Innovation Contests with Distinct Approaches

Simon Block

University of Bonn

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Innovation Contests

- Principal sets up a *contest* to induce agents to create an *innovation*.
- Principal's objective: successful innovation (not just effort).
- Common uncertainty: is innovation feasible?
- Agents engage in a race:
  - fixed quality standard
  - variable date of discovery

Distinct Approaches

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Example: Vaccines can be categorized into distinct approaches.

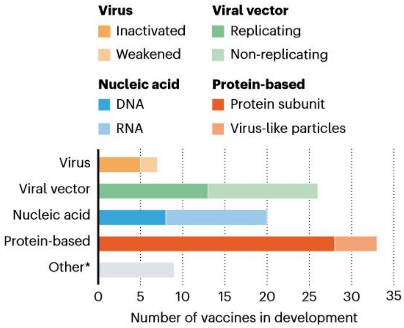

Figure 1: *from "The race for coronavirus vaccines" (Ewan Callaway, nature news feature, 28.4.20)*

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Example: Different technologies to remove CO_2 from the atmosphere.

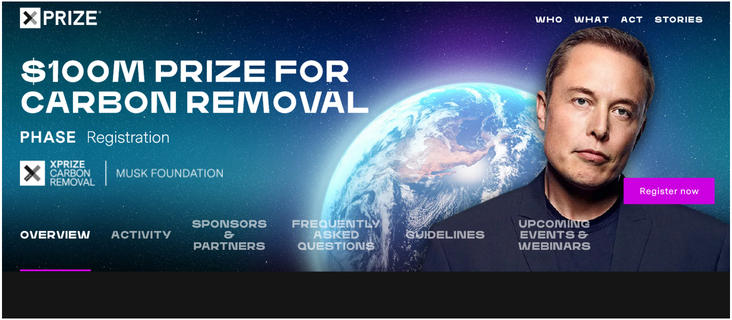

Figure 2: *$100M XPRIZE for a carbon removal technology* *(site: xprize.org)*

Correlated Successes

Figure 3: *Stock prices BioNTech and Moderna, November 2020*

- **9.Nov:** Biontec ↑↑, Moderna ↑
- **16.Nov:** Biontec ↓, Moderna ↑

Preview of Model and Results

How do approaches differ?

1. *Viability*: different costs, and probabilities of success.
2. *Correlation* of successes within approaches.
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- *How to identify the efficient assignment of agents to approaches?*
  With a greedy algorithm if costs are equal.
- *How can a desired behavior be implemented?*
  Contest with approach-specific prizes.

Related Literature

- *Innovation contests*:
  Halac, Kartik and Liu (2017), Choi (1991),
  Malueg and Tsutsui (1997) and many others
- *Contests with distinct approaches*:
  Letina (2016), Letina and Schmutzler (2019),
  Akcigit and Liu (2016), Cabral (2001) and others
- *Portfolio choice theory*:
  Chade and Smith (2006), Olszewski and Vohra (2016),
  Shorrer (2019)

Model: Agents and Approaches

- N identical, risk-neutral agents; 2 Periods.
- K distinct *approaches* $\{a_1, ..., a_K\} =: \mathcal{A}$.
- Unobservable state of the world: $(\theta_{a_1}, ..., \theta_{a_K}) \in \{\text{Good}, \text{Bad}\}^K$.

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1. Each agent chooses
   - to follow an approach $a \in \mathcal{A}$,
   - or to abstain.
2. All agents following $a \in \mathcal{A}$
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- Actions and successes publicly observed.

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- **Equilibrium**:
  - Perfect Bayesian equilibrium in pure strategies,
  - potentially asymmetric.

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- Holding $P_a \lambda_a$ fixed:
  - $\lambda_a \uparrow \Longrightarrow \rho_a \uparrow$
  - $P_a \uparrow \Longrightarrow \lambda_a \downarrow \Longrightarrow \rho_a \downarrow$
- Extremes:
  - $\lambda_a = 1 \implies \rho_a = 1$
  - $P_a = 1 \implies \rho_a = 0$

Correlation Matters

Example 1:

- 2 agents: 1 and 2
- 2 approaches: A and B:
  - identical costs: $c_A = c_B = c$
  - A more viable: $P_B \lambda_B = \frac{1}{4} < P_A \lambda_A = \frac{2}{3}$
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Insight

An approach-independent contest may induce inefficient equilibrium behavior.

Efficient Assignment

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Proposition 1

If *all approaches have equal costs,* then *a* greedy algorithm *that*

- *always adds the approach with the highest marginal benefit,*
- *until no approach has a positive marginal benefit,*
- *or until all agents are assigned*

identifies the social optimum.

Pseudo marginal social benefit: Denote by $mb_{a,i}(\pi)$ the *hypothetical* marginal social benefit of

an additional agent following a,

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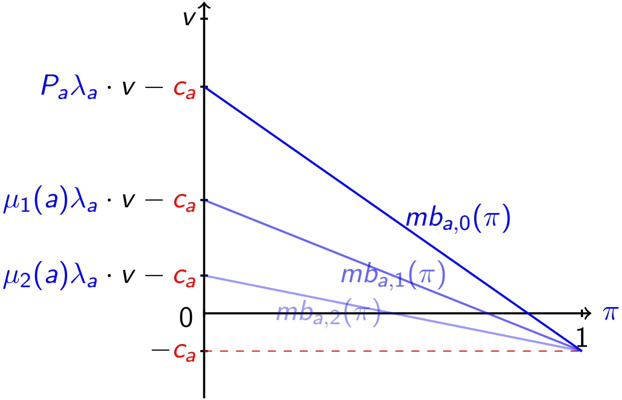
$$mb_{a,i}(\pi) := \mu_i(a)\lambda_a(1-\pi)\cdot v - c_a$$



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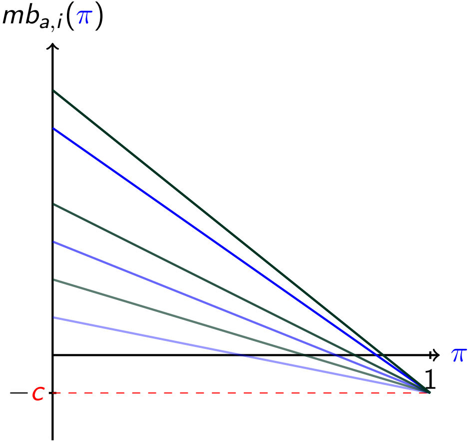

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If $c_a > 0$ *and* $P_a \lambda_a > 0$ *for all approaches,* then

the principal can **uniquely**[a] *implement* **any** *action profile,*

and extract (almost) the entire social surplus at the same time,

by selecting suitable approach-specific prizes $w_{a_1}, \ldots, w_{a_k}$.

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Corollary

It is optimal for the principal to implement the social optimum.

Two Periods: crowding-out

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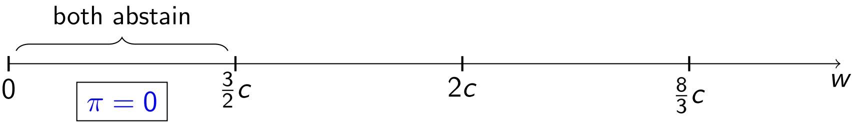

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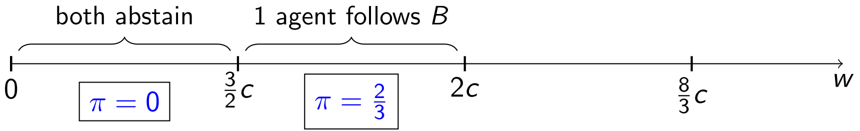

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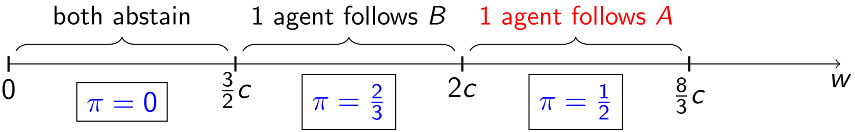

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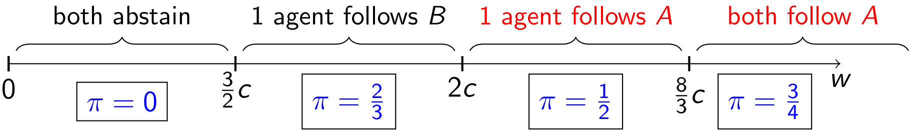

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Equilibria for varying w:

w		
0 to $\frac{3}{2}c$	both abstain	$\pi = 0$
$\frac{3}{2}c$ to $2c$	1 agent follows B	$\pi = \frac{2}{3}$
$2c$ to $\frac{8}{3}c$	1 agent follows A	$\pi = \frac{1}{2}$
above $\frac{8}{3}c$	both follow A	$\pi = \frac{3}{4}$

Insights

- Increasing w can cause decrease in probability of success. (crowding-out)

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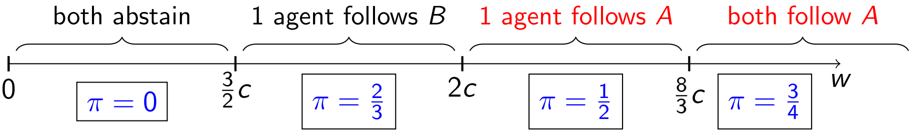

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- Even for large w, equilibrium behavior can be inefficient.

Summary

- Approach-independent contests are potentially inefficient.
  Possible causes:
  1. High correlation on most viable approaches.
  2. Crowding-out effect.

- When costs are equal, a greedy algorithm determines the efficient assignment. (Otherwise this is a hard problem.)

- Approach-specific prizes: strong tool to implement desired behavior in the static case. (Can be extended to dynamic setting if v and N are large enough.)