The Coherence Side of Rationality

Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts*

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Abstract

We develop a theory of forecast coherence in production and use it to evaluate the rules-of-thumb managerial textbooks propose to help firms make rational forecasts about multiple own variables. We rationalize some rules as second-best optimal responses to noisy signals and obtain testable predictions linking incoherence, rules-of-thumb, and performance. Incoherence arises from 'narrow thinking'—intra-personal frictions in coordinating forecasts across variables—and operates via rules-of-thumb's use. Using the Duke Survey of top executives of large US corporations, we find support for our model's predictions. Our results imply about one-half of CFOs make incoherent forecasts of own output and inputs' growth.

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I Introduction

Coherence, "the consistency of the elements of the person's judgment" (Hammond (2007), p. xvi), is one of the two standards of rationality, together with accuracy.[1] When forecasting multiple variables at the same time, coherence requires the forecaster to assess all the connections among the variables under consideration, with the purpose of delivering rational forecasts. Accuracy, on the other hand, requires that forecasts of individual variables are not systematically different from realizations ex post. While there are numerous studies of forecast accuracy,[2] coherence has received much less attention. In this paper we provide a theory of forecast coherence in a firm production setting and evidence on the extent to which top financial executives make (in)coherent forecasts of their own firm's output and input growth.

Coherence, or lack thereof, may be consequential for firms. To allocate resources within the firm, top executives regularly make detailed internal forecasts, aka 'firm plans'. When preparing such plans, they typically start from output by making a sales revenue forecast (aka 'top line' forecast), and then proceed to forecast all other balance sheet variables, including capital and labor expenditures. This is a challenging multidimensional forecasting problem, which requires executives to draw on their knowledge of the firm's production possibilities and budget constraint in order to deliver accurate and coherent forecasts. To see this, consider a firm aiming at doubling its output: such a firm will likely need a lot more capital and labor, lest the desired output proves unattainable. Ignoring the firm's production possibility and/or budget constraint may thus imply the use of a suboptimal mix of capital and labor and may be costly to the firm.

Forecast coherence with respect to the firm's technology and budget provides a normative benchmark for rational internal plans, at least in static or stable environments. Dynamic evolution and disruptions such as unanticipated aggregate shocks or technological innovations may call for adaptation and, thus, incoherence relative to

[1] Coherence can be traced back at least to Aristotle (see Fogelin (2003)). Tversky and Kahneman (1981), p. 453, write, "the definition of rationality has been much debated, but there is general agreement that rational choices should satisfy some elementary requirements of consistency and coherence." See also Tversky and Kahneman (1974), Sen (1993), Becker (1996), Posner (2014).

[2] See, e.g., Tversky and Kahneman (1974), and Benjamin (2019) for a recent review.

initial plans. Although we believe that the implications of dynamics and disruptions for coherence deserve closer analysis, in this paper we consider a static model of firm production and we examine cross sectional variation in forecast coherence in a sample period characterized by the absence of major shocks or disruptions.

Planning and internal forecasting underlie all resource allocation and investment decisions inside the firm and are still not well understood (Graham, 2022). Managerial textbooks and case studies acknowledge the challenges of making plans about multiple variables at the same time and provide rules of thumb to help firms make rational forecasts.[3] To the best of our knowledge, none of these rules have been assessed to date, be it theoretically or empirically. We do so in this paper.

We consider a taxonomy of five rules.[4] Four of them implicitly incorporate a coherence concern, because they prescribe to forecast an input's rate of growth as some function of the forecasted output growth. One exception is the "plain growth forecast" rule, forecasting an input's growth rate, e.g., capital, by projecting that input's past growth rate into the future without considering information about its relation to output and the other inputs. This rule is reminiscent of 'narrow bracketing' behavior of decision makers who, when considering multiple related choices (e.g., consumption bundles), make each choice in isolation (e.g., Thaler (1985), Read et al. (1999), and Lian (2021)). By disregarding how budget constraint and utility function tie their choice variables together, narrow bracketers obtain lower utility than broad bracketers.[5] We note a parallel in the firm context, as Chief Financial Officers (CFOs) make plans about multiple firm's variables related through the production technology and budget constraint. Narrow bracketing in firm plans could induce incoherent, suboptimal allocation of resources to capital expenditures while ignoring labor costs, or vice versa.

Studying coherence is challenging, because it requires eliciting elements external

[3]See, e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), and Koller et al. (2020).

[4]These are (R1) plain growth, (R2) proportion of sales, (R3) economies of scale, (R4) industry based, and (R5) disaggregated; see Welch (2017), p. 593. We describe them in detail in Section II.

[5]One mechanism underlying narrow bracketing in consumption is mental accounting (Tversky and Kahneman (1981), Thaler (1985)), whereby agents hold a separate mental account for each decision, as opposed to a single budget constraint for their total expenditures.

to individuals' choice behavior, including objectives, values, or norms.[6] There are two reasons why our setting may be more suitable to study coherence. First, firm executives have a good sense of their firm's technology and agree on their firm's objective (e.g., profit maximization). Second, prior behavioral and experimental research has studied coherence using 'consistency' benchmarks from propositional logic (modus ponens/tollens) and probability theory (e.g., conjunction rule, or Bayes' rule). These universal, domain-general criteria are not cast as an optimization problem and have been criticized.[7] In this paper, we study forecast coherence among optimizing agents.

We develop a theory of forecast coherence in firm production. The forecaster minimizes expected inaccuracy subject to a coherence constraint. A normative version of our theory provides a benchmark of first-best coherent (and accurate) forecasts. We then present a positive theory where we study the forecasts of agents who observe noisy signals of inputs and output. We rationalize some rules of thumb (but not others) as second-best optimal responses to noisy signals and we obtain testable predictions linking incoherence, rules of thumb, and corporate performance. In our theory, incoherence arises from 'narrow thinking', namely, intra-personal frictions in coordinating forecasts of multiple contemporaneous variables, and operates via the use of rules of thumb.

Using the Duke Survey of top executives of large- and mid-size US corporations (see Ben-David et al. (2013) and Graham (2022)), we present evidence of the extent to which top financial executives make (in)coherent forecasts of their own firm's contemporaneous output and input growth. Besides being finance professionals, most of these forecasters are top financial executives and CFOs that are actively involved in setting the investment and financing policies of their firms, which allows us to jointly assess corporate forecasts and corporate policies by linking the Duke Survey to Compustat balance sheet data.

In our data, a popular rule of thumb is the "pure proportion of sales" forecast rule, assigning to each item, e.g., capital expenditures, the same growth rate forecasted for

[6] Sen (1993), p. 495, argues that there is no way of determining whether an individual's choice function is coherent or not "without referring to something external to choice behavior (such as objectives, values, or norms)". Tversky and Kahneman (1974), p. 1130, similarly argue that to fully assess coherence of probability judgments one should elicit "the entire web of beliefs held by the individual. Unfortunately there can be no simple formal procedure for assessing [this]".

[7] "(N)orms of coherence are crucial in the service of an organism's goals" (Arkes et al. (2016), p. 31).

sales (i.e., output). This popularity is consistent with the teachings of managerial and consulting textbooks and case studies (e.g., Koller et al. (2020), Luehrman and Heilprin (2009), and Stafford and Heilprin (2011)), and with the fact that this rule is simple to implement, potentially intuitive, and seemingly incorporates a coherence concern by anchoring the forecast of each input to that of output growth. Indeed, one can express this rule as a mean regression of capital expenditures growth on sales growth with a zero intercept and a unit slope. For this reason, we refer to it as the 'sales anchoring' rule.

It is useful to compare the 'sales anchoring' rule to the "economies of scale" rule, implemented by estimating a best linear predictor under square loss (OLS) of each balance sheet item's growth on contemporaneous sales growth using Compustat data. Clearly, the two rules are equivalent iff the estimated intercept is zero and the estimated slope is one. In the data, however, the estimated intercept is 0.217, significantly larger than zero, and the estimated slope is 1.055. To give some context, consider a firm aiming to increase total sales revenues by 5%. Under a 'sales anchoring' rule, this firm should forecast an increase in capital expenditures by 5%, whereas under an "economies of scale" rule, the same firm should forecast an increase in capital expenditures by 27%. Critically, using the simple, intuitive, and much advertised 'sales anchoring' rule implies a much smaller rate of increase in capital expenditures relative to sales than it is observed in the data, because this rule disregards the fixed component, which is empirically relevant and sizable.[8]

Similarly, we find that the 'narrow bracketing' rule yields forecasts of capital expenditures growth that are consistently below the realizations, because this rule ignores the connection between capital expenditures and sales.[9]

We use our framework to evaluate the rules of thumb that managerial textbooks have proposed to help managers make balance-sheet forecasts. In our model, two inputs (capital and labor) combine to produce output according to a standard production technology. The

[8]The fixed component is empirically relevant and sizable in all industries also when estimating the univariate regression by industry.

[9]It should be possible to do better than the 'narrow bracketing', 'sales anchoring', or "economies of scale" rules, even among the simple linear rules of managerial textbooks. In fact, we show later both theoretically and empirically that more sophisticated rules involving a multivariate regression or an industry-based regression do better. In this paper we do not address the broader question of what is the best method to provide contemporaneous forecasts beyond those suggested by managerial textbooks. Using flexible prediction methods (non-parametric regressions, machine learning algorithms) with big data sets will likely yield superior forecasts to any of the rules that we evaluate in this paper.

optimal forecasts of inputs and output coherently reflect both the production technology and the budget constraint. In the first best, we establish that a forecaster who is asked to produce forecasts of growth rates of output and inputs should provide forecasts that are linked cross-sectionally by parameters reflecting the contribution of capital and labor to the firm's production technology and budget constraint.

We establish conditions under which the rules of thumb of managerial textbooks yield optimal coherent forecasts. We find that some (but not all) of these rules of thumb represent an optimal second-best forecast rule when CFOs observe noisy signals about the firm's inputs and outputs. In particular, 'narrow bracketing' forecasts projecting past capital expenditures into the future are second-best optimal in the limit in which the CFO observes infinitely noisy signals about the output and the other input (e.g., labor). Conversely, "economies of scale" forecasts projecting the univariate connection between capital and output are second-best optimal if the CFO observes an infinitely noisy signals about the other input, but an informative signal about output.

To assess our model empirically, we develop a continuous, ex ante measure of managerial incoherence given by the (absolute value of the) orthogonal distance between the actual forecast and the theoretically optimal coherent one. This distance measure is predetermined relative to corporate performance and can thus be used to assess our model's predictions. Consistent with our model's predictions, we find that (1) the 'narrow bracketing' rule is the most distant from the optimal coherent forecast, followed by the 'sales anchoring' rule and (2) corporate performance (ROA) correlates negatively with managerial incoherence and is lowest for firms whose CFOs provide 'narrow bracketing' and 'sales anchoring' forecasts. We also show that the use of incoherent rules of thumb correlates negatively with investment spending and positively with leverage.

We then examine corporate performance, investment, and debt around the date when the CFO takes office for the subset of CFOs who disclose their identity. We use hand-collected data to track these critical event dates and perform an event-study analysis. Our results show that performance decreases in the years following the start date of an incoherent CFO, and such decline is larger, the more incoherent the CFO. In the same spirit, investment spending declines in the years following the start date of a 'narrow-bracketing' CFO. Although, given our data, we cannot to conclude that these empirical relationships are necessarily causal, these results are consistent with our model.

Our results so far imply that at least 48% of the CFOs in our sample make incoherent forecasts by using incoherent rules of thumb, including the 'narrow bracketing' and the 'sales anchoring' forecast rules. While these results are consistent with the partial ranking of the rules of thumb predicted by our model, we emphasize that these results do hot hinge on our model's assumptions about the firm's environment and technology and simply reflect the CFOs' use of particular rules of thumb.

In the final part of the paper, we ask what we can learn from our data by relying more directly on assumptions about the firm's environment and technology. We start by considering a general empirical formulation of a production function in which inputs and state variables are allowed to affect output with firm-specific lags and in a time-varying, firm-specific manner.[10] We note that in this setting coherent forecasts of firm-level inputs and output should be cross-sectionally linked in the same way as their realizations. Therefore, computing forecast errors by subtracting forecasts from realizations differences away any firm-level observed and unobserved heterogeneity known to, or predictable by, the firm at the time of the forecast. This yields a cross-sectional relation between the forecast error of output and the contemporaneous forecast errors of inputs, where the parameters are the loadings on the inputs.

Based on the above, we discuss two plausible ex post coherence restrictions on contemporaneous forecast errors of output and inputs. First, under free disposal the loadings of each input's forecast error should be weakly larger than zero. Second, under no increasing returns the loadings of each input's forecast error should be lower than one. The two shaded areas in Figure 1 depict these coherence regions; Section V works out the details. Figure 2 plots the joint empirical distribution of the contemporaneous forecast errors of output and capital in our data. By overlaying Figure 2 on Figure 1, we find that 52% of CFOs violate these restrictions, a result in the same ballpark as the 48% of CFOs using incoherent rules of thumb.

The cross-sectional relationship among forecast errors just discussed further suggests

[10]In this formulation output and inputs can enter the regression in levels, growth rates, logs, or other transformations that preserve linearity in the regression parameters. The details are in Section V.

some regression tests to assess coherence and distinguish it from accuracy. Intuitively, while accuracy requires that for each individual variable the forecast errors are on average zero, coherence requires that the cross sectional relation between the forecast errors of the output and inputs mirror the relation between output and inputs in realizations. When implementing these tests, we reject coherence on average and we also reject accuracy of capital expenditures, but we cannot reject accuracy of sales forecasts.

Finally, we introduce additional structure and derive formal statistical tests of coherence and accuracy at the individual CFO level. We implement these tests and reject the null of coherence for 56% of CFOs in our sample. In sum, while relying on different assumptions and degrees of stringency, all our results point to about one half of CFOs providing incoherent forecasts of simultaneous balance sheet variables.

Our individual-level tests enable us to disentangle coherence from accuracy. At the individual level, accuracy requires that forecast errors are 'sufficiently' close to zero, whereas coherence requires that forecast errors of output and inputs are 'sufficiently' close to one another. Figure 3 provides a graphical representation of this intuition for a hypothetical firm with a capital loading of one third. In our data, allowing the loading on the capital input to vary by industry, we find that 12% of CFOs are incoherent but accurate, whereas 13% of CFOs are inaccurate but coherent. Disentangling coherence and accuracy is crucial as, under some conditions, coherence can be assessed ex ante.

These additional tests also help rule out alternative interpretations of our findings. First, one concern might be that incoherence about different variables might reflect the different extent to which these variables are under the control of the CFO. While a CFO has ultimate authority on the firm's capital expenditures, a firm's sales depend also on a number of external factors, including consumer behavior and product market competition. If so, one might expect CFOs to be relatively more accurate about capital expenditures than about sales. This is not what we find. To the contrary, while CFOs are reasonably accurate in forecasting sales, reflecting their incentives (e.g., see Graham (2022)), CFOs are much less accurate in forecasting capital expenditures, reflecting the fact their capital expenditure forecast is incoherently linked to their sales forecast.

Second, incoherence is different from both overconfidence and optimism. We are able to control directly for measures of overconfidence and optimism as developed in the literature (Ben-David et al., 2013) and we find that incoherence is associated with lower investment spending, whereas overconfidence and optimism are associated with more aggressive investment spending.

Third, while our setting is a static one and we examine a sample period without major disruptions, aggregate shocks, etc., one could argue that incoherence might represent an optimal response to an idiosyncratic shock at the firm level. Upon observing such a shock, a CFO might realize that to achieve the planned sales growth the prior investment plan is no longer sufficient, and thus invest more than planned. If so, however, one would expect this type of incoherence to correlate positively with performance. Instead, we find that on average incoherence correlates negatively with performance. These findings suggest that incoherent firms may leave money on the table by using a suboptimal mix of inputs, consistent with recent work on behavioral firms making inefficient choices, see DellaVigna (2018), DellaVigna and Gentzkow (2019), and Strulov-Shlain (2022). Our results are also consistent with recent work on narrow thinking in consumption, see Lian (2021), and suggest that narrow thinking may drive incoherent firm forecasts.

The paper proceeds as follows. Section II presents our model. Section III describes the data. Section IV presents our empirical results. Section V derives our statistical tests and discusses alternative interpretations of our findings. Section VI discusses our contribution to the literature, and Section VII concludes.

II Theoretical Framework

When preparing firm plans, CFOs typically start from output by making a sales revenue forecast (aka top line forecast) for a number years, and then proceed to make forecasts of all other balance sheet variables, including capital and labor expenditures (e.g., see Welch (2017) and Graham (2022)). Therefore, CFOs face a challenging multidimensional forecasting problem, which requires making forecasts of multiple balance sheet items that are individually accurate and collectively coherent with one another.

Managerial textbooks and case studies propose a number of rules of thumb to make corporate forecasts. Welch (2017), p. 593-594, provides the following taxonomy:

(R1) A **plain growth** forecast, projecting the past growth rates of each individual item. Welch (2017) implements this rule by computing the average of the two most recent past annual growth rates and taking this average as the predictor of future growth.

(R2) A pure **proportion of sales** forecast, forecasting each item, e.g., capital expenditures, as a fixed proportion of sales. Welch (2017) implements this rule by assigning each item the same growth rate as sales. One can express this rule as a mean regression of capital expenditures growth on sales growth in which the intercept is zero and the slope is one.

(R3) An **economies-of-scale** forecast, positing for each item a fixed component and a variable component, the latter itself a proportion of sales. Welch (2017) implements this rule by estimating a best linear predictor under square loss (i.e., univariate mean linear regression) of each balance sheet item's growth on contemporaneous sales growth using Compustat data. The estimated regression intercept is the fixed component and the estimated slope multiplied by the sales forecast is the variable component.

(R4) An **industry-based** forecast, drawing on information from other firms in the same industry. Welch (2017) implements this rule exactly as (R3), but using only data from firms in the same industry as the firm under consideration.

(R5) A **disaggregated** forecast, recognizing that each item may comove not only with sales but also with the other items. Welch (2017) implements this rule by expanding the specification of the (R3) regression to include additional contemporaneous items and using all Compustat data.

As discussed above, we refer to (R1) as the narrow bracketing rule and to (R2) as the sales anchoring rule. The literature has not reached a consensus on which of the above rules, if any, constitutes best practice. For example, Ruback (2004) advocates using methods (R2) and (R3). Harvard Business School case studies typically suggest (R1), (R2), and (R4), e.g., see Luehrman and Heilprin (2009) and Stafford and Heilprin (2011). Koller et al. (2020) advocate (R2), writing that "*net Property, Plant and Equipment should be* *forecast as a percentage of revenues*" (p. 286).[11] Titman and Martin (2016) describe a method akin to (R3) and Holthausen and Zmijewski (2020) one akin to (R5).

This literature lacks a formal framework designed to offer guidance as to whether the above methods are equivalent to one another or differ along key dimensions and, if so, which method is best and under what conditions.

In this section, we address this gap. We have two objectives. First, in Subsection *A.* we develop a normative theoretical framework to derive rational coherent corporate forecasts. Second, in Subsection *B.* we develop a positive framework describing how CFOs make second-best optimal forecasts in the presence of noisy signals about the firm's technology. Our framework nests the above rules of thumb and shows conditions under which some of them emerge as second-best optimal.

A. A Benchmark Model of Optimal Corporate Forecasts

Consider a firm aiming to maximize its profits, $\Psi = p_y y - p_1 x_1 - p_2 x_2$, under a budget constraint, $p_1 x_1 + p_2 x_2 = Z$, where y is the output, x_1 and x_2 are input quantities (capital and labor), p_1 and p_2 are the input prices, the output price p_y is normalized to 1, and Z is a real-valued budget constraint. Output is generated by a general class of constant elasticity of substitution (CES) production functions,

$$y = f(x_1, x_2) = \left(\frac{a}{a+b} x_1^{\xi} + \frac{b}{a+b} x_2^{\xi} \right)^{\frac{a+b}{\xi}}, \tag{1}$$

where $\nu \equiv a + b > 0$ are parameters governing the returns to scale (constant for $\nu = 1$, increasing for $\nu > 1$, and decreasing for $\nu < 1$), and the elasticity of substitution between x_1 and x_2 is $\chi = \frac{1}{1-\xi}$. We assume that factor-augmenting productivities are constant over time and we normalize them to one.[12] We also assume that the technological relationship

[11] In turn, Koller et al. (2020) indicate that depreciation should be forecasted as a proportion of net Property, Plant, and Equipment (PPE), and that capital expenditures should be calculated by summing the projected increase in net PPE to depreciation (p. 286-287). As a result of this chain of calculations, capital expenditures is forecasted as a percent of revenues, and thus the forecast of capital expenditures growth should equal the forecast of sales revenues growth, as in (R2). Similarly, Harvard Business School case studies (e.g., Luehrman and Heilprin (2009) and Stafford and Heilprin (2011)) provide case solutions in which the forecast of capital expenditures growth equals the forecast of sales revenues growth.

[12] This is without loss of generality because in our setting a TFP shock would be observationally equivalent to input price shocks in the same direction.

is stable over time and not subject to aggregate shocks.[13] This formulation is quite general (Moysan and Senouci, 2016) and it nests a number of widely used specifications as special cases.[14] Finally, denote $\log p_i = \pi_i$, where $i = 1, 2$, and assume for now input prices are i.i.d., $\{\pi_{i,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_i^2)$, with $corr(\pi_1, \pi_2) = \rho_{1,2}$.[15]

Consider a forecaster who at time t issues a vector of forecasts, $\mathbf{F}_t$, of the future realization of a vector, $\mathbf{x}_{t+1}$, to minimize a quadratic loss function,

$$\min_{\mathbf{F}_t} \mathbb{E}\left[(\mathbf{x}_{t+1} - \mathbf{F}_t)^2 |\Omega_t\right],$$

where $\mathbf{x}_{t+1} = (y_{t+1}, x_{1,t+1}, x_{2,t+1})$, Ω_t denotes the information set at t, which includes knowledge of the production function, (1), and at solution $\mathbf{F}_t^* = \mathbb{E}[\mathbf{x}_{t+1}|\Omega_t] \equiv \mathbb{E}_t[\mathbf{x}_{t+1}]$. In words, the forecaster minimizes expected inaccuracy subject to a coherence constraint. This formulation allows to nest rules of thumb (R1)-(R5) because forecasts under those rules are conditional means, which are the best predictor under quadratic loss.

A.1 Optimal Forecasts

Proposition 1 (Inequality). *Forecast coherence requires that the forecasts of output and inputs, $\mathbb{E}_t[y_{t+1}]$, $\mathbb{E}_t[x_{1,t+1}]$, and $\mathbb{E}_t[x_{2,t+1}]$, satisfy an inequality, whose direction depends on whether the CES production function is concave or convex. For $\xi \leq 1$ and $a + b \leq 1$, the CES function is concave and forecast coherence requires*

$$\mathbb{E}_t[y_{t+1}] \leq f(\mathbb{E}_t[x_{1,t+1}], \mathbb{E}_t[x_{2,t+1}]) = \left(\frac{a}{a+b}\mathbb{E}_t[x_{1,t+1}]^{\xi} + \frac{b}{a+b}\mathbb{E}_t[x_{2,t+1}]^{\xi}\right)^{\frac{a+b}{\xi}}. \quad (2)$$

[13]This is plausible because we focus on cross-sectional differences in coherence across forecasters, and we implement most of our tests over 2001-2007 at the peak of the 'great moderation', a time when aggregate volatility was not a concern.

[14]For $\chi \to +\infty$ the inputs are perfect substitutes and the production function is linear; for $\chi \to 0$ there is no substitution and the production function is Leontieff; and for $\chi = 1$ we have a Cobb-Douglas. The empirical literature suggests as plausible a range $\chi \in (0.5, 1]$ (e.g., see Berndt (1976) and Oberfield and Raval (2021)), implying $\xi \in (-1, 0]$.

[15]While our theory can be readily extended to the general case of n inputs, we shall focus on the case of a production function $F(K, L)$ with two inputs, capital (K) and labor (L), as it allows a tight mapping with our data. In principle, we could consider a production function $F(K, L, M)$ with three inputs, capital (K), labor (L), and materials (M). However, CFOs are asked to forecast future sales and future expenditures on capital and wages, but not materials. Similarly, Compustat contains data on realized sales, capital expenditures, and wages, but not materials.

For $\xi \geq 1$ and $a+b \geq 1$, the CES function is convex and coherence requires

$$\mathbb{E}_t\left[y_{t+1}\right] \geq f\left(\mathbb{E}_t\left[x_{1,t+1}\right], \mathbb{E}_t\left[x_{2,t+1}\right]\right) = \left(\frac{a}{a+b}\mathbb{E}_t\left[x_{1,t+1}\right]^{\xi} + \frac{b}{a+b}\mathbb{E}_t\left[x_{2,t+1}\right]^{\xi}\right)^{\frac{a+b}{\xi}}. \quad (3)$$

All Proofs are in the Appendix.

Proposition 1 provides a first restriction on contemporaneous forecasts that a coherent forecaster must satisfy. In the Online Appendix we examine empirically how many CFOs report forecasts satisfying the coherence requirement of Proposition 1.

In general, the CES is a non-linear function of the inputs. Because the rules of thumb (R1)-(R5) are instead linear, and one of our objectives is to rationalize these rules of thumb at least in a second-best sense, we shall now focus on the limit case of $\xi \to 0$, corresponding to a Cobb-Douglas production function,

$$\lim_{\xi \to 0}\left(\frac{a}{a+b}x_1^{\xi} + \frac{b}{a+b}x_2^{\xi}\right)^{\frac{a+b}{\xi}} = x_1^a \cdot x_2^b.$$

Corollary 1 (Cobb-Douglas). *In the limit case in which $\xi \to 0$,*

$$\mathbb{E}_t \log\left[y_{t+1}\right] = a \cdot \mathbb{E}_t \log\left[x_{1,t+1}\right] + b \cdot \mathbb{E}_t \log\left[x_{2,t+1}\right].$$

Similarly,

$$\mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] = a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right].$$

Because the Cobb-Douglas production function is linear in logs, the coherence requirement of Proposition 1 holds with equality, both for forecasts expressed in levels and in growth rates.

A.2 Optimal Forecasts and Rules of Thumb

We now consider the forecasting problem in the case a and b are unknown to the forecaster.

Proposition 2. *If parameters a and b are unknown, a forecaster can estimate them using a linear projection operator, with the forecasted variables in logs.*

Corollary 2. *In a multivariate linear projection,* $\mathbb{E}_t \log\left[x_{1,t+1}\right] = \alpha + \beta_1 \cdot \mathbb{E}_t \log\left[y_{t+1}\right] +$

$\beta_2 \cdot \mathbb{E}_t \log[x_{2,t+1}]$, *the parameters are*

$$\alpha = \mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0, \qquad \beta_1 = \frac{1}{a}, \qquad \beta_2 = -\frac{b}{a},$$

where $\mathbb{E}\log[y_{t+1}]{=}\mu_y$ *and* $\mathbb{E}\log[x_{i,t+1}]{=}\mu_i$*, for* $i = 1, 2$*, are the unconditional means. The same result obtains with the variables in growth rates.*

Corollary 2 rationalizes how a rule of thumb akin to (R5) described above delivers the first-best optimal forecast. Specifically, Corollary 2 prescribes implementing (R5) using data on output growth and labor growth forecasts to provide forecasts of capital growth and using parameters derived from a linear projection of the firm's input on the output and the other input. Note also that (R5), as well as (R1)-(R4), are defined in the managerial education literature as linear functions of growth rates (not in logs). Our analysis implies that such linear rules will be correct up to a first-order Taylor approximation. Importantly, Corollary 2 holds both in levels and in growth rates.

Corollary 3. *In a univariate linear projection,* $\mathbb{E}_t\log[x_{1,t+1}] = \alpha + \beta \cdot \mathbb{E}_t \log[y_{t+1}]$*, the parameters are*

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}\mu_y, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_1^2}.$$

In a univariate linear projection with the variables in growth rates, $\mathbb{E}_t\log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] = \alpha + \beta \cdot \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right]$*, the parameters are*

$$\alpha = 0, \qquad \beta = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}.$$

Corollary 3 shows that (R3), which makes use of information on the output and one input but neglects the other input, in general yields different forecasts than (R5). Therefore (R3) yields in general incoherent forecasts. We now examine a special case in which (R3) yields coherent forecasts.

Corollary 4. *If* $\rho_{1,2} = 1$ *and* $\sigma_1^2 = \sigma_2^2 = \sigma_{1,2} = \sigma^2$*, then for a linear regression in growth rates,* $\mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right] = \alpha + \beta \cdot \mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] + e_{i,t+1}$*, with* $i = 1, 2$*, we have* $\alpha > 0 \iff 0 < \beta < 1 \iff \nu < 1$*. The same is true for i.i.d. shocks, setting* $\gamma_i = 0\ \forall i$*.*

Corollary 4 shows that (R3) can be optimal under rather special circumstances, that is, when input prices are perfectly correlated and thus there is no added benefit from a multivariate rule like (R5) relative to the univariate rule (R3).

Furthermore, Corollary 4 indicates that rule (R2), which amounts to setting $\mathbb{E}_t\left[\frac{x_{i,t+1}}{x_{i,t}}\right] = \mathbb{E}_t\left[\frac{y_{t+1}}{y_t}\right]$, is optimal when $\alpha = 0$ and $\beta = 1$, that is, under constant returns to scale $\nu = 1$. In this case, (R2) is exactly equivalent to (R3). If returns to scale are not constant, rule (R2) is suboptimal. More generally, whenever $\rho_{1,2} \in (-1, 1)$ and $\sigma_1^2 \neq \sigma_2^2$, both (R3) and (R2) yield incoherent forecasts and the forecaster would do better by relying on information provided by all inputs and the output.

Finally, rule (R1) amounts to projecting past information of the input being forecasted, while disregarding information about the output and the other input. Because (R1) treats each item in isolation, we interpret (R1) as an example of narrow bracketing. In general, (R1) amounts to setting the forecast of $x_{i,t+1}$ equal to the average of k past growth rates, $\log F_{i,t}^{R1} = \frac{1}{k}\sum_{j=1}^{k} \log \frac{x_{i,t+1-j}}{x_{i,t-j}}$. Welch (2017) advocates $k = 2$. We establish:

Corollary 5 (Losses Under Narrow Bracketing Forecasts). *Under (R1) and $k \to +\infty$, $\mathbb{E}_t\left[L_{t+1}^{R1}\right] = \mathbb{E}_t\left[L_{t+1}^{o}\right] + \left[(1-\gamma_i)\,\pi_{i,t}\right]^2 > \mathbb{E}_t\left[L_{t+1}^{o}\right]$ for $\gamma_i < 1$, where $\mathbb{E}_t\left[L_{t+1}^{R1}\right]$ and $\mathbb{E}_t\left[L_{t+1}^{o}\right]$ denote the expected losses under (R1) and the optimal forecast, respectively.*

Corollary 5 implies that (R1) is optimal if and only if one uses a lot of data, $k \to +\infty$, and if the input price follows a random walk, $\gamma_i = 1$, otherwise it is strictly inferior.

Under optimal forecasts individuals should be broad bracketers, as they should compute forecasts taking into account the structure of the firm's problem and all available data on inputs, x_i's, and output, y. Under narrow bracketing, individuals ignore the structure of the firm's problem and, when forecasting growth of item x_i, they examine data about x_i in isolation and disregard data on items $x_{\neg i}$ and y.

In reality, narrow bracketing could be a second-best optimal response to imperfect information. Moreover, individuals may be producing forecasts between the two extremes of broad and narrow bracketing. They may be better informed about the price of input 1 and have difficulty accessing information about the price of input 2.

Following Lian (2021), in the next subsection we capture these possibilities by introducing noisy signals, and we recast the forecasting problem under narrow bracketing as multiple selves playing an incomplete information, common interest game.[16] With two inputs, capital and labor, the CFO "capital self" makes forecasts of capital expenditures growth by observing imprecise signals of output and labor growth. Conversely, the CFO "labor self" makes forecasts of labor expenditures growth by observing imprecise signals of output and capital growth. In equilibrium, each self does not perfectly know other selves' signals (states of mind) and, thus, makes forecasts with imperfect knowledge of other selves' forecasts. In this sense, narrow thinking reflects intra-personal frictions in coordinating multiple forecasts.

B. A Model of Bracketing in Corporate Forecasts

We consider a forecaster self that makes a forecast for input 1, $F \log x_1$, to minimize a quadratic loss function,

$$\min_{F \log x_1} \mathbb{E}\left[(\log x_1 - F \log x_1)^2 | \Omega \right],$$

where for simplicity we drop the time subscript, t, because the problem is stationary.[17] The firm's production technology is $y = x_1^a x_2^b$ and the budget constraint is $p_1 x_1 + p_2 x_2 = Z$. We assume that the forecaster observes two noisy signals, $\eta_y = \log y + \epsilon_y$ and $\eta_2 = \log x_2 + \epsilon_2$, where $\epsilon_y \sim \mathcal{N}\left(\mu_y, s_y^2\right)$ and $\epsilon_2 \sim \mathcal{N}\left(\mu_2, s_2^2\right)$.

Proposition 3. *The optimal forecast of input x_1 given signals η_y and η_2 is*

$$\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y \left(\eta_y - \mu_y\right) + \beta_2 \left(\eta_2 - \mu_2\right),$$

where

$$\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2 + s_2^2}}, \quad \beta_2 = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.$$

Proposition 3 shows that, upon observing signals η_y and η_2, the optimal forecast of input x_1 is given by a linear projection of (the deviation of the signals from the prior

[16] In the multiple selves literature, multiple selves have conflicting interests (e.g., Marschak and Radner (1972), Piccione and Rubinstein (1997), and Benabou and Tirole (2002)); whereas in Lian (2021) and in our paper they have common interests. Despite common interests, since different selves do not share their information, they have difficulty in coordinating their decisions.

[17] This formulation is without loss of generality as it can be cast in terms of generic inputs i and $\neg i$.

means of) output y and input x_2. In such a linear projection, the constant term is the prior mean of x_1 and the slope coefficients are functions of the fundamental uncertainty and of the precision of the signals. Proposition 3 rationalizes rule of thumb (R5) as the optimal coherent forecast also in a second-best sense. Here, Proposition 3 clarifies that in a second-best world the accuracy of this linear projection will depend on the precision of the signals. Note that in our model the forecaster makes forecasts based on different, non-nested information, that is, in the sense of Blackwell, neither input i's signal is more informative than input $\neg i$'s signal nor input $\neg i$'s signal is more informative than input i's signal (see also Lian (2021)). Next, we examine a number of special cases.

Corollary 6 (Narrow Bracketing). *When* $s_y^2, s_2^2 \rightarrow +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1$.

When both signals are infinitely noisy, the optimal forecast of input x_1 ignores the signals and instead projects the prior mean μ_1 into the future. Corollary 6 rationalizes rule of thumb (R1) as an optimal forecast when the CFO observes infinitely noisy signals about the output and the other input.

Corollary 7 (Univariate Projections). *When* $s_2^2 \rightarrow +\infty$ *and* $0 < s_y^2 < +\infty$, *the optimal forecast is* $\mathbb{E}\left[\log x_1 | \eta_y, \eta_2\right] = \mu_1 + \beta_y\left(\eta_y - \mu_y\right)$, *where* $\beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}$.

When the signal of the other input is infinitely noisy and the signal of the output is noisy but informative, the optimal forecast of x_1 is a univariate linear projection of (the deviation from the prior mean of) the output, where the intercept is still the prior mean of x_1 and the slope is a function of the fundamental uncertainty and of the precision of the signal. This corollary rationalizes (R3) as an optimal forecast when the CFO observes infinitely noisy signals about the other input. Note that nothing necessarily implies $\mu_1 = 0$ and $\beta_y = 1$; hence, (R2) is in general not optimal, even in a second-best world.

Finally, (R4) can be thought of as a version of (R3) in which the linear projection is estimated for a subsample of firms in the same industry as the firm under consideration. On the one hand, firms in an industry might differ from firms in other industries, for example, because a_j or b_j are specific to industry j, in which case (R4) may be superior to (R3). On the other hand, using a smaller sample might hurt the performance of the linear projection, in which case (R3) may be superior to (R4). We will evaluate these possibilities empirically in the next section.

For completeness, we consider also the case in which both signals are infinitely precise.

Corollary 8 (Precise Signals). *When* $s_y^2, s_2^2 \to 0$, *the optimal forecast is* $\mathbb{E}[\log x_1|\eta_y, \eta_2] = \frac{1}{a}(\eta_y - b\eta_2)$.

To sum up, our theory implies that ex ante forecasts that differ from the normative benchmark are incoherent. Therefore, the main empirical prediction of our model is that profits, $\Psi = p_y y - p_1 x_1 - p_2 x_2$, should decrease on average with the extent of incoherence. Furthermore, our theory delivers a ranking of the rules of thumb proposed in managerial textbooks, (R5) $\succeq$ (R4)-(R3) $\succeq$ (R2)-(R1), and a key additional prediction, namely, the mechanism through which incoherence arises is the use of certain rules of thumb, most notably, (R1) and (R2).[18] In the next section, we evaluate empirically these predictions.

III Data

We use two main sources of data, one on CFO expectations and one on firm realizations. CFO expectations come from the Duke Survey, run by John Graham and Campbell Harvey and launched in July 1996. Each quarter, the study surveys between 2,000 and 3,000 CFOs, asking their views about the US economy and corporate policies, as well as their expectations of future firm performance and operational plans. The usual response rate is 5% to 8%; most responses arrive within the first two days of the survey invitation date. Since the end of the 1990s, the survey consistently asks respondents their expectations of the future twelve-month growth of key corporate variables, including revenues, capital expenditures, employment, and earnings.[19]

Our data comprises 72 quarterly surveys conducted between March 2001 and December 2018. We observe corporate forecasts as a single number per variable, which

[18]Specifically, Proposition 3 says that the optimal ex ante coherent forecast is a specific version of (R5), because it implies the use of an optimal mix of inputs. Corollary 6 implies that (R1) is the most extreme—among those considered—deviation from (R5) when all signals are infinitely noisy. Corollary 7 implies that (R2) uses information about the output, but in a way that is not optimal. Corollary 7 further implies that (R3) and (R4) use optimally the information about the output but ignore the other input. As a result, (R5) should be the optimal coherent forecast rule; (R1) and (R2) should be the worst; (R3) and (R4) should be better than either (R1) or (R2), but not necessarily approximating (R5).

[19]Historical surveys and aggregated responses can be accessed at https://cfosurvey.fuqua.duke.edu/.

we interpret as the CFO's expected value, corresponding to the firm's base case scenario. For many firms, the base case is the only scenario that gives rise to fleshed out forecasts in their internal planning process.[20]

Forecasts are elicited for all variables jointly as follows:

Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, −2%, etc.) [Leave blank if not applicable] Revenues: _____; Capital spending: _____; R&D spending: _____; Technology spending: _____; Prices of your product: _____; Earnings: _____; Cash on balance sheet: _____; Number of domestic full-time employees: _____; Wage: _____; Dividends: _____. Advertising: _____. Share repurchases: _____.

Figure A1 of the Online Appendix displays an actual screenshot of the above questions. Table 1 reports summary statistics on CFO twelve-month ahead growth forecasts (Panel A) and on growth realizations in a matched Duke-Compustat sample (Panel B).[21]

Firm realizations come from Compustat, which extracts the information from the Security and Exchange Commission (SEC)-required public filing of financial statements. Compustat covers all publicly traded firms across all sectors of the US economy since 1955. We exclude firms with negative assets and we winsorize at the 1% level.

When matching Duke and Compustat data there are four sources of attrition: (1) due to privacy restrictions associated with these data, not all Duke respondents report their firm ID, so they cannot be matched to Compustat; (2) not all Duke respondents give forecasts about all variables in each survey; (3) not all variables elicited in the Duke survey have a precise counterpart in Compustat, namely, technology spending, outsourced employees, health spending, productivity, product prices, and share repurchases; and (4) not all variables for which there is a precise counterpart in Compustat have full coverage, chiefly among those, wages are missing for about 90% of Compustat firms and R&D and

[20]Firms that internally consider additional scenarios typically consider a downside scenario to plan for contingencies and an upside scenario to lay out stretch goals. However, these additional scenarios are often developed in less detail than the base case and do not necessarily lead to fleshed-out forecasts. See Graham (2022), p. 1997, for more details.

[21]We match the Duke and Compustat datasets by firm ID, implying that for some firm-year pairs there might be multiple CFO forecasts. Table A1 in the Appendix reports the same statistics in the full Compustat population.

advertising expenditures are also missing for a large fraction of Compustat firms.[22]

Table A2 in the Appendix reports summary statistics on the full Compustat sample and on the matched Duke-Compustat sample. Firms in the Duke data are on average larger than Compustat firms in terms of sales and assets. Firms in the Duke data are also more profitable and hoard more cash than Compustat firms, but are otherwise similar in terms of market-to- book ratio, investment, and leverage. These patterns broadly concur with prior work using the Duke data (e.g., Ben-David et al. (2013)).

Comparing Panel A and B of Table 1 shows that CFOs are on average slightly more optimistic about output (i.e., revenues), although the medians of forecasts and realizations are quite close to one another, consistent with the observation in Graham (2022) that CFOs care about getting revenues forecasts right. Conversely, CFOs' forecast of capital expenditures are systematically low, with the distribution of capital expenditures realizations shifted to the right relative to the distribution of the corresponding forecasts.

IV Empirical Analysis

In this section, we present our main empirical findings. In Subsection *A.*, we establish which rules of thumb are reflected in CFOs forecasts, we develop our continuous measure of incoherence, and we investigate how the use of rules of thumb correlates with incoherence. In Subsection *B.*, we examine how incoherence and the rules of thumb correlate with firm performance. In Subsection *C.*, we examine how the use of rules thumb correlate with firm investment and debt policy. In Subsection *D.*, we investigate how firm behavior changes around the years in which CFOs take office.

[22]The matched Duke-Compustat sample mostly refers to the earlier part of the sample, until about 2011Q4. This is not a problem since we will conduct most of our regression analysis in the pre-financial crisis period. Points (1) and (2) imply a potential selection problem. If anything, however, our respondents are positively selected among those more likely to give coherent and accurate forecasts of all variables, under the assumption that missing forecasts reflect lack of knowledge about the variables. Points (3) and (4) imply that our analysis of forecast errors needs to be limited to variables for which there is full coverage in both Duke and Compustat.

A. Rules of Thumb Indicators and Ex Ante Incoherence

We characterize in our data the rules of thumb used by CFOs to produce corporate forecasts. We focus on forecasts about sales revenues (i.e., output) and capital expenditures because they have a clear mapping with theory and we observe their realizations in Compustat. In the Online Appendix we report data on other items.

We consider the five rules of thumb discussed in Section II. Whenever a rule of thumb can be implemented in multiple ways, we follow Welch (2017). Therefore, (R1) uses the average growth of the past two years as the forecast. (R2) uses the same growth rate for sales revenues and capital expenditures, equivalent to using a univariate regression model, Sales Growth $= \alpha + \beta \cdot$ Capital Expenditures Growth $+ \varepsilon$, where $\alpha = 0$ and $\beta = 1$. (R3) uses the same regression, but estimates it in the population of Compustat firms. (R4) estimates the same regression by industry. We do so at the 1-digit SIC code level to make sure that each industry has enough observations.[23] Finally, (R5) expands on the above by using a multivariate regression model.

We begin by examining how the rules' implementation following Welch (2017) looks like using Compustat data over 2000-2019. Table 2 reports the results. The top row of Table 2 shows that under (R1) firms set their capital expenditures (CapEx henceforth) forecast only considering past average CapEx growth, and the estimated coefficient is -0.089, precisely estimated, indicating mean reversion in CapEx growth. That is, a firm experiencing high CapEx growth in the past two years under (R1) should set a low one-year ahead forecast of CapEx growth, regardless of any contemporaneous sales revenue growth goal, or other inputs' growth patterns.

The second row of Table 2 reports estimates of a univariate regression model, CapEx Growth $= \alpha + \beta \cdot$Sales Growth$+\varepsilon$, where we find $\widehat{\alpha}$ =0.217 and $\widehat{\beta}$ =1.055, precisely estimated. There are two implications of these estimates. First, these estimates indicate how to implement (R3) using Compustat data, and imply that a firm wishing to achieve a 5% sales growth rate should under (R3) forecast a CapEx growth rate of 1.055 x 5% +

[23]SIC 1-digit codes roughly correspond to the following sectors: Agriculture, forestry, and fishing; Mining; Construction; Manufacturing; Transportation, communications, and public utilities; Wholesale trade; Retail trade; Finance, insurance, real estate; Services; and Public administration. SIC codes also allow a close mapping to the analogous classification of the Bureau of Economic Analysis.

21.7% = 26.7%. Second, these estimates imply that (R2) provides (much) lower CapEx growth forecasts relative to sales than observed in the data. In fact, the same firm wishing to achieve a 5% sales growth rate should under (R2) forecast a CapEx growth rate of 5%. The reason of the discrepancy is that, while the slope 1.055 is statistically indistinguishable from 1 at standard significance levels, (R2) uses an intercept of 0, whereas in the data the intercept is positive and large.[24]

The subsequent ten rows report results from estimating the same regression separately for the ten industries defined by one-digit SIC codes, describing the implementation of (R4). In all ten industries the intercept is positive and statistically significantly larger than 0, ranging from 0.163 to 0.402. Furthermore, the slope of sales ranges from 0.859 to 2.050. These results confirm that in all industries using (R2) implies setting a (much) lower CapEx growth forecast relative to sales growth than observed in the data.

Finally, the last row reports results from a multivariate model designed to describe the implementation of (R5). Welch (2017) presents a number of examples with multiple regressors without specifying which variables to include, possibly because no benchmark theory is available in the literature. Our model of Section II indicates that the full multivariate model delivering the optimal rational coherent forecast should include the firm's inputs, as follows:

$$\text{CapEx Growth} = \alpha + \beta \cdot \text{Sales Growth} + \theta \cdot \text{Labor Costs Growth} + \gamma \cdot \text{Materials Growth} + \varepsilon.$$

However, as previously mentioned, Compustat does not provide information on materials and provides only scant information about wages. Furthermore, to measure incoherence one needs to observe both forecasts and realizations of both output and all input variables, implying that using capital expenditures and wages as our input variables we end up with about 50 observations, too few to allow for a meaningful empirical analysis. On the other hand, there are two variables containing information about the operating costs of the goods sold for which we do observe both forecasts and realizations, namely Earnings,

[24]Using (R2) also leads to underpredicting the growth of other inputs relative to (R3), although by a smaller amount in the case of measures of direct costs. For example, our regression results imply that a firm aiming at a 5% sales growth should under (R3) forecast a 12.7% advertising growth and a 7.7% wage growth.

corresponding in our model to $F(K,L) - p_L L$, and Advertising, a specific operating cost.[25]

Therefore, in the main text we replace Labor Costs Growth with Earnings Growth, because we observe both realizations and forecasts of earnings for a large number of firms. The bottom row of Table 2 shows that in this case $\widehat{\alpha}$ =0.217, $\widehat{\beta}$ =1.042, and $\widehat{\theta}$ =0.018, precisely estimated. We take these estimates as representing both Rule (R5) and as characterizing the rational coherent benchmark. In the Online Appendix we replace Labor Costs Growth with Advertising Growth, in which case Table A3, Column 7 shows that $\widehat{\alpha}$ =0.178, $\widehat{\beta}$ =0.996, and $\widehat{\theta}$ =0.178, all precisely estimated.

We can now assign a unique type to each CFO. We do so in two steps. First, for each CFO we compute the orthogonal distance between the actual forecast and that implied by each of the five rules of thumb (R1)-(R5). Second, for each CFO we compute the minimum distance among those five distances and we assign a type, $\tau = 1, .., 5$, corresponding to the rule that is closest to the actual forecast.[26]

Table 3 shows that, among the 396 CFOs for whom we observe the identity and the joint forecasts and realizations of all variables, a plurality of about 40% makes a forecast that is closest to the sales anchoring rule, (R2), 27% of whom use exactly (R2). This is perhaps not surprising, since (R2) is a simple rule to implement, as it entails assigning the same forecast to the two items under consideration. About 15% of CFOs are closest to the rational coherent rule, (R5), and 7.6% of CFOs are closest to the narrow bracketing one, (R1). Finally, about 11% are closest to (R3) and 27% to (R4). These results underscore the large heterogeneity in forecasting rules used by the CFOs, reflecting the fact that providing coherent forecasts is a challenging task and the managerial education literature has not achieved a consensus in recommending either rule of thumb.

In Table 3 the average distance between the actual forecast and the attributed

[25] Alternatively, Compustat has extensive coverage of the cost of goods sold (COGS). The COGS item bundles together all expenses directly attributable to the production of the goods sold by the firm and includes materials and intermediate inputs, labor cost, energy, and so on. However, the Duke survey does not ask CFOs to forecast COGS, or other non-labor operating expenses, with the exception of advertising.

[26] We compute the distance measure relative to the sales forecast because CFOs are more accurate in their sales forecast than in their forecasts of any other item, implying that doing so allows us to focus on incoherence and minimize concerns that inaccuracy might confound our results. In the Online Appendix, we perform a number of robustness tests. Table A4 finds similar results when using advertising instead of earnings to measure (R5). Table A5 and Table A6 report an even larger incidence of (R2) when distance is measured relative to the earnings forecast and the capital expenditures forecast, respectively.

rule is small, 0.033. This average discrepancy masks heterogeneity across CFOs as documented by the standard deviation of 0.059. Small discrepancies likely reflect minor differences in reporting (e.g., truncation or rounding) or in implementation of the rules (e.g., averaging across a number of past CapEx growth observations other than two in (R1) or using samples of different size in estimating (R3)-(R5)). Larger discrepancies raise the possibility that the CFO is using a different method altogether. To investigate this issue, we compute the fraction of observations for which the distance between the actual forecast and that generated by the attributed rule of thumb falls within a pre-specified interval. We find that in more than 80% of cases, the closest rule of thumb among the five considered is within a $+/-0.05$ interval around the actual CFO forecast. Using the tighter $+/-0.025$ interval around the actual CFO forecast captures close to 60% of CFOs. By way of comparison the standard deviation of revenues growth forecasts is 0.271 (see Panel A of Table 1). Therefore, the width of the $+/-0.05$ interval is about one third of such standard deviation, and that of the $+/-0.025$ interval is about one sixth. These results indicate that our classification is plausible and most likely these CFOs are actually using the rule of thumb we attribute them. To account for the possibility that some CFOs use a different method than the five rules considered, we re-define our CFOs types by limiting ourselves to the cases in which the theoretical rule falls within a $+/-0.05$ interval around the actual forecast, and by defining a residual category, (R6), in which we combine together all CFOs whose closest attributed rule is outside such an interval from their actual forecast. In this residual category (R6) there are 18.7% of CFOs. Below we use this alternative categorization in robustness analyses.

We now compute our ex ante measure of incoherence as the orthogonal distance between the three-dimensional point corresponding to the forecasts of sales growth, capital expenditure growth, and the growth of our proxy for labor costs, and the hyperplane corresponding to (R5). We implement (R5) via a multivariate regression of sales growth on capital expenditures growth and the growth of the same proxy for labor costs,

$$y_{i,t} = \beta_0 + \beta_1 x_{1i,t} + \beta_2 x_{2i,t} + \varepsilon_{i,t}. \tag{4}$$

Specifically, we define

$$\text{Incoherence}_{i,t} = \frac{\left| F_{i,t}\left[y_{i,t+1}\right] - \widehat{\beta}_1 F_{i,t}\left[x_{1i,t+1}\right] - \widehat{\beta}_2 F_{i,t}\left[x_{2i,t+1}\right] - \widehat{\beta}_0 \right|}{\sqrt{1^2 + \widehat{\beta}_1^{\,2} + \widehat{\beta}_2^{\,2}}}, \tag{5}$$

where $\widehat{\beta}_0$, $\widehat{\beta}_1$, $\widehat{\beta}_2$ are the estimated coefficients of (4).

When we proxy $x_{2i,t}$ with the growth of earnings, i.e., net income, as in the bottom row of Table 2, we end up with 396 CFOs in our final sample. In the Online Appendix, where we proxy $x_{2i,t}$ with advertising growth using the estimated coefficients reported in column 8 of Table A3, we end up with 130 CFOs. In the latter case, we obtain similar results. Proxying $x_{2i,t}$ with wages growth results in a sample of about 50 CFOs, too few to allow for a meaningful analysis.

Rather than the specific variable used to proxy for labor and other operating costs, what turns out to be important is moving from a univariate regression of sales growth on capital expenditures growth to a multivariate regression including some proxy of labor costs. That is, it is important that a forecaster making forecasts about individual items takes into account the contemporaneous relationship between multiple items.

Our model predicts a pecking order of rules of thumb: (R5) is first best optimal (see Corollary 2); narrow-bracketing (R1) should be the farthest from the optimal one (see Corollary 6), and economies of scale (R3) should be somewhere in between, dominating sales anchoring (R2) but dominated by (R5) (see Corollary 7).

We evaluate these predictions by regressing our ex ante measure of incoherence on dummy variables for the CFO type. Columns 1 through 4 of Table 4 present estimates of univariate regressions including one dummy at the time, while column 5 presents the full specification where (R5) is used as the reference group. Consistent with our theory, the estimates in column 5 show that (R1) is the farthest away from (R5), followed by (R2). Both (R1) and (R2) deliver significantly different forecasts from the optimal rule (R5), implying the highest incoherence. By contrast, (R3) and (R4) deliver forecasts that are on average statistically indistinguishable from the rational coherent one.

Robustness analyses yield similar results. In Table A7 of the Online Appendix we use advertising instead of earnings to measure (R5), and we find that (R1) is the most distant forecast from (R5), followed by (R2), and both (R1) and (R2) are significantly different from (R5). Using the alternative categorization (R1)-(R6) described above, we find that forecasts in the residual category (R6) are, as expected, the most distant from (R5). Importantly, the relative ranking of (R1)-(R4) is unchanged, with (R1) being most distant from (R5), followed by (R2) and (R3)-(R4). As in the baseline case, only (R1) and (R2) are statistically significantly different from (R5).

Next, we explore how our ex ante measure of incoherence varies with CFOs personal characteristics and firm characteristics. We also include the Optimism and Miscalibration measures of Ben-David et al. (2013) to examine how incoherence is related to those. Panel A of Table 5 shows descriptive statistics: 45% of the CFOs in our sample has an MBA, 9% are females, and on average they are 50.4 years old and have been on the job for 4.3 years. Average firm size is 2.5 billion dollars of sales, average market-to-book ratio is 1.685, and 64% of sample firms pay a dividend. These figures are in line with those reported in prior work (e.g., Ben-David et al. (2013)).

Panel B of Table 5 reports our regression results. It shows that incoherence is lowest at the intermediate age range, 41-50, suggesting that experience may help CFOs form more coherent forecasts, but also that coherence declines at older ages. Incoherence is unrelated to optimism or miscalibration, consistent with the idea from psychology that incoherence and overconfidence are different constructs. Perhaps the most interesting finding is that having an MBA does not correlate with incoherence, likely reflecting the twin facts that some rules of thumb are quite simple, thus CFOs may come up with them on their own, and that there is no consensus in MBA textbooks on which of the different possible rules of thumb should be used. In fact, as we have just seen in Table 4, the rules of thumb perform very differently in terms of forecast coherence.[27] Finally, larger firms and firms paying dividends have a lower incoherence, although the statistical significance is marginal. These results suggest that in stable and more predictable environments it might be easier to come up with more coherent forecasts.

[27] CFO tenure does not correlate with incoherence. This, combined with the fact that longer tenured CFOs delegate less (Graham et al., 2015), suggests that incoherence is unrelated to delegation of authority.

B. Incoherence, Rules of Thumb, and Firm Performance

We now examine the main prediction of our theoretical model: corporate performance should decrease with managerial incoherence, to the extent that incoherence implies the use of a suboptimal mix of inputs. We estimate the regression model,

$$\mathrm{ROA}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \mathrm{Incoherence}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where i indexes the CFO-firm pair, j indexes the industry, t indexes time, the dependent variable $\mathrm{ROA}_{i,j,t}$ is the percent return on the firm's assets, λ_j are industry fixed effects, δ_t are survey fixed effects, and $X_{i,j}$ includes firm-level controls—firm size, market-to-book, and dividends—and CFO-level variables such as miscalibration and optimism, measured at both short- and long-term horizons. Based on our model, we hypothesize $\beta < 0$. We also assume that the technological relationship is stable over time and not subject to aggregate shocks. Therefore, for this part of our empirical analysis we limit ourselves to the 2001-2007 period, to abstract from the impact of the financial crisis, which is arguably an aggregate shock and it has been documented to have an impact on managerial beliefs (e.g., Boutros et al. (2020)). Furthermore, the 2001-2007 period was the peak of the 'great moderation', a time when aggregate volatility was not a concern. We compute bootstrap standard errors following Cameron et al. (2008) and we cluster them at the firm level. Given the above and the fact that we have no source of exogenous variation in incoherence, the empirical results should simply be interpreted as correlations.

Table 6 reports our results. Column 1 indicates that a one standard deviation increase in incoherence (0.079 from Table 4) is associated with a 3-percent lower ROA. This correlation is significant at the 5% level. Columns 2 and 3 show that the results are robust and quantitatively stable when we condition on measures of managerial miscalibration and optimism. Column 4 shows that the results are also stable when we include industry and survey fixed effects. Columns 5-8 report the same specifications when we add firm-level regressors. There is some attrition so sample size shrinks reflecting the availability of regressors, but the main result remains statistically significant and quite stable.

Next, we investigate the extent to which the previous results reflect the use of different rules of thumb. We estimate a specification similar to the previous one, but instead of incoherence we include dummies for CFO types, corresponding to the use of rules of thumb (R1)-(R4), so our results should be interpreted relative to the corporate performance of firms whose CFOs use (R5). Our model predicts that performance should be lowest for CFOs using the narrow-bracketing rule (R1). Table 7 reports our results. Consistent with our model, Table 7 shows that in all specifications (R1) is associated with the lowest corporate performance, with an estimated coefficient that implies a 5%-to-6% lower ROA for firms whose CFO uses (R1) relative to firms whose CFO uses (R5). These are very large differences in economic terms. With respect to the other rules, Table 7 shows both (R2) and (R3) are associated with a 2%-3% lower ROA relative to (R5), whereas performance of firms whose CFOs use (R4) is indistinguishable from the performance of those firms whose CFOs use (R5).

In a robustness analysis, using the (R1)-(R6) categorization described above, we find that forecasts in the residual category (R6) are associated with a 4% lower Return on Asset (ROA), and the results of (R1)-(R4) are very similar to our baseline case just discussed. These results indicate that CFOs whose forecast is quite distant from all the five rules are unlikely to use a more sophisticated or 'better' approach not accounted for in Welch (2017)'s rules of thumb, further supporting our use of (R5) as the ex ante optimal rule of thumb. We conclude that, consistent with our model, corporate performance correlates negatively with incoherence and is on average lowest for firms whose CFOs use the narrow-bracketing rule of thumb.

C. *Incoherence, Rules of Thumb, and Corporate Policies*

We now examine the channels underlying the observed negative correlation between incoherence and performance, and between the narrow-bracketing rule of thumb and performance. According to our theoretical model, incoherence reflects the use of a suboptimal mix of inputs, which leads to lower earnings than it would otherwise be possible given the firm's technology and budget constraint. Given our previous empirical results and the way the rules (R1) and (R2) are constructed, we conjecture that one way through which the suboptimal mix of inputs may come up is by planning a lower investment spending than that needed to achieve the sales revenue growth target. We investigate this conjecture by estimating the following regression model:

$$\mathrm{Y}_{i,j,t} = \alpha + \lambda_j + \delta_t + \beta \cdot \text{Rules of Thumb}_{i,j,t} + \theta \cdot X_{i,j,t} + \varepsilon_{i,j,t},$$

where Rules of $\text{Thumb}_{i,j,t}$ is a vector of binary indicators for the four rules (R1)-(R4), and the dependent variable $\mathrm{Y}_{i,j,t}$ in columns (1)-(3) of Table 8 is the ratio of capital expenditures divided by assets, and then in columns (4)-(6) of Table 8 the ratio of corporate long-term book debt divided by assets. Table 8 reports the results. Columns 1 and 2 of Table 8 show that both (R1) and (R2) are associated with 1.3%-1.6% lower levels of capital expenditures relative to (R5). The difference is large in economic terms, and for (R2) also statistically significant. Columns 4 and 6 of Table 8 show also that (R1) and (R2) are associated with 4% and 9% higher leverage relative to (R5). Again, for (R2) the difference is statistically significant. Interestingly, our results are robust to conditioning for miscalibration and optimism and, consistent with Ben-David et al. (2013), we find that miscalibration and optimism are correlated with higher investment spending, underscoring that incoherence and miscalibration are different phenomena. These results show that the most incoherent rules of thumb (R1) and (R2) are associated with lower investment and higher leverage and suggest that, in line with our theory, managerial incoherence comes with suboptimal investment and financing policies.

D. Change in Firm Behavior when CFOs Take Office

We now search for hints about the direction of causality. On the one hand, high incoherence might lead to lower investment and lower performance. Alternatively, lower investment levels might induce CFOs to be incoherent and forecast too high a growth in sales revenue. Relatedly, incoherent CFOs might be selected, or might self select to work in firms with low investment spending and poor performance.

To shed light on the direction of causality, we exploit within-firm variation across time. We examine how corporate performance, investment, and leverage evolve in the years surrounding a CFO's hiring. We extract the dates when CFOs join firms from

Execucomp and Boardex data and supplement this information by hand-collecting data from 10-K filings. A CFO is considered to take office in a firm when he or she first signs the financial reports. We match corporate performance, investment, leverage, and characteristics from Compustat for the year of taking office. The dependent variables in our regressions are the difference in average ROA, investment, and leverage between the two years following the CFO taking office and the two years prior to the event.

Table 9 presents our results. Column 1 shows that corporate performance declines following the appointment of an incoherent CFO, in particular, a CFO who uses (R1). The use of the narrow-bracketing rule of thumb is associated with a 2.2% lower investment intensity in the two years after that CFO takes office relative to an average investment intensity of 4.5 percentage points. On the other hand, we find no change in corporate leverage around the years an incoherent CFO takes office.

Although we cannot rule out reverse causality, our findings are consistent with CFO incoherence and the use of a narrow-bracketing rule of thumb leading to a decrease in corporate investment and a decrease in corporate performance.

V Statistical Tests of Forecast Coherence

Our results so far indicate that about one half of CFOs use incoherent rules of thumb, including (R1) and (R2), and that the use of such rules correlates negatively with corporate performance and investment spending. Intuitively, these rules imply an association of capital expenditures growth and sales growth forecasts that is much lower than the association observed in realizations. These empirical results do not depend on the theoretical assumptions of Section II; in particular, they do not hinge on the specific form of the production function.

In our empirical analysis so far we have used an ex ante notion of incoherence, ensuring incoherence is predetermined relative to our outcomes of interest (firm performance, investment spending, etc.). Next, we combine forecasts and realizations, and we show how to use forecast errors to disentangle (in)coherence from (in)accuracy ex post. To do so, we add more structure to our framework and we derive novel tests of coherence, which also shed light on potential alternative interpretations of our findings.

A. Disentangling Coherence and Accuracy: Forecast Errors

We begin by considering an empirical formulation of a generic production function,

$$y^f_{t+1} = \alpha + \alpha^f + \sum_{i=1}^{n} \beta^f_i x^f_{i,t+1} + \sum_{i=1}^{n}\sum_{s=0}^{t} \delta^f_{i,t-s} x^f_{i,t-s} + \sum_{j=1}^{m}\sum_{s=0}^{t} \gamma^f_{j,t-s} z^f_{j,t-s} + \varepsilon^f_{t+1}, \quad (6)$$

where f indexes firms, i inputs, and z any relevant state variable (e.g., inventory, cash, etc.). Inputs and state variables are allowed to affect output with firm-specific lags and in a time-varying, firm-specific manner, that is, loadings $\delta^f_{i,t-s}$ and $\gamma^f_{j,t-s}$ can vary both by firm and over time. Note y^f_{t+1} and $x^f_{i,t+1}$ could be in levels, growth rates, or logs, or other transformations that preserve linearity in the regression parameters. Notably, coherent forecasts of these firm-level variables should be cross-sectionally linked in a similar fashion as their realizations in (6), that is,

$$\mathbb{E}_t\left[y^f_{t+1}\right] = \alpha + \alpha^f + \sum_{i=1}^{n} \beta^f_i \mathbb{E}_t\left[x^f_{i,t+1}\right] + \sum_{i=1}^{n}\sum_{s=0}^{t} \delta^f_{i,t-s} x^f_{i,t-s} + \sum_{j=1}^{m}\sum_{s=0}^{t} \gamma^f_{j,t-s} z^f_{j,t-s}. \quad (7)$$

Computing forecast errors by subtracting (7) from (6), $\mathrm{FE}_t\left[y^f_{t+1}\right] = y^f_{t+1} - \mathbb{E}_t\left[y^f_{t+1}\right]$, at the *individual firm level* yields

$$\mathrm{FE}_t\left[y^f_{t+1}\right] = \sum_{i=1}^{n} \beta^f_i \mathrm{FE}_t\left[x^f_{i,t+1}\right] + \varepsilon^f_{t+1}, \quad (8)$$

further implying that the forecast errors associated to coherent forecasts of output and inputs should also be cross-sectionally linked by the parameters of the production function, this time only in terms of the loadings on the contemporaneous inputs, β^f_i. Differently from the forecasts equation (7), the forecast errors equation (8) does not depend on any firm-level unobserved heterogeneity known to, or predictable by, the firm at the time of forecast, as such heterogeneity gets differenced away when subtracting the forecasts from the realizations. This result holds for any production function admitting a separable representation with respect to its inputs, possibly after log-linearization, as in (6).

This discussion suggests two intuitive restrictions on forecast errors of output and inputs. First, as long as inputs contribute positively to output, that is, input loadings are positive, one would expect forecast errors of output and each input to be positively associated. Second, as long as no individual input has increasing returns, that is, input loadings are not larger than one, one would expect forecast errors to lie between the 45 degree line and the horizontal axis. See the shaded areas in Figure 1.

Figure 2 reports the joint distribution of forecast errors of sales and capital expenditures in our data.[28] While the regression slope is positive as expected, 42% of CFOs report forecast errors of sales and capital expenditures of opposite sign, that is, these observations fall outside the first restriction of positive loadings. For an additional 10.4% of CFOs the forecast errors, while of the same sign, imply an input loading larger than one, violating the second restriction. Therefore, more than 50% of CFOs report contemporaneous forecast errors that violate intuitive coherence bounds. We find similar results for additional pairs of forecast errors, including among others sales and earnings.

Building on this intuition, below we introduce additional structure and derive a series of formal statistical tests of coherence. In the next subsection V.$B.$ we discuss regression tests, and in the following subsection V.$C.$ we derive individual-level tests.

B. *Disentangling Coherence and Accuracy: Regression tests*

Besides providing coherence restrictions, Equation (8) also illustrates the connection between forecast accuracy and forecast coherence, and how to distinguish between them. Intuitively, while forecast accuracy requires that for each individual variable being forecasted the forecast errors are on average zero, that is, Avg $\text{FE}_t\left[y^f_{t+1}\right] = 0$ and Avg $\text{FE}_t\left[x^f_{i,t+1}\right] = 0$ for each input i, forecast coherence across variables entering the production function requires that the forecast errors of the output and inputs are "close to one another," in the sense of equation (8).

[28]In our model, we take our input x_1 to be capital, that is, the stock of physical capital, K, typically measured by the balance sheet item Property, Plant, and Equipment, whereas in our empirical implementation we use balance sheet item Capital Expenditures, that is, I. In the Appendix we show that in general the forecast errors of capital expenditures growth should equal the forecast errors of capital growth, $\text{FE}[\Delta I_{t+1}] = \text{FE}[\Delta K_{t+1}]$. The intuition is the same as in (8), namely, computing forecast errors differences away any heterogeneity known to, or predictable by, the firm at the time of the forecast.

These observations suggest regression-based tests of accuracy and coherence. Accuracy can be assessed by testing for each variable separately that the mean of the forecast errors is zero. It can also be tested by regressing realizations on forecasts for each variable separately. Under rationality, one expects a unit slope and, under no aggregate shocks, a zero intercept.[29]

Coherence, on the other hand, can be tested by regressing forecast errors of output on forecast errors of inputs, as per Equation (8). Under rationality, one expects the slope of the forecast error of each input to equal the loadings of that input in (6).

C. *Disentangling Coherence and Accuracy: CFO-level tests*

Adding further structure to our setting of Section II enables us to derive individual-level tests of coherence and accuracy that refine the intuition and formalize the restrictions of subsections V.*A.* and V.*B.*, while maintaining analytical tractability. We assume a unit elasticity of substitution, $\xi \to 0$, and an AR(1) process for the log of input prices. That is, $\pi_{i,t+1} = \gamma_i \pi_{i,t} + \epsilon_{i,t+1}$, with $0 < \gamma_i < 1$ ($\gamma_i = 0$ denotes the i.i.d. case), where the error terms are i.i.d., normally distributed, and uncorrelated, $\{\epsilon_{1,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_1^2)$, $\{\epsilon_{2,t}\}_{t\geq 1} \sim \mathcal{N}(0, \sigma_2^2)$, and $\{\epsilon_{1,t}\}_{t\geq 1} \perp \{\epsilon_{2,t}\}_{t\geq 1}$. The latter assumption provides a way to capture the different conditions and uncertainty firms face, which cannot be directly assessed based on the point forecasts observed in our data.

Proposition 4 (Individual-Level Test Statistics). *If $\xi \to 0$, $\rho_{1,2} = 0$, and $p_1 x_1 + p_2 x_2 = Z$, under the null hypothesis of coherent forecasts it holds that*

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1) \tag{9}$$

and

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1), \tag{10}$$

where $\text{FE}_t \log y_{t+1} = \log y_{t+1} - \mathbb{E}_t \log y_{t+1}$ *and* $\text{FE}_t \log x_{1,t+1} = \log x_{1,t+1} - \mathbb{E}_t \log x_{1,t+1}$.

[29] There is a long tradition in economics of empirical tests of rational expectations of this type, e.g., see Muth (1961), Lovell (1986), Ivaldi (1992), and for a review Pesaran and Weale (2006). For more recent tests involving higher moments, see Crossley et al. (2021) and D'Haultfoeuille et al. (2021).

Proposition 4 derives two test statistics at the individual CFO level. These statistics have an intuitive interpretation. Under the null of coherence, the forecasts of the output and of one input cannot be "too far" from each other, as in (9). Similarly, the forecast errors of the output and of one input cannot be "too far" from each other, as in (10).[30] While Proposition 4 is illustrated for the case of two inputs, the Proof in the Online Appendix also solves the general case with n inputs. Relative to the fairly atheoretical Equation (8) that requires observing forecast errors on all n inputs, Proposition 4 only requires to observe forecast errors of $n-1$ inputs.

Additionally, under Proposition 4, accuracy of the output forecast implies $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y} \sim \mathcal{N}(0,1)$ and, similarly, accuracy of the forecast of a generic input, x_i, implies $\frac{\text{FE}_t \log x_{i,t+1}}{\sigma_i} \sim \mathcal{N}(0,1)$. These conditions provide simple individual-level tests of forecast accuracy. Therefore, once again, our FE-based tests enable us to distinguish forecast accuracy (based on the latter statistics) from forecast coherence (based on the C2-stat in (10)).

We now depict the theoretical connection between the concepts of forecast accuracy and forecast coherence within our current framework. We illustrate this connection in Figure 3 for a firm whose input has a loading of $\frac{1}{3}$. There are four conceptual cases, corresponding to the four areas of the figure. In the first area, the forecasts are both accurate and coherent. This occurs when both $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero and also close to each other. In the second area, $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero but quite close to each other, so the forecasts are inaccurate but coherent. In the third area, both $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$ and $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$ are close to zero but sufficiently apart from each other, so the forecasts are accurate but incoherent. In the fourth area, $\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1}$, $\frac{\text{FE}_t \log y_{t+1}}{\sigma_y}$, or both are statistically different from zero and also far apart from each other, so the forecasts are both inaccurate and incoherent.

Much research in psychology and elsewhere has been cast in terms of whether the accuracy or the coherence paradigm is the correct one; see Hammond (2007) and references therein. These interpretations are incomplete or even misleading. By clarifying their

[30] Relative to the intuitive restrictions discussed in subsection V.A., Proposition 4 clarifies that forecast errors of output and input with opposite signs do not necessarily reflect incoherence, as long as they are 'close enough' to each other. On the other hand, forecast errors of the same sign can reflect incoherence, as long as they are sufficiently far from each other.

theoretical connection, Figure 3 shows that coherence and accuracy are two distinct but related concepts, both of which are necessary to understand rationality of individual forecasts. Figure 3 further shows that one needs an analytical framework nesting both accuracy and coherence to disentangle them and to understand their relationship.

D. Test Implementation and Results

In this subsection we implement our tests from subsections V.*B.* and V.*C.*. In the Online Appendix we also implement the inequality condition we derived in Proposition 1.

Regression tests. We now implement our regression tests from subsection V.*B.* Recall that to implement these tests we need the quantities $\text{FE}_t \log x_{1,t+1}$ and $\text{FE}_t \log y_{t+1}$. These quantities are not directly observed in our data, as in the Duke Survey the forecasts are not elicited in logs. Hence, we use the following transformation, written for a generic x:

$$\mathbb{E}_t \log x_{t+1} = \log \mathbb{E}_t x_{t+1} - \frac{1}{2} V_t \log x_{t+1}, \tag{11}$$

where $V_t \log x_{t+1}$ is the conditional variance of $\log x$. Therefore, $V_t \log x_{i,t+1} = \sigma_i^2 = (1 - \gamma_i^2)\, V \log x_{i,t+1}$ for $i = 1, 2$, where $V \log x_{i,t+1}$ is the unconditional variance and γ_i is the coefficient of an AR(1) regression of $\log x_{i,t}$. The conditional variance of the output is then $V_t \log y_{t+1} = a^2\sigma_1^2 + b^2\sigma_2^2$. We compute a and b using the universe of industries from the Bureau of Economic Analysis.

Table 10 reports the results. To test for accuracy, we regress $\text{FE}_t \log x_{1,t+1}$ on a constant (column 1) and $\text{FE}_t \log y_{t+1}$ on a constant (column 2). Column 1 shows that the constant term is significantly different from zero at the 10% level, whereas column 2 shows that the constant term is insignificantly different from zero. These results indicate that CFO forecasts are on average inaccurate about capital expenditures growth, whereas they are on average accurate about sales growth. The latter is consistent with the observations in Graham (2022) that top executives care the most about their sales forecasts.

To test for coherence, we first estimate the regression,

$$\text{FE}_t \log y_{t+1} = \alpha + \beta_1 \cdot \text{FE}_t \log x_{1,t+1} + \beta_2 \cdot \text{FE}_t \log x_{2,t+1} + \varepsilon_{t+1}, \tag{12}$$

based on Equation (8), whereby under forecast coherence one would expect α=0, β_1=a, and β_2=b, namely, the slope coefficients should equal the capital and labor shares, respectively. Column 3 of Table 10 reports the results. We find $\widehat{\alpha}$ =0.46, statistically different from 0 at the 5% level. We also find $\widehat{\beta_1}$ =0.113, marginally significant at the 10% level, and significantly smaller than the capital share of 0.4 at all significance levels. Finally, $\widehat{\beta_2}$ =0.023, insignificantly different from zero and thus smaller than the labor share. Therefore, we reject the null of coherence. However, requiring observations of forecasts and realizations of wages implies that we end up with only 51 observations.

To address this data limitation, we take advantage of Proposition 4, which implies that the coherence test based on Equation (8) can be implemented using only $n-1$ inputs. Therefore, in column 4 we implement the regression test without wages. We now find $\widehat{\alpha}$ not significantly different from zero, and $\widehat{\beta_1}$ =0.135, precisely estimated and significantly smaller than the capital share of 0.4. In fact, we reject the null for any estimate of the capital share of 0.30 or above. When we allow the capital share to vary by industry, we reject the null of coherence for 7 out of 13 industries, representing 86% of the total observations (not shown). Furthermore, we find $\widehat{\beta_2}$ to be insignificantly different from zero. For completeness, in column 5 we estimate Equation (8) without capital expenditures, and we also find the coefficient on wages to be insignificantly different from zero.

In sum, the results in Table 10 indicate that CFOs are on average incoherent and inaccurate about capital expenditures, while being on average accurate about sales. These results suggest that accuracy and coherence are not only distinct concepts in theory but also in our data. We now dig deeper in this direction by implementing our individual-level tests below, which aim at disentangling coherence and accuracy at the individual level.

Individual-level tests. To test for coherence, we focus on the C2-stat because it does not require observing the second input (labor), budget (Z), or other firm-level characteristics. To implement the C2-stat, for each CFO we observe four items, two forecasts and two realizations, and we estimate three parameters, a, b, and σ_2, from aggregate industry data from the Bureau of Economic Analysis and Bureau of Labor Statistics. As a result, our C2-stat is distributed according to a Student t distribution with $N - K = 4 - 3 = 1$ degree of freedom. Table 11 reports our results. Panel A of Table 11 shows that for 55.7% of CFOs in our sample we reject the null hypothesis of coherence at the 95% confidence level.[31] This result parallels our previous findings that 48% of CFOs use incoherent rules of thumb, including (R1) and (R2), and the use of these rules correlates negatively with performance. This result further parallels our previous finding that 52% of CFOs violate the restrictions discussed in subsection V.A.

Panel A of Table 11 confirms that, by contrast, CFOs are fairly accurate with respect to their output forecasts. In fact, we reject the null of accuracy for output forecasts at the 95% confidence level for 27.2% of CFOs. Panel A also confirms that CFOs are substantially less accurate with respect to their capital expenditures forecasts, as we reject the null of accuracy in capital expenditures forecasts for 47.9% of CFOs in our sample. When considering output and capital input forecasts together, we reject the null of accuracy for 57% of CFOs in our sample.

These results allow us to reject an alternative interpretation of our previous findings, that is, that CFOs wield control over these variables to a different extent. While CFOs have ultimate authority over their firm's capital expenditures, their firm's sales depend also on external factors, including customer behavior and product market competition. Thus, one might expect CFOs to be more accurate about capital expenditures than about sales. This is the opposite of what we find. In fact, CFOs are much less accurate in forecasting capital expenditures, reflecting the fact that their capital expenditure forecast is incoherently linked to their sales forecast.

Panel B of Table 11 assesses coherence and accuracy together. It shows that 31.1% of CFOs in our sample are both coherent and accurate; 13.2% are coherent but inaccurate; 12.0% are accurate but incoherent; and the remaining 43.7% are both incoherent and inaccurate (all at the 95% confidence level).[32] The summary statistics of the cross-sectional distribution of our calculated C2-stat and of the forecast errors for output and capital input are shown in Panel C.

[31] At the 99% confidence level, we reject the null of coherence for 7.7% of CFOs. The difference between the rejection regions at 95% and 99% confidence level reflects the distribution of the Student t with one degree of freedom.

[32] The figures at the 99% confidence level are 89.4%, 2.9%, 3.4%, and 4.3%, respectively.

One concern with these results is the extent to which the computed C2-stat, as well as the accuracy statistics, are sensitive to the uncertainty coming from estimating the parameters a, b, and σ_2.[33] To address this concern, we perform a non-parametric bootstrap procedure, as follows. For each CFO, we generate 1,000 bootstrap repetitions of the C2-stat.[34] Using these 1,000 replications, we compute the fraction of cases out of 1,000 for which we reject the null of coherence at the 95% and 99% confidence levels. Hence, for each CFO and confidence level, the computed statistic is a number between 0 and 1, where 0 means that the null of coherence was never rejected across all 1,000 repetitions and 1 means that the null of coherence was rejected for all 1,000 repetitions.

In Figure 4, we plot the value of this statistic (on the vertical axis) against its empirical cumulative distribution function across CFOs (on the horizontal axis). The top plot refers to the calculation of the statistic at the 95% confidence level, while the bottom plot refers to its calculation at the 99% confidence level. The graph on the top shows that for about 40% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 15% of CFOs the null of coherence is never rejected; and for the remaining 45% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. Consistent with the results in Table 11, the proportion of CFOs for whom the null of coherence is rejected more than half of the times is approximately 55%.[35] We conclude that our results in Table 11 are robust to estimation uncertainty.

[33]To be precise, the C2-stat in (10) depends on a, b, and σ_2 *directly*, where σ_2 is the square root of the conditional variance of the log of x_2 at $t+1$ given the information set at t. Furthermore, the C2-stat depends *indirectly* on σ_1 (defined analogously to σ_2) through the forecast error of the log of x_1, which depends on (11). Finally, the C2-stat further depends indirectly on σ_y through the forecast error of the log of y, which in turn depends on a, b, σ_1^2, and σ_2^2.

[34]Specifically, for each of the 15 BEA industries and pair of consecutive years between 1987 and 2018, we resample observations with replacement 1,000 times (aka bootstrap replications). At each replication, we obtain an estimate of σ_1 based on the residual sum of squares (RSS) of the regression of total capital compensation on its lag and an estimate of σ_2 based on the RSS of the regression of total labor compensation on its lag, using cluster bootstrap with 6 clusters corresponding to the following year windows: 1988-1992, 1993-1997, 1998-2002, 2003-2007, 2008-2012, and 2013-2018. We additionally generate bootstrap estimates of a and b as the capital and labor shares of total factor compensation. Endowed with bootstrap estimates of a, b, σ_1^2, and σ_2^2, we derive corresponding estimates for σ_y^2, for the forecast errors of the log of output (y) and of the log of capital input (x_1), and thus for the C2-stat.

[35]The graph on the bottom of Figure 4 shows that for slightly less than 10% of CFOs the null of coherence is rejected in all bootstrap repetitions; for about 60% of CFOs the null of coherence is never rejected across all bootstrap repetitions; and for the remaining 30% of CFOs the fraction of rejections across bootstrap repetitions is strictly between 0 and 1. The proportion of CFOs for whom the null of coherence is rejected more than half of the times is slightly over 10%, that is, slightly higher but in the same ballpark as the proportion of rejections calculated in Table 11.

Discussion. Our results in subsections IV.*A.* and IV.*B.* imply that at least 48% of the CFOs in our data make incoherent forecasts by using incoherent rules of thumb, including the 'narrow bracketing' and the 'sales anchoring' forecast rules (R1) and (R2). These results are consistent with our theory's predictions that (R1) and (R2) should be the most distant from the ex ante coherent one, (R5), and that (R1) and (R2) should yield inferior performance. At the same time, these empirical results do not depend on our model's assumptions about the firm's environment and technology of Section II, as they simply reflect the CFOs' ex ante use of particular rules of thumb.

In subsection V.*A.*, we have discussed some plausible ex post coherence restrictions on contemporaneous forecast errors of output and inputs, and we have found that such restrictions are violated by 52% of CFOs in our data.

In subsection V.*B.*, we have discussed some regression tests of accuracy and coherence. We have found that CFOs are on average incoherent in their joint forecasts of capital expenditures and sales growth; they are on average inaccurate in their forecasts of capital expenditures growth; but they are on average accurate in their forecasts of sales growth. These results already point to the fact that coherence and accuracy are not only two distinct theoretical concepts but they are also different constructs in the data.

In subsection V.*C.*, we have introduced additional structure and derived a series of formal statistical tests of accuracy and coherence at the individual level. We have rejected the null of coherence at the 95% level for 56% of CFOs in our data. We have also found that 13.2% of CFOs are coherent but inaccurate and 12.0% are accurate but incoherent. These results confirm empirically the theoretical prior that coherence and accuracy are two distinct standards of rationality.

All of the above sets of results, while relying on different setups and assumptions, paint a consistent picture indicating that about one half of CFOs provide incoherent forecasts of their firm's contemporaneous output and input growth.

VI Related Literature

Coherence and Accuracy Requirements of Rationality. The psychology literature has long recognized that rationality in probabilistic judgments and forecasts involves both accuracy (sometimes called 'correspondence') and coherence (sometimes called 'consistency').[36] This literature typically maintains that accuracy and coherence are separate properties, but has struggled to provide a formal framework or direct evidence to assess such a claim.[37] Rather, the literature has focused on predicting systematic inaccuracy from specific violations of statistics and probability laws.

In a series of famous experiments, Tversky and Kahneman (1971, 1974, 1983) document systematic misconceptions of statistics and probability theory, including the law of large numbers, the conjunction rule, the law of total probability, and Bayes' theorem. Since then, a number of theories and experiments have shown how the extent of such misconceptions predicts systematic inaccuracy in specific prediction tasks, see, e.g., Rabin (2002), Benjamin et al. (2016), Wright et al. (1994), Zhu et al. (2022), Berg et al. (2022).

Tversky and Kahneman (1974) have famously labeled this research program as "heuristics and biases", whereby the use of heuristics generates systematic and predictable forecast errors (see also Thaler (2018)). More recently, however, a number of authors have recognized that at least some of these results can also be cast in terms of the coherence-accuracy framework. For example, Hammond (1996), Tentori et al. (2013), Arkes et al. (2016), Jönsson and Shogenji (2019), and others discuss how Tversky and Kahneman (1983)'s conjunction fallacy can be understood as a violation of coherence with respect to probability laws. Similar arguments can be made with respect to the disjunction fallacy and violations of Bayes' theorem. Therefore, documenting a form of incoherence such as the violation of the law of total probability or Bayes' theorem in one

[36] See, e.g., Hammond (1990, 1996), Osherson et al. (1994), Gigerenzer et al. (1999), Mandel (2005), Gigerenzer and Gaissmaier (2011), Lee and Zhang (2012), Arkes et al. (2016).

[37] Ethical and political philosophers and legal scholars have often seen incoherence in the context of human institutions such as moral and legal systems as concerning, e.g., Raz (1994), Rawls (1999), Dworkin (1986), Sunstein et al. (2002), and Fogelin (2003). A theoretical literature in philosophy develops axiomatic definitions of coherence using probability theory, e.g., see Schippers (2014) and references therein. A general theme in this literature is to define two propositions as coherent with each other if they are positively correlated according to some suitably defined measure of correlation. In the data, however, multiple forecasts may be correlated for reasons other than coherence. Furthermore, none of these papers disentangle coherence from accuracy.

domain typically serves the purpose of predicting future systematic inaccuracy in another domain, without aiming at disentangling coherence from accuracy.

Relative to this literature, our key contribution is to provide a formal framework in a forecasting setting in which coherence and accuracy are defined with respect to the same forecasting task, which allows us to jointly assess forecast accuracy and coherence. We show that to do so one needs both theory and data. In terms of theory, an economic model provides a benchmark against which to judge coherence. This is similar to the use of probability theory for assessing coherence of probabilistic judgments, but crucially economic theory allows us to nest forecast coherence and accuracy in a setting with optimizing agents. In terms of data, observing both forecasts and realizations—and thus forecast errors—allows us to jointly assess accuracy and coherence, and also to disentangle them at the individual level. Accuracy is assessed by testing whether forecast errors of each variable are 'sufficiently' close to zero; coherence is assessed by testing whether forecast errors of different variables (inputs and output in our setting) are 'sufficiently' close to one another. In both cases, the extent of 'sufficiently' is pinned down by economic theory. In our data, we find that 12% of individuals are incoherent but accurate, and 13% of individuals are inaccurate but coherent.

Disentangling coherence and accuracy is crucial, because—at least under some conditions—coherence can be assessed ex ante. This is a similar intuition to that of the debiasing research program in psychology (e.g., Fischhoff (1982)), with the key difference that in our framework one can use economic theory and regression analysis to determine the ex ante coherent rule. In fact, the managerial education literature (e.g., Ruback (2004), Titman and Martin (2016), Welch (2017), Holthausen and Zmijewski (2020), and Koller et al. (2020)) recognizes the challenge of forecasting many firm variables at the same time and provides a number of rules of thumb, without however relying on theory or evidence to guide the choice among them. We show both theoretically and empirically that not all rules of thumb are equivalent to one another. To the contrary, while some rules of thumb do come close enough to the ex ante optimal coherent forecast, others provide incoherent forecasts, particularly the 'sales anchoring' and the 'narrow bracketing' rules.

Bracketing. Lab experiments and empirical research in psychology and behavioral economics show that decision makers often narrowly bracket and make interrelated decisions in isolation (e.g., Tversky and Kahneman (1981), Read et al. (1999), Rabin and Weizsäcker (2009), and Ellis and Freeman (2020)). To rationalize this evidence, Thaler (1985, 2018) and Heath and Soll (1996) argue that individuals hold a mental account of each decision without considering their interdependence implied by the budget constraint and the marginal rates of substitution in the utility function.

Economic theories of narrow bracketing and mental accounting include Barberis et al. (2006), Rabin and Weizsäcker (2009), Hastings and Shapiro (2013, 2018), and Lian (2021).[38] Our theory is closest to Lian (2021), who models a narrow thinker making consumption decisions about individual goods with imperfect knowledge of the other goods and faces difficulties at coordinating multiple decisions, thereby endogenizing narrow bracketing.

We add to this literature by developing the first model of bracketing in a firm setting featuring a production technology, and by providing field evidence of narrow bracketing in a sample of senior financial executives directly involved with corporate forecasts and production decisions. Similar to Lian (2021), there are no explicit mental budgets, which avoids the need to take a stand on where such mental budgets come from. Moreover, our agent also makes decisions and forecasts based on different, non-nested information, which differentiates both us and Lian (2021) from the rational inattention literature (e.g., Sims (2003), Mackowiak and Wiederholt (2009), Matějka and McKay (2015), and Kőszegi and Matějka (2020)), in which different decisions are made based on the same nested information.[39] Unlike Lian (2021), we study a production model where the interconnection among different decisions come from the budget constraint and the production technology,

[38] These models study the monetary gambles, stock market participation, and consumption decisions of narrow-bracketing agents. They show that narrow bracketing can lead to stochastically dominated choices, including low stock market participation, stochastically dominated gambles, and suboptimal consumption bundles.

[39] Rational inattention models also use noisy signals to capture decision makers' inability to incorporate all relevant information when making each decision. Related to rational inattention but using a deterministically imperfect perception of fundamentals rather than noisy signals, Gabaix (2014, 2019) develops a sparsity model in which, similar to the rational inattention approach, the sparse agent's multiple decisions are made based on the same, imprecise perception of the fundamental. However, in all these models, different decisions are made based on the same nested information.

which delivers novel predictions about corporate forecasts and corporate performance, which we then test in our data.

Survey Expectations of Firms. Our paper is also related to the recent and growing empirical literature studying beliefs and forecasts of corporate top executives both about the macroeconomy and about own variables.[40] Ben-David et al. (2013) and Boutros et al. (2020) show that top executives are miscalibrated, as they provide probability distributions of stock market returns that are too narrow, consistent with managerial overconfidence (see also Campello et al. (2010), Campello et al. (2011, 2012) on financial constraints and the financial crisis). Bloom et al. (2021) show that forecasting firms' own variables is even harder than forecasting the aggregate economy. Gennaioli et al. (2016) show that corporate investment plans as well as actual investment are explained by CFOs' expectations of earnings growth. Graham (2022) documents that the revenue growth forecast is most important in terms of its consequences for the firm and its plans (see also Altig et al. (2022)). Bachmann and Bayer (2013, 2014) find that the dispersion and volatility of expectations and expectation errors are countercyclical. We confirm that firms make on average accurate sales growth forecasts, but we also show that expectations of other variables, for example capital expenditures, are much less predictive of realized growth rates, consistent with incoherence. We add to this literature by documenting heterogeneity in the way in which corporate managers provide forecasts of multiple balance-sheet items at the same time. We are able to directly control for the measures of overconfidence and optimism used in this literature and we show that, unlike overconfidence that predicts more aggressive corporate investment spending, incoherence correlates with reduced investment spending and increased leverage, consistent with the idea from psychology that incoherence and overconfidence are different constructs.

[40] See selected chapters on firm expectations in the recent Handbook of Economic Expectations, e.g., Born et al. (2023) and Candia et al. (2023).

VII Conclusion

We develop a theory of forecast coherence in firm production, which yields a benchmark of first-best coherent forecasts. In a positive version of our theory, incoherence arises from 'narrow thinking'—intra-personal frictions in coordinating forecasts of multiple contemporaneous variables—and operates via the use of rules of thumb. We rationalize some (but not all) of the rules of thumb proposed by managerial textbooks as second-best optimal responses to noisy signals about the firm's inputs and output.

Using the Duke Survey of top executives, we document substantial heterogeneity in the use of rules of thumb. Consistent with the predictions of our model, firm performance correlates negatively with incoherence, and is on average lowest for firms whose CFOs provide 'narrow bracketing' and 'sales anchoring' forecasts. We emphasize that this first set of empirical results reflects the CFO's ex ante use of particular rules of thumb, and not the theoretical assumptions of our model.

We then introduce a series of ex post coherence restrictions, regression tests, and individual-level tests, all based on forecast errors. While relying on different setups and assumptions, all our empirical results paint a consistent picture: about one half of CFOs provide incoherent forecasts of their firm's contemporaneous output and input growth.

These results reflect the fact that using different rules of thumb can generate very different capital expenditures forecasts given the same sales growth forecast; recall the comparison of (R2) sales anchoring with (R3) economies of scale. These results further reflect the lack of consensus in managerial textbooks and case studies and the lack of theory and evidence to distinguish among a plethora of competing rules of thumb.

An important takeaway of our paper is that simple, intuitive, and much advertised rules of thumb such as those we have dubbed sales anchoring and narrow bracketing perform poorly, and thus should not be part of the toolkit provided to future managers.

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Figure 1: **Coherence Restrictions on Contemporaneous Forecast Errors**

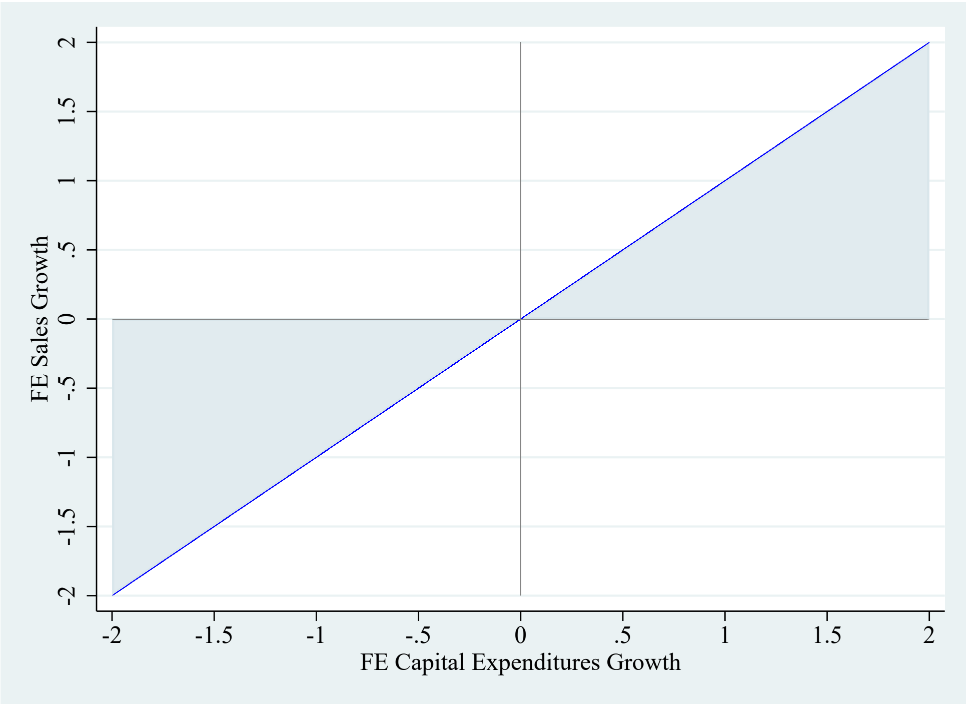

Figure 2: **Contemporaneous Forecasts Errors of Output and Capital**

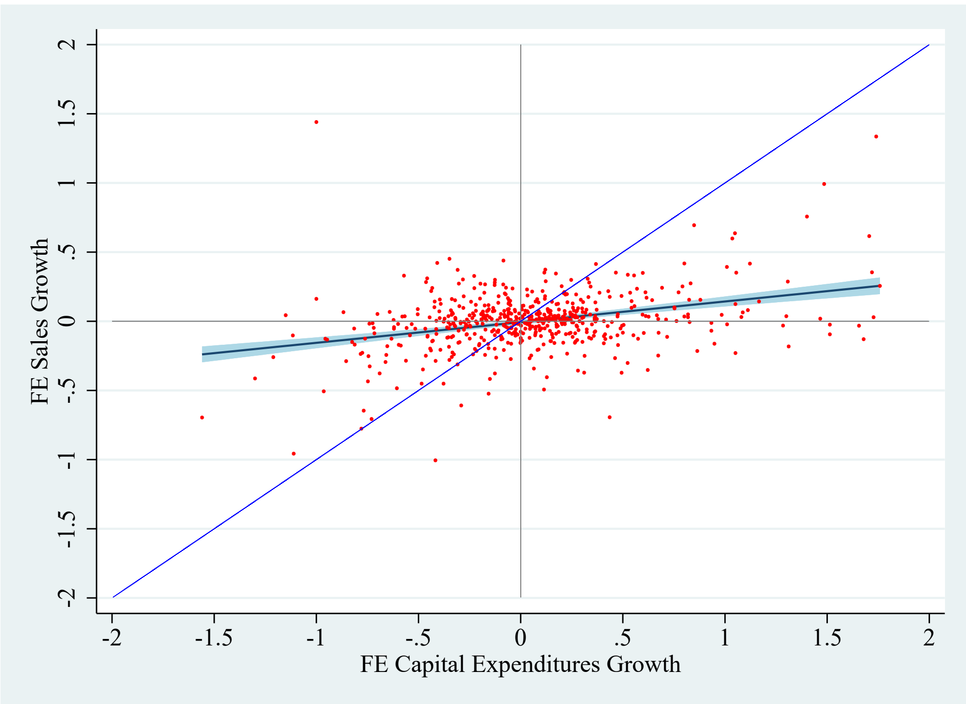

Figure 3: **(In)Coherence and (In)Accuracy Areas**

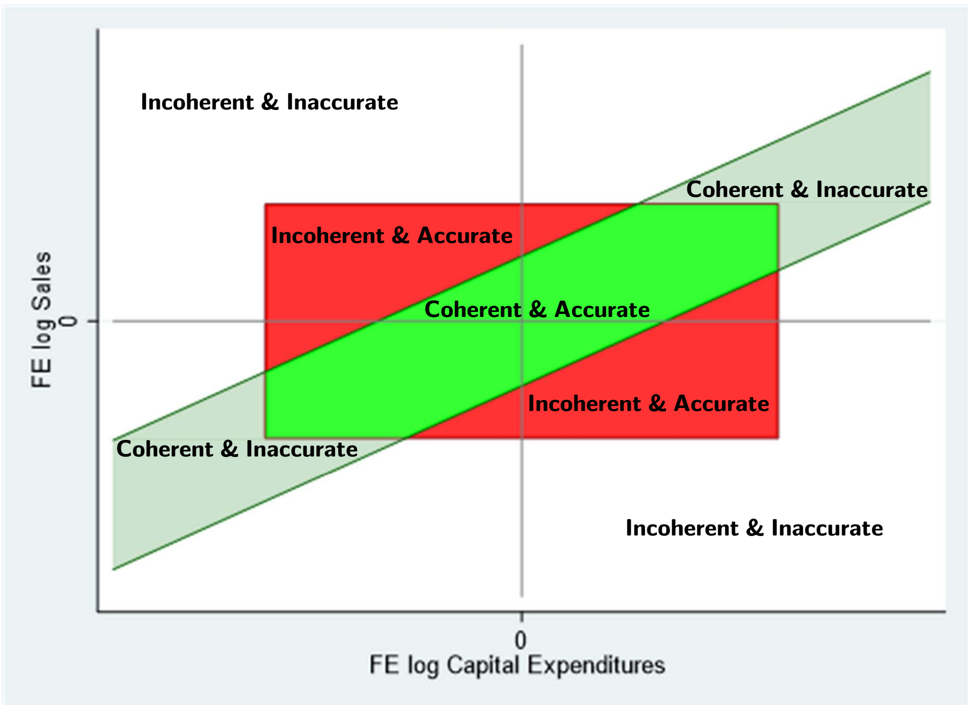

Figure 4: **Bootstrap of Coherence Test Statistic C2-stat**

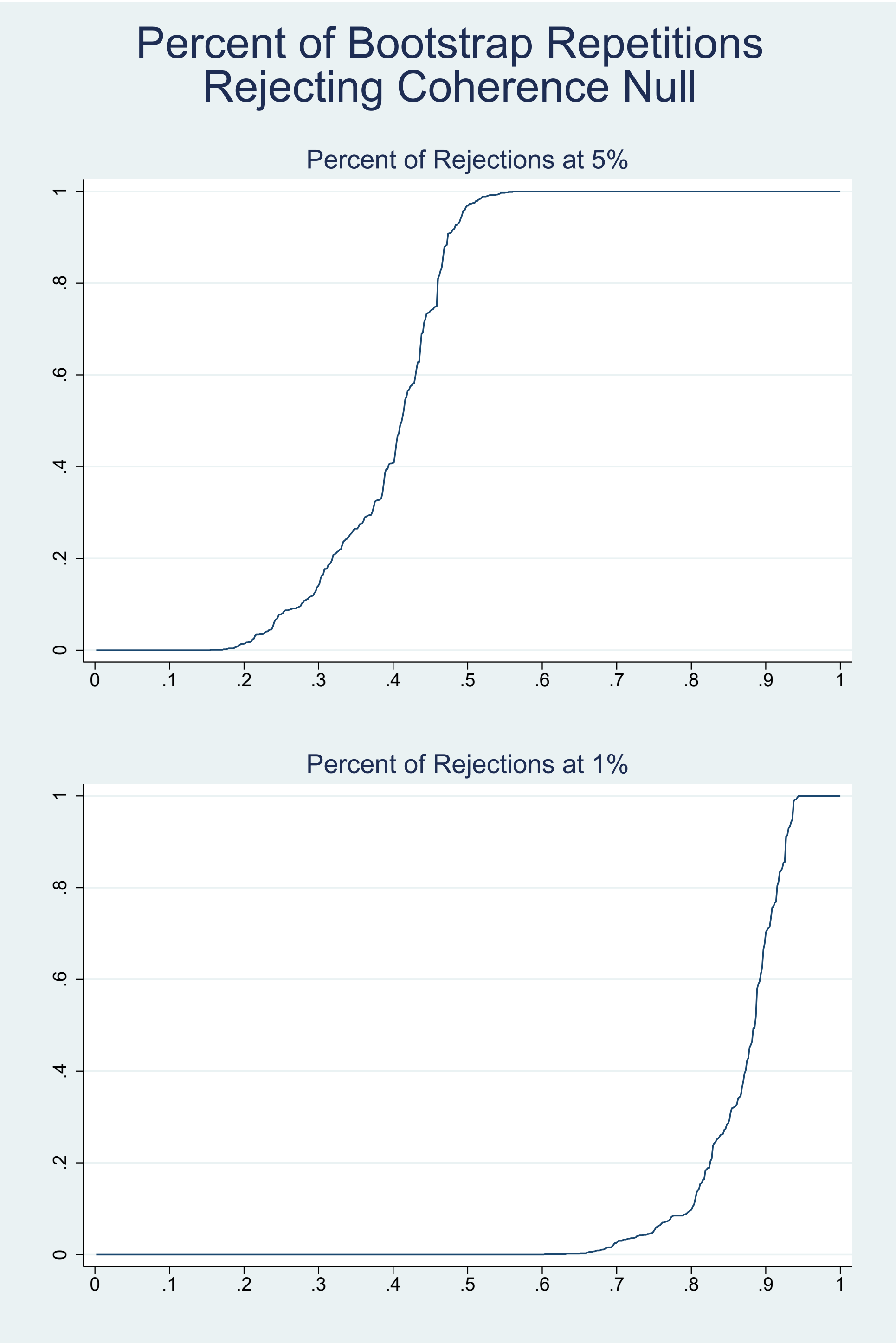

Table 1: **CFO Growth Forecasts and Realizations of Selected Balance Items**

Panel A – CFO Growth Forecasts (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Expected Growth in Revenues and in Earnings						
Revenues	9.30	27.13	-5.00	5.00	20.00	14,490
Earnings	11.00	42.37	-10.00	5.00	30.00	25,472
Expected Growth in Capital-Related Expenditures						
Capital Expenditures	8.11	43.90	-15.00	3.00	25.00	25,305
R & D	4.51	21.65	0.00	0.00	15.00	8,325
Technology Spending	6.68	28.02	-5.00	3.00	20.00	22,404
Expected Growth in Labor-Related Costs						
Wages	3.90	12.41	0.00	3.00	7.00	27,472
Employees	3.95	30.16	-5.00	1.00	10.00	25,471
Outsourced Employees	3.74	21.19	0.00	0.00	10.00	10,990
Health Spending	8.59	11.65	1.00	8.00	15.00	25,064
Expected Growth in Productivity, Product Prices, and Advertising						
Productivity	3.91	9.38	0.00	3.00	10.00	18,197
Product Prices	2.08	8.22	-3.00	2.00	7.00	24,499
Advertising	4.75	21.83	-5.00	2.00	15.00	20,989
Expected Growth in Cash Holdings and Corporate Payout						
Cash	5.02	38.56	-20.00	0.00	20.00	16,876
Dividends	4.54	30.52	0.00	0.00	15.00	5,227
Share Repurchases	1.55	24.40	0.00	0.00	5.00	5,487

Panel B – Realizations, Matched Compustat-Duke Sample (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Actual Growth in Revenues and in Earnings						
Revenues	6.80	21.32	-13.56	5.23	27.25	14,549
Earnings	-10.36	307.02	-161.71	2.36	124.59	14,580
Actual Growth in Capital-Related Expenditures						
Capital Expenditures	15.87	67.07	-42.59	3.96	75.70	13,770
R & D	7.09	29.57	-19.85	4.27	33.33	6,456
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Labor-Related Costs						
Wages	7.02	14.65	-7.23	5.35	22.10	2,836
Employees	2.98	16.95	-11.88	1.19	17.71	14,359
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Actual Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	8.03	42.14	-26.76	2.79	38.46	5,735
Actual Growth in Cash Holdings and Corporate Payout						
Cash	35.42	132.66	-46.26	5.76	113.78	14,520
Dividends	12.68	52.88	-12.22	6.15	38.44	8,762
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table 2: **Cross Sectional Regressions for Rules of Thumb using Compustat Data**

	Intercept	Slope 1	Slope 2	R^2	N Obs
Rule 1 ('narrow bracketing')	0.316***	-0.089***		0.004	74,413
	(0.041)	(0.016)			
Rule 3 ('economies of scale')	0.217***	1.055***		0.081	100,441
	(0.025)	(0.036)			
Rule 4 ('industry based')					
SIC 0	0.330***	2.050***		0.097	358
	(0.156)	(0.344)			
SIC 1	0.243***	0.950***		0.115	8,983
	(0.045)	(0.072)			
SIC 2	0.243***	0.859***		0.050	14,777
	(0.026)	(0.054)			
SIC 3	0.186***	1.188***		0.104	24,852
	(0.024)	(0.058)			
SIC 4	0.180***	0.925***		0.064	14,398
	(0.022)	(0.091)			
SIC 5	0.163***	1.281***		0.081	10,266
	(0.027)	(0.121)			
SIC 6	0.402***	0.963***		0.036	7,477
	(0.041)	(0.062)			
SIC 7	0.202***	1.162***		0.105	14,673
	(0.039)	(0.090)			
SIC 8	0.198***	1.216***		0.088	3,911
	(0.023)	(0.128)			
SIC 9	0.222***	1.288***		0.123	746
	(0.058)	(0.182)			
Rule 5	0.217***	1.042***	0.018***	0.082	100,040
	(0.025)	(0.036)	(0.004)		

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.

Table 3: **Minimum Distance of Sales Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.033	0.058	0.030	0.019	0.031	0.043
Std. Dev.	0.059	0.100	0.064	0.017	0.035	0.069
Frac. Zeros	0.106	0.000	0.268	0.000	0.000	0.000
P10	0.000	0.008	0.000	0.005	0.002	0.003
P25	0.007	0.015	0.000	0.006	0.007	0.008
P50	0.019	0.028	0.014	0.010	0.023	0.023
P75	0.036	0.064	0.035	0.028	0.048	0.043
P90	0.071	0.114	0.071	0.048	0.072	0.089
N of Observations	396	30	157	43	107	59
Fraction	1.000	0.076	0.396	0.109	0.270	0.149

Notes: Cross-sectional analysis with 396 CFOs.

Table 4: **Incoherence and Rules of Thumb: Distance from Optimal Forecast**

	(1)	(2)	(3)	(4)	(5)
Rule 1 (‘narrow bracketing’)	0.081***				0.104***
	(0.014)				(0.016)
Rule 2 (‘sales anchoring’)		0.039***			0.053***
		(0.008)			(0.011)
Rule 3 (‘economies of scale’)			-0.055***		-0.020
			(0.012)		(0.014)
Rule 4 (‘industry based’)				-0.027***	0.010
				(0.009)	(0.012)
Constant	0.066***	0.057***	0.079***	0.080***	0.043***
	(0.004)	(0.005)	(0.004)	(0.005)	(0.009)
R^2	0.071	0.057	0.045	0.023	0.175
N observations	396	396	396	396	396

Summary Statistics of the dependent variable	
Mean	0.073
Std. Dev.	0.079
P10	0.012
Median	0.059
P90	0.139

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table 5: **Incoherence and CFO & Firm Characteristics**

Panel A – Summary statistics

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
CFO has MBA	0.452	0.498	0.000	0.000	1.000	396
Age	50.43	6.937	42.00	50.00	60.00	396
Tenure	4.273	4.099	0.000	3.000	9.000	396
Gender	0.088	0.284	0.000	0.000	0.000	396
Miscalibration ST	0.035	0.920	-1.166	0.329	0.985	360
Optimism ST	0.052	0.981	-0.918	-0.077	1.285	373
Miscalibration LT	0.039	0.979	-1.095	0.262	0.917	362
Optimism LT	0.033	1.088	-1.008	-0.078	1.077	374
Firm size	7.829	2.296	4.898	7.805	10.61	396
Market-to-Book	1.685	0.900	0.998	1.393	2.731	364
Dividends	0.636	0.482	0.000	1.000	1.000	396

Panel B – Incoherence and CFO characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
CFO has MBA	0.005					0.007
	(0.009)					(0.011)
Tenure > Median	0.008					0.006
	(0.008)					(0.011)
Age 40-		-0.011				-0.025
		(0.022)				(0.029)
Age 41-50		-0.027*				-0.038**
		(0.016)				(0.019)
Age 51-60		-0.024				-0.030*
		(0.017)				(0.017)
Gender		0.002				0.005
		(0.010)				(0.012)
Miscalibration ST			-0.012			-0.014
			(0.008)			(0.010)
Optimism ST			-0.012			-0.010
			(0.007)			(0.009)
Miscalibration LT				-0.005		-0.002
				(0.004)		(0.006)
Optimism LT				0.001		0.005
				(0.004)		(0.007)
Firm size					-0.006*	-0.005*
					(0.003)	(0.003)
Market-to-Book					0.011	0.014
					(0.013)	(0.014)
Dividends					-0.015	-0.020
					(0.012)	(0.013)
Constant	0.043*	0.078***	0.052*	0.046**	0.137**	0.159***
	(0.022)	(0.025)	(0.027)	(0.021)	(0.036)	(0.049)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.111	0.112	0.154	0.126	0.162	0.227
N of Observations	396	396	360	362	364	332

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Robust standard errors are reported in parenthesis.

Table 6: **Incoherence and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Incoherence	-0.377**	-0.378**	-0.360**	-0.396**	-0.399**	-0.386**	-0.317*	-0.307*
	(0.157)	(0.179)	(0.162)	(0.162)	(0.186)	(0.169)	(0.192)	(0.181)
Miscalibration ST		0.003			0.001		-0.001	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.000			0.000		0.001	
		(0.006)			(0.006)		(0.005)	
Miscalibration LT			0.004			0.002		0.001
			(0.005)			(0.005)		(0.005)
Optimism LT			0.008			0.007		0.009
			(0.006)			(0.006)		(0.006)
Firm size							0.009***	0.009**
							(0.003)	(0.003)
Market-to-Book							0.028**	0.027*
							(0.014)	(0.015)
Dividends							0.022*	0.023*
							(0.012)	(0.013)
Constant	0.069***	0.069***	0.068***	0.054***	0.056***	0.057***	-0.131***	-0.123***
	(0.011)	(0.011)	(0.011)	(0.014)	(0.020)	(0.019)	(0.047)	(0.0471)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.046	0.042	0.047	0.071	0.064	0.068	0.177	0.185
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 7: **Rules of Thumb and Corporate Performance (Return on Assets)**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Rule 1 ('narrow bracketing')	-0.057**	-0.061**	-0.059**	-0.051**	-0.059**	-0.055**	-0.053**	-0.051**
	(0.022)	(0.025)	(0.024)	(0.023)	(0.025)	(0.025)	(0.026)	(0.025)
Rule 2 ('sales anchoring')	-0.026*	-0.027*	-0.023	-0.023	-0.028*	-0.024	-0.034	-0.031
	(0.0138)	(0.015)	(0.015)	(0.015)	(0.017)	(0.016)	(0.021)	(0.019)
Rule 3 ('economies of scale')	-0.031*	-0.036*	-0.034*	-0.027	-0.037*	-0.034	-0.047**	-0.045**
	(0.017)	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.023)	(0.022)
Rule 4 ('industry based')	-0.012	-0.010	-0.010	-0.008	-0.008	-0.007	-0.012	-0.011
	(0.012)	(0.014)	(0.014)	(0.013)	(0.014)	(0.015)	(0.015)	(0.015)
Miscalibration ST		0.001			-0.001		-0.002	
		(0.005)			(0.005)		(0.004)	
Optimism ST		0.001			0.000		0.001	
		(0.006)			(0.005)		(0.005)	
Miscalibration LT			0.003			0.002		0.001
			(0.006)			(0.005)		(0.004)
Optimism LT			0.007			0.006		0.008
			(0.006)			(0.006)		(0.005)
Firm size							0.010***	0.009***
							(0.004)	(0.004)
Market-to-Book							0.028**	0.028*
							(0.014)	(0.015)
Dividends							0.029**	0.030**
							(0.013)	(0.014)
Constant	0.065***	0.066***	0.064***	0.040***	0.045**	0.046*	-0.147***	-0.137***
	(0.011)	(0.012)	(0.013)	(0.015)	(0.019)	(0.028)	(0.046)	(0.050)
Industry FE	No	No	No	Yes	Yes	Yes	Yes	Yes
Survey FE	No	No	No	Yes	Yes	Yes	Yes	Yes
R^2	0.014	0.014	0.019	0.033	0.031	0.034	0.165	0.170
N of CFOs	311	282	284	311	282	284	263	265
N of Firms	277	252	254	277	252	254	235	237
N of Observations	468	423	428	468	423	428	396	401

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 8: **Rules of Thumb and Corporate Policies**

	Investment			Leverage		
	(1)	(2)	(3)	(4)	(5)	(6)
Rule 1 ('narrow bracketing')	-0.016	-0.014	-0.015	0.055	0.041	0.047
	(0.011)	(0.011)	(0.012)	(0.092)	(0.101)	(0.092)
Rule 2 ('sales anchoring')	-0.013**	-0.015**	-0.012	0.093*	0.098	0.092*
	(0.006)	(0.007)	(0.008)	(0.053)	(0.060)	(0.053)
Rule 3 ('economies of scale')	-0.007	-0.011	-0.010	-0.023	-0.015	-0.027
	(0.008)	(0.010)	(0.010)	(0.073)	(0.091)	(0.084)
Rule 4 ('industry based')	-0.003	-0.003	-0.003	-0.004	0.005	0.001
	(0.007)	(0.008)	(0.008)	(0.045)	(0.050)	(0.046)
Miscalibration ST		0.001			0.012	
		(0.003)			(0.024)	
Optimism ST		0.002			-0.006	
		(0.003)			(0.019)	
Miscalibration LT			0.002			0.013
			(0.002)			(0.018)
Optimism LT			0.004*			-0.010
			(0.002)			(0.017)
Firm size	-0.002	-0.002	-0.002	0.012	0.011	0.012
	(0.002)	(0.002)	(0.002)	(0.017)	(0.019)	(0.018)
Market-to-Book	0.006**	0.006*	0.006*	-0.082***	-0.081***	-0.081***
	(0.003)	(0.003)	(0.003)	(0.020)	(0.021)	(0.022)
Dividends	0.000	0.004	0.004	0.037	0.044	0.043
	(0.009)	(0.008)	(0.008)	(0.072)	(0.076)	(0.080)
Constant	0.044*	0.043*	0.050**	0.568**	0.666***	0.620**
	(0.025)	(0.023)	(0.023)	(0.249)	(0.228)	(0.249)
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Survey FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.210	0.223	0.230	0.069	0.062	0.066
N of Observations	437	397	402	437	397	402

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 9: **Change in Performance and Corporate Policies when New CFOs Take Office**

	Change in ROA		**Change in Investment**		**Change in Leverage**	
	(1)	(2)	(3)	(4)	(5)	(6)
Incoherence	-1.633*		-0.049		-0.047	
	(0.989)		(0.045)		(1.115)	
Rule 1 ('narrow bracketing')		-0.274		-0.022*		-0.011
		(0.213)		(0.012)		(0.231)
Rule 2 ('sales anchoring')		-0.000		-0.003		-0.201
		(0.036)		(0.008)		(0.199)
Rule 3 ('economies of scale')		-0.057		-0.008		-0.110
		(0.051)		(0.012)		(0.153)
Rule 4 ('industry based')		0.019		0.001		-0.070
		(0.048)		(0.009)		(0.118)
Firm characteristics	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.391	0.192	0.024	0.017	0.053	0.042
N of Observations	142	142	140	140	146	146

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively. Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table 10: **Regression Tests of Accuracy and Coherence**

	FE of Log CapEx Growth	FE of Log Sales Growth			
	(1)	(2)	(3)	(4)	(5)
FE of Log CapEx Growth			0.113*	0.135***	
			(0.063)	(0.032)	
FE of Log Wages Growth			0.023		0.019
			(0.309)		(0.321)
Constant	-0.042*	-0.009	0.046**	-0.004	0.033
	(0.025)	(0.009)	(0.023)	(0.009)	(0.022)
R^2	0.000	0.000	0.108	0.127	0.018
N observations	359	359	51	359	52

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table 11: **The Coherence and Accuracy Sides of Rationality**

Panel A – Separate Assessment of Coherence and Accuracy (Percent of Rejections of Null)

Significance level α	Coherence Sales-CapEx	Accuracy Sales	Accuracy CapEx	Accuracy Both
	(1)	(2)	(3)	(4)
5%	55.7%	27.2%	47.9%	57.0%
1%	7.7%	1.8%	6.4%	7.1%

Panel B – Joint Assessment of Coherence and Accuracy

Significance level α	Coherent + Accurate	Coherent + Inaccurate	Incoherent + Accurate	Incoherent + Inaccurate
	(1)	(2)	(3)	(4)
5%	31.1%	13.2%	12.0%	43.7%
1%	89.4%	2.9%	3.4%	4.3%

Panel C – Test Statistics: Summary Statistics

	Mean	Std.Dev.	P05	Median	P95	N Obs.
C-statistic	-0.193	4.846	-8.335	-0.135	7.871	560
FE Sales	-0.538	19.07	-23.53	0.554	22.24	563
FE CapEx	-0.988	31.28	-54.18	1.186	41.20	560

Notes: In Panel B, Accuracy means both accurate; and inaccuracy means at least one inaccurate. Critical values are those of the t-student with one degree of freedom, +/-12.706 at the 5% and +/-63.657 at the 1%. Sales are the output. Capital Expenditures (CapEx) are input 1. Labor Expenditures are input 2 (unobserved).

Online Appendix with Supplementary Material for The Coherence Side of Rationality: Rules of thumb, narrow bracketing, and managerial incoherence in corporate forecasts

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Not for Publication

A Proofs

Proof of Proposition 1. Recall that for a concave function, f, it holds that $\mathbb{E}\left[f(x)\right] \leq f\left(\mathbb{E}\left[x\right]\right)$. Assume $\xi \leq 1$ and start by assuming that $a+b=1$. The CES function f is homogeneous of degree one because, for a scalar λ, we have that

$$f(\lambda \mathbf{x}) = \left[a(\lambda x_1)^{\xi} + b(\lambda x_2)^{\xi}\right]^{\frac{1}{\xi}} = \lambda \left(a x_1^{\xi} + b x_2^{\xi}\right)^{\frac{1}{\xi}}.$$

Furthermore, note that f is also quasiconcave because it is a monotone transformation of a concave function. In fact,

$$f = g^{\frac{1}{\xi}},$$

and to see that g is concave, compute its Hessian, H_g,

$$H_g = \begin{bmatrix} \frac{\partial^2 g}{\partial x_1^2} & \frac{\partial^2 g}{\partial x_2 \partial x_1} \\ \frac{\partial^2 g}{\partial x_1 \partial x_2} & \frac{\partial^2 g}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} a\xi(1-\xi)x_1^{\xi-2} & 0 \\ 0 & b\xi(1-\xi)x_2^{\xi-2} \end{bmatrix}.$$

Since H_g is negative semi-definite, we can conclude that g is concave.

Now, let $a+b \leq 1$. We have that $f = \left(g^{\frac{1}{\xi}}\right)^{a+b}$, where g is concave, as shown above. Then, f is a concave increasing function of a concave function, from which we can conclude that f is concave, which proves the first part of the proposition. The second part of the proposition on convexity follows very similar arguments.
QED

Proof of Corollary 1. Here we prove the statement in growth rates (the one in levels follows similar steps). In a Cobb-Douglas for $\xi \to 0$, assuming without loss of generality that the constraint is binding, the solution for optimal input quantities are

$$x_1^* = \frac{Z}{p_1}\frac{a}{a+b}, \qquad x_2^* = \frac{Z}{p_2}\frac{b}{a+b}.$$

Therefore, for $i = 1, 2$ we have

$$\begin{aligned} \log\left[\frac{x_{i,t+1}}{x_{i,t}}\right] &= \log\left[\frac{p_{i,t}}{p_{i,t+1}}\right] \\ \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{p_{1,t}}{p_{1,t+1}}\right] + b \cdot \log\left[\frac{p_{2,t}}{p_{2,t+1}}\right]. \end{aligned}$$

Putting these together, we obtain

$$\begin{aligned} \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right] \\ \mathbb{E}_t \log\left[\frac{y_{t+1}}{y_t}\right] &= a \cdot \mathbb{E}_t \log\left[\frac{x_{1,t+1}}{x_{1,t}}\right] + b \cdot \mathbb{E}_t \log\left[\frac{x_{2,t+1}}{x_{2,t}}\right]. \end{aligned}$$

QED

Proof of Proposition 2. The Proof follows directly from the observation that in our setting the conditional expectation function is

$$\mathbb{E}\left[y|x_1,x_2\right] = \mathbb{E}\left[y\right] + \beta_1\left(x_1 - \mathbb{E}\left[x_1\right]\right) + \beta_2\left(x_2 - \mathbb{E}\left[x_2\right]\right),$$

where the parameters can be derived by the Frisch-Waugh-Lovell theorem, as

$$\beta_1 = \frac{cov\left(y,x_1\right) - \left(\frac{cov(x_1,x_2)cov(y,x_2)}{var(x_2)}\right)}{var\left(x_1\right) - \frac{cov(x_1,x_2)^2}{var(x_2)}},\ \ \beta_2 = \frac{cov\left(y,x_2\right) - \left(\frac{cov(x_1,x_2)cov(y,x_1)}{var(x_1)}\right)}{var\left(x_2\right) - \frac{cov(x_1,x_2)^2}{var(x_1)}},$$

and where $var\left(x_1\right)$, $var\left(x_2\right)$, $cov\left(y,x_1\right)$, and $cov\left(y,x_2\right)$ are functions of parameters a and b.
QED

Proof of Corollary 2. In levels,

$$\mathbb{E}\left[\log x_1|\log y,\log x_2\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right) + \beta_2\left(\log x_2 - \mu_2\right),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\log y,\log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log x_2)}{var(\log x_2)}\right)}{var\left(\log y\right) - \frac{cov(\log y,\log x_2)^2}{var(\log x_2)}},$$

$$\beta_2 = \frac{cov\left(\log x_2,\log x_1\right) - \left(\frac{cov(\log y,\log x_2)cov(\log x_1,\log y)}{var(\log y)}\right)}{var\left(\log x_2\right) - \frac{cov(\log y,\log x_2)^2}{var(\log y)}}.$$

In detail, we have:
$var\left(\log x_2\right) = \sigma_2^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$,
$cov\left(\log x_2,\log x_1\right) = 0$,
$cov\left(\log y,\log x_1\right) = cov\left(a\log x_1 + b\log x_2,\log x_1\right) = a\sigma_1^2$,
$cov\left(\log y,\log x_2\right) = cov\left(a\log x_1 + b\log x_2,\log x_2\right) = b\sigma_2^2$.
Substituting yields

$$\begin{aligned}
\mathbb{E}\left[\log x_1|\log y,\log x_2\right] &= \mu_1 + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 - \frac{b^2\sigma_2^4}{\sigma_2^2}}\left(\log y - \mu_y\right) + \frac{-\left(\frac{b\sigma_2^2 a\sigma_1^2}{a^2\sigma_1^2+b^2\sigma_2^2}\right)}{\sigma_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2}}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b\sigma_2^2 a\sigma_1^2}{\sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right) - b^2\sigma_2^4}\left(\log x_2 - \mu_2\right) \\
&= \mu_1 + \frac{1}{a}\left(\log y - \mu_y\right) - \frac{b}{a}\left(\log x_2 - \mu_2\right).
\end{aligned}$$

where $\mu_1 - \frac{1}{a}\mu_y + \frac{b}{a}\mu_2 = 0$ follows by Corollary 1. Proving the statement in growth rates follows similar steps.
QED

Proof of Corollary 3. In levels,

$$\mathbb{E}\left[\log x_1|\log y\right] = \mu_1 + \beta_y\left(\log y - \mu_y\right),$$

where coefficients equal

$$\alpha = \mu_1 - \beta_y \mu_y, \qquad \beta_y = \frac{cov\left(\log y, \log x_1\right)}{var\left(\log y\right)}.$$

We have:
$cov\left(\log x_2, \log x_1\right) = 0$,
$cov\left(\log y, \log x_1\right) = cov\left(a \log x_1 + b \log x_2, \log x_1\right) = a\sigma_1^2$,
$var\left(\log y\right) = a^2\sigma_1^2 + b^2\sigma_2^2$.
Substituting yields

$$\alpha = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y, \qquad \beta_y = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2},$$

and thus

$$\mathbb{E}\left[\log x_1 | \log y\right] = \mu_1 - \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\mu_y + \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2}\left(\log y - \mu_y\right).$$

Proving the statement in growth rates follows similar steps and intercept α is differenced away. QED

Proof of Corollary 4. Consider the regression

$$\frac{y_{t+1}}{y_t} = \alpha + \beta \frac{x_{i,t+1}}{x_{i,t}} + e_{t+1}.$$

Denoting variable at optimum with superscripts *, we have

$$\begin{aligned}
\beta &= \frac{cov\left(\frac{y_{t+1}}{y_t}, \frac{x^*_{i,t+1}}{x^*_{i,t}}\right)}{var\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)} = \frac{\mathbb{E}\left[\frac{y_{t+1}}{y_t} \cdot \frac{x^*_{i,t+1}}{x^*_{i,t}}\right] - \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] \cdot \mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]}{\mathbb{E}\left[\left(\frac{x^*_{i,t+1}}{x^*_{i,t}}\right)^2\right] - \left(\mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right]\right)^2}, \\
\alpha &= \mathbb{E}\left[\frac{y_{t+1}}{y_t}\right] - \beta \mathbb{E}\left[\frac{x^*_{i,t+1}}{x^*_{i,t}}\right].
\end{aligned}$$

Recall that

$$\left\{\begin{pmatrix}\pi_{1,t} \\ \pi_{2,t}\end{pmatrix}\right\} \overset{iid}{\sim} \mathcal{N}_2\left(\mathbf{0}, \begin{bmatrix}\sigma_1^2 & \sigma_{1,2} \\ \sigma_{1,2} & \sigma_2^2\end{bmatrix}\right).$$

Under the assumption that $\sigma_1^2 = \sigma_2^2 = \sigma^2$, for every t we have that the Pearson correlation coefficient $\rho_{1,2} = 1$, from which it follows that $\pi_{1,t} = \pi_{2,t} + c$ almost surely, where c is a constant.[41] As a result, the setting can be recast as one in which prices are constant and the budget Z is stochastic because

$$\begin{aligned}
\frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{p_{1,t}}{p_{1,t+1}} \overset{\text{a.s.}}{=} \frac{p_{2,t}}{p_{2,t+1}} = \frac{x^*_{2,t+1}}{x^*_{2,t}} \\
\frac{y_{t+1}}{y_t} &= a\frac{p_{1,t}}{p_{1,t+1}} + b\frac{p_{2,t}}{p_{2,t+1}} \overset{\text{a.s.}}{=} (a+b)\frac{p_{1,t}}{p_{1,t+1}}
\end{aligned}$$

[41] To see this, suppose that X, Y are two random variables such that $\rho\left(X, Y\right) = 1$. Let $V = X - \mathbb{E}\left[X\right]$ and $W = Y - \mathbb{E}\left[Y\right]$. We have $\mathbb{E}\left[\left(V - W\right)^2\right] = var\left(X\right) + var\left(Y\right) - 2cov\left(X, Y\right) = 0$, so that $V \overset{\text{a.s.}}{=} W$, from which the result $\pi_{1,t} = \pi_{2,t} + c$ follows.

and

$$\left\{\frac{x^*_{i,t+1}}{x^*_{i,t}}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2$$
$$\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right).$$

In fact, denote $z_t = \log\left(Z\right) \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \sigma^2\right)$, and assume p_1 and p_2 constant for all t. We have

$$\begin{aligned}\frac{x^*_{1,t+1}}{x^*_{1,t}} &= \frac{x^*_{2,t+1}}{x^*_{2,t}} = e^{z_{t+1}-z_t} \\ \frac{y_{t+1}}{y_t} &= e^{(a+b)(z_{t+1}-z_t)}\end{aligned}$$

and

$$\left\{\frac{x^*_{i,t+1}}{x^*_{i,t}}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\sigma^2\right), \quad i = 1, 2$$
$$\left\{\frac{y_{t+1}}{y_t}\right\} \overset{\text{iid}}{\sim} \mathcal{N}\left(0, 2\left(a+b\right)^2\sigma^2\right),$$

as it was with stochastic prices. Therefore, for clarity from now on we drop the subscript i. Now, recalling that $\nu \equiv a+b$, that $cov\left(z_{t+1}, z_t\right) = \gamma\frac{\sigma^2}{1-\gamma^2}$, and that for any scalar, c, we have

$$\mathbb{E}\left[e^{c(z_{t+1}-z_t)}\right] = e^{\frac{1}{2}c^2 var(z_{t+1}-z_t)} = e^{\frac{1}{2}c^2\left(2\frac{\sigma^2}{1-\gamma^2} - 2\gamma\frac{\sigma^2}{1-\gamma^2}\right)} = e^{c^2\left(\frac{\sigma^2}{1+\gamma}\right)},$$

we obtain the expressions

$$\begin{aligned}\beta &= \frac{e^{\left[(\upsilon+1)^2\frac{\sigma^2}{1-\gamma}\right]} - e^{\left[(\upsilon^2+1)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(4\frac{\sigma^2}{1-\gamma}\right)} - e^{\left(2\frac{\sigma^2}{1-\gamma}\right)}} = \frac{e^{\left[(\upsilon^2+2\upsilon)\frac{\sigma^2}{1-\gamma}\right]} - e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(3\frac{\sigma^2}{1-\gamma}\right)} - e^{\left(\frac{\sigma^2}{1-\gamma}\right)}} \\ \alpha &= \frac{e^{\left[(\upsilon^2+2)\frac{\sigma^2}{1-\gamma}\right]} - e^{\left[(\upsilon^2+2\upsilon)\frac{\sigma^2}{1-\gamma}\right]}}{e^{\left(2\frac{\sigma^2}{1-\gamma}\right)} - 1}.\end{aligned}$$

We can then directly verify the coimplications of the Corollary, that is,

$$\begin{aligned}\beta &< 1 \Longleftrightarrow e^{\left[(\upsilon^2+2\upsilon)\frac{\sigma^2}{1-\gamma}\right]} - e^{\left[\upsilon^2\frac{\sigma^2}{1-\gamma}\right]} < e^{\left(3\frac{\sigma^2}{1-\gamma}\right)} - e^{\left(\frac{\sigma^2}{1-\gamma}\right)} \Longleftrightarrow \upsilon < 1 \\ \alpha &> 0 \Longleftrightarrow e^{\left[(\upsilon^2+2)\frac{\sigma^2}{1-\gamma}\right]} > e^{\left[(\upsilon^2+2\upsilon)\frac{\sigma^2}{1-\gamma}\right]} \Longleftrightarrow \upsilon < 1,\end{aligned}$$

which also holds for i.i.d. shocks, that is, for $\gamma = 0$.
QED

Proof of Corollary 5. We have

$$\log\frac{x_{i,t+1}}{x_{i,t}} = \log\frac{1/p_{i,t+1}}{1/p_{i,t}} = \pi_{i,t} - \pi_{i,t+1}.$$

Denote $\log F_t^0$ the optimal forecast of log input $x_{i,t}$ growth. We have

$$\log F_t^o = \mathbb{E}_t \left[\log \frac{x_{i,t+1}}{x_{i,t}} \right] = (1 - \gamma_i)\, \pi_{i,t}.$$

Under the optimal forecast, the forecast error will be minus the innovation of the log price shock,

$$\log \frac{x_{i,t+1}}{x_{i,t}} - \mathbb{E}_t \left[\log \frac{x_{i,t+1}}{x_{i,t}} \right] = -\epsilon_{i,t+1} | \Omega_t \sim \mathcal{N}(0,1),$$

so that the loss and the expected loss under the optimal forecast, L_{t+1}^o and $\mathbb{E}_t \left[L_{i,t+1}^o \right]$, are

$$\begin{aligned} L_{t+1}^o &= \epsilon_{i,t+1}^2 = \sigma_i^2 \frac{1}{\sigma_i^2} \epsilon_{i,t+1}^2 \\ \mathbb{E}_t \left[L_{i,t+1}^o \right] &= \sigma_i^2 \mathbb{E}_t \left[\frac{1}{\sigma_i^2} \epsilon_{i,t+1}^2 \right] = \sigma_i^2, \end{aligned}$$

where the last equality follows from $\epsilon_{i,t+1}^2 = \sigma_i^2 = \sigma_i^2 \frac{1}{\sigma_i^2} \epsilon_{i,t+1}^2$, and $\frac{1}{\sigma_i^2} \epsilon_{i,t+1}^2 | \Omega_t \sim \chi^2$ with mean 1.

Under the narrow-bracketing rule (R1), $\log F_{i,t}^{R1} = \frac{1}{k} \sum_{j=1}^{k} \log \frac{x_{i,t+1-j}}{x_{i,t-j}}$, the forecast error in logs is

$$\log \frac{x_{i,t+1}}{x_{i,t}} - \log F_{i,t}^{R1}.$$

There are several possibilities. If $k = 1$, $\log F_{i,t}^{R1} = \log \frac{x_{i,t}}{x_{i,t-1}}$, then the forecast error is $\log \frac{x_{i,t+1}}{x_{i,t}} - \log \frac{x_{i,t}}{x_{i,t-1}} = -\left[\epsilon_{i,t+1} - (1-\gamma_i)\, \pi_{i,t} - (\pi_{i,t} - \pi_{i,t-1}) \right]$, and

$$\mathbb{E}_t \left[L_{i,t+1}^{R1} \right] = \sigma_i^2 \left(1 + \frac{\left[(1-\gamma_i)\, \pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1}) \right]^2}{\sigma_i^2} \right) = \mathbb{E}_t \left[L_{i,t+1}^o \right] + \left[(1-\gamma_i)\, \pi_{i,t} + (\pi_{i,t} - \pi_{i,t-1}) \right]^2.$$

For a general k, one obtains

$$\mathbb{E}_t \left[L_{i,t+1}^{R1} \right] = \mathbb{E}_t \left[L_{i,t+1}^o \right] + \left[(1-\gamma_i)\, \pi_{i,t} + \frac{1}{k} \sum_{j=1}^{k} (\pi_{i,t+1-j} - \pi_{i,t-j}) \right]^2.$$

For $k \to \infty$,

$$\lim_{k \to \infty} \mathbb{E}_t \left[L_{i,t+1}^{R1} \right] = \mathbb{E}_t \left[L_{i,t+1}^o \right] + \left[(1-\gamma_i)\, \pi_{i,t} \right]^2.$$

QED

Proof of Proposition 3. The Proposition is stated in the text for the case of $\rho_{1,2} = 0$. Here we prove the Proposition for the general case with correlated prices, i.e., for a generic value of $\rho_{1,2} \in [0.1]$. We have that

$$\mathbb{E} \left[\log x_1 | \eta_y, \eta_2 \right] = \mu_1 + \beta_y (\eta_y - \mu_y) + \beta_2 (\eta_2 - \mu_2),$$

where coefficients equal

$$\beta_y = \frac{cov\left(\eta_y, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_2)}{var(\eta_2)}\right)}{var\left(\eta_y\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_2)}}, \quad \beta_2 = \frac{cov\left(\eta_2, \log x_1\right) - \left(\frac{cov(\eta_y,\eta_2)cov(\log x_1,\eta_y)}{var(\eta_y)}\right)}{var\left(\eta_2\right) - \frac{cov(\eta_y,\eta_2)^2}{var(\eta_y)}}.$$

We have:
$cov\left(\eta_y, \log x_1\right) = cov\left(a \log x_1 + b \log x_2 + \epsilon_y, \log x_1\right) = a\sigma_1^2 + b\rho_{1,2}$,
$cov\left(\eta_2, \log x_1\right) = \rho_{1,2}$,
$cov\left(\eta_y, \eta_2\right) = cov\left(a \log x_1 + b \log x_2 + \epsilon_y, \log x_2 + \epsilon_2\right) = b\sigma_2^2 + a\rho_{1,2}$,
$var\left(\eta_y\right) = a^2\sigma_1^2 + b^2\sigma_2^2 + \sigma_y^2 + 2ab\rho_{1,2}$,
$var\left(\eta_2\right) = \sigma_2^2 + s_2^2$.
Substituting yields

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2 + b\rho_{1,2} - \frac{\rho_{1,2}\left(a\rho_{1,2}+b\sigma_2^2\right)}{\sigma_2^2+s_2^2}}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 + 2ab\rho_{1,2} - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)^2}{\sigma_2^2+s_2^2}} \\
\beta_2 &= \frac{\rho_{1,2} - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2+b\rho_{1,2}\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2+2ab\rho_{1,2}}}{\sigma_2^2 + s_2^2 - \frac{\left(a\rho_{1,2}+b\sigma_2^2\right)^2}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2+2ab\rho_{1,2}}}
\end{aligned}$$

For $\rho_{1,2} = 0$, we obtain

$$\begin{aligned}
\beta_y &= \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}}, \\
\beta_2 &= \frac{-\frac{\left(a\rho_{1,2}+b\sigma_2^2\right)\left(a\sigma_1^2\right)}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}}{\sigma_2^2 + s_2^2 - \frac{b^2\sigma_2^4}{a^2\sigma_1^2+b^2\sigma_2^2+s_y^2}} = -\frac{ab\sigma_1^2\sigma_2^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2} \times \frac{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2}{\left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right) - b^2\sigma_2^4} \\
&= \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)}.
\end{aligned}$$

QED

Proof of Corollary 6.

$$\begin{aligned}
\lim_{s_y,s_2\to+\infty} \beta_y &= \lim_{s_y,s_2\to+\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = 0, \\
\lim_{s_y,s_2\to+\infty} \beta_2 &= \lim_{s_y,s_2\to+\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.
\end{aligned}$$

QED

Proof of Corollary 7.

$$\lim_{s_2 \to +\infty} \beta_y = \lim_{s_2 \to +\infty} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2},$$

$$\lim_{s_2 \to +\infty} \beta_2 = \lim_{s_2 \to +\infty} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = 0.$$

QED

Proof of Corollary 8.

$$\lim_{s_y, s_2 \to 0} \beta_y = \lim_{s_y, s_2 \to 0} \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2 - \frac{b^2\sigma_2^4}{\sigma_2^2+s_2^2}} = \frac{a\sigma_1^2}{a^2\sigma_1^2 + b^2\sigma_2^2 + - \frac{b^2\sigma_2^4}{\sigma_2^2}} = \frac{1}{a},$$

$$\lim_{s_y, s_2 \to 0} \beta_2 = \lim_{s_y, s_2 \to 0} \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \left(\sigma_2^2 + s_2^2\right)\left(a^2\sigma_1^2 + b^2\sigma_2^2 + s_y^2\right)} = \frac{ab\sigma_1^2\sigma_2^2}{b^2\sigma_2^4 - \sigma_2^2\left(a^2\sigma_1^2 + b^2\sigma_2^2\right)} = -\frac{b}{a}.$$

QED

Proof of Proposition 4. To derive the test statistic, note that we have $\mathbb{E}_t \log y_{t+1} = a\mathbb{E}_t \log x_{1,t+1} + b\mathbb{E}_t \log x_{2,t+1}$ and $\log x_{2,t+1} = \log \frac{b}{a+b} Z - \pi_{2,t+1}$, implying that the CFO forecast for input 2 is

$$\mathbb{E}_t \log x_{2,t+1} = \log \frac{b}{a+b} Z - \gamma_2 \pi_{2,t}. \tag{13}$$

Note that (13) depends on the technological parameters, a and b, and budget, Z, which we assume are known to the CFO at the time of the forecast and are stable over time. From (13) we obtain that

$$\frac{\mathbb{E}_t \log x_{2,t+1} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1).$$

We can then derive our test statistic C1-stat based on the joint forecasts of the first input and output by recalling that $\log y = a \log x_1 + b \log x_2$ as follows:

$$\text{C1-stat} \equiv \frac{\frac{\mathbb{E}_t \log y_{t+1} - a\mathbb{E}_t \log x_{1,t+1}}{b} - \log \frac{b}{a+b} Z}{\gamma_2 \sigma_2} \sim \mathcal{N}(0,1),$$

where the distribution here is obtained under the null hypothesis of coherent forecasts.

To derive our C2-stat, we start by defining the forecast error of a generic variable x forecasted at t and realized at $t+1$ as the difference between the realization and the forecast, $\text{FE}_t x_{t+1} = x_{t+1} - \mathbb{E}_t x_{t+1}$. We then have that

$$\begin{aligned}
\text{FE}_t \log x_{2,t+1} &= \log x_{2,t+1} - \mathbb{E}_t \log x_{2,t+1} \\
&= \log \frac{b}{a+b} Z - \pi_{2,t+1} - \mathbb{E}_t \left[\log \frac{b}{a+b} Z - \pi_{2,t+1}\right] \\
&= -\text{FE}_t \pi_{2,t+1} = -\epsilon_{2,t+1}.
\end{aligned}$$

As a result, the forecast error of the log of the second input is the negative of the innovation of the second log-price process. It follows that

$$\frac{\text{FE}_t \log x_{2,t+1}}{\sigma_2} \sim \mathcal{N}(0,1).$$

Noting that $\text{FE}_t \log y_{t+1} = a\text{FE}_t \log x_{1,t+1} + b\text{FE}_t \log x_{2,t+1}$, we obtain our C2-stat,

$$\text{C2-stat} \equiv \frac{\text{FE}_t \log y_{t+1} - a\text{FE}_t \log x_{1,t+1}}{\sigma_2 b} \sim \mathcal{N}(0,1).$$

QED

Coherence Test Statistic: Multiple Inputs Case. Here we generalize our C2-stat to a multivariate case with N inputs. For this subsection, the production setting is

$$y = \prod_{i=1}^{N} x_i^{a_i}$$

$$\boldsymbol{p}'\boldsymbol{x} = Z,$$

where $\boldsymbol{p}$ and $\boldsymbol{x}$ are the column vectors of factor prices and quantities, respectively. As in the $N = 2$ case we have a linear relationship between the logs of inputs and output,

$$\log y = \sum_{i=1}^{N} a_i \log x_i,$$

where the same equation holds for the forecast errors. Analogously to the bivariate case, we have that $\text{FE}_t \log x_{1,t+1} = -\epsilon_{1,t+1}$ so that

$$\frac{\text{FE}_t \log x_{1,t+1}}{\sigma_1} \sim \mathcal{N}(0,1).$$

Then, using the linear relationship between the logs of inputs and output we obtain our generalized C2-stat,

$$\frac{\text{FE}_t \log y_{t+1} - \sum_{i=2}^{N} a_i \text{FE}_t \log x_{i,t+1}}{\sigma_1 a_1} \sim \mathcal{N}(0,1).$$

Proof of Equivalence of Forecast Error of Investment and Forecast Error of Capital. In our model, we take our input x_1 to be capital, that is, the stock of physical capital, K, typically measured by the balance sheet item Property, Plant, and Equipment, whereas in our empirical implementation we use balance sheet item capital expenditures, that is, I. Here we discuss the relation between investment growth (ΔI) and capital growth (ΔK), and we show that in general forecast errors of investment growth should equal the forecast errors of capital growth, $\text{FE}[\Delta I_{t+1}] = \text{FE}[\Delta K_{t+1}]$. To do so, denote end-of-period capital, K_{t+1}, and consider the law of motion:

$$K_{t+1} = K_t (1-\delta) + I_{t+1}$$

where K_t is beginning-of-period capital, I_{t+1} denotes per-period capital expenditures, and δ is the depreciation rate. Rearranging:

$$I_{t+1} = \Delta K_{t+1} + \delta K_t$$

where $\Delta K_{t+1} = K_{t+1} - K_t$. Therefore,

$$\Delta I_{t+1} = I_{t+1} - I_t = \Delta K_{t+1} + \delta K_t - \Delta K_t - \delta K_{t-1} = \Delta K_{t+1} - \Delta K_t + \delta \Delta K_t$$

and then

$$\Delta I_{t+1} = \Delta K_{t+1} - (1 - \delta) \Delta K_t$$

Similarly, in expectations it holds that:

$$\mathbb{E} [\Delta I_{t+1}] = \mathbb{E} [\Delta K_{t+1}] - (1 - \delta) \Delta K_t$$

Finally, subtracting expectations from realizations we obtain forecast errors, $\text{FE}[\Delta I_{t+1}] = \Delta I_{t+1} - \mathbb{E} [\Delta I_{t+1}]$, and $\text{FE}[\Delta K_{t+1}] = \Delta K_{t+1} - \mathbb{E} [\Delta K_{t+1}]$, respectively, which must satisfy the relationship:

$$\text{FE} [\Delta I_{t+1}] = \text{FE} [\Delta K_{t+1}]$$

That is, the forecast error of capital should equal the forecast error of investment. The intuition is the same of Section V.A., namely computing forecast errors differences away any heterogeneity known to, or predictable by, the firm at the time of the forecast.
QED

B Implementation of Inequality Condition

Inequality test. We implement the inequality restriction in Proposition 1, developed under a general CES function. We view this inequality as imposing on the data as little restriction as possible. We compute a and b using the universe of industries from the Bureau of Economic Analysis and find that $a + b \leq 1$. Furthermore, the elasticity of substitution between capital and labor in the US economy, denoted with χ, is typically found to be between 0.5 and 1 (e.g., see Berndt (1976) and Oberfield and Raval (2021)), where $\chi = 1$ defines the Cobb-Douglas production function. Therefore, the CES function is weakly concave and thus inequality (2) is the relevant one. We account for heterogeneity by allowing a and b to vary by industry and by presenting our results for three different values of the elasticity of substitution between capital and labor, $\chi = 0.5, 0.7, 0.9$. We implement our inequality restriction both in levels and in growth rates.[42]

Table A9 reports our results. Panel A shows that most CFOs' forecasts violate the inequality restriction of Proposition 1. In levels, almost all CFOs give joint forecasts of capital, labor, and output that jointly violate the inequality. However, as just discussed, results in levels should be seen with caution, as they refer to a much smaller sample given the limitations of Compustat data on wages. In growth rates, about 73% of CFOs' forecasts violate the inequality. These results are quite stable across different values of the elasticity of substitution between capital and labor. If anything, moving toward $\chi = 1$ (Cobb-Douglas) appears to give a slightly better shot at CFOs to give coherent forecasts, perhaps because much business teaching uses examples based on the Cobb-Douglas.

Panel B reports summary statistics of the difference between the left-hand side and the right-hand side of the inequality of Proposition 1. Most CFOs forecast a growth of output that is larger than the output growth implied by feeding into a CES production function the CFOs' forecasts of capital and labor input growth.

[42]We observe CFO forecasts of growth rates, not of levels. Moreover, while we observe the CFO forecast of the growth rate of labor expenditures, $\mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$, for a large sample, in Compustat we observe few realizations of $x_{2,t+1}$. Therefore, when we compute $\mathbb{E}_t [x_{2,t+1}] = x_{2,t} \cdot \mathbb{E}_t \frac{[x_{2,t+1}]}{[x_{2,t}]}$ to implement the inequality restriction in levels, we end with much fewer observations in levels than in growth rates.

C Online Appendix - Further Figures and Tables

Figure A1: **Survey Questions of Firm Forecasts**

4. Relative to the previous 12 months, what will be your company's PERCENTAGE CHANGE during the next 12 months? (e.g., +3%, -2%, etc.) [Leave blank if not applicable]	
% Prices of your products	% Technology spending
% Overtime	% Earnings
% Advertising/Marketing spending	% Revenues
% Number of employees	% Inventory
% Productivity (output per hour worked)	% M&A activity
% Wages/Salaries	% Capital spending
% Health care costs	% Dividends

Table A1: **Growth Realizations of Selected Balance Items**

Realizations in Compustat (percent)

	Mean	Std. Dev.	Q10	Median	Q90	N Obs.
Growth in Revenues and in Earnings						
Revenues	13.38	35.96	-16.64	6.89	46.24	105,866
Earnings	-16.21	432.06	-207.66	-3.92	174.42	105,841
Growth in Capital-Related Expenditures						
Capital Expenditures	35.71	132.99	-56.41	5.26	129.28	100,633
R & D	15.91	53.09	-25.24	6.75	57.64	40,715
Technology Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Labor-Related Costs						
Wages	10.57	24.22	-9.57	6.94	31.96	29,491
Employees	6.18	25.30	-14.29	2.07	29.17	107,435
Outsourced Employees	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Health Spending	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Growth in Productivity, Product Prices, and Advertising						
Productivity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Product Prices	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Advertising	19.12	80.07	-37.35	4.39	71.33	34,251
Growth in Cash Holdings and Corporate Payout						
Cash	76.55	308.36	-57.50	5.23	184.62	103,833
Dividends	18.74	97.74	-56.43	5.42	60.56	54,841
Share Repurchases	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table A2: **Summary Statistics**

Panel A – Matched Duke-Compustat sample

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.845	1.629	0.875	1.402	4.157	15,929
ROA	0.025	0.235	-0.154	0.037	0.167	16,591
Sales	10,617.34	28,482.18	53.66	2,043.96	49,545.00	17,799
Log(sales)	7.591	2.093	4.022	7.629	10.813	17,757
Assets	37,698.73	187,568.6	74.12	2,894.43	113,960.0	17,799
Log(assets)	7.993	2.221	4.306	7.971	11.644	17,799
Book Leverage	0.411	1.755	0.000	0.371	0.910	17,733
Capital Expenditure	0.045	0.058	0.001	0.031	0.134	16,200
R & D	0.060	0.202	0.000	0.025	0.211	9,043
Cash Flow	0.302	13.075	-1.127	0.413	3.005	17,010
Cash	4.895	62.328	0.014	0.563	15.188	17,269
Advertising	0.025	0.043	0.000	0.009	0.098	6,729
Dividends	0.117	1.420	0.000	0.060	0.349	17,391
Dividends (0/1)	0.607	0.488	0.000	1.000	1.000	17,391

Panel B – Compustat data

	Mean	Std. Dev.	P05	Median	P95	N Obs.
Market-to-book	1.843	2.526	0.708	1.297	4.489	105,769
ROA	0.006	0.279	-0.273	0.021	0.187	123,155
Sales	3,521.58	15,220.67	17.992	315.011	14,687.00	127,307
Log(sales)	5.911	2.062	2.890	5.753	9.595	127,307
Assets	12,626.69	98,056.55	24.147	597.555	30,241.99	140,894
Log(assets)	6.529	2.170	3.184	6.393	10.317	140,894
Book Leverage	0.408	22.120	0.000	0.362	0.996	139,264
Capital Expenditure	0.063	0.159	0.000	0.032	0.212	108,909
R & D	0.075	0.135	0.000	0.027	0.290	52,955
Cash Flow	0.050	0.261	-0.222	0.061	0.256	118,905
Cash	0.192	0.396	0.002	0.081	0.685	111,475
Advertising	0.034	0.103	0.000	0.009	0.131	39,813
Dividends	0.144	7.286	0.000	0.000	0.557	122,340
Dividends (0/1)	0.431	0.495	0.000	0.000	1.000	122,340

Table A3: **Additional Cross Sectional Regressions in Compustat Data for Rules of Thumb**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
CapEx Growth$_{t-1 \text{ to } t}$	-0.060***									
	(0.010)									
Avg. CapEx Growth$_{t-1 \text{ to } t}$		-0.089***								-0.089***
		(0.016)								(0.040)
Sales Growth$_{t-1 \text{ to } t}$			1.055***				1.042***	0.996***	0.652***	0.702***
			(0.036)				(0.036)	(0.045)	(0.060)	(0.178)
Wages Growth$_{t-1 \text{ to } t}$				1.036***					0.589***	0.380***
				(0.082)					(0.081)	(0.159)
Earnings Growth$_{t-1 \text{ to } t}$					0.053***		0.018***			-0.034
					(0.005)		(0.004)			(0.024)
Advertising Growth$_{t-1 \text{ to } t}$						0.317***		0.178***		0.263***
						(0.021)		(0.016)		(0.076)
Constant	0.327***	0.316***	0.217***	0.215***	0.355***	0.274***	0.217***	0.178***	0.192***	0.166***
	(0.042)	(0.041)	(0.025)	(0.026)	(0.039)	(0.034)	(0.025)	(0.027)	(0.023)	(0.034)
R^2	0.004	0.004	0.081	0.047	0.005	0.042	0.082	0.096	0.063	0.074
N observations	85,838	74,413	100,441	15,955	100,040	33,202	100,040	33,202	15,955	3,013

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table A4: **Minimum Distance of Sales Forecasts from Rules of Thumb: Robustness to Alternative Definition of Rule 5**

	All	R1	R2	R3	R4	R5
Mean	0.029	0.040	0.025	0.030	0.041	0.023
Std. Dev.	0.039	0.031	0.044	0.031	0.056	0.016
Frac. Zeros	0.146	0.000	0.365	0.000	0.000	0.000
P10	0.000	0.005	0.000	0.006	0.002	0.004
P25	0.005	0.017	0.000	0.009	0.003	0.009
P50	0.016	0.031	0.007	0.022	0.023	0.019
P75	0.035	0.064	0.032	0.040	0.065	0.031
P90	0.071	0.094	0.071	0.074	0.122	0.048
P95	0.106	0.094	0.106	0.094	0.222	0.049
N of Observations	130	9	52	30	18	21
Fraction	1.000	0.069	0.400	0.231	0.138	0.162

Notes: Cross-sectional analysis with 130 CFOs.

Table A5: **Minimum Distance of Earnings Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.026	0.045	0.021	0.028	0.032	0.031
Std. Dev.	0.033	0.054	0.028	0.046	0.035	0.030
Frac. Zeros	0.197	0.000	0.356	0.000	0.000	0.000
P10	0.000	0.002	0.000	0.002	0.005	0.004
P25	0.004	0.003	0.000	0.008	0.007	0.009
P50	0.014	0.027	0.014	0.010	0.017	0.015
P75	0.035	0.064	0.035	0.017	0.050	0.052
P90	0.068	0.101	0.057	0.099	0.071	0.073
P95	0.101	0.177	0.085	0.182	0.111	0.090
N of Observations	396	24	219	35	48	70
Fraction	1.000	0.061	0.553	0.088	0.121	0.177

Notes: Cross-sectional analysis with 396 CFOs.

Table A6: **Minimum Distance of CapEx Forecasts from Rules of Thumb**

	All	R1	R2	R3	R4	R5
Mean	0.064	0.079	0.054	0.161	0.079	0.104
Std. Dev.	0.099	0.150	0.072	0.189	0.121	0.083
Frac. Zeros	0.106	0.000	0.146	0.000	0.000	0.000
P10	0.000	0.009	0.000	0.006	0.005	0.012
P25	0.014	0.022	0.014	0.008	0.029	0.039
P50	0.035	0.046	0.035	0.089	0.039	0.090
P75	0.071	0.076	0.071	0.321	0.062	0.157
P90	0.141	0.144	0.127	0.488	0.357	0.173
P95	0.220	0.264	0.163	0.508	0.390	0.293
N of Observations	396	58	288	14	25	11
Fraction	1.000	0.147	0.727	0.035	0.063	0.028

Notes: Cross-sectional analysis with 396 CFOs.

Table A7: **Incoherence and Rules of Thumb: Robustness to Alternative Definition of Rule 5**

	(1)	(2)	(3)	(4)	(5)
Rule 1 ('narrow bracketing')	0.099***				0.134***
	(0.022)				(0.025)
Rule 2 ('sales anchoring')		0.018			0.053***
		(0.012)			(0.016)
Rule 3 ('economies of scale')			-0.024*		0.023
			(0.014)		(0.018)
Rule 4 ('industry based')				0.003	0.044**
				(0.018)	(0.020)
Constant	0.058***	0.058***	0.071***	0.065***	0.023*
	(0.006)	(0.008)	(0.007)	(0.007)	(0.014)
R^2	0.126	0.009	0.014	-0.008	0.175
N observations	130	130	130	130	396
Summary Statistics of the dependent variable					
Mean	0.065				
Std. Dev.	0.069				
P10	0.012				
Median	0.045				
P90	0.153				

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels.

Table A8: **Incoherence, Rules of Thumb, and Corporate Performance: Robustness Using Alternative Definition of Rule 5**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Incoherence	-0.279*	-0.335**	-0.320**	-0.317**			
	(0.156)	(0.151)	(0.159)	(0.152)			
Rule 1 (‘narrow bracketing’)					-0.062**	-0.071*	-0.071*
					(0.031)	(0.042)	(0.042)
Rule 2 (‘sales anchoring’)					0.009	0.012	0.014
					(0.032)	(0.035)	(0.035)
Rule 3 (‘economies of scale’)					0.018	0.023	0.020
					(0.037)	(0.037)	(0.038)
Rule 4 (‘industry based’)					0.020	0.020	0.026
					(0.033)	(0.039)	(0.036)
Miscalibration ST		0.009				0.004	
		(0.005)				(0.007)	
Optimism ST		0.011				0.009	
		(0.011)				(0.011)	
Miscalibration LT			0.014				0.012
			(0.010)				(0.011)
Optimism LT			0.008				0.007
			(0.011)				(0.011)
Constant	0.051***	0.051***	0.048***	0.014	0.024	0.015	0.015
	(0.011)	(0.011)	(0.012)	(0.013)	(0.031)	(0.034)	(0.034)
Industry FE	No	No	No	Yes	No	No	No
Survey FE	No	No	No	Yes	No	No	No
R^2	0.055	0.075	0.073	0.140	0.017	0.020	0.026
N of Observations	136	122	121	136	136	122	121

Notes: *, **, *** denote two-tailed significance at the 10%, 5%, and 1% levels, respectively.
Standard errors are bootstrapped following Cameron, Gelbach, and Miller (2008) and clustered at the firm level.

Table A9: **Violations of Coherence Inequality Restrictions**

Panel A – Inequality Test of Coherence

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
% Incoherent	100.00	100.00	99.07
% Coherent	0.00	0.00	0.93
% Total	100.00	100.00	100.00
N. Obs.	107	107	107
Inequality in Growth Rates			
% Incoherent	73.31	73.14	72.96
% Coherent	26.69	26.86	27.04
% Total	100.00	100.00	100.00
N. Obs.	577	577	577

Panel B – Summary stats of difference, LHS − RHS

	$\chi = 0.5$	$\chi = 0.7$	$\chi = 0.9$
Inequality in Levels			
Mean	15,419.59	15,201.16	15,033.78
Std. Dev.	28,574.83	28,233.56	27,990.89
Q10	213.3352	207.60	194.3495
Median	3,252.30	3228.92	3,205.96
Q90	39,737.28	39,505.75	39,360.82
N. Observations	107	107	107
Inequality in Growth Rates			
Mean	0.047	0.044	0.042
Std. Dev.	0.122	0.125	0.128
Q10	-0.061	-0.067	-0.067
Median	0.034	0.033	0.033
Q90	0.164	0.163	0.162
N. Observations	577	577	577

Note: χ denotes the elasticity of substitution between capital and labor.