kappa}}$ for all $n < i \leq \hat{\kappa} < j$. By the assumption in this subcase, we see that $r_i = 0$ for $n < i \leq \hat{\kappa}$. Then the promises profile $\boldsymbol{r}$ is of the reaching across the aisle type.

Consider the second subcase where $u_{n+1} = \cdots = u_{\widehat{\kappa}} = u_{\widehat{\kappa}+1}$. By Proposition 7 Part 1, we have $r_i \geq 0$ for $n < i \leq \hat{\kappa}$ and $r_{\hat{\kappa}+1} \leq 0$. Then $v_i^{\boldsymbol{r}} \geq u_i = u_{\hat{\kappa}+1} \geq v_{\hat{\kappa}+1}^{\boldsymbol{r}}$. By Proposition 7 Part 2, we have $v_{\hat{\kappa}+1}^{\boldsymbol{r}} \geq v_i^{\boldsymbol{r}}$, and thus we must have $v_{\hat{\kappa}+1}^{\boldsymbol{r}} = v_i^{\boldsymbol{r}}$. Thus, $r_i = 0$ for $n < i \leq \hat{\kappa}$, and therefore $\boldsymbol{r}$ is of the reaching across the aisle type promise.

We next prove the only if part of statement 2 in Proposition 5. To do so, we assume that either statement $2.a$ or statement $2.b$ is false, and construct an equilibrium promises profile $\boldsymbol{r} \in \mathcal{E}$ where some promises recipients are reform supporters. Note that, when either statement $2.a$ or statement $2.b$ is false, together with (1) and the assumption that $0 < G^S \leq \Delta U_{\hat{\kappa}}$, one of the following two statements must be true:

$$u_{n+1} < u_{\hat{\kappa}} \quad \text{and} \quad 0 < G^S \leq \Delta U_{\hat{\kappa}}; \tag{69}$$

$$u_{n+1} = \cdots = u_{\hat{\kappa}} < u_{\hat{\kappa}+1} \quad \text{and} \quad 0 < G^S < \Delta U_{\hat{\kappa}}. \tag{70}$$

Now let $0 < \varepsilon < G^S$ and we modify the equilibrium promises profile in (51) as follows:

$$\begin{aligned} r_i &:= -\frac{G^S - \varepsilon}{U^S} u_i, \ i \in \mathcal{C}^S; \quad r_{n+1} := \varepsilon; \\ r_j &:= 0, \ n+1 < j \leq \hat{\kappa}; \quad r_k := -\frac{G^S}{\Delta U_{\widehat{\kappa}}}[u_k - u_{\widehat{\kappa}}], \ k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R. \end{aligned} \tag{71}$$

It can be directly checked that $\boldsymbol{r} \in \mathcal{P}$ and satisfies conditions (18) and (20) from Proposition 7, so to establish that $\boldsymbol{r}$ is an equilibrium, we only need to prove (19).

In the sub-case (69), assume further that $\varepsilon < u_{\hat{\kappa}} - u_{n+1}$. Then one can check that

$$v_i^{\boldsymbol{r}} \leq 0 \leq v_j^{\boldsymbol{r}} \leq u_{\hat{\kappa}} \leq v_k^{\boldsymbol{r}}, \quad \text{for all } i \leq n < j \leq \hat{\kappa} < k.$$

In the sub-case (70), assume further that $\varepsilon < [1 - \frac{G^S}{\Delta U_{\hat{\kappa}}}][u_{\hat{\kappa}+1} - u_{\hat{\kappa}}]$. Then using $u_{n+1} = u_{\widehat{\kappa}}$ one can see

$$v_i^{\boldsymbol{r}} \leq 0 \leq v_j^{\boldsymbol{r}} \leq u_{\hat{\kappa}} + \varepsilon \leq u_{\hat{\kappa}} + [1 - \frac{G^S}{\Delta U_{\hat{\kappa}}}][u_{\hat{\kappa}+1} - u_{\hat{\kappa}}] \leq v_k^{\boldsymbol{r}}, \quad \text{for all } i \leq n < j \leq \hat{\kappa} < k.$$

To sum up, in all subcases, the promises profile $\boldsymbol{r}$ defined in (71) also satisfies the condition (19) of Proposition 7 when ε is small enough, and as a result, $\boldsymbol{r} \in \mathcal{E}$. We conclude then by observing that since $r_{n+1} > 0$ the reform supporter $n+1$ is a promise recipient, and therefore the equilibrium $\boldsymbol{r}$ has some circle the wagon transfers.

We now prove statement 3 in Proposition 9.

We first show the if part. Fix an arbitrary $\boldsymbol{r} \in \mathcal{E}$. By Proposition 8, we have $r_i \geq 0$ and $u_i \leq v_i^{\boldsymbol{r}} \leq u_*$ for $n+1 \leq i \leq \hat{\kappa}$. However, since we assume $u_{n+1} = \cdots = u_{k_*-1} = u_*$ here, we must have $r_i = 0$ for $n+1 \leq i \leq \hat{\kappa}$. Since $r_j = -(u_j - u_*) \geq 0$ for $j \geq k_*$, we see that the promises profile $\boldsymbol{r}$ is of the reaching across the aisle type.

We now prove the only if part using the contrapositive. Equivalently, recalling (1), we assume $u_{n+1} < u_*$ and we shall construct an $\boldsymbol{r} \in \mathcal{E}$ which has some circle the wagon type transfers. Let $0 < \varepsilon < \mathcal{T}_*$ and we modify the promises profile described in (59) as follows:

$$\begin{aligned} r_i &:= -\frac{\mathcal{T}_* - \varepsilon}{U^S} u_i, \ i \leq n; \qquad r_{n+1} := \varepsilon; \\ r_k &:= 0, \ n+1 < k < k_*; \quad r_j := -[u_k - u_*], \ j \geq k_*. \end{aligned} \tag{72}$$

Assume further that $\varepsilon < u_* - u_{n+1}$, then one can check that

$$v_i^{\boldsymbol{r}} \leq 0 \leq v_j^{\boldsymbol{r}} \leq u_* = v_k^{\boldsymbol{r}}, \quad \text{for all } i \leq n < j \leq \hat{\kappa} < k.$$

It can be checked that $\boldsymbol{r} \in \mathcal{P}$ and satisfies all the other requirements in Proposition 8. We conclude by observing that $r_{n+1} > 0$, and therefore the equilibrium promises profile $\boldsymbol{r} \in \mathcal{E}$ constructed in (72) has some circle the wagon transfers. $\square$

An equilibrium selection algorithm

In this subsection, we propose an algorithm describing how to select a sequence of incremental promises that reach an equilibrium after a finite number of steps. Unlike the algorithm from Example 5, the algorithm described in this section dictates in each step how to proceed for the next step and does not leave room for multiple choices. Importantly, the algorithm achieves an equilibrium for all distributions of *ex ante* intensities; this covers all cases that we have discussed in this paper.

To describe the algorith, we introduce an operator Φ as follows. Given an intensity profile $\boldsymbol{u}$ satisfying (1), with $u_1 < u_I$, let $k_1(\boldsymbol{u}) \leq k_2(\boldsymbol{u})$ be such that, abbreviating as k_1, k_2 for notational simplicity,

$$u_1 = \cdots = u_{k_1} < u_{k_1+1}, \quad u_{k_2} < u_{k_2+1} = \cdots = u_I. \tag{73}$$

We then define, for any $X > 0$,

$$\begin{aligned} \Phi(\boldsymbol{u}, X) := \boldsymbol{v}, \quad \text{where} \quad v_i = u_i,\ k_1 < i \leq k_2, \quad \text{and} \\ v_1 = \cdots = v_{k_1} := u_1 + \frac{X}{k_1}, \qquad v_{k_2+1} = \cdots = v_I := u_I - \frac{X}{I - k_2}. \end{aligned} \tag{74}$$

We first establish the following simple lemma, whose proof is postponed to the end of Appendix B.

Lemma 7. *Denote $v := \Phi(\boldsymbol{u}, X)$.*

(i) For any $X > 0$, $\boldsymbol{r} := \boldsymbol{v} - \boldsymbol{u} \in \mathcal{P}$.

(ii) Assume $X \leq \min\big(k_1(u_{k_1+1} - u_{k_1}), (I - k_2)(u_{k_2+1} - u_{k_2})\big)$, then $v_1 \leq \cdots \leq v_I$.

(iii) For the X in (ii), if $v_1 < v_I$, then $k_1(\boldsymbol{u}) \leq k_1(\boldsymbol{v}) \leq k_2(\boldsymbol{v}) \leq k_2(\boldsymbol{u})$.

We now present the algorithm more formally:

Step 0 (Initialization). If $\sum_{i=1}^{\widehat{\kappa}} u_i \geq 0$, $\boldsymbol{u}$ is already stable, so $\boldsymbol{r} = (0, \cdots, 0)$ is the unique equilibrium. We thus assume $\sum_{i=1}^{\widehat{\kappa}} u_i < 0$, and set $\boldsymbol{v}^0 := \boldsymbol{u}$.

Step 1. Given $\boldsymbol{v}^l$ satisfying $v_1^l \leq \cdots \leq v_I^l$ and $\sum_{i=1}^{\widehat{\kappa}} v_i^l < 0$, we define $\boldsymbol{v}^{l+1}$ recursively as follows until $\sum_{i=1}^{\widehat{\kappa}} v_i^{l+1} \geq 0$: denoting $k_1 := k_1(\boldsymbol{v}^l)$, $k_2 := k_2(\boldsymbol{v}^l)$ as in (73),

$$\boldsymbol{v}^{l+1} := \Phi(\boldsymbol{v}^l, X_{l+1}) \quad \text{where} \quad X_{l+1} := \min\big(k_1(v_{k_1+1}^l - v_{k_1}^l), (I - k_2)(v_{k_2+1}^l - v_{k_2}^l)\big),$$

We remark that by Lemma 7 (ii) $\boldsymbol{v}^{l+1}$ remains ordered.

Step 2. We now assume $\sum_{i=1}^{\widehat{\kappa}} v_i^l < 0 \leq \sum_{i=1}^{\widehat{\kappa}} v_i^{l+1}$ for some l. Then we set

$$\boldsymbol{v}^* := \Phi(\boldsymbol{v}^l, X^*) \quad \text{where } 0 < X^* \leq X_{l+1} \text{ is determined by } \sum_{i=1}^{\widehat{\kappa}} v_i^* = 0,$$

and stop the algorithm.

We note that, in each recursion of Step 1, we will have either $v_{k_1+1}^{l+1} = v_{k_1}^{l+1}$ or $v_{k_2+1}^{l+1} = v_{k_2}^{l+1}$, namely $\boldsymbol{v}^{l+1}$ will have at least one more tied component than $\boldsymbol{v}^l$, so Step 1 will stop after finitely many recursions. Moreover, clearly $\Phi(\boldsymbol{v}^l, X)$ is continuous and strictly monotone in X, so the transfer X^* in Step 2 is unique. The following proposition shows that the above algorithm implements an equilibrium.

Proposition 10. *The promises profile $\boldsymbol{r}^* := \boldsymbol{v}^* - \boldsymbol{u}$ produced after a finite number of steps from the above algorithm is an equilibrium and $v_1^* \leq \cdots \leq v_I^*$ with $v^* \equiv v^{\boldsymbol{r}^*}$. Moreover, denoting by $k_1^* = k_1(\boldsymbol{v}^*)$ and $k_2^* = k_2(\boldsymbol{v}^*)$. the committee members defined as in* (73)*, the following properties hold true:*

(i) Members of the coalition of promisers $\{k_2^*+1, \cdots, I\}$, make individual promises $r_i \leq 0$ and get equalized ex post intensities $v_i^* = \overline{v} > 0$, for all $i > k_2^*$.

(ii) Members of the the coalition of promisees $\{1, \cdots, k_1^*\}$, receive individual transfers $r_i \geq 0$ and get equalized ex post intensities $v_i^* = \underline{v} \leq 0$, for all $i \leq k_1^*$.

(iii) Members of the coalition $\{k_1^* + 1, \cdots, k_2^*\}$ make no promises, that is, $r_i = 0$ and $v_i^* = u_i$ for all $k_1^* + 1 \leq i \leq k_2^*$.

(iv) Among all equilibria, the promises profile $\boldsymbol{r}^*$ produced by the algorithm generates the ex post intensity $\boldsymbol{v}^*$ with the smallest dispersion. That is, for any $\boldsymbol{r} \in \mathcal{E}$ such that $\boldsymbol{r} \neq \boldsymbol{r}^*$, we have

$$\min_i v_i^{\boldsymbol{r}} \leq v_1^*, \quad \max_i v_i^{\boldsymbol{r}} \geq v_I^*, \quad \textit{and} \quad \max_i v_i^{\boldsymbol{r}} - \min_i v_i^{\boldsymbol{r}} > v_I^* - v_1^*.$$

Proposition 10 shows that the algorithm that we describe in this subsection converges after a finite number of steps to a unique equilibrium. The algorithm has an egalitarian underpinning: in each step, the utility transfers flow from the members with the highest intensities to the members with the lowest intensities while preserving the ordering of the *interim* intensities. Perhaps not surprisingly, property (iv) of Proposition 10 shows that the resulting equilibrium is the one with lowest dispersion. The algorithm is in fact an equilibrium selection that comes with a dynamic implementation based on transfers between members of the two tails of the distribution of intensities. There may be other algorithms based on other principles that will select other types of equilibria.

We now give the proof of Lemma 7 and that of Proposition 10.

Proof of Lemma 7:

(i) By (74), we have

$$\begin{aligned}\sum_{i\in\mathbb{I}} r_i &= \sum_{i=1}^{k_1} r_i + \sum_{i=k_1+1}^{k_2} r_i + \sum_{i=k_2+1}^{I} r_i \\ &= \sum_{i=1}^{k_1} \frac{X}{k_1} + \sum_{i=k_1+1}^{k_2} 0 + \sum_{i=k_2+1}^{I} (-\frac{X}{I-k_2}) = X - X = 0.\end{aligned}$$

That is, $\boldsymbol{r} \in \mathcal{P}$.

(ii) Recall $u_1 = u_{k_1}$. Since $X \le k_1(u_{k_1+1} - u_{k_1})$, we have

$$v_{k_1} = u_{k_1} + \frac{X}{k_1} \le u_{k_1} + (u_{k_1+1} - u_{k_1}) = u_{k_1+1} = v_{k_1+1}. \tag{75}$$

Similarly, recalling $u_I = u_{k_2+1}$, since $X \le (I-k_2)(u_{k_2+1} - u_{k_2})$, then

$$v_{k_2+1} = u_{k_2+1} - \frac{X}{I-k_2} \ge u_{k_2+1} - (u_{k_2+1} - u_{k_2}) = u_{k_2} = v_{k_2}. \tag{76}$$

Now since $v_i = u_i$ for $k_1 < i \le k_2$, by (1) we have $v_{k_1+1} \le \cdots \le v_{k_2}$. Then we obtain

$$v_1 = \cdots = v_{k_1} \le v_{k_1+1} \le \cdots \le v_{k_2} \le v_{k_2+1} = \cdots = v_I. \tag{77}$$

(iii) Inequalities (77) imply $k_1(\boldsymbol{u}) = k_1 \le k_1(\boldsymbol{v}) \le k_2(\boldsymbol{v}) \le k_2 = k_2(\boldsymbol{u})$. □

Proof of Proposition 10:

In the case $\sum_{i=1}^{\widehat{\kappa}} u_i \ge 0$ (there are no blocking coalitions), the algorithm stops at Step 0. All the statements are obvious.

Henceforth we assume $\sum_{i=1}^{\widehat{\kappa}} u_i < 0$ in this proof. Note that $\sum_{i=1}^{I} u_i > 0$, then we must have $u_1 < 0 < u_I$, and thus $k_1(\boldsymbol{u})$ and $k_2(\boldsymbol{u})$ are well defined.

For all l in Step 1 such that $\sum_{i=1}^{\widehat{\kappa}} v_i^l < 0$, by Lemma 7 (iii) we see that $k_1(\boldsymbol{v}^l)$ is weakly increasing in l and $k_2(\boldsymbol{v}^l)$ is weakly decreasing in l. Moreover, by (74) we have

$$v_i^{l+1} > v_i^l,\ i \le k_1(\boldsymbol{v}^l);\quad v_k^{l+1} = v_k^l,\ k_1(\boldsymbol{v}^l) < k \le k_2(\boldsymbol{v}^l);\quad v_j^{l+1} < v_j^l, j > k_2(\boldsymbol{v}^l).$$

Then, since $\boldsymbol{v}^0 = \boldsymbol{u}$, by induction on l we see that

$$v_i^{l+1} > u_i,\ i \le k_1(\boldsymbol{v}^l);\quad v_k^{l+1} = u_k,\ k_1(\boldsymbol{v}^l) < k \le k_2(\boldsymbol{v}^l);\quad v_j^{l+1} < u_j, j > k_2(\boldsymbol{v}^l).$$

In particular, for the l in Step 2, we have $k_1(\boldsymbol{v}^l) \le k_1^* \le k_2^* \le k_2(\boldsymbol{v}^l)$, and

$$v_i^* > u_i,\ i \le k_1(\boldsymbol{v}^l);\quad v_k^* = u_k,\ k_1(\boldsymbol{v}^l) < k \le k_2(\boldsymbol{v}^l);\quad v_j^* < u_j, j > k_2(\boldsymbol{v}^l).$$

This implies that

$$r_i > 0,\ i \le k_1(\boldsymbol{v}^l);\quad r_i = 0,\ k_1(\boldsymbol{v}^l) < i \le k_1^*;\quad r_k = 0,\ k_1^* < k \le k_2^*;$$
$$r_j = 0,\ k_2^* < j \le k_2(\boldsymbol{v}^l);\quad r_j < 0,\ j > k_2(\boldsymbol{v}^l).$$

This proves (iii), and by the definition of k_1^*, k_2^* in (73), we have $v_1^* = \cdots = v_{k_1^*}^*$, and $v_{k_2^*+1}^* = \cdots = v_I^*$. Note further that $\sum_{i=1}^{\widehat{\kappa}} v_i^* = 0 < \sum_{i=1}^{I} v_i^*$ and by Lemma 7 $\boldsymbol{v}^*$ is ordered, so we must have $\boldsymbol{v}_1^* \le 0 < v_I^*$. This completes the proof for (i) and (ii).

Moreover, again since $\sum_{i=1}^{\widehat{\kappa}} v_i^* = 0$ and $\boldsymbol{v}^*$ is ordered, by setting $\mathcal{C} = \{1, \cdots, \widehat{\kappa}\}$ in Lemma 4 (ii) we see that $\boldsymbol{r}^* \in \mathcal{S}_0$. Let $\underline{k}_*, \overline{k}_*$ follow the definition in (38) from Lemma 3, using $\boldsymbol{r}^*$. By (i)-(iii) it is clear that

$$\underline{k}_* \le k_1^* \le k_2^* < \overline{k}_* \quad \text{and} \quad \underline{k}_* \le n \le k_2^*.$$

We next prove $\boldsymbol{r}^* \in \mathcal{E}$ and (iv) in three cases.

Case 1: $|\mathcal{C}^R| < \kappa$, and thus $n = I - |\mathcal{C}^R| \ge \widehat{\kappa}$. By (i) it is clear that $u_i \ge v_i^* > 0$ for $i > k_2^*$, then $n \le k_2^*$ and by (ii) and (iii) we have $v_i^* \le 0$ for all $i \le n$. On the other hand, since $\sum_{i=1}^{\widehat{\kappa}} v_i^* = 0$ and $\widehat{\kappa} \le n$, then we must have $v_i^* = 0$ for all $i \le \widehat{\kappa}$. Since $\boldsymbol{v}^*$

is ordered, we actually have $v_i^* = 0$, and thus $r_i > 0$ for all $i \leq n$. Then $n \leq \underline{k}_*$ and in fact they are equal. Then

$$\mathcal{T}_{\boldsymbol{r}^*} = \sum_{i=1}^{\underline{k}_*} r_i^* = \sum_{i=1}^{n} r_i^* = \sum_{i \in \mathcal{C}^S} (-u_i) = U^S.$$

Now by Proposition 5 we see that $\boldsymbol{r}^* \in \mathcal{E}$.

Moreover, let $\boldsymbol{r} \in \mathcal{E}$ be an arbitrary equilibrium. By Proposition 5 Part 1 we have

$$\sum_{i > k_2^*} (u_i - v_I^*) = U^S = \mathcal{T}_{\boldsymbol{r}} = \sum_{i > n} (-r_i) \geq \sum_{i > k_2^*} (u_i - v_i^{\boldsymbol{r}}).$$

Here we used the fact that $k_2^* \geq n$. This clearly implies that $\max_{i \in \mathbb{I}} v_i^r \geq v_I^*$, and if equality holds, we must have $v_i^r = v_I^*$ for all $i > k_2^*$ and $r_i = 0$ for all $n < i \leq k_2^*$. Similarly, by Proposition 5 Part 2, in both cases we have

$$\sum_{i \in \mathcal{C}^S} (v_1^* - u_i) = \sum_{i \in \mathcal{C}^S} r_i^* = U^S = \mathcal{T}_r = \sum_{i \in \mathcal{C}^S} r_i = \sum_{i \in \mathcal{C}^S} (v_i^r - u_i).$$

Then clearly $\min_{i \in \mathbb{I}} v_i^r \leq v_1^*$, and if equality holds, we must have $v_i^r = v_1^*$ for all $i \in \mathcal{C}^S$. The same logic applies to maximum of $v_i^r - u_i$, directly implying $\max_{i \in \mathbb{I}} v_i^{\boldsymbol{r}} - \min_{i \in \mathbb{I}} v_i^{\boldsymbol{r}} \geq v_I^* - v_1^*$. Now if the two sides are actually equal, then $\max_{i \in \mathbb{I}} v_i^r = v_I^*$ and $\min_{i \in \mathbb{I}} v_i^r = v_1^*$, thus $v_i^r = v_1^*$ for $i \in \mathcal{C}^S$, $r_i = 0$ for $n < i \leq k_2^*$, and $v_i^r = v_I^*$ for $i > k_2^*$. This exactly means $\boldsymbol{r} = \boldsymbol{r}^*$. So (iv) holds in this case.

Case 2: $\mid \mathcal{C}^R \mid \geq \kappa$ and $k_2^* \geq \widehat{\kappa}$. This implies that $\underline{k}_* \leq n < \widehat{\kappa} \leq k_2^*$. Note that $r_i = 0$ for $\underline{k}_* < i \leq k_2^*$ and $\sum_{i=1}^{\widehat{\kappa}} v_i^* = 0$, then

$$\mathcal{T}_{\boldsymbol{r}^*} = \sum_{i=1}^{\underline{k}_*} r_i^* = \sum_{i=1}^{\widehat{\kappa}} r_i^* = \sum_{i=1}^{\widehat{\kappa}} (v_i^* - u_i) = -\sum_{i=1}^{\widehat{\kappa}} u_i = G^S.$$

Note that by Proposition 7 we see that the minimum total promises transfer is G^S in the case $0 < G^S \leq \Delta U_{\widehat{\kappa}}$, and by Proposition 8 it is larger than G^S in the case $\Delta U_{\widehat{\kappa}} < G^S$. So we must be in the case $0 < G^S \leq \Delta U_{\widehat{\kappa}}$ and $\boldsymbol{r}^* \in \mathcal{E}$.

Moreover, let $\boldsymbol{r} \in \mathcal{E}$ be an arbitrary equilibrium. By Proposition 7 Part 1 we have

$$\sum_{i>k_2^*}(u_i - v_I^*) = \mathcal{T}_{\boldsymbol{r}^*} = G^S = \mathcal{T}_r = \sum_{i>\widehat{\kappa}}(-r_i) \geq \sum_{i>k_2^*}(-r_i) = \sum_{i>k_2^*}(u_i - v_i^r).$$

This clearly implies that $\max_{i\in\mathbb{I}} v_i^r \geq v_I^*$, and if equality holds, we must have $v_i^r = v_I^*$ for all $i > k_2^*$ and $r_i = 0$ for all $\widehat{\kappa} < i \leq k_2^*$. Similarly, by Proposition 7 Part 2 we have

$$\sum_{i=1}^{k_1^*}(v_1^* - u_i) = \sum_{i=1}^{k_1^*} r_i^* = \mathcal{T}_{r^*} = G^S = \mathcal{T}_r = \sum_{i=1}^{\widehat{\kappa}} r_i \geq \sum_{i=1}^{k_1^*} r_i = \sum_{i=1}^{k_1^*}(v_i^r - u_i).$$

Then clearly $\min_{i\in\mathbb{I}} v_i^r \leq v_1^*$, and if equality holds, we must have $v_i^r = v_1^*$ for all $i \leq k_1^*$ and $r_i = 0$ for all $k_1^* < i \leq \widehat{\kappa}$. (iv) holds in this case by subtracting the minimum inequality from the maximum.

Case 3: $\mid \mathcal{C}^R \mid \geq \kappa$ and $k_2^* < \widehat{\kappa}$. By the analysis in *Case 2*, we must have $\Delta U_{\widehat{\kappa}} < G^S$, and thus we are in the situation of Proposition 8. Note that, since $\sum_{i=1}^{\widehat{\kappa}} v_i^* = 0$,

$$\sum_{i>k_2^*}(u_i - v_i^*) = \sum_{i>k_2^*}(-r_i^*) = \mathcal{T}_{\boldsymbol{r}^*} = \sum_{i\leq k_2^*} r_i^* = \sum_{i\leq k_2^*}(v_i^* - u_i) = -\sum_{i=k_2^*+1}^{\widehat{\kappa}} v_i^* - \sum_{i\leq k_2^*} u_i.$$

This implies that, recalling (55),

$$\sum_{i=1}^{I} u_i = \sum_{i=\widehat{\kappa}+1}^{I} v_i^* = (\kappa - 1)v_I^*, \quad \text{and thus} \quad v_I^* = \frac{1}{\kappa - 1}\sum_{i=1}^{I} u_i = u_*.$$

This implies immediately that $k_2^* = k_*$ and $\mathcal{T}_{\boldsymbol{r}^*} = \mathcal{T}_*$, where k_* and $\mathcal{T}_*$ are as in Proposition 8. Therefore, $\boldsymbol{r}^* \in \mathcal{E}$.

Moreover, let $\boldsymbol{r} \in \mathcal{E}$ be an arbitrary equilibrium. By Proposition 8 Part 1 we see that $\max_{i\in\mathbb{I}} v_i^r = u_* = v_I^*$. On the other hand, recalling again that $k_* = k_2^* \geq k_1^*$, by Proposition 8 Part 2 we have

$$\sum_{i=1}^{k_1^*}(v_1^* - u_i) = \sum_{i=1}^{k_1^*} r_i^* = \mathcal{T}_{\boldsymbol{r}^*} = \mathcal{T}_* = \mathcal{T}_{\boldsymbol{r}} = \sum_{i=1}^{k^*} r_i \geq \sum_{i=1}^{k_1^*} r_i = \sum_{i=1}^{k_1^*}(v_i^{\boldsymbol{r}} - u_i).$$

Then clearly $\min_{i\in\mathbb{I}} v_i^{\boldsymbol{r}} \leq v_1^*$, and if equality holds, we must have $v_i^{\boldsymbol{r}} = v_1^*$ for all $i \leq k_1^*$ and $r_i = 0$ for all $k_1^* < i \leq k_2^*$. Putting the maximum and minimum together shows that (iv) holds in this case as well. □