Beliefs, Aggregate Risk, and the U.S. Housing Boom

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Introduction

- U.S. housing boom of the 2000s: shift in credit conditions + optimistic beliefs
- Kaplan et al. (2020): beliefs quantitatively more important
  - exogenous shocks orthogonal to credit conditions
  - calibration implies a housing boom once a generation

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- Kaplan et al. (2020): beliefs quantitatively more important
  - exogenous shocks orthogonal to credit conditions
  - calibration implies a housing boom once a generation
- Revisiting how beliefs are modeled because house prices rebounded to a new high in 2022 along with optimism
  - propose endogenous instead of exogenous beliefs to address why there was a shift
  - attribute shift to incomplete information about the evolution of house prices

Introduction

- GE life-cycle model with incomplete markets, aggregate risk, and incomplete info.
  - Kaplan et al. (2020), Favilukis et al. (2017), Hoffman (2016)
- Unknown evolution of house prices via adaptive learning leads to persistently positive forecast errors give rise to endogenously optimistic beliefs
  - consistent with survey evidence from Kindermann et al. (2022) and a novel empirical proxy from the Michigan Survey
- Matches the time path, standard deviation, and autocorrelation of aggregate house prices throughout the 2000s in addition to the increase
- Muted direct impact of looser credit conditions on higher house prices
  - credit conditions account for rise in homeownership rate and mortgage leverage
- (Literature Review)

A Simple Model

- Households value housing at price p and consumption goods at price 1

$$U(c, h') = c + \phi h', \quad s.to. \quad c + ph' = ph$$

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$$p = \phi + \mathbb{E}[p']$$

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$$p = \frac{\phi + a^0}{1 - a^1}$$

- Let beliefs be given as $p' = a^0 + a^1 p$, then prices p increase via ϕ or $\boldsymbol{a} = (a^0, a^1)'$

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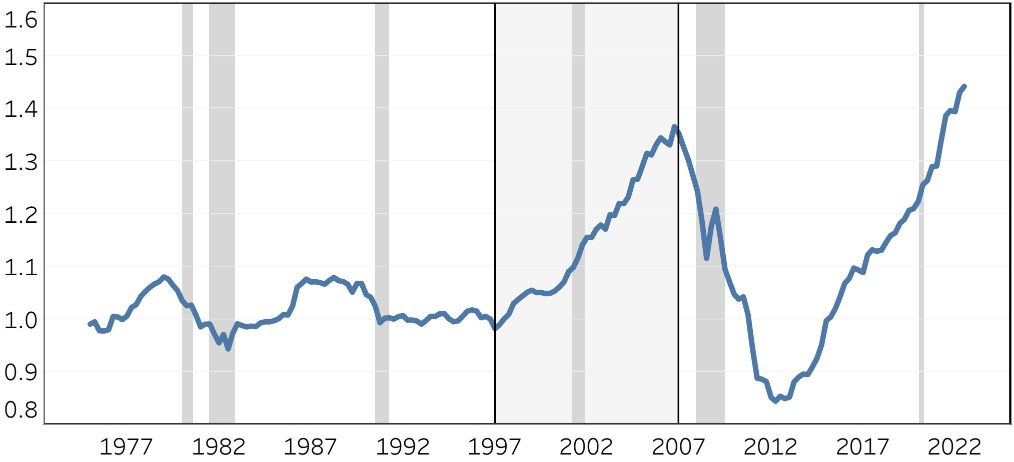
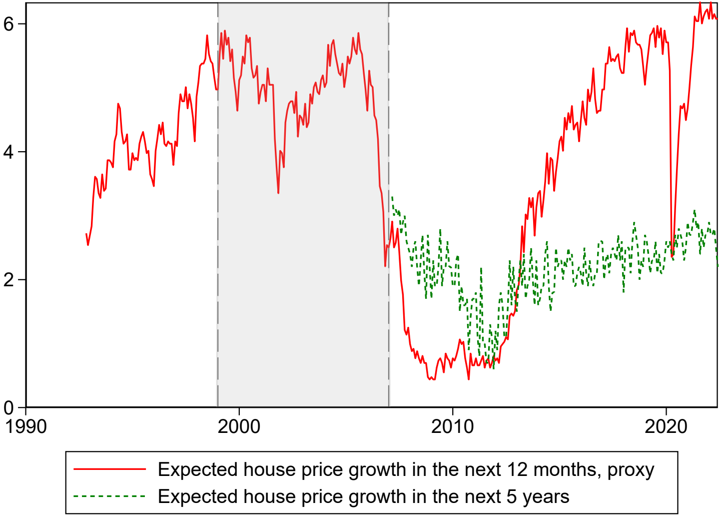
- Let beliefs be given as $p' = a^0 + a^1 p$, then prices p increase via ϕ or $\boldsymbol{a} = (a^0, a^1)'$
- Optimistic beliefs as a driver of higher house prices requires co-moving p and $\boldsymbol{a}$, the opposite of rational expectations

Real FHFA National House Prices, 1=1997



Expectations: Highest During Booms



Source: The University of Michigan Survey of Consumers, authors' calculations

Environment

- Notation
  - h' $\implies$ next period h
  - lowercase: individuals
  - uppercase: aggregate $\implies$ $H' = \int h' d\mu$
- Nests Kaplan et al. (2020), beliefs endogenous instead of exogenous
- Continuum of heterogeneous finitely lived renters and homeowners aged j
  - non-separable preferences over housing and consumption, utility premium for homeowners (preferences details)
  - bequest in final period of life (bequest details)

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- Standard income endowment while working subject to taxes $\mathcal{T}(y)$ ‣ (income tax details)

$$\log y = \underbrace{\log \Theta}_{\text{aggregate stochastic}} + \underbrace{\chi_j}_{\text{deterministic life-cycle}} + \underbrace{\epsilon}_{\text{idiosyncratic stochastic}}$$

Environment

Incomplete markets

- One-period liquid financial instruments b at risk-free price q_b for all households
- rental housing $\tilde{h}$ costs ρ, renter individual states: $\{b, \epsilon, j\}$
- homeowners finance housing h at price p via multi-period mortgages m at price q_j
- homeowner individual states: $\{h, b, m, \epsilon, j\}$

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Beliefs

- full knowledge of exogenous shocks $\{Z, \epsilon\}$
- bounded rationality: approximate the distribution μ, $\mathcal{Z} = \{Z, Z'\}$

$$\mu' = \Gamma_\mu(\mu, \mathcal{Z}) \quad \iff \log p'(\mu, \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(\mu, Z)$$

Households' Problem

Renters

- Rent: $c + \rho(\mu, Z)\tilde{h}' + q_b b' \leq b + y - \mathcal{T}(y, 0)$
- Own: $c + q_b b' + p_h(\mu, Z)h' + \kappa^m \leq b + y - \mathcal{T}(y, 0) + q_j(\boldsymbol{x}', y; \mu, Z)m'$

subject to, ▸ m'_{min}

loan-to-value (LTV)	$m' \leq \theta^{LTV} p h'$
payment-to-income (PTI)	$m'_{min} \leq \theta^{PTI} y$

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Howeowners

- Sell house: $b^n = b + \underbrace{(1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m}_{\text{liquidated housing equity}}$

Households' Problem

Renters

- Rent: $c + \rho(\mu, Z)\tilde{h}'_0 + q_b b' \leq b + y - \mathcal{T}(y, 0)$
- Own: $c + q_b b' + p_h(\mu, Z)h' + \kappa^m \leq b + y - \mathcal{T}(y, 0) + q_j(\boldsymbol{x}', y; \mu, Z)m'$

subject to, ▸ m'_{min}

$$\begin{aligned} \text{loan-to-value (LTV)} \qquad m' &\leq \theta^{LTV} ph' \\ \text{payment-to-income (PTI)} \qquad m'_{min} &\leq \theta^{PTI} y \end{aligned}$$

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- Sell house: $b^n = b + \underbrace{(1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m}_{\text{liquidated housing equity}}$
- Default and rent smallest housing unit $\tilde{h}'_0$ with a utility penalty

Households' Problem

Renters

- Rent: $c + \rho(\mu, Z)\tilde{h}' + q_b b' \leq b + y - \mathcal{T}(y, 0)$
- Own: $c + q_b b' + (\delta_h + \tau_h)p(\mu, Z)h + (1 + r_m)m \leq b + y - \mathcal{T}(y, m) + m'$

subject to, ▸ m'_{min}

$$\begin{aligned} \text{HELOC} \qquad & -b' \leq \theta^{HELOC} p(\mu, Z)h \\ \text{Min. payment} \qquad & m_{min} \leq (1 + r_m)m - m' \end{aligned}$$

Howeowners

- Sell house: $b^n = b + \underbrace{(1 - \delta_h - \tau_h - \kappa_h)p(\mu, Z)h - (1 + r_m)m}_{\text{liquidated housing equity}}$
- Default and rent smallest housing unit $\tilde{h}'_0$ with a utility penalty
- Stay in house and pay mortgage

Households' Problem

Renters

- Rent: $c + \rho(\mu, Z)\tilde{h}' + q_b b' \leq b + y - \mathcal{T}(y, 0)$
- Own:

$$c + q_b b' + \kappa^m + (\delta_h + \tau_h) p(\mu, Z) h + (1 + r_m) m \leq b + y - \mathcal{T}(y, m) + q_j(\boldsymbol{x}', y; \mu, Z) m'$$

subject to, ▸ m'_{min}

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- Sell house: $b^n = b + \underbrace{(1 - \delta_h - \tau_h - \kappa_h) p(\mu, Z) h - (1 + r_m) m}_{\text{liquidated housing equity}}$
- Default and rent smallest housing unit $\tilde{h}'_0$ with a utility penalty
- Stay in house and or refinance

Housing demand

▸ (closing the model) ▸ (RCE definition)

- Asset pricing equation for homebuyers, $\mathcal{Z} = \{Z, Z'\}$

$$p(\mu, Z) \leq \frac{U_{h'}}{U_c} + \mathbb{E}_{Z',\epsilon'|Z,\epsilon}[\mathcal{M}_b'(1 - \delta_h - \tau_h - \mathbb{1}_{h' \neq h''})p'(\mu, \mathcal{Z})]$$

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- Exogenous beliefs: news shock to housing preferences pushes up prices
  - households speculate demand may increase and buy today
  - front load, i.e. skip the starter home

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- Krusell and Smith (1998) solution method

$$\log p'(\mu, \mathcal{Z}) = a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p(\mu, Z)$$

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- Standard Krusell and Smith (1998) solution method (algorithm details)

$$\log p'(\mu, \mathcal{Z}) = \underbrace{a^0_{\mathcal{Z}} + a^1_{\mathcal{Z}} \log p_{\mathcal{Z}_t}}_{\boldsymbol{a}_{\mathcal{Z}} \boldsymbol{x}_{\mathcal{Z}_t}}$$

$$\boldsymbol{a}_{\mathcal{Z}} = \left(\sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \boldsymbol{x}'_{\mathcal{Z}_t}\right)^{-1} \sum_{t=1}^{T} \boldsymbol{x}_{\mathcal{Z}_t} \log p'_{\mathcal{Z}_t}$$

Housing demand ▸ (closing the model) ▸ (RCE definition)

- Asset pricing equation for homebuyers, $\mathcal{Z} = \{Z, Z'\}$

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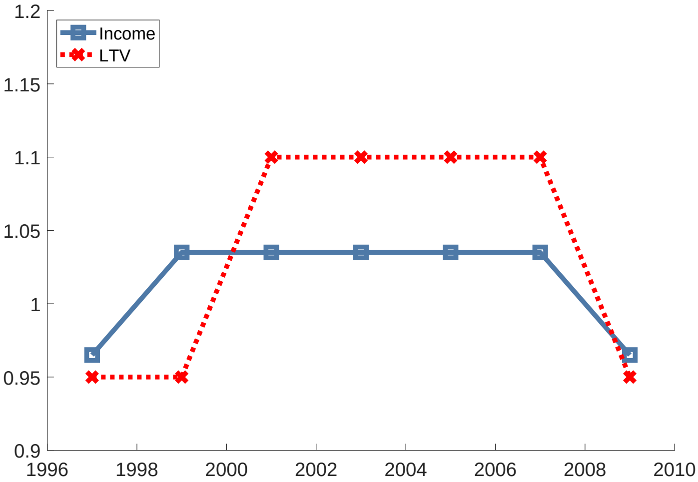
- Lagged belief updating Krusell and Smith (1998) solution method ▸ (algorithm details)

$$\log p'(\mu, \mathcal{Z}) = \underbrace{a^0_{\mathcal{Z}_t} + a^1_{\mathcal{Z}_t} \log p_{\mathcal{Z}_t}}_{\boldsymbol{a}_{\mathcal{Z}_t} \boldsymbol{x}_{\mathcal{Z}_t}}$$

$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{t-1} + g_t \boldsymbol{x}_{t-2} \underbrace{\left(\log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{t-1}\right)}_{e_{t-1}}$$

- Agents adjust beliefs with incoming information
- Evolution of house prices *un*known in economic states without historical precedent
  - $t - 1$ information in belief formation avoids simultaneity when determining p_t

Housing Boom Simulation: Aggregate Shocks



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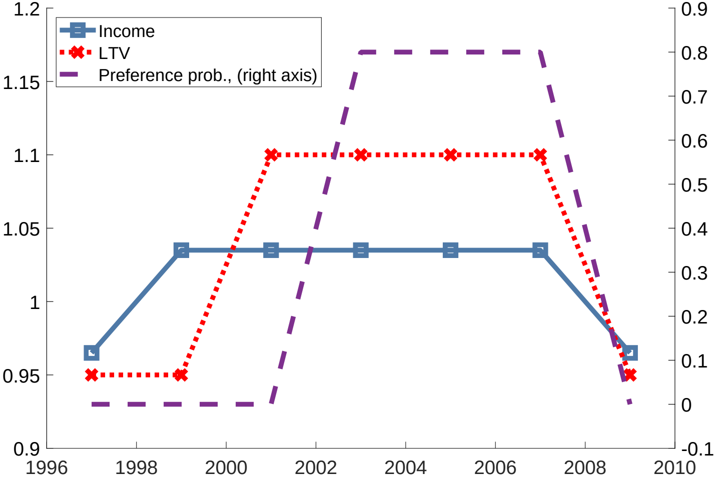
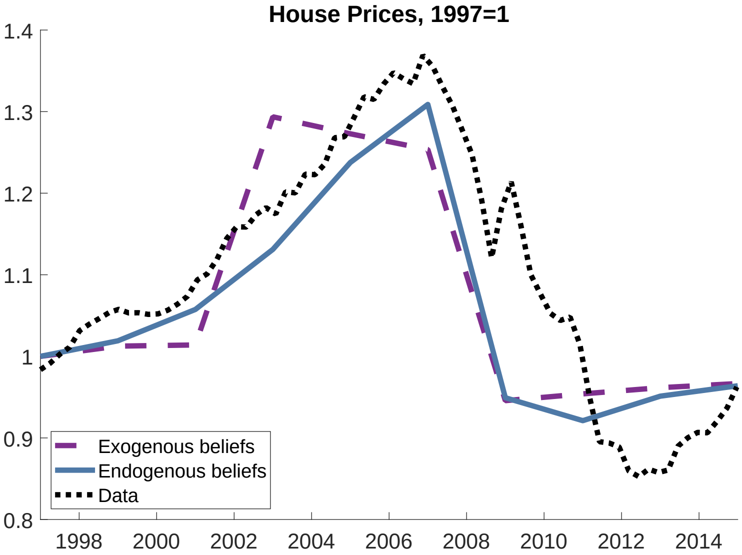


- Exogenous beliefs: housing preference parameter transitions to a "news" state where there is a non-zero probability it can increase

Housing Boom Simulation Results

(solution details)



Learning Calibration

- Learning begins at the onset of the housing boom when aggregate income transitions to high state and credit conditions loosen one-time

$$\boldsymbol{a}_{\mathcal{Z}_t} = \boldsymbol{a}_{t-1} + g_t \boldsymbol{x}_{t-2} \underbrace{\left(\log p_{t-1} - \boldsymbol{x}'_{t-2}\boldsymbol{a}_{t-1}\right)}_{e_{t-1}}$$

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- Initial coeffs.: known non-boom values match the initial forecast error, $\boldsymbol{a}_0 = \boldsymbol{a}^{tight}_{Z_{high}, Z'_{high}}$

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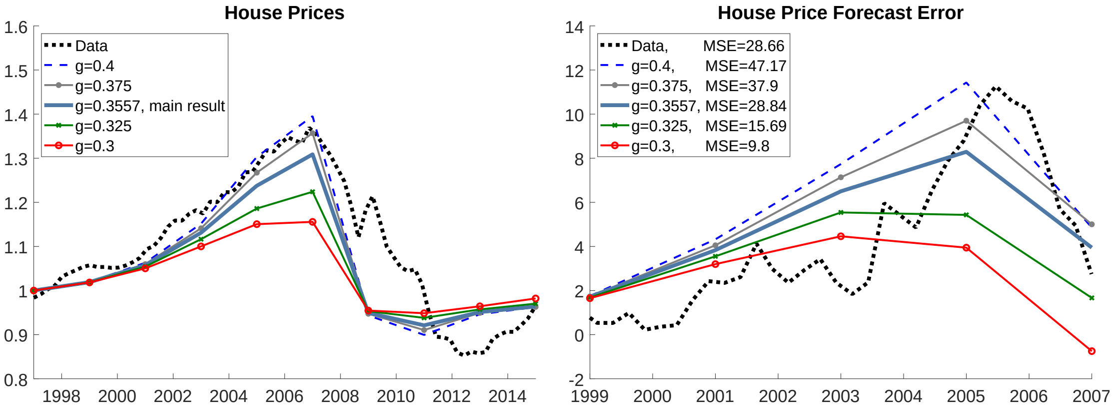
- Initial coeffs.: known non-boom values match the initial forecast error, $\boldsymbol{a}_0 = \boldsymbol{a}^{tight}_{Z_{high}, Z'_{high}}$
- Mixed gain follows Marcet and Nicolini (2003) and Milani (2014)

$$g_t = \begin{cases} g & \text{in boom} \\ 1/t & \text{non-boom} \end{cases}$$

- $g = 0.3557$, minimized difference of house price mean squared forecast errors e_{t-1} from the model and an empirical proxy as in Caines (2020)
- t=100, only sensitive for small $t < 40$

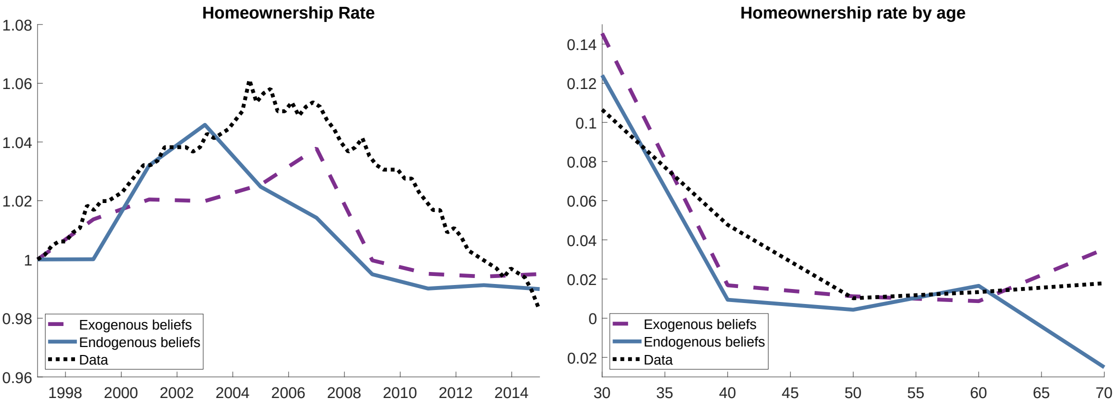
Housing Boom Simulation Results

▸ (beliefs proxy)



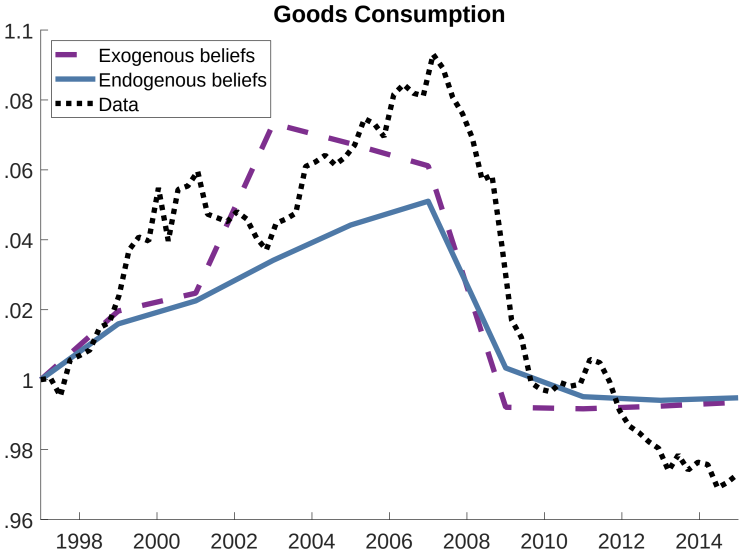
Housing Boom Simulation Results

(life-cycle calibration)



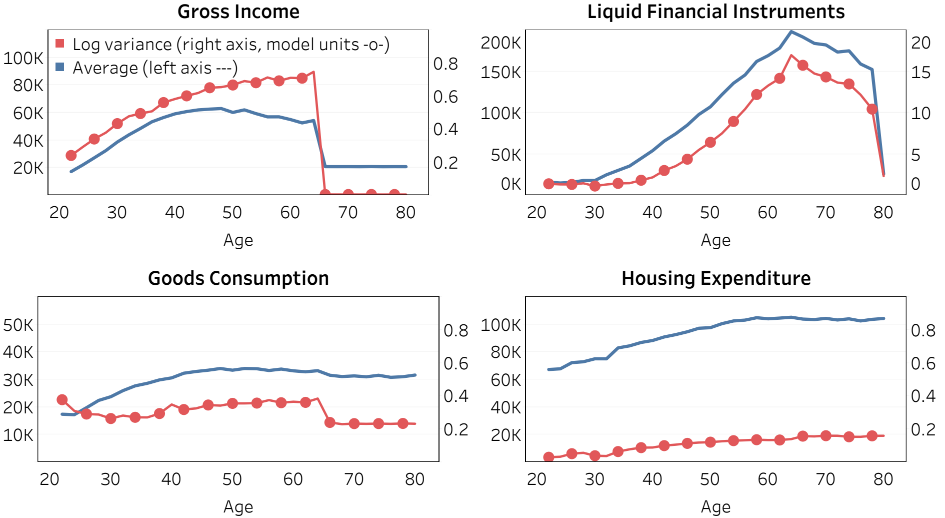
Housing Boom Simulation Results

▸ (other quantities) ▸ (alternative simulations) ▸ (belief convergence)



Life-Cycle Results (1998 $) follow Kaplan et al. (2020)

▸ (parameters) ▸ (calibration)



Conclusion

- *Why* beliefs about future house prices shifted in the 2000s
- Looser credit conditions in an economic expansion lacked precedent
  - learning about the evolution of house prices gives rise to persistently positive forecast errors and endogenously optimistic beliefs
  - consistent with Kindermann et al.'s (2022) survey and a novel empirical proxy
- Endogenous beliefs
  - match the time path, standard deviation, and autocorrelation of aggregate house prices in the 2000s boom
  - allow for housing booms more frequent than once a generation

Functional Forms

- Preferences

$$U_j(c,s) = e_j \frac{\left[(1-\phi)c^{1-\gamma} + \phi s^{1-\gamma}\right]^{\frac{1-\sigma}{1-\gamma}}}{1-\sigma}$$

- McClement's scale (e_j)
- Risk aversion (σ), intertemporal elast. of substitution (γ), housing preference (ϕ)

Functional Forms

- Warm glow bequest: De Nardi (2004)

$$\mathbb{E}_0 \sum_{j=1}^{J} \beta^{j-1} U_j(c_j, s_j) + \beta^J \psi \frac{(b' + (1 - \delta_h - \tau_h - \kappa_h) p' h + \underline{b})^{1-\sigma}}{1-\sigma}$$

- Strength of bequest motive (ψ), bequests as luxuries ($\underline{b}$)

Functional Forms

- Taxes: Heathcote et al. (2017)

$$\mathcal{T}(y) = y - \tau_y^0 (y)^{1-\tau_y^1}$$

- τ_y^0: average level of taxation, τ_y^1: degree of progressiveness

Functional Forms

- constant amoritization: $m'_{min} = \dfrac{r_m(1+r_m)^{J-j}}{(1+r_m)^{J-j}-1} m'$

Literature Review

Credit conditions

- Account for housing boom: Mian and Sufi (2009, 2017), Liu et al. (2013), Cox and Ludvigson (2021), Greenwald and Guren (2021), Arslan et al. (2022), Greenwald (2018), Justiniano et al. (2019)
- Tough to reconcile: Albanesi et al. (2017), Adelino et al. (2018), Foote et al. (2012), Kiyotaki et al. (2011)

Optimistic beliefs

- Empirical evidence: Case and Shiller (1988, 2004), Case et al. (2012), Soo (2018), Armona et al. (2019), Ben-David et al. (2019), De Stefani (2021)
- Adaptive learning: Caines (2020), Kindermann et al. (2022), Adam et al. (2012), Boz and Mendoza (2014), Kuang (2014)
- Why did beliefs shift? Chodorow-Reich et al. (2021), Howard and Liebersohn (2022), Adam et al. (2022)

Closing the Model

- Competitive final goods firms
$$Y = \Theta(Z) N_c \quad \Longrightarrow \quad w = \Theta(Z)$$
- Construction firms solve:
$$\max_{N_h} \left\{ \; p(\mu, Z)[\Theta(Z) N_h]^{\alpha} \bar{L}^{1-\alpha} - w N_h \right\}$$
$$H_h = (\alpha p)^{\frac{\alpha}{1-\alpha}} \bar{L}, \quad \text{since } \Theta(Z) = w$$
- Labor inelastically supplied and mobile across sectors
- Rental rate determined by competitive rental sector that owns housing:
$$\rho(\mu, Z) = \Xi + p(\mu, Z) - (1 - \delta_h - \tau_h)\mathbb{E}_{Z', \epsilon' | Z, \epsilon}[q_b p'(\mu', Z')]$$
- Mortgages priced by lenders' zero-profit conditions:
$$q_j(x'; \mu, Z) = -\zeta + \frac{\mathbb{E}_{Z', \epsilon' | Z, \epsilon}}{(1 + r_m) m'} \begin{cases} (1 + r_m) m' & \text{if sell/refi} \\ (1 - \delta_h^d - \tau_h - \kappa_h) p'(\mu', Z') h' & \text{if default} \\ (1 + r_m) m' - m'' + q_{j+1}(b'', h'', m'', y', \mu', Z') m'' & \text{if pay} \end{cases}$$

Recursive Competitive Equilibrium

- Income endowments y to households aged j
- Prices for loans, houses, rental houses, wages $\{q_j(\mu, Z), p(\mu, Z), \rho(\mu, Z), w(\mu, Z)\}$
- Government parameters for the loan-to-value, payment-to-income, HELOCs, land permits, taxes, and social security $\{\theta^{LTV}(Z), \theta^{PTI}(Z), \theta^{HELOC}(Z), \bar{L}, \mathcal{T}(y, m), \tau_h, \rho_{SS}\}$
- Perceived laws of motion for the state space $\mu' = \Gamma_\mu(\mu, Y, Y')$ where $\mu = \mu^r + \mu^h$
- Value & policy functions solve the hhs' problem. Firms maximize profits, markets clear

$$
\begin{aligned}
\text{Assets}: &\quad \int_{\mathcal{X}^h} b' d\mu^h + \int_{\mathcal{X}^r} b' d\mu^r = B' \\
\text{Mortgages}: &\quad \int_{\mathcal{X}^h} m' d\mu^h = M' \\
\text{Rentals}: &\quad \int_{\mathcal{X}^r} \tilde{h}' d\mu^r + \int_{\mathcal{X}^h} \tilde{h}' d\mu^h = \tilde{H}' \\
\text{Housing}: &\quad \tilde{H}' - (1-\delta_h)\tilde{H} + \int_{\mathcal{X}^h} h' d\mu^h = H_h - \delta_h \int_{\mathcal{X}^h} h d\mu^h + \\
&\qquad \ldots \int_{\mathcal{X}^h} h[\mathbb{1}_{sell} + \mathbb{1}_{default}(1 - \delta_h^d + \delta_h)] d\mu^h + \int_{\mathcal{X}^h} \mathbb{1}_{bequest} h' d\mu^h \\
\text{Labor}: &\quad \int (\chi_j + \epsilon) d\mu_{J_{work}} = N_c + N_h \\
\text{Gov't}: &\quad \mathcal{T}(y, m) + \tau_h p(\mu, Z) \int_{\mathcal{X}^h} h d\mu^h + [p(\mu, Z) H_h - w(\mu, Z) N_h] = \rho_{ss} \int_{\mathcal{X}} y_{ret} d\mu_{\mathcal{J}_{ret}} + G \\
\text{Net exp.}: &\quad (\rho(\mu, Z) - \Xi)\tilde{H}' + \int_{\mathcal{X}^r} [b - q_b b'] d\mu^r + \int_{\mathcal{X}^h} [b - q_b b' \mathbb{1}_{[b'>0]} - \left(r_b(1+\iota)\right)^{-1} b' \mathbb{1}_{[b'<0]}] d\mu^h + \\
&\qquad \ldots \int_{\mathcal{X}^h} [(1 + r_m)m + q_j(\mathbf{x}', y; \mu, Z) m'] d\mu^h - p(\mu, Z)[\tilde{H}' - (1 - \delta_h - \tau_h)\tilde{H}] = NX \\
\text{ARC}: &\quad \int_{\mathcal{X}^h} c d\mu^h + \int_{\mathcal{X}^r} c d\mu^r + G + NX + \Xi \tilde{H}' = Y - \kappa p(\mu, Z) \int_{\mathcal{X}^h} h(\mathbb{1}_{sell} + \mathbb{1}_{default}) d\mu^h - \\
&\qquad \ldots \iota r_b \int_{\mathcal{X}^h} (m + b\mathbb{1}_{\{b<0\}}) d\mu^h - (\zeta + \kappa^m) \int_{\mathcal{X}^h} m'(\mathbb{1}_{buy} + \mathbb{1}_{refi}) d\mu^h
\end{aligned}
$$

Solution Method: Benchmark Krusell and Smith (1998)

1. Define grid over p and guess coefficients $\boldsymbol{a}_{\mathcal{Z}}$ to forecast p' for each $\mathcal{Z} = \{Z, Z'\}$
2. Solve individual borrowers' problem: value function iteration with grid search
3. Simulate $\{Z_t\}_{t=1}^{5000}$ agg. realizations once, and individual policy functions for N$=150000$ each period (Details)
5. Aggregate policy functions for rental and owner-occupied housing each period:

$$\tilde{H}_{t+1}(p, Z_t) = \frac{1}{N} \sum_{i=1}^{N} \tilde{h}'(b_{i,t}^n, \epsilon_{i,t}, j_{i,t}; p, Z_t)$$
$$H_{t+1}(p, Z_t) = \frac{1}{N} \sum_{i=1}^{N} h'(b_{i,t}^n, h_{i,t}, m_{i,t}, \epsilon_{i,t}, j_{i,t}; p, Z_t)$$

6. Compute excess demand functions for housing to find market clearing price p_t^*

$$H_{t+1}(p_t^*, Z_t) + \tilde{H}_{t+1}(p_t^*, Z_t) = H_h + (1 - \delta_h)[H_t + \tilde{H}_t]$$

7. Interpolate all individual policy functions at p_t^*
8. Partition $\{p_t^*\}_{t=burn}^{5000}$ into $\mathcal{Z} = \{Z, Z'\}$ sub-samples, compute new OLS coefficients

$$\log p_{t+1}^* = a_{\mathcal{Z}}^{0,new} + a_{\mathcal{Z}}^{1,new} \log p_t^*$$

9. Repeat until $\{\boldsymbol{a}_{\mathcal{Z}}\} \approx \{\boldsymbol{a}_{\mathcal{Z}}^{new}\}$

Auctioneer Iteration: Chipeniuk et al. (2022)

10. Solve households' problem setting the p grid to select values of p_t^* obtained from simulation in step 6 with the converged coefficients $\boldsymbol{a}_{\mathcal{Z}}$ obtained in step 9
11. If $|p_t^{*,new} - p_t^*| < e^{-4}$ stop, otherwise
12. Compute new coefficients
$$\log p_{t+1}^{*,new} = a_{\mathcal{Z}}^{0,*,new} + a_{\mathcal{Z}}^{1,*,new} \log p_t^{*,new}$$
13. Repeat steps 11-12 until convergence

1. Define grids over $\{p, p'(Z'_{low}), p'(Z'_{high})\} \implies$ parallelizable
2. Set initial beliefs to low/tight $\boldsymbol{a}_{\mathcal{Z}_0} = \boldsymbol{a}_{Z_{l,l}}$ and compute time t coefficients:
$$\boldsymbol{a}_{\mathcal{Z}_t}(p) = \boldsymbol{a}_{\mathcal{Z}_{t-1}} + g_t \boldsymbol{R}_{\mathcal{Z}_t} \boldsymbol{x}_{t-2}(\log p_{t-1} - \boldsymbol{x}'_{t-2} \boldsymbol{a}_{\mathcal{Z}_{t-1}})$$
3. Simulate individual housing policy functions for $\text{N}=150000$ and aggregate
$$\tilde{H}_{t+1}(p, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t) = \frac{1}{N} \sum_{i=1}^{N} \tilde{h}'(b^n_{i,t}, \epsilon_{i,t}, j_{i,t}; p, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t)$$
$$H_{t+1}(p, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t) = \frac{1}{N} \sum_{i=1}^{N} h'(b^n_{i,t}, h_{i,t}, m_{i,t}, \epsilon_{i,t}, j_{i,t}; p, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t)$$

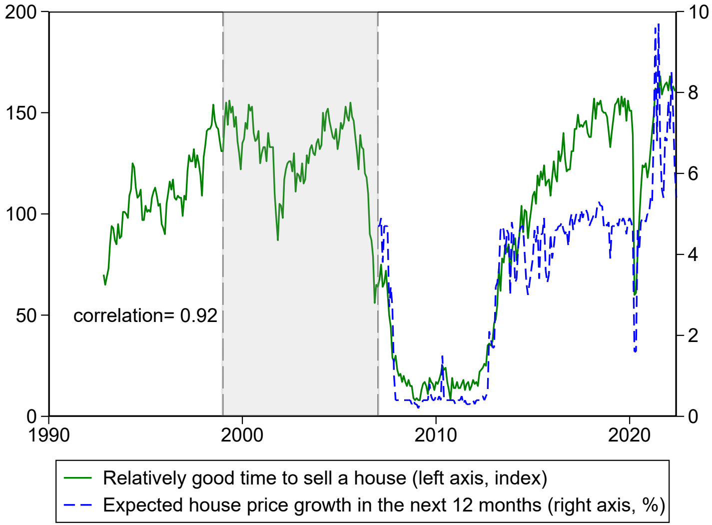
$$\log \boldsymbol{p}_{Z_t} = \left(\exp\{a^0_{Z_t, Z'_{low}} + a^1_{Z_t, Z'_{low}} \log p^*_t(Z_t)\}, \exp\{a^0_{Z_t, Z'_{high}} + a^1_{Z_t, Z'_{high}} \log p^*_t(Z_t))\}\right)'$$

4. Compute excess demand functions for housing to find market clearing price p^*_t
$$H_{t+1}(p^*_t, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t) + \tilde{H}_{t+1}(p^*_t, \boldsymbol{p}_{\mathcal{Z}_t}, Z_t) = H_h + (1 - \delta_h)[H_t + \tilde{H}_t]$$
5. Interpolate individual policy functions at p^*_t to see that $\boldsymbol{a}_{\mathcal{Z}_t}$ are close to their known values

Simulation of Individual Borrower States

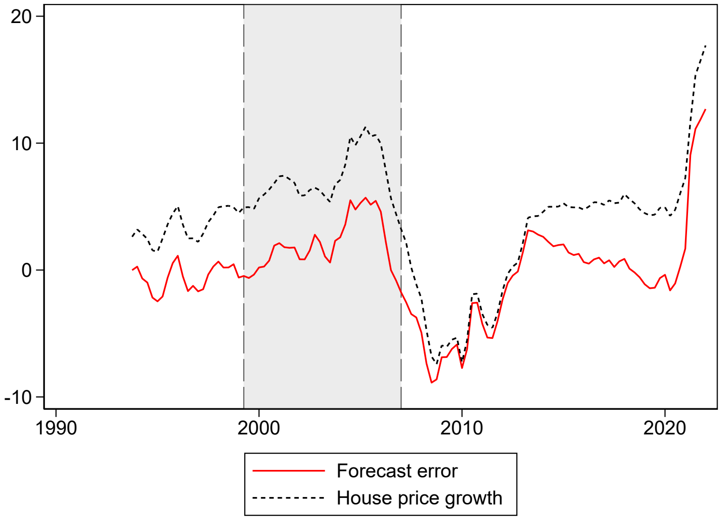
- All households start economic life $j = 1$ as renters. In $t = 1$, $b = 0$ for all hhs
- Initial income ϵ and housing, h or $\tilde{h}$, correlated and follows Kaplan et al. (2020)
- Initial ages drawn from a uniform dist. $j_0 \sim \mathcal{U}\{1, \ldots, J\}$
- After household i exits, i.e. $j_{i,t} = J$, a new household replaced that agent that agent with $j_{i,t+1} = 1$
  - households start life with no debt $m = 0$ as renters
  - receive liquid wealth b from liquidated bequested housing, inherit a random draw correlated with their individual income

Minimizing the House Price Forecast Error: Beliefs Proxy

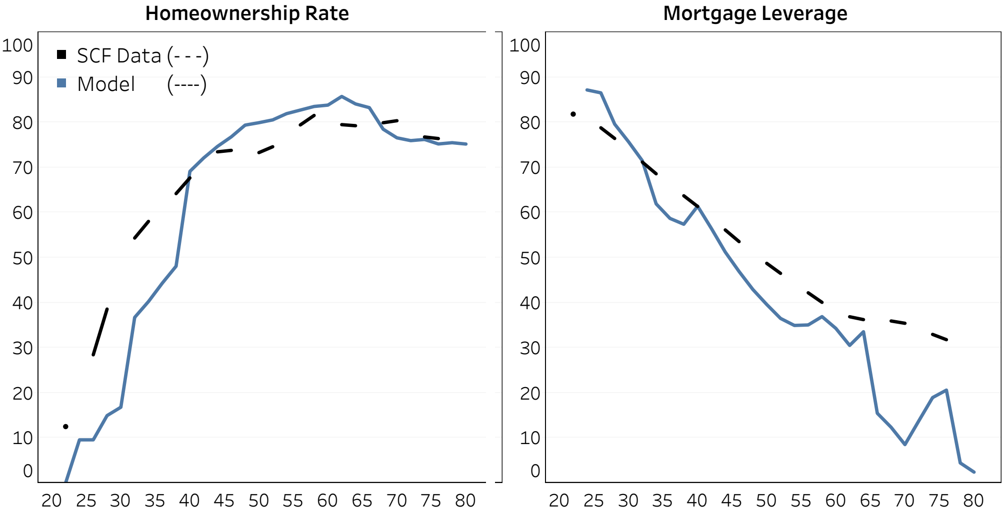


Responses from the University of Michigan Surveys of Consumers

Minimizing the House Price Forecast Error: Beliefs Proxy



Life-Cycle Results, Percentage Points follow Kaplan et al. (2020)



Parameter Values, 1=$52K 1998 SCF average annual income

Demographics			
Maximum age	J	30	N
Retirement age	J^{ret}	23	N
Preferences			
Inverse elasticity of substitution	γ	0.8	N
Risk aversion	σ	2	N
Discount factor	β	0.967	Y
Strength of bequest motive	ψ	100	N
Extent of bequests as a luxuries	$\underline{b}$	7.7	N
Taste for housing	ϕ	0.13	N
Additional utility from owning	ω	1.015	N
Utility cost of foreclosure	ξ	0.8	N
Individual income			
Deterministic income	$\{\chi_j\}$	Kaplan & Violante (2014)	N
Annual persistence, ind. income	ρ_ϵ	0.97	Y
Annual st. dev., ind. income	σ_ϵ	0.20	Y
Initial st. dev., ind. income	σ_{ϵ_0}	0.42	Y
Distribution of bequest to new hhs	$b_{j=1} = b'_{j=J}$	Kaplan & Violante (2014)	N

Parameter Values, 1=$52K 1998 SCF Average Annual Income

Housing			
Owner-occupied housing unit sizes	$\mathcal{H}$	$\{1.5, 1.92, 2.46, 3.15, 4.03, 5.15\}$	N
Rental housing unit sizes	$\tilde{\mathcal{H}}$	$\{1.125, 1.5, 1.92\}$	N
Depreciation rate of housing	δ_h	0.015	Y
Housing loss in foreclosure	δ_h^d	0.22	Y
Housing transaction cost	κ_h	0.07	N
Operating cost of rental company	Ξ	0.003	N
Housing supply elasticity	$\alpha/(1-\alpha)$	1.5	N
New land permits	$\bar{L}$	0.311	N
Financial instruments			
Risk-free interest rate	r	0.025	Y
Interest rate wedge on borrowing	ι	0.33	N
Maximum HELOC	θ^{HELOC}	0.2	N
Government			
Property tax on housing	τ_h	0.01	Y
Income tax function	τ_y^0, τ_y^1	0.75,0.151	N
Mortgage interest deduction	ϱ	0.75	N
Social Security replacement rate	ρ_{SS}	0.42	N

The model period is two years and annualized values are noted in the final column with a Y.

Parameter Values, 1=$52K 1998 SCF Average Annual Income

Interpretation	Parameter	Value
Aggregate income Θ(Z)		
Aggregate income	$\{\Theta(high), \Theta(low)\}$	$\{1.035, 0.965\}$
Transition probability	$\pi^{\Theta}_{h,h} = \pi^{\Theta}_{l,l}$	0.9
Aggregate credit conditions $\mathcal{C}(Z)$		
Loan-to-value ratio	$\{\theta^{LTV}(loose), \theta^{LTV}(tight))\}$	$\{1.1, 0.95\}$
Payment-to-income ratio	$\{\theta^{PTI}(loose), \theta^{PTI}(tight)\}$	$\{0.5, 0.25\}$
Fixed origination cost	$\{\kappa^{m}(loose), \kappa^{m}(tight)\}$	$\{\$1,200, \$2,000\}$
Proportional origination cost	$\{\zeta(loose), \zeta(tight)\}$	$\{0.006, 0.010\}$
Beliefs/learning		
Constant gain	$g_t = g$	0.3557
Least-squares gain	t	100
Initial coefficients	$\boldsymbol{a}_0$	$\boldsymbol{a}_{Z_{high}, Z'_{high}}$
Normalization matrix	$R_{\mathcal{Z}_t}$	$\boldsymbol{I}$

The model period is two years and values are not annualized.

Targeted Calibration, 1998 SCF via Kaplan et al. (2020)

Moment	Parameter	Empirical Value	Model Value
Agg. net worth/annual agg. labor income	β	5.5	4.9
Median ratio of net worth to labor income	β	1.2	1.2
Median net worth: age 75/age 50	ψ	1.55	1.48
% of bequests in bottom 1/2 of wealth dist.	$\underline{\flat}$	0	0
Housing/total cons. expenditures	ϕ	0.16	0.16
Aggregate homeownership rate	ω	0.66	0.68
Foreclosure rate	ξ	0.005	0.001
P10 housing/total net worth of owners	$\min \mathcal{H}$	0.11	0.13
P50 housing/total net worth of owners	$\#\mathcal{H}$	0.5	0.32
P90 housing/total net worth of owners	gap $\mathcal{H}$	0.95	0.76
Average sized owned/rented house	$\min \tilde{\mathcal{H}}$	1.5	1.6
Average earnings of owners to renters	$\#\tilde{\mathcal{H}}$	2.1	2.7
Annual fraction of houses sold	κ_h	0.1	0.09
Homeownership rate of < 35 y.o.	Ξ	0.39	0.33
Employment in construction sector	$\bar{L}$	0.05	0.04

Housing Segmentation Assumptions, Percentage Points

House Size	Data Owners	Benchmark Model	No seg.	Full seg.	Partial seg.	Smaller size 1	Larger size 1
1	9		19		19		
2	24	55	23		22	46	49
3	25	9	23		24	17	14
4	18	13	13	89	12	13	13
5	10	17	16	2	16	18	18
6	9	4	5	9	5	5	4
7	6	2	1	0	1	1	1

House Size	Data Renters	Benchmark Model	No seg.	Full seg.	Partial seg.	Smaller size 1	Larger size 1
1	51	79	76	75	76	73	77
2	28	14	14	12	14	18	14
3	11	7	6	13	10	9	9
4	5		2				
5	2		2				
6	1		1				
7	2		0				

Untargeted Calibration

Parameter	Empirical Value	Model Value
Fraction of homeowners w/mortgage	0.66	0.64
Fraction of homeowners w/HELOC	0.06	0.02
Aggr. mortgage debt/housing value	0.42	0.46
P10 LTV ratio for mortgages	0.15	0.01
P50 LTV ratio for mortgages	0.57	0.48
P90 LTV ratio for mortgages	0.92	0.84
Share of NW held by bottom quintile	0	0
Share of NW held by middle quintile	0.05	0.09
Share of NW held by top quintile	0.81	0.67
Share of NW held by top 10 percent	0.7	0.42
Share of NW held by top 1 percent	0.46	0.06
P10 house value/earnings	0.9	0.93
P50 house value/earnings	2.1	1.8
P90 house value/earnings	5.5	4.1

Empirical values from 1998 Survey of Consumer Finances via Kaplan et al. (2020)

Computational Details: Benchmark

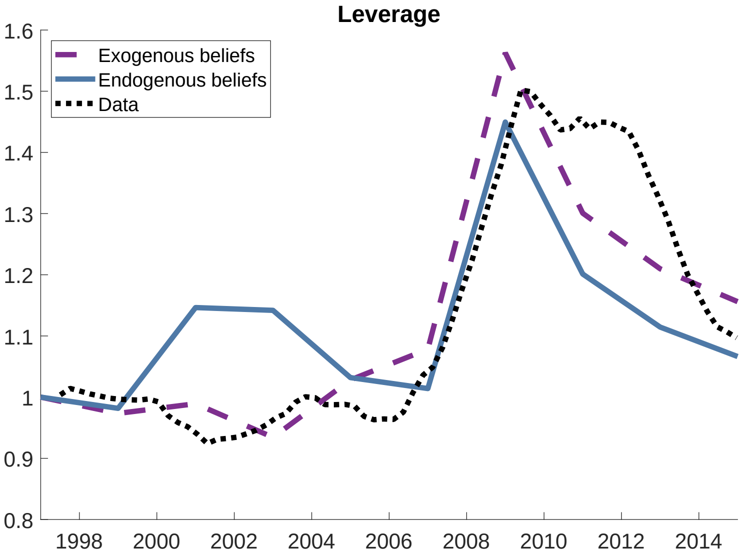
- Solved on Indiana University's Carbonate supercomputer, FRB Cluster, University of Dallas Big-Tex.

Number of grid points	
House prices (p)	13
Liquid financial instruments (b)	26
Mortgages	22

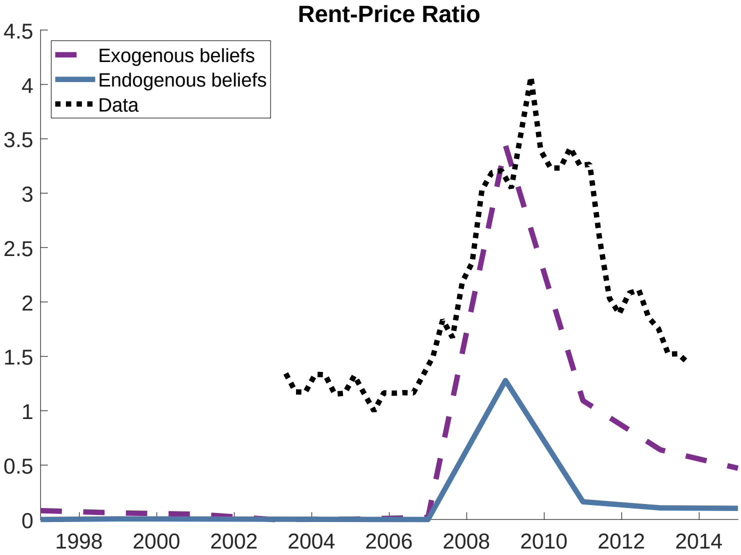
- Approximate law of motion of the distribution, $R^2 = 0.9999$

$$\begin{aligned}
\log P'_{high,high} &= -0.070 + 0.888 \log p \\
\log P'_{high,low} &= -0.084 + 0.888 \log p \\
\log P'_{low,high} &= -0.059 + 0.889 \log p \\
\log P'_{low,low} &= -0.073 + 0.889 \log p
\end{aligned}$$

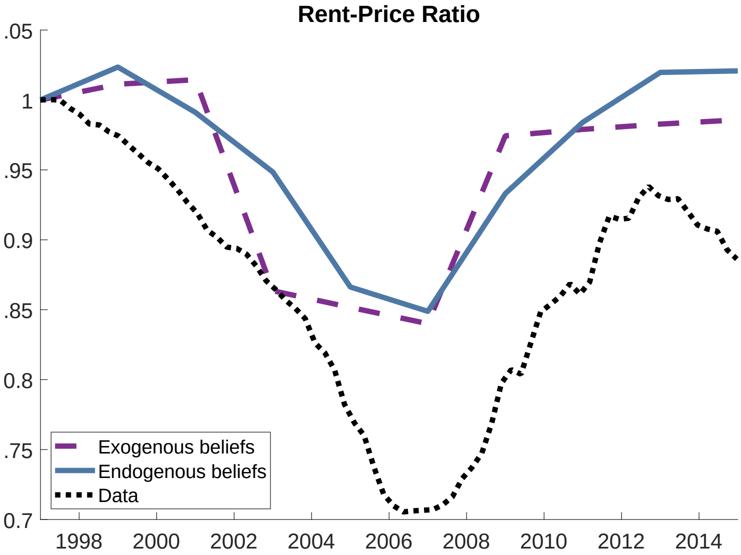
Housing Boom Simulation Results



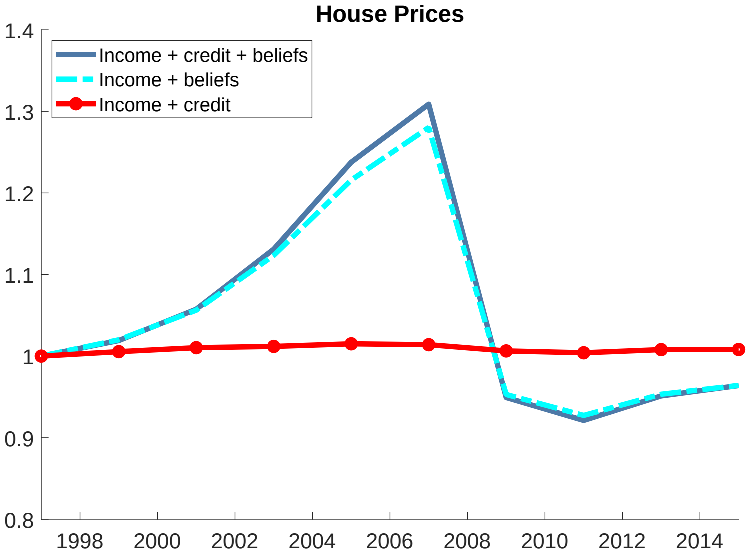
Housing Boom Simulation Results



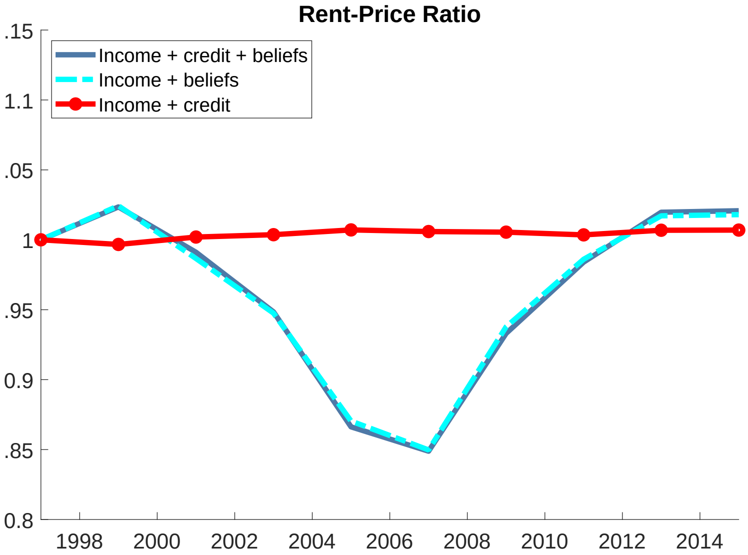
Housing Boom Simulation Results



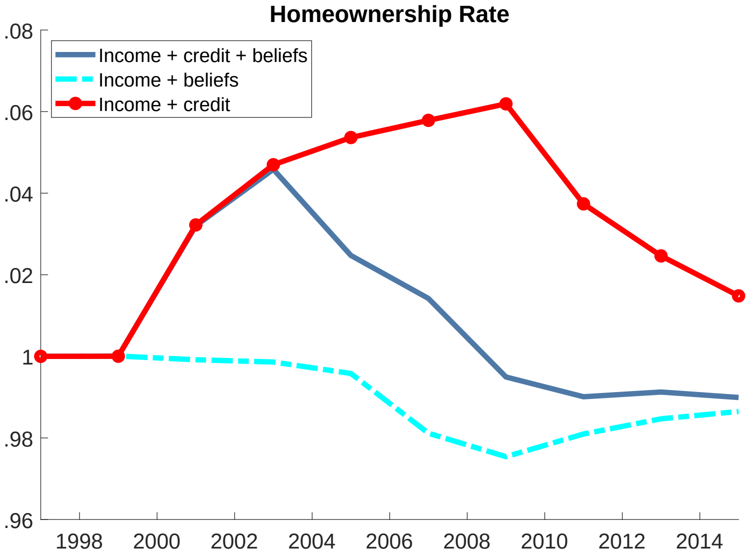
Alternative Housing Boom Simulation Results



Alternative Housing Boom Simulation Results



Alternative Housing Boom Simulation Results



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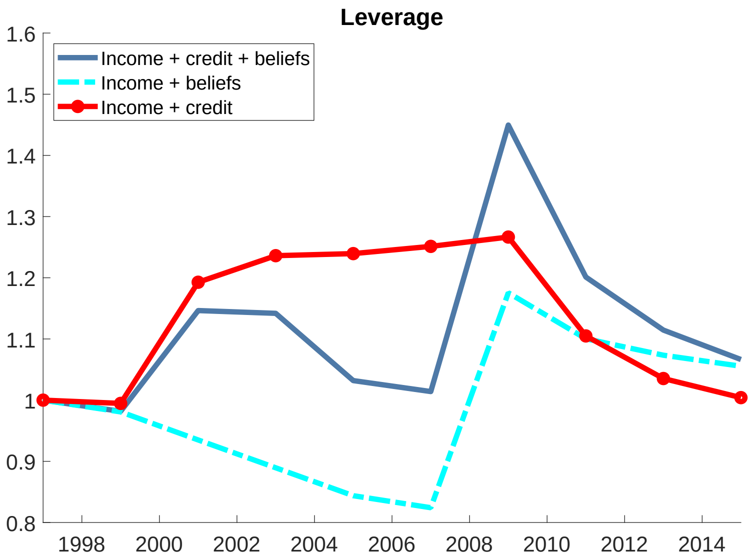

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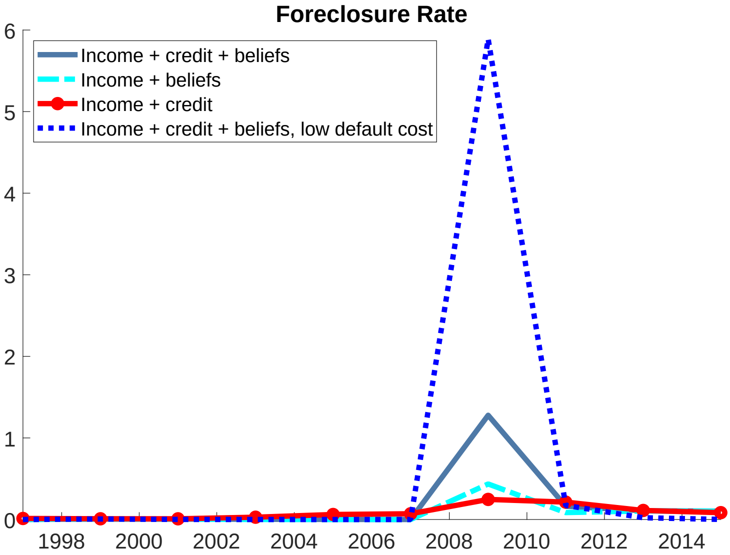

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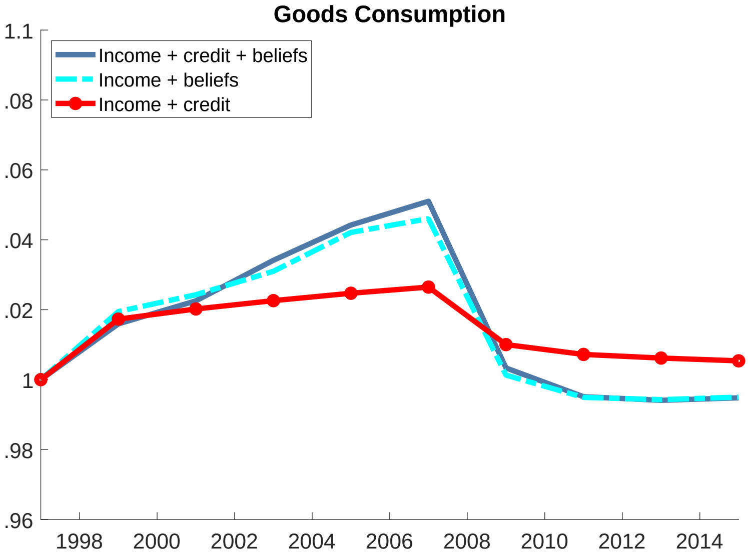

Beliefs Convergence