Improving output gap estimation—a bottom-up approach

Alexander Rathke and Sina Streicher

KOF Swiss Economic Institute, ETH Zurich

Introduction

Introduction

- Output gaps are usually estimated at the **national** and **supra-regional** level, for example for an economic area or a monetary union

Introduction

- Output gaps are usually estimated at the **national** and **supra-regional** level, for example for an economic area or a monetary union
- Current models are silent about the underlying driving factors of the **business cycle** and **potential growth**

Introduction

- Output gaps are usually estimated at the **national** and **supra-regional** level, for example for an economic area or a monetary union
- Current models are silent about the underlying driving factors of the **business cycle** and **potential growth**
- Monitoring the business cycle of individual sectors can lead to more targeted and efficient policy

Introduction

- Output gaps are usually estimated at the **national** and **supra-regional** level, for example for an economic area or a monetary union
- Current models are silent about the underlying driving factors of the **business cycle** and **potential growth**
- Monitoring the business cycle of individual sectors can lead to more targeted and efficient policy
- Sector cycles and trends useful for economic forecasting

$\Rightarrow$ Multidimensional state space model which estimates the aggregate output gap and long-term growth **consistent** with the **dynamics** of the various **sectors** of the economy

Model setup

Model setup

Measurement equation

Model setup

Measurement equation

Output

$y_t = \tau_t + g_t$

$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$

- g_t is the output gap
- g_{it} is the gap in sector i
- y_{it} is output in sector i

Model setup

Measurement equation

Output

$$y_t = \tau_t + g_t$$

$$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$$

Labor market

$$u_t = \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}$$

$$e_t = \tau_{et} + \Psi_e(L)\, g_t + c_{et}$$

$$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t}$$

- g_t is the output gap
- g_{it} is the gap in sector i

- y_{it} is output in sector i
- u_t is the unemployment rate
- e_{it} is employment in sector i

Model setup

Measurement equation

Output

$y_t = \tau_t + g_t$

$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$

Labor market

$u_t = \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}$

$e_t = \tau_{et} + \Psi_e(L)\, g_t + c_{et}$

$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t}$

Phillips curve

$\pi_t = \tau_{\pi t} + \delta\,(u_t - \tau_{ut}) + c_{\pi t}$

- g_t is the output gap
- g_{it} is the gap in sector i
- y_{it} is output in sector i
- u_t is the unemployment rate
- e_{it} is employment in sector i
- π_t is inflation

Model setup

Measurement equation

Output

$$y_t = \tau_t + g_t$$

$$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$$

Labor market

$$u_t = \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}$$

$$e_t = \tau_{et} + \Psi_e(L)\, g_t + c_{et}$$

$$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t}$$

Phillips curve

$$\pi_t = \tau_{\pi t} + \delta\,(u_t - \tau_{ut}) + c_{\pi t}$$

State equation

- g_t is the output gap
- g_{it} is the gap in sector *i*

- y_{it} is output in sector *i*
- u_t is the unemployment rate
- e_{it} is employment in sector *i*
- π_t is inflation

Model setup

Measurement equation

Output

$$y_t = \tau_t + g_t$$

$$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$$

Labor market

$$u_t = \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}$$

$$e_t = \tau_{et} + \Psi_e(L)\, g_t + c_{et}$$

$$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t}$$

Phillips curve

$$\pi_t = \tau_{\pi t} + \delta\,(u_t - \tau_{ut}) + c_{\pi t}$$

State equation

Trends

$$\tau_{\cdot t} = \tau_{\cdot t-1} + \mu_{\cdot t-1} + \varepsilon_{\tau_{\cdot t}}$$

$$\mu_{\cdot t} = \mu_{\cdot t-1} + \varepsilon_{\mu_{\cdot t}}$$

$$\tau_{\pi t} = \tau_{\pi t-1} + \varepsilon_{\tau_\pi t}$$

- g_t is the output gap
- g_{it} is the gap in sector i

- y_{it} is output in sector i
- u_t is the unemployment rate
- e_{it} is employment in sector i
- π_t is inflation

Model setup

Measurement equation

Output

$y_t = \tau_t + g_t$

$y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it}$

Labor market

$u_t = \tau_{ut} + \Psi_u(L)\, g_t + c_{ut}$

$e_t = \tau_{et} + \Psi_e(L)\, g_t + c_{et}$

$e_{it} = \tau_{e_i t} + \Psi_{e_i}(L)\, g_{it} + c_{e_i t}$

Phillips curve

$\pi_t = \tau_{\pi t} + \delta\,(u_t - \tau_{ut}) + c_{\pi t}$

State equation

Trends

$\tau_{\cdot t} = \tau_{\cdot t-1} + \mu_{\cdot t-1} + \varepsilon_{\tau \cdot t}$

$\mu_{\cdot t} = \mu_{\cdot t-1} + \varepsilon_{\mu \cdot t}$

$\tau_{\pi t} = \tau_{\pi t-1} + \varepsilon_{\tau_\pi t}$

Cycles

$\Phi_\cdot(L)\, c_{\cdot t} = \varepsilon_{c \cdot t}$

$\Phi_g(L)\, g_t = \varepsilon_{gt}$

- g_t is the output gap
- g_{it} is the gap in sector i

- y_{it} is output in sector i
- u_t is the unemployment rate
- e_{it} is employment in sector i
- π_t is inflation

Aggregation constraints

Aggregation constraints

Output

$$Y_t = \sum_{i=1}^{n} w_{ti}^{p} Y_{it},$$

Aggregation constraints

Output

$$Y_t = \sum_{i=1}^{n} w_{ti}^{p} Y_{it},$$

implies

$$\Delta y_t = \sum_{i=1}^{n} w_{it}^{nom} \Delta y_{it},$$

$$\mu_t = \sum_{i=1}^{n} w_{it}^{nom} \mu_{it}$$

Aggregation constraints

Output

$$Y_t = \sum_{i=1}^{n} w_{ti}^{p} Y_{it},$$

implies

$$\Delta y_t = \sum_{i=1}^{n} w_{it}^{nom} \Delta y_{it},$$
$$\mu_t = \sum_{i=1}^{n} w_{it}^{nom} \mu_{it}$$

where

- $y_t = \ln Y_t$
- w_{ti}^{p} denotes relative prices at $t-1$
- w_{it}^{nom} denotes nominal output weights at $t-1$

Aggregation constraints

Output

$$Y_t = \sum_{i=1}^{n} w_{ti}^{p} Y_{it},$$

implies

$$\Delta y_t = \sum_{i=1}^{n} w_{it}^{nom} \Delta y_{it},$$
$$\mu_t = \sum_{i=1}^{n} w_{it}^{nom} \mu_{it}$$

where

- $y_t = \ln Y_t$
- w_{ti}^{p} denotes relative prices at $t-1$
- w_{it}^{nom} denotes nominal output weights at $t-1$

Employment

$$E_t = \sum_{i=1}^{n} E_{it}$$

implies

$$\Delta e_t = \sum_{i=1}^{n} w_{it}^{e} \Delta e_{it},$$
$$\mu_{et} = \sum_{i=1}^{n} w_{it}^{e} \mu_{e_i t},$$

where

- $e_t = \ln E_t$
- w_{ti}^{e} denotes employment weights at $t-1$

Estimation

State-space representation:

Observation equation: $\tilde{\boldsymbol{y}}_t = \boldsymbol{Z}_t \boldsymbol{x}_t$

State equation: $\boldsymbol{x}_t = \boldsymbol{T}_t \boldsymbol{x}_{t-1} + \boldsymbol{R}_t \varepsilon_t,$ $\varepsilon_t \sim \mathcal{N}(0, \boldsymbol{Q}_t)$

Estimation

State-space representation:

Observation equation: $\tilde{\boldsymbol{y}}_t = \boldsymbol{Z}_t \boldsymbol{x}_t$

State equation: $\boldsymbol{x}_t = \boldsymbol{T}_t \boldsymbol{x}_{t-1} + \boldsymbol{R}_t \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \boldsymbol{Q}_t)$

Gibbs-sampling:

Unknown parameters Θ, unknown states $\boldsymbol{x}_t$, $t = 1, \ldots, T$

Draw $\Theta^{(k)} | (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)^{(k-1)}$

Draw $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)^{(k)} | \Theta^{(k)}$

Estimation

State-space representation:

Observation equation: $\tilde{\boldsymbol{y}}_t = \boldsymbol{Z}_t \boldsymbol{x}_t$

State equation: $\boldsymbol{x}_t = \boldsymbol{T}_t \boldsymbol{x}_{t-1} + \boldsymbol{R}_t \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \boldsymbol{Q}_t)$

Gibbs-sampling:

Unknown parameters Θ, unknown states $\boldsymbol{x}_t$, $t = 1, \ldots, T$

Draw $\Theta^{(k)} | (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)^{(k-1)}$

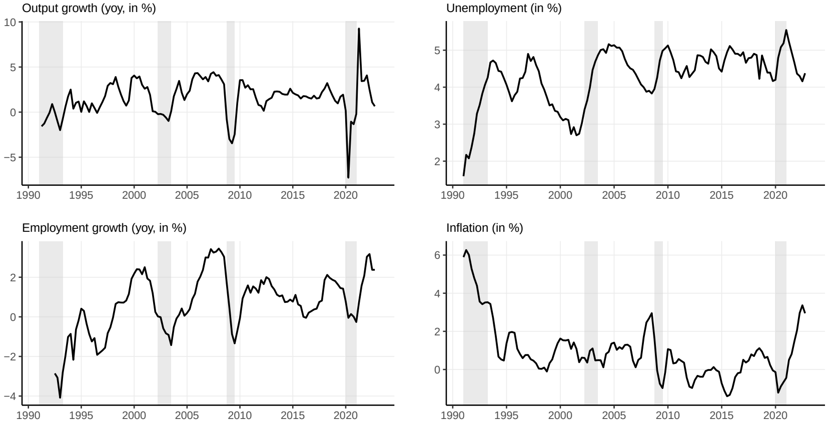
Draw $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T)^{(k)} | \Theta^{(k)}$

Priors

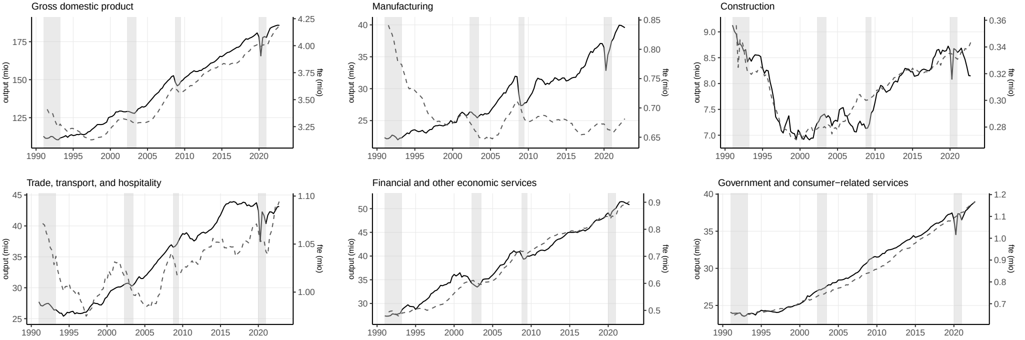
Loadings, autoregressive parameters: Normal ($\mu = 0, \sigma^2 = 1000$)

Variances: Inverse-gamma

Data: growth, unemployment, inflation

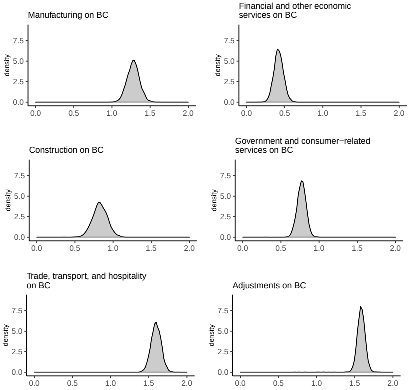


Data: Output and full-time equivalent employment



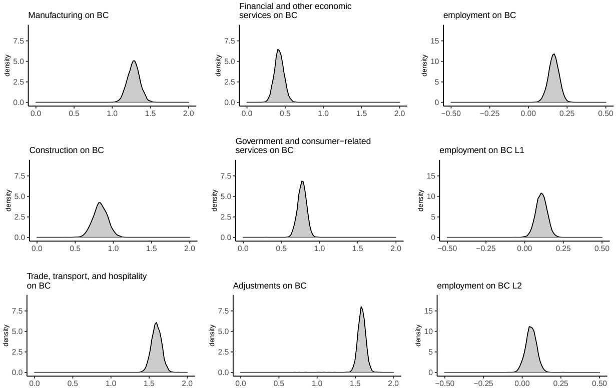
The solid lines (left axes) show quarterly output in million 2019 CHF and the dashed lines (right axes) depict full-time equivalent (fte) employment in million.

Prior and posterior distributions: output loadings



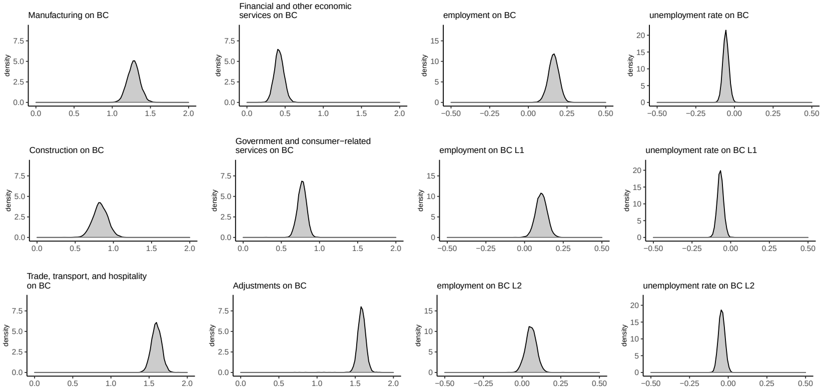
BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30’000 draws where the first 6’000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Prior and posterior distributions: output loadings



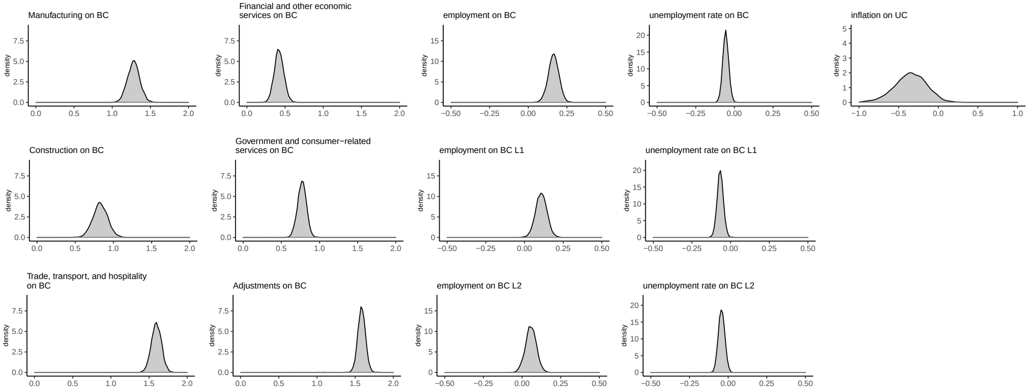
BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Prior and posterior distributions: output loadings



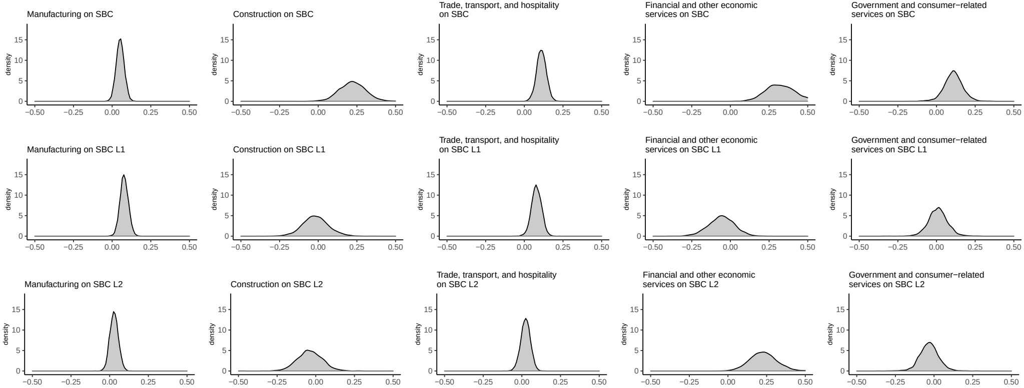
BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30’000 draws where the first 6’000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Prior and posterior distributions: output loadings



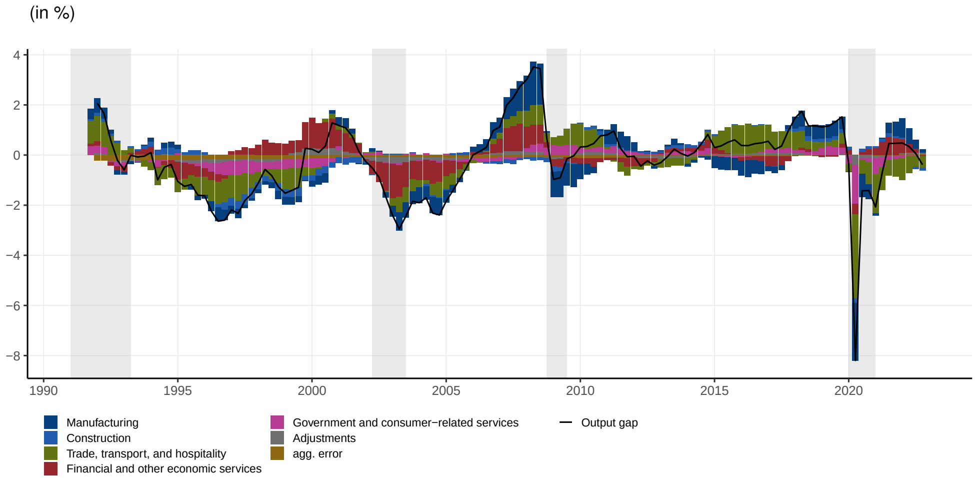
BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Prior and posterior distributions: employment loadings



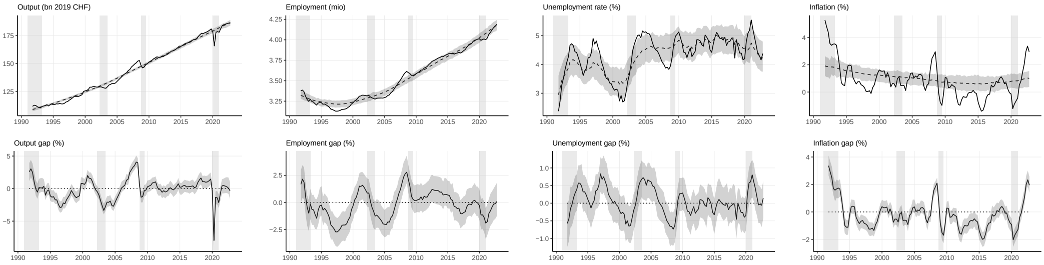
SBC denotes the sector business cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.

Output gap decomposition



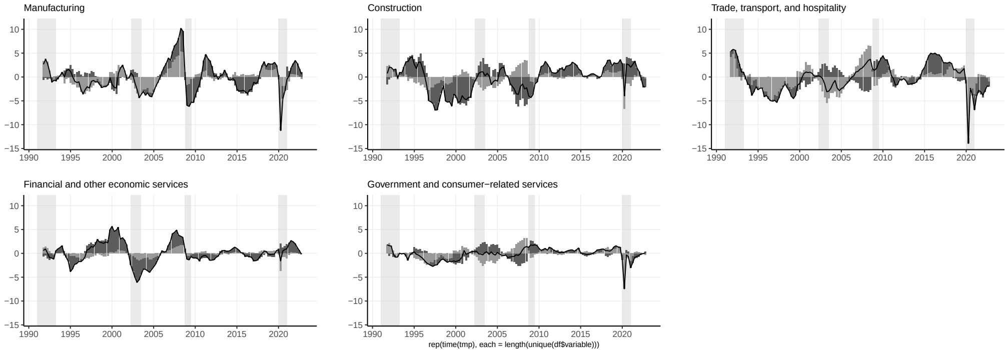
Aggregate trends and cycles

Output | Employment | Unemployment rate | Inflation



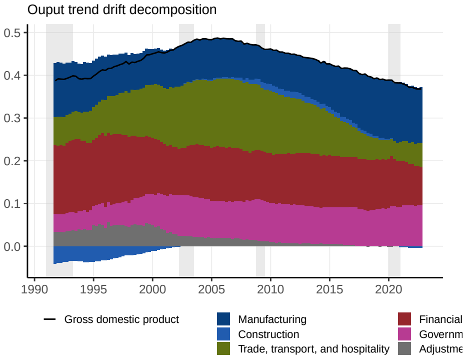
The original data are solid and the trends dashed (top). The estimated cycles are solid (bottom). The shaded areas indicate 68% HPDI.

Sector output decomposition



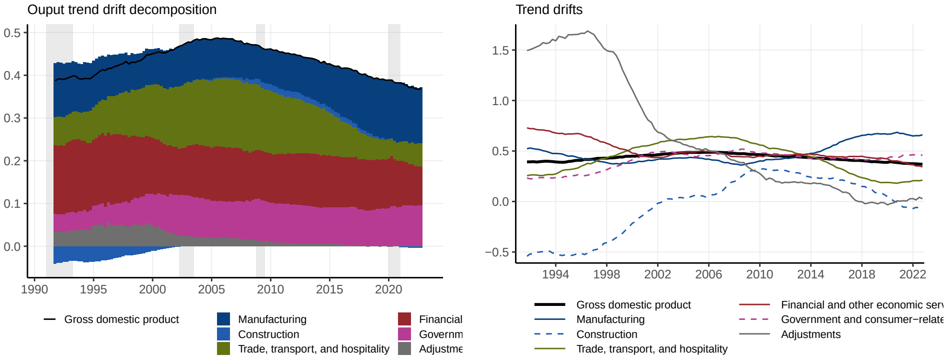
Sector gaps in %. Light shaded areas represent sector-specific output cycle contributions and dark areas those of idiosyncratic output and employment cycles, respectively.

Drifts



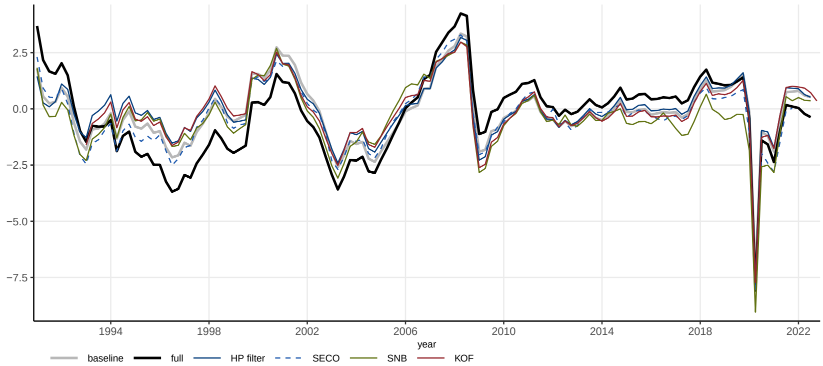
Ouput trend drift decomposition
0.5
0.4
0.3
0.2
0.1
0.0
1990
1995
2000
2005
2010
2015
2020
Gross domestic product
Manufacturing
Construction
Trade, transport, and hospitality
Financial
Governm
Adjustme

Drifts



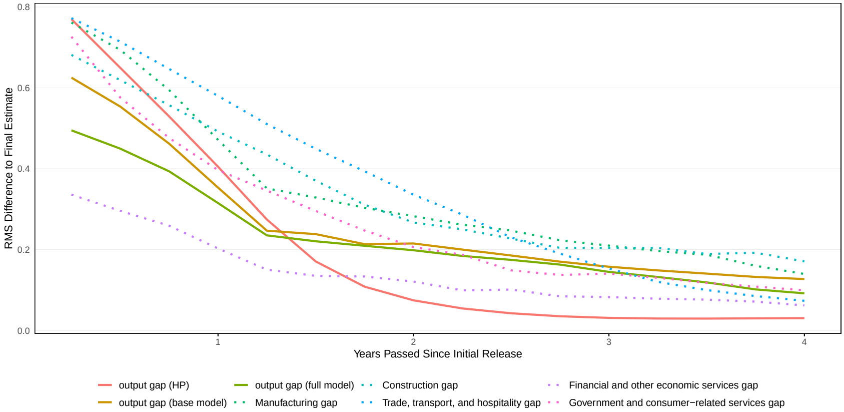
Trend growth rate decomposition (left column) and sector trends (right column) for output %.

Model comparison



Output gaps are in %. The output gaps from SECO, SNB, and KOF are each based on a production function approach. The smoothing constant for the HP filter gap is 1600

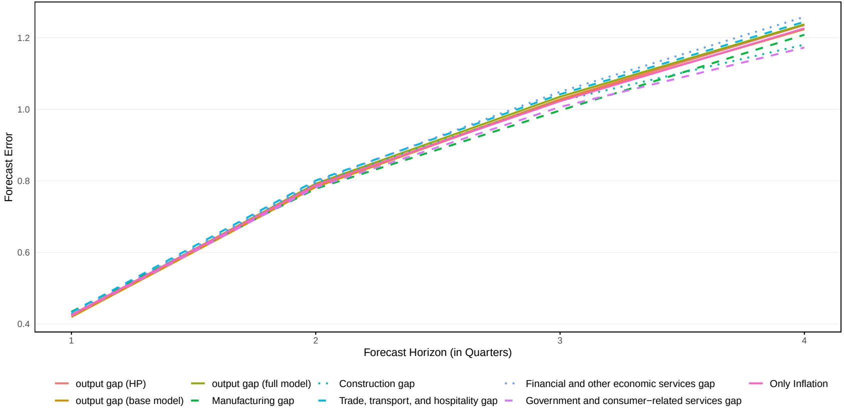
Pseudo real-time analysis: Revisions



Pseudo real-time analysis: Inflation forecasting performance

Autoregressive distributed lag Phillips curve forecasting equation (out-of-sample):

$$\pi_{t+h} - \pi_t = \alpha + \sum_{p=1}^{n_\pi} \beta_p \Delta\pi_{t-p} + \sum_{p=1}^{n_y} \beta_p \gamma_p y^t_{t-p,i} + \varepsilon_{t+h}$$



Conclusion and outlook

- Swiss data very informative in estimating the model
- consistent trends and cycles useful for forecasting models

Conclusion and outlook

- Swiss data very informative in estimating the model
- consistent trends and cycles useful for forecasting models

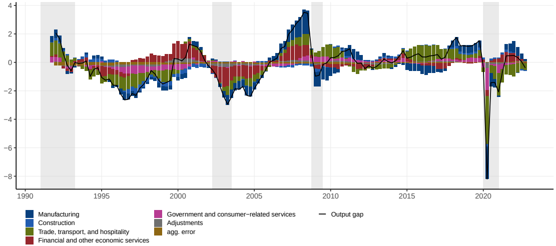
Going forward...

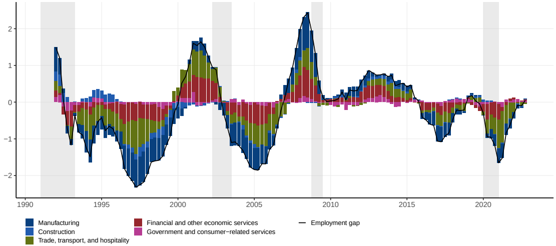
- extend real-time comparison (alternative models)
- real-time (sector) output and PPI forecasting exercise
- adding other indicators (e.g. expenditure side)
- setting an anchor to stabilize the current edge
- sensitivity to aggregation level (more sub-sectors)

Thanks for your attenion!

Appendix

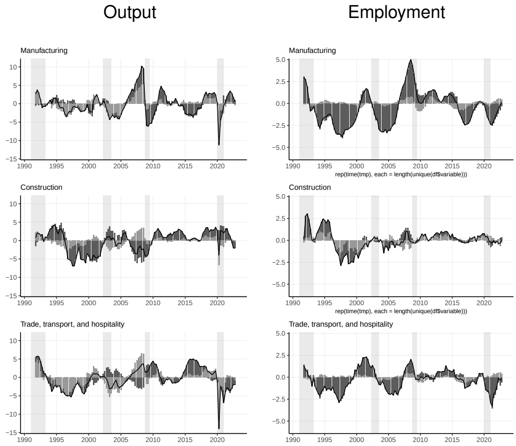
Output and employment gap decomposition





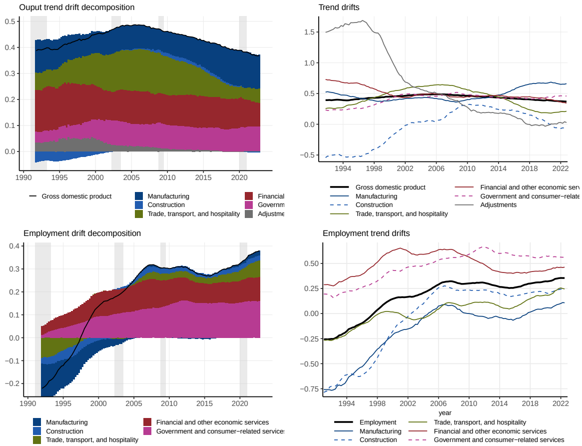
Gaps and contributions are in %.

Sector output and employment cycle decomposition



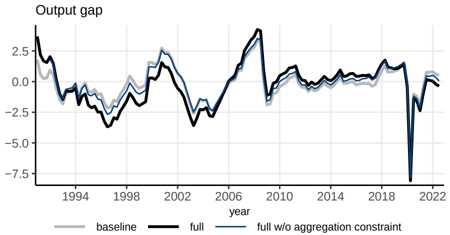
Sector gaps in %. Light shaded areas represent sector-specific output cycle contributions and dark areas those of idiosyncratic output and employment cycles, respectively.

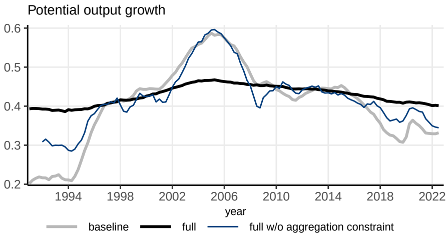
Drifts

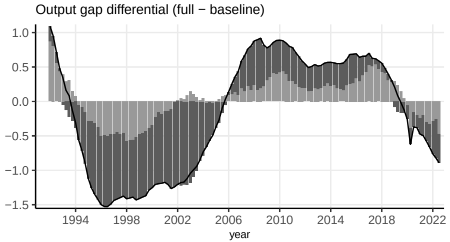


Trend growth rate decomposition (left column) and sector trends (right column) for output (top) and employment in %.

Sources for model differences







	sectors	agg. constraints
full	x	x
full w/o agg. constraint	x	–
baseline	–	–

Bottom panel: contributions of sectors (light shaded) and those of the aggregation constraints (dark shaded)

Data

- Real gross domestic product (sport-event adjusted), SECO
- Real gross value added before adjustments (sport-event adjusted), SECO
- Full time equivalent employment, FSO (JOBSTAT)
- ILO unemployment rate, FSO
- Consumer Price Index excluding oil, FSO
- Period: 1990Q1–2022Q3, quarterly frequency

SECO	Swiss State Secretariat for Economic Affairs
FSO	Swiss Federal Statistical Office
JOBSTAT	Job Statistic
ILO	International Labor Organization

Table: Structure of sectors

Sector	Sub-sectors	NOGA
Manufacturing	Agriculture, forestry and fishing	01-03
	Mining and quarrying	05-09
	Manufacturing	10-33
	Electricity, gas, steam and air conditioning supply	35
	Water supply, sewerage, waste management and remediation activities	36-39
Construction	Construction	41-43
Trade, transport and hospitality	Trade, repair of motor vehicles and motorcycles	45-57
	Transportation and storage; Information and communication	49-53; 58-63
	Accommodation and food service activities	55-56
Financial and other economic services	Financial service activities	64
	Insurance service activities	65
	Real estate, professional, scientific and technical activities; Administrative and support service activities	68-57; 77-82
Government and consumer-related services	Public administration and defense; compulsory social services	84
	Education	85
	Human health and social work activities	86-88
	Arts, entertainment and recreation	90-93
	Other service activities	94-96
	Activities of housholds as employers and producers for own use	97-98
Adjustments	Taxes on products	
	Subsidies on products	

Estimation: priors

Name	Support	Density	Parameters	
β_i, δ	$\mathbb{R}$	Normal	$\mu = 0,$	$\sigma^2 = 1000$
$(\psi_0, \psi_1, \psi_2)'$	$\mathbb{R}^3$	Normal	$\mu = (0, 0, 0)'$,	$\sigma^2 = 1000\mathbf{I}_3$
$(\phi_1, \phi_2)'$	$\mathbb{R}^2 \times I_{\phi \in S_\phi}$	Normal	$\mu = (0, 0)'$,	$\sigma^2 = 1000\mathbf{I}_2$
σ_c^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4$
σ_μ^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4\lambda^{-1}$
σ_τ^2	$(0, \infty)$	Inverse-gamma	$\nu = 6,$	$s = 4\lambda^{-1}$

Notes: $I_{\phi \in S_\phi}$ denotes the indicator function and S_ϕ the stationary region of an AR(2) process. All indices are suppressed for the sake of readability. The normal distribution is parametrized via mean and variance, the inverse-gamma distribution via degrees of freedom ν and location s with mean $s/\nu-2$. The smoothing constant λ is set to 100.