Informed Entry in First-price Auctions

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Introduction

- Nonparticipation: Not all eligible/interested bidders submit bids. Affects cost of procurement.
- Entry costs (Samuelson, 1985); Uncertainty (Levin and Smith, 1994).
- Affiliated entry model with entry costs & informative signals (Ye, 2007; Marmer, Shneyerov, and Xu, 2013; Gentry and Li, 2014; Chen, Gentry, Li, and Lu, 2020).

Affiliated Model of Entry

- Potential bidders don't know their values before entry.
- Receive private (noisy) signals.
- Can pay entry cost to learn their values and become active bidders.
- Only active bidders bid.

Example: TxDoT highway maintenance procurement auctions (Li and Zheng, 2009)

- A project is advertised: Engineer's estimate, brief description.
- Interested firms request plans: plan-holders = potential bidders.
- Sealed bids submitted, lowest bid wins.
- Not all potential bidders submit bids ($< 30\%$).
- Identities of active bidders are released only after.

This paper

- Role of bidders' informedness / signals.
- Does the auctioneer prefer more informed bidders?
- Should the auctioneer release more information?
- Not clear: More information may discourage entry.

Contributions

- Signals' informativeness: Information & Cutoff effects.
- Information: Entering bidders tend to be more efficient (lower values).
- Cutoff: Changes equilibrium entry. May reduce or increase entry and cost of procurement.

Empirical findings

- Data: signals are moderately informative. Spearman rank corr. 0.47.
- Entry costs are between $3\% - 6\%$ of engineer's estimate.
- More informative signals typically reduce entry.
- Optimal (least procurement cost) signals can be informative, uninformative, or in between (nontrivial level of informativeness). Depends on entry costs, number of potential bidders, etc.
- Optimal informativeness can save up to 10% of engineer's estimate.

Procurement: Infinite bid strategy

- Positive probability of being the only bidder.
- Strategy if no reserve price (maximum allowed bid): Bid "infinity".
- TxDoT: Reserve price (engineer's estimate) is not binding: $14\% - 49\%$ of winning bids are above.

TxDoT: Missing auctions with a single active bidder

- Only 2% of auctions have a single bid, but should be at least 28% (given the entry probs.)
- Government may reject bids "in the best interests of the State".
- Assumption: At least two bids are required! Contract is canceled otherwise. Rules out the infinite bid strategy.
- Empirical finding: Switching to binding reserve price saves up to 32% of procurement cost.

Model

- $N \geq 2$ risk-neutral potential bidders.
- Bidders independently draw private values V and signals S from

$$F_{V,S}(v,s) = C(F(v), F_S(s)),$$

where

- $C(\cdot,\cdot) =$ Copula.
- $F(\cdot) =$ Marginal CDF of values.
- $F_S(\cdot) =$ Marginal CDF of signals.

Good news assumption

- Smaller signals imply stochastically smaller values:

  For all $s_1 \leq s_2$,

  $$F_{V|S}(\cdot, s_1) \geq F_{V|S}(\cdot, s_2).$$

  (First-order stochastic dominance under larger signals.)

- Entry strategy: Enter if $S \leq s_N$ (entry cutoff, determined in equilibrium).

- Entry probability: $p_N = \Pr(S \leq s_N) = F_S(s_N)$.

Winning probability

- Active bidder with value v.
- Prob of entry p.
- Prob of winning:

$$H(v \mid p, N) = \Lambda(v \mid p)^{N-1} - (1-p)^{N-1}.$$

- $\Lambda(v \mid p) = 1 - C(F(v), p)$. (Prob competitor doesn't enter or draws value below v.)
- $(1-p)^{N-1}$. (Prob of no competitors - auction is canceled.)

Equilibrium

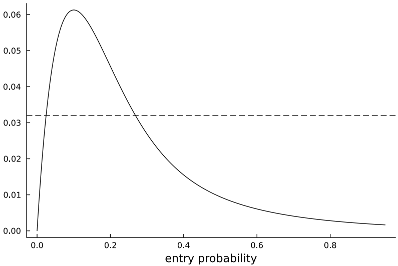
- Bidding strategy

$$\beta(v \mid p, N) = v + \int_{v}^{\bar{v}} \frac{H(u \mid p, N)}{H(v \mid p, N)} du.$$

- Equilibrium entry: Marginal bidder's ($S = s_N$) expected revenue $=$ Entry cost κ.
- Equilibrium entry probability solves:

$$\int_{\underline{v}}^{\bar{v}} (\beta(v \mid p_N, N) - v) H(v \mid p_N, N) dF_{V|S}(v \mid F_S^{-1}(p_N)) = \kappa,$$

Marginal bidder's expected revenue (———) and entry cost (- - - - - -)



- From TxDoT data for auctions with $N = 14$.
- Nonmonotonicity of revenue due to at least 2 bids requirement.
- Two* equilibria. The left equilibrium is unstable.

Expected cost of procurement conditional on at least 2 active bidders

- Conditional cost of procurement: Expected winning bid.

$$K(p_N \mid n \geq 2) = \left(N \int_{\underline{v}}^{\bar{v}} \Lambda(v \mid p_N)^{N-1} \left(1 - \frac{N-1}{N} \Lambda(v \mid p_N)\right) dv \right.$$
$$\left. + \underline{v} - \bar{v} \Pr(n < 2 \mid p_N) \right) / \Pr(n \geq 2 \mid p_N),$$

- n = number of active bidders.
- $\Pr(n \geq 2 \mid p_N) = 1 - (1 - p_N)^N - N p_N (1 - p_N)^{N-1}$.
- Cost of procurement depends on p_N and

  $\Lambda(v \mid p) = 1 - C(F(v), p)$.

Parametric copula

- Signals are unobserved = the model is not identified fully nonparametrically.
- Assume $C(F(v), p) = C(F(v), p; \theta_0)$; $C(\cdot, \cdot; \theta)$ is known. (Discussed in Gentry and Li, 2014.)
- Assume $\partial C(x, y; \theta)/\partial\theta \geq 0$ (positive ordering).
- Under positive ordering and higher θ:
  - Stronger association between V and S (more informative signals).
  - Distribution of values conditional on entry is less stochastically dominant, entering bidders tend to have smaller values.

Signals' informativeness and procurement cost

$$\frac{dK(\theta, p_N \mid n \geq 2)}{d\theta} = \underbrace{\frac{\partial K(\theta, p_N \mid n \geq 2)}{\partial \theta}}_{\text{information effect}} + \underbrace{\frac{\partial K(\theta, p_N \mid n \geq 2)}{\partial p} \cdot \frac{\partial p_N}{\partial \theta}}_{\text{cutoff effect}}.$$

- Two effects: Information and Cutoff.
- Information: Change in informativeness, same entry probability
- Cutoff: Different equilibrium entry probability.

Information effect

$$\frac{\partial K(\theta, p \mid n \geq 2)}{\partial \theta} = -\frac{N(N-1)}{\Pr(n \geq 2 \mid p)} \times \int_{\underline{v}}^{\bar{v}} \frac{\partial C(F(v), p; \theta)}{\partial \theta} \Lambda(v \mid p, \theta)^{N-2} C(F(v), p; \theta) dv \leq 0.$$

- Stochastically lower values for entering bidders under the same p.
- Lower cost of procurement.

Cutoff effect: Undetermined

$$\frac{\partial K(\theta, p_N \mid n \geq 2)}{\partial p} \cdot \frac{\partial p_N}{\partial \theta}$$

- Because of $\Pr(n \geq 2 \mid p_N)$,

$$\frac{\partial K(\theta, p_N \mid n \geq 2)}{\partial p} \lesseqgtr 0.$$

- Because of $\partial C_2(F(v), p; \theta)/\partial \theta \lesseqgtr 0$,

$$\frac{\partial p_N}{\partial \theta} \lesseqgtr 0.$$

Unconditional cost of procurement

- Assume: If auction fails, procurement cost is the maximum value $\bar{v}$.
- Unconditional cost of procurement:

$$K(\theta, p_N) = K(\theta, p_N \mid n \geq 2) \cdot \Pr(n \geq 2 \mid p_N) + \bar{v} \cdot \Pr(n < 2 \mid p_N).$$

- Information effect: $\frac{\partial K(\theta,p)}{\partial \theta} < 0$.
- $\frac{\partial K(\theta,p)}{\partial p} < 0$ (more entry, lower procurement cost).
- Cutoff effect is still undetermined: $\frac{\partial p_N}{\partial \theta} \lesseqgtr 0$.

Identification

- Data: All submitted bids and number of potential bidders N for many iid auctions.
- Entry prob. p_N from data.

Identification of inverse bidding strategy

- Follows Guerre, Perrigne, and Vuong (2000) and Marmer, Shneyerov, and Xu (2013).
- Inverse bidding strategy: $\beta^{-1}(b \mid p_N, N) =$

$$b - \frac{1}{p_N(N-1)g(b \mid N)}\left(1 - p_N \cdot G(b \mid N) - \frac{(1-p_N)^{N-1}}{\left(1 - p_N \cdot G(b \mid N)\right)^{N-2}}\right).$$

- $b =$ bid.
- $G =$ CDF of bids.
- $g =$ PDF of bids.
- Values V are identified conditional on entry.

Identification

- CDF of values conditional on entry is identified from V's.
- Denote it $F^*(v \mid p_N) \equiv \Pr(V \leq v \mid S \leq F_S^{-1}(p_N))$.
- Assume: Uncond. distribution of values is independent of N.
- Model's restrictions: For all N and v,

$$p_N F^*(v \mid p_N) = C(F(v), p_N; \theta_0).$$

Identification of θ

$$p_N F^*(v \mid p_N) = C(F(v), p_N; \theta_0)$$

- Define $F(v; p_N, \theta) \equiv C^{-1}(p_N F^*(v \mid p_N), p_N; \theta)$.
- Concentrate out F: For all N and v,

$$p_N F^*(v \mid p_N) = C(F(v; p_N, \theta_0), p_N; \theta_0)$$

Identification of the entry cost κ

Entry cost is identified given F and θ:

$$\kappa = \int_{\underline{v}}^{\bar{v}} (\beta(v \mid p_N, N) - v) H(v \mid p_N, N) dF_{V|S}(v \mid F_S^{-1}(p_N))$$
$$= \int_{\underline{v}}^{\bar{v}} C_2(F(v), p_N) H(v \mid p_N, N) dv.$$

Estimation

- $\hat{F}(v;\theta) \equiv \frac{1}{|\mathcal{N}|}\sum_{N\in\mathcal{N}} C^{-1}(\hat{p}_N \hat{F}^*(v \mid \hat{p}_N), \hat{p}_N, \theta)$.
- $\hat{\Delta}(v, N) \equiv \hat{p}_N \hat{F}^*(v \mid \hat{p}_N) - C(\hat{F}(v;\theta), \hat{p}_N; \theta)$.
- W weights (efficient).

$$\hat{\theta} = \arg\min_{\theta\in\Theta} \sum_{N\in\mathcal{N}} \sum_{N'\in\mathcal{N}} \int\int \hat{\Delta}(v, N) W(v, N; v', N') \hat{\Delta}(v', N') dv dv'.$$

Data: Li and Zheng (2009), TxDoT 2001-2003

Potential bidders	**9**	**10**	**12**	**13**	**14**
Number of auctions	15	15	16	11	10
Number of bids	40	41	43	41	40
Mean Eng.est. ($)	104,813	89,489	113,838	84,025	77,493
Mean Bid/Eng.est.	1.068	1.004	1.106	1.037	1.057
Min Bid/Eng.est.	0.815	0.721	0.799	0.703	0.722
Max Bid/Eng.est.	1.106	1.207	1.249	1.148	1.124

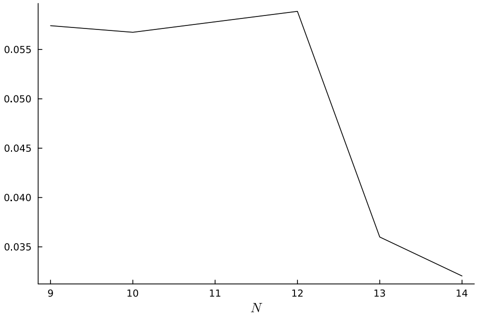
- Normalized bids by engineer's estimate.
- Due to sample sizes, we only use $N = 9, 10, 12, 13, 14$ (at least 40 bids for each N).
- Only jobs with one item (mowing)
- No state or interstate highway jobs.

Results: Signals' informativeness

	Estimate	**95% confidence interval**
Copula parameter θ	3.21	[2.44, 3.98]
	(0.39)	
Spearman correlation ρ	0.47	[0.38, 0.56]
	(0.09)	

- Frank copula.
- Estimated support of F: $[0.47, 1.56]$.
- Moderately informative signals.

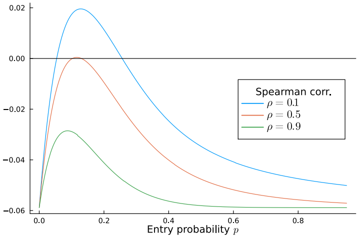
Estimated entry cost



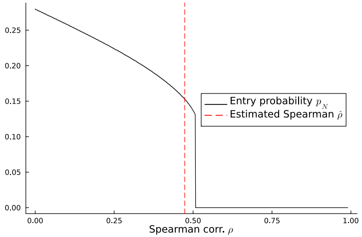
- Negatively associated with N.
- Between 3.2% – 5.9% of Engineer's estimate.
- $\$2,485 - \$6,023$.

Counterfactuals for $N = 12$

(a) Marginal bidder's profit from entry



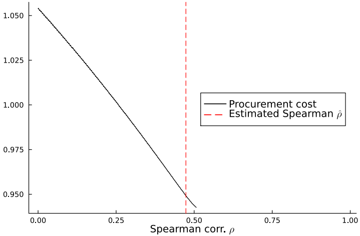
(b) Entry probability



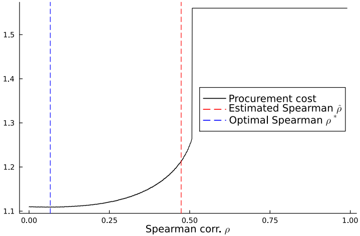
- Expected profit of the marginal entering bidder decreases with signals' informativeness.
- No entry once it reaches Spearman corr. ≈ 0.5.

Counterfactual cost of procurement

(a) Conditional on receiving at least two bids



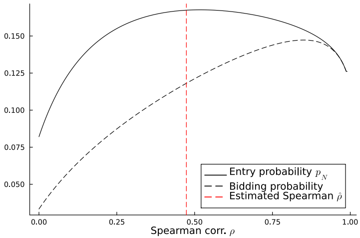
(b) Unconditional



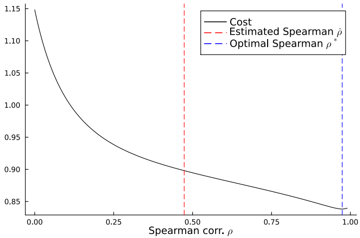
- Spearman 0.47 is nearly optimal for cond. procurement cost. Doesn't take into account auction failure.
- Unconditional: Optimal Spearman corr. 0.08. Reduces procurement cost by $\approx 10\%$.

Format change: Binding reserve price, 1+ bidders

(a) Entry and bidding probabilities



(b) Unconditional procurement cost



- Reserve price $= 1$ (engineer's estimate).
- Non-monotone entry and bidding prob.'s.
- Optimal Spearman corr. 0.97.

Counterfactual unconditional costs

Format	Signals' informativeness (Spearman corr.)	Unconditional procurement cost
no reserve price, 2+ bids (data)	0.47	1.22
binding reserve price, 1+ bids	0.47	0.90
binding reserve price 1+ bids	0.97	0.84

- Binding reserve price $= 1.0$ (engineer's estimate).
- Switching to binding reserve price saves 32%.
- Reducing signals' informativeness saves additional 6%.