Improving output gap estimation—a bottom-up approach

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Introduction

Output gaps are usually estimated at the national and supra-regional level, for example for an economic area or a monetary union. Current models are silent about the underlying driving factors of the business cycle and potential growth. Monitoring the business cycle of individual sectors can lead to more targeted and efficient policy. Sector cycles and trends useful for economic forecasting.

⇒ Multidimensional state space model which estimates the aggregate output gap and long-term growth consistent with the dynamics of the various sectors of the economy.
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• Sector cycles and trends useful for economic forecasting

⇒ Multidimensional state space model which estimates the aggregate output gap and long-term growth consistent with the dynamics of the various sectors of the economy
Model setup

- $y_t$ is output
- $\pi_t$ is inflation
- $u_t$ is the unemployment rate
- $e_t$ is employment in sector $i$
- $g_t$ is the output gap
- $g_{it}$ is the gap in sector $i$
- $y_{it}$ is output in sector $i$
- $\tau$ are trends
Model setup

Measurement equation
Model setup

Measurement equation

Output

\[ y_t = \tau_t + g_t \]

\[ y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it} \]

- \( g_t \) is the output gap
- \( g_{it} \) is the gap in sector \( i \)
- \( y_{it} \) is output in sector \( i \)
Model setup

Measurement equation

Output
\[ y_t = \tau_t + g_t \]
\[ y_{it} = \tau_{it} + g_{it} = \tau_{it} + \beta_i g_t + c_{it} \]

Labor market
\[ u_t = \tau u_t + \psi_u (L) g_t + c u_t \]
\[ e_t = \tau e_t + \psi_e (L) g_t + c e_t \]
\[ e_{it} = \tau_{e_i t} + \psi_{e_i} (L) g_{it} + c_{e_i t} \]

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Labor market
\[ u_t = \tau_{ut} + \psi_u (L) g_t + c_{ut} \]
\[ e_t = \tau_{et} + \psi_e (L) g_t + c_{et} \]
\[ e_{it} = \tau_{e_{it}} + \psi_{e_i} (L) g_{it} + c_{e_{it}} \]

Phillips curve
\[ \pi_t = \tau_{\pi t} + \delta (u_t - \tau_{ut}) + c_{\pi t} \]

- \( g_t \) is the output gap
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\[ y_t = \tau_t + g_t \]
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\[ u_t = \tau_{ut} + \psi_u(L) g_t + c_{ut} \]
\[ e_t = \tau_{et} + \psi_e(L) g_t + c_{et} \]
\[ e_{it} = \tau_{ei,t} + \psi_{ei} (L) g_{it} + c_{ei,t} \]

Phillips curve
\[ \pi_t = \tau_{\pi t} + \delta (u_t - \tau_{ut}) + c_{\pi t} \]

State equation

Trends
\[ \tau_t = \tau_{t-1} + \mu_t - 1 + \epsilon_{\tau,t} \]
\[ \mu_t = \mu_{t-1} + \epsilon_{\mu,t} \]
\[ \tau_{\pi t} = \tau_{\pi t-1} + \epsilon_{\tau_{\pi} t} \]

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\[ \pi_t = \tau_{\pi t} + \delta (u_t - \tau_{ut}) + c_{\pi t} \]

State equation

Trends
\[ \tau_{.t} = \tau_{.t-1} + \mu_{.t-1} + \varepsilon_{\tau_{.t}} \]
\[ \mu_{.t} = \mu_{.t-1} + \varepsilon_{\mu_{.t}} \]
\[ \tau_{\pi t} = \tau_{\pi t-1} + \varepsilon_{\tau_{\pi t}} \]

Cycles
\[ \Phi_c (L) c_t = \varepsilon_{c_{.t}} \]
\[ \Phi_g (L) g_t = \varepsilon_{gt} \]

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Aggregation constraints

\[ Y_t = \sum_{i=1}^{n} w_{pi} Y_{it} \]

implies

\[ \Delta y_t = \sum_{i=1}^{n} w_{nomi} \Delta y_{it} \]

\[ \mu_t = \sum_{i=1}^{n} w_{nomi} \mu_{it} \]

where

- \( y_t = \ln Y_t \)
- \( w_{pi} \) denotes relative prices at \( t-1 \)
- \( w_{nomi} \) denotes nominal output weights at \( t-1 \)

Employment

\[ E_t = \sum_{i=1}^{n} E_{it} \]

implies

\[ \Delta e_t = \sum_{i=1}^{n} w_{ei} \Delta e_{it} \]

\[ \mu_{et} = \sum_{i=1}^{n} w_{ei} \mu_{ei t} \]

where

- \( e_t = \ln E_t \)
- \( w_{ei} \) denotes employment weights at \( t-1 \)
Aggregation constraints

Output

\[ Y_t = \sum_{i=1}^{n} w_{pi} Y_{it}, \]

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\[ E_t = \sum_{i=1}^{n} E_{it}, \]

\[ \Delta e_t = \sum_{i=1}^{n} w_{ei} \Delta E_{it}, \]

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Aggregation constraints

Output

\[ Y_t = \sum_{i=1}^{n} w_t^p Y_{it}, \]

implies

\[ \Delta y_t = \sum_{i=1}^{n} w_{it}^{nom} \Delta y_{it}, \]
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\[ Y_t = \sum_{i=1}^{n} w^p_{it} Y_{it}, \]

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\[ E_t = \sum_{i=1}^{n} E_{it} \]

implies

\[ \Delta e_t = \sum_{i=1}^{n} w^e_{it} \Delta e_{it}, \]

\[ \mu_{et} = \sum_{i=1}^{n} w^e_{it} \mu_{et}, \]

where

- \( e_t = \ln E_t \)
- \( w^e_{it} \) denotes employment weights at \( t - 1 \)
Estimation

State-space representation:

Observation equation: \( \tilde{y}_t = Z_t x_t \)

State equation: \( x_t = T_t x_{t-1} + R_t \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, Q_t) \)
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Gibbs-sampling:

Unknown parameters \( \Theta \), unknown states \( x_t, t = 1, \ldots, T \)

Draw \( \Theta^{(k)} | (x_1, \ldots, x_T)^{(k-1)} \)

Draw \( (x_1, \ldots, x_T)^{(k)} | \Theta^{(k)} \)
Estimation

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Priors

Loadings, autoregressive parameters: Normal (\( \mu = 0, \sigma^2 = 1000 \))

Variances: Inverse-gamma
Data: growth, unemployment, inflation

Output growth (yoy, in %)

Unemployment (in %)

Employment growth (yoy, in %)

Inflation (in %)
Data: Output and full-time equivalent employment

The solid lines (left axes) show quarterly output in million 2019 CHF and the dashed lines (right axes) depict full-time equivalent (fte) employment in million.
Prior and posterior distributions: output loadings

BS denotes the business cycle (output gap) and UC the unemployment cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.
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Prior and posterior distributions: employment loadings

SBC denotes the sector business cycle. The posterior densities are based on 30'000 draws where the first 6'000 draws are discarded. Of the remaining draws, all but each 10th draw are dropped.
Aggregate trends and cycles

The original data are solid and the trends dashed (top). The estimated cycles are solid (bottom). The shaded areas indicate 68% HPDI.
Sector output decomposition

Sector gaps in %. Light shaded areas represent sector-specific output cycle contributions and dark areas those of idiosyncratic output and employment cycles, respectively.
Trend growth rate decomposition (left column) and sector trends (right column) for output %.
Model comparison

Output gaps are in %. The output gaps from SECO, SNB, and KOF are each based on a production function approach. The smoothing constant for the HP filter gap is 1600.
Pseudo real-time analysis: Revisions

![Graph showing RMS Difference to Final Estimate over Years Passed Since Initial Release]

- output gap (HP)
- output gap (full model)
- Construction gap
- Financial and other economic services gap
- output gap (base model)
- Manufacturing gap
- Trade, transport, and hospitality gap
- Government and consumer−related services gap
Pseudo real-time analysis: Inflation forecasting performance

Autoregressive distributed lag Phillips curve forecasting equation (out-of-sample):

$$\pi_{t+h} - \pi_t = \alpha + \sum_{p=1}^{n_\pi} \beta_p \Delta \pi_{t-p} + \sum_{y=1}^{n_y} \beta_y \gamma y^t_{t-p,y} + \epsilon_{t+h}$$
Conclusion and outlook

• Swiss data very informative in estimating the model
• consistent trends and cycles useful for forecasting models
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Going forward...
• extend real-time comparison (alternative models)
• real-time (sector) output and PPI forecasting exercise
• adding other indicators (e.g. expenditure side)
• setting an anchor to stabilize the current edge
• sensitivity to aggregation level (more sub-sectors)
Thanks for your attention!
Output and employment gap decomposition

Gaps and contributions are in %.
Sector output and employment cycle decomposition

Output

- Manufacturing
- Construction
- Trade, transport, and hospitality

Employment

- Manufacturing
- Construction
- Trade, transport, and hospitality

Output

- Financial and other economic services

Employment

- Financial and other economic services

Sector gaps in %. Light shaded areas represent sector-specific output cycle contributions and dark areas those of idiosyncratic output and employment cycles, respectively.
Drifts

Trend growth rate decomposition (left column) and sector trends (right column) for output (top) and employment in %.
Sources for model differences

Output gap

Output gap differential (full − baseline)

Potential output growth

bottom panel: contributions of sectors (light shaded) and those of the aggregation constraints (dark shaded)
Data

- Real gross domestic product (sport-event adjusted), SECO
- Real gross value added before adjustments (sport-event adjusted), SECO
- Full time equivalent employment, FSO (JOBSTAT)
- ILO unemployment rate, FSO
- Consumer Price Index excluding oil, FSO
- Period: 1990Q1–2022Q3, quarterly frequency

SECO Swiss State Secretariat for Economic Affairs
FSO Swiss Federal Statistical Office
JOBSTAT Job Statistic
ILO International Labor Organization

Table: Structure of sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>Sub-sectors</th>
<th>NOGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>Agriculture, forestry and fishing</td>
<td>01-03</td>
</tr>
<tr>
<td></td>
<td>Mining and quarrying</td>
<td>05-09</td>
</tr>
<tr>
<td></td>
<td>Manufacturing</td>
<td>10-33</td>
</tr>
<tr>
<td></td>
<td>Electricity, gas, steam and air conditioning supply</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Water supply, sewerage, waste management and remediation activities</td>
<td>36-39</td>
</tr>
<tr>
<td>Construction</td>
<td>Construction</td>
<td>41-43</td>
</tr>
<tr>
<td>Trade, transport and hospitality</td>
<td>Trade, repair of motor vehicles and motorcycles</td>
<td>45-57</td>
</tr>
<tr>
<td></td>
<td>Transportation and storage; Information and communication</td>
<td>49-53; 58-63</td>
</tr>
<tr>
<td></td>
<td>Accommodation and food service activities</td>
<td>55-56</td>
</tr>
<tr>
<td>Financial and other economic services</td>
<td>Financial service activities</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>Insurance service activities</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>Real estate, professional, scientific and technical activities; Administrative and support service activities</td>
<td>68-57; 77-82</td>
</tr>
<tr>
<td>Government and consumer-related services</td>
<td>Public administration and defense; compulsory social services</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>Education</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>Human health and social work activities</td>
<td>86-88</td>
</tr>
<tr>
<td></td>
<td>Arts, entertainment and recreation</td>
<td>90-93</td>
</tr>
<tr>
<td></td>
<td>Other service activities</td>
<td>94-96</td>
</tr>
<tr>
<td></td>
<td>Activities of households as employers and producers for own use</td>
<td>97-98</td>
</tr>
<tr>
<td>Adjustments</td>
<td>Taxes on products</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Subsidies on products</td>
<td></td>
</tr>
</tbody>
</table>
Estimation: priors

<table>
<thead>
<tr>
<th>Name</th>
<th>Support</th>
<th>Density</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_i, \delta$</td>
<td>$\mathbb{R}$</td>
<td>Normal</td>
<td>$\mu = 0, \sigma^2 = 1000$</td>
</tr>
<tr>
<td>$(\psi_0, \psi_1, \psi_2)'$</td>
<td>$\mathbb{R}^3$</td>
<td>Normal</td>
<td>$\mu = (0,0,0)', \sigma^2 = 1000I_3$</td>
</tr>
<tr>
<td>$(\phi_1, \phi_2)'$</td>
<td>$\mathbb{R}^2 \times I_{\phi \in S_\phi}$</td>
<td>Normal</td>
<td>$\mu = (0,0)', \sigma^2 = 1000I_2$</td>
</tr>
<tr>
<td>$\sigma^2_c$</td>
<td>(0, $\infty$)</td>
<td>Inverse-gamma</td>
<td>$\nu = 6, s = 4$</td>
</tr>
<tr>
<td>$\sigma^2_\mu$</td>
<td>(0, $\infty$)</td>
<td>Inverse-gamma</td>
<td>$\nu = 6, s = 4\lambda^{-1}$</td>
</tr>
<tr>
<td>$\sigma^2_\tau$</td>
<td>(0, $\infty$)</td>
<td>Inverse-gamma</td>
<td>$\nu = 6, s = 4\lambda^{-1}$</td>
</tr>
</tbody>
</table>

Notes: $I_{\phi \in S_\phi}$ denotes the indicator function and $S_\phi$ the stationary region of an AR(2) process. All indices are suppressed for the sake of readability. The normal distribution is parametrized via mean and variance, the inverse-gamma distribution via degrees of freedom $\nu$ and location $s$ with mean $s/\nu - 2$. The smoothing constant $\lambda$ is set to 100.