Is it Better to be First?
Search with Endogenous Information Acquisition

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Classical Search Overview

Classical search in Simon (1955), Weitzman (1979):

- DM sequentially samples independent r.v.
- Binary technology for resolving uncertainty
- Fixed cost for item
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DM has **too passive** learning strategy: *sequential interview design*

Search & Endogenous Info

Endogenous info (RI): flexible learning at cost

Our Search & RI:

- DM sequentially samples iid alternatives
- **Flexible learning** about alternative
- **Endogenous, history-dependent** cost for alternative
- DM may learn nothing about alternative, **no recall**

DM *conducts endogenous interviews sequentially*

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- *Our results:* generalization of decreasing threshold rule via flexible information

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Optimal Interview Design: is it possible to characterise the optimal search rule?

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Order Discrimination: does the order of inspection influences the probability of choice?

- Better to be first: flexible learning (Steiner et al. 2021); search (Armstrong, 2017)
- Mixed evidence: attention discrimination (Bartos et al. 2016); anectodical evidence (reddit.com)
- *Our results:* depends on cost function

Flexible Learning

- Extremes: no-learning or full learning
- Our DM:
  - obtains **noisy** estimate about quality of candidate
  - controls whole *distribution* of **noise**
  - choice is stochastic

General Model

- T candidates: iid binary r.v. $\theta_t = \{0, 1\}$ with $P(\theta_t = 1) = \mu$
- DM learns at stage t about candidate t
- Learning
  - posterior distribution p_t: $p_t \in \Delta[0, 1] : \int_{[0,1]} x dp_t(x) = \mu.$
  - cost for p_i: $C(p_t) = k \int_{[0,1]} c(x) dp_t(x).$
- DM chooses one candidate without recall
- DM's payoff: expected posterior minus cost of learning

Reduction to Collection of Static Problems

- no recall $\Rightarrow$
- no state variable in dynamic problem $\Rightarrow$
- in period t after learning two actions available: *choose* candidate / *continue search* with **exogenous** outside option V_{T-t} $\Rightarrow$
- collection of static problems with different outside options

Difficulty and Optimality

Definition 1

Interview (x^L, x^H) is *more difficult* then (y^L, y^H) if

$$x^L > y^L, x^H > y^H$$

Optimality of interviews with *decreasing difficulties* as feature of flexible learning

Satisficing with Endogenous Information

- Restricted Model:
  - $\mu = 0.5$
  - Function $c(x)$ is symmetric around μ, $c(\mu) = 0$
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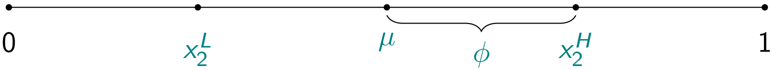

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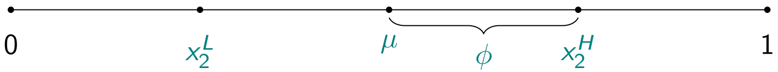

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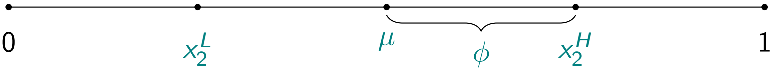

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- Optimal ϕ is independent from T: satisficing threshold

Harder Test is Better

- Fix symmetric posteriors and increase both by δ: $\Delta x_t^H = \Delta x_t^L = \delta$
- Probabilities and rewards change: $p_t^H \downarrow$, $x_t^H \uparrow$, $p_t^L \uparrow$

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- First-order effect of reward:

$$p_t^H \Delta x_t^H + \Delta p_t^H V_{T-t} - \Delta p_t^H x_t^H =_{[\text{Symmetry, } \mu = 0.5]}$$
$$= \Delta p_t^H (V_{T-t} - 0.5) > 0$$

It is better to increase both posteriors!

Main Theorem

Theorem 1

Let $c(x)$ be convex, twice differentiable then

$$x_t^L, x_t^H \text{ are decreasing in } t$$

Moreover,

$$\lim_{t\to\infty} x_t^L = \mu, \qquad \lim_{t\to\infty} x_t^H = x'$$

Optimal Posteriors

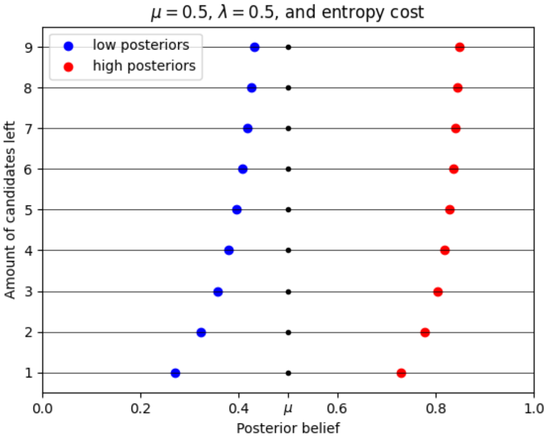

Figure: Optimal posterior beliefs given the amount of candidates left

Properties of Optimal Learning

1. x_t^H dynamics: cherry-picking
2. x_t^L dynamics: *rational* procrastination, postponing info acquisition
3. x_t^H & x_t^L dynamics: interview t is *more difficult* than interview $t-1$

Better to be First?

better to be first

hired if success

better to be last

the easiest interview

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Proposition 1

Let $c(x) = k(x - \mu)^2$ then

unconditional probabilities of choice P_t are *decreasing* in t

It is better to be first!

Cherry Picking and Lemon Dropping

- $T = 2$
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Proposition 2

If $c'(x)$ is concave on $(0, y)$ and $c'(x)$ is convex on $(y, 1)$ then exists y', y'' such that

$$\text{if } \mu \in (y'', y) \text{ then } P_2 > P_1;$$
$$\text{if } \mu \in (y, y') \text{ then } P_1 > P_2$$

- Condition for $c'(x)$ holds for entropy cost:
  if $\mu > 0.5$ DM chooses first more often, if $\mu < 0.5$ DM chooses second more often

Beyond $T = 2$ case

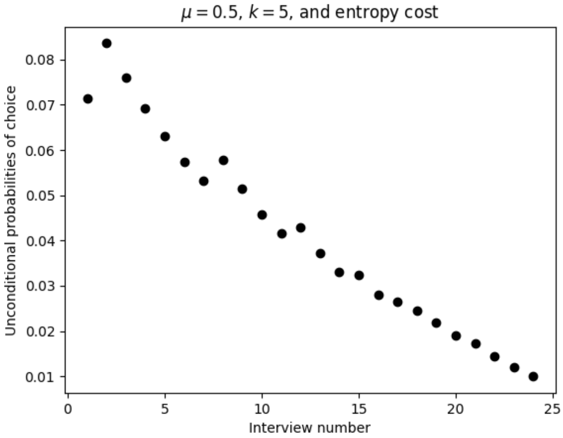

Figure: Non-monotone unconditional probabilities

Conclusion

Optimal search rule: DM *decreases* difficulty of interviews

1. Cherry-picking
2. Postponing info acquisition

Discrimination: depends of cost function properties

1. if $c'''(x) = 0$ better to be first
2. if $c'''(x) \neq 0$ and $T = 2$ attention discrimination