Limited (energy) supply, sunspots, and monetary policy

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The views expressed herein are the authors'. They do not necessarily reflect the views of the Board of Governors, the Federal Reserve System, or the ECB.
recent years have seen monumental changes in the macro environment
  - shortages of (imported) inputs, e.g. energy, metals, ...
  - more to come: labor shortages from aging, climate transition, ...
  - volatile prices for inputs $\iff$ sensitivity to local demand

should *supply constraints* on *input factors* affect our thinking about monetary policy?
Our paper in one slide: input factor scarcity and monetary policy

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- should supply constraints on input factors affect our thinking about monetary policy?

- **main result** (theory): factor shortages raise the risk of self-fulfilling fluctuations
  - if high prices induce a redistribution of incomes from low- to high-MPC agents
  - elasticities, factor size, ownership (heterogeneity), consumption/production factor, fiscal
  - policy: firmer focus of central bank on price stability or input prices
• New Keynesian open economy model [next slide]

• new feature: energy supply is inelastic, flexible price clears market

• for the theory, “energy” is just a shorter label for “factor in inelastic supply”
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• input factor scarcity $\rightarrow$ energy-price-activity feedback loop
  • high energy price, core inflation, interest rates and economic activity; but low GDP
  • a part of AD increases with energy prices & is insensitive to interest rates
  • high energy prices reflect high demand in a supply-constrained environment, not a shock
Model
Model – birds-eye view of the economy

- two-country New Keynesian model as in Blanchard and Galí (2009)
  - Home imports **energy** from Foreign in exchange for goods
  - Foreign can accumulate net foreign assets

- heterogeneous households consume goods & **energy**, supply labor
  - savers: permanent income
  - spenders: unit MPC, hand-to-mouth

- firms use labor and **energy**, New Keynesian setup

- government consists of monetary and fiscal policy:
  - monetary policy: controls nominal rate, potentially responds to **energy** price
  - fiscal policy: potentially excess **energy** price subsidies, redistribute firms’ dividends
Model – most important equations of the model

- energy market clearing: \( \xi_E = (1 - \lambda)C_{S,E,t} + \lambda C_{H,E,t} + E_t \)

- goods market clearing: \( Y_{G,t} = (1 - \lambda)C_{S,G,t} + \lambda C_{H,G,t} + X_{G,t} \)

- foreign demand: \( X_{G,t} = f(\text{energy revenues}_t, \text{savings}_{t-1}) \)
  - parameterized with marginal propensities to demand exports out of both components
  + foreign budget: \( P_{G,t}X_{G,t} - [B_t - R_{t-1}B_{t-1}] = P_{E,t}\xi_E \)

- further equations:
  - households: savers’ Euler eq., labor supply schedules, CES cons. allocation, budgets
  - firms: CES production, PPI Phillips curve, energy and labor demand
Paper-and-pencil
Paper-and-pencil – in just one slide

- simplified model has usual **three-equation representation**, IS curve can invert
  - even for domestic representative household model

- “conventional” slope features Taylor principle
- “unconventional” slope requires (much) **stronger response**
• simplified model has usual **three-equation representation**, IS curve can invert
  • even for domestic representative household model

• “conventional” slope features Taylor principle
• “unconventional” slope requires (much) **stronger response**

• room for **self-fulfilling energy-price-activity feedback loop** if
  • unwilling to **substitute intertemporally** (high $\sigma$) of inelastic labor supply (high $\phi$)
  • flat Phillips curve (low $\epsilon/\psi$)
  • important **share of energy** in costs (high $\alpha$) or hard to **substitute energy** (low $\theta$)

• household heterogeneity **amplifies** effect of scarce energy
Calibration
Calibration – calibration strategy for energy

• theoretical channel established paper-and-pencil → can it matter quantitatively?

• today, all energy is imported → match expenditure shares for imported energy
• target share of scarce fossil energy imports in German primary energy usage
  • natural gas and (some) coal at 2022 prices
Calibration – parameters that govern indeterminacy

• energy-related parameters
  \( \eta \): elasticity of substitution between energy and goods in consumption (0.1)
  \( \gamma \): energy consumption as share of GDP (5%)
  \( \bar{e} \): subsistence energy consumption (25%)
  \( \theta \): elasticity of substitution between energy and labor in production (0.1)
  \( \alpha \): energy production as share of GDP (10%)
  \( \mu_{F,1} \): Foreign’s MPC out of energy revenues (0.25)
  \( \tau^c_E, \tau^f_E \): excessive-energy-price subsidies for firms and households (33%)

• important non-energy parameters
  \( \psi \): price adjustment costs match slope of NKPC (0.1)
  \( \varphi \): inverse Frisch elasticity of labor supply (3)
  \( \sigma \): inverse elasticity of intertemporal substitution (3)
  \( \lambda \): share of spenders (0.22)
Quantitative results
Sunspot belief of high energy prices under baseline policy

• 20% sunspot increase in wholesale energy prices → marginal costs and core inflation increase
• CB increases interest rates → savers’ consumption falls
• but aggregate demand does not (due to foreign demand & hand-to-mouths’ demand)
• output rises, GDP falls
non-fundamental belief does not only affect aggregate economic activity but also the distribution of incomes → what is the role of MPCs?

- Taylor principle holds if Foreign’s MPC does not exceed 0.19 (solid)
- absent MPC heterogeneity in home, the feedback loop would arise only when the Foreign MPC exceeds 0.58 (dashed)
Drivers of the feedback loop: energy consumption or production?

- supply shortages may primarily affect the supply of goods for consumption or of factors of production, in baseline: both → what is the role of each?

- Taylor principle is violated as soon as firms’ expenditure share of energy exceeds six percent of GDP (solid)

- energy use in consumption dampens the feedback loop (dashed)
Drivers of the feedback loop: fiscal-monetary interaction

- feedback loop arises when high demand for goods comes with high energy prices, and if these do not substantially dampen demand → what is the role of subsidies?

- energy subsidy to households considerably supports feedback loop, high gradient (dashed)

- energy subsidy to firms of less importance, low gradient (solid)
core inflation: feedback loop arises for $\phi \Pi \leq 9.23$

- **headline inflation**: determinacy if $\phi \Pi > 1$
  - intuition: headline inflation contains energy prices, thereby, reflects firms' cost pressures
  - fails if energy consumption share is low or energy subsidy for consumers is high

- **input price inflation**: determinacy if $\phi \Pi > 1$
  - intuition: rigidity prevents firms to pass on their rising costs, directly stabilize them
  - independent of energy consumption share or energy subsidy
  - alternatively: core plus energy price inflation, determinacy if $\phi_{PE} > 0.01$
Conclusion
Conclusion – input factor shortages raise risk of self-fulfilling fluctuations

- environment with inelastic supply of an imported production factor
  - external demand positively linked to price of imported good
  - domestic absorption less interest sensitive due to subsidies and heterogeneity

- energy-price-activity feedback loop
  - high energy prices reflect high demand in a supply-constrained environment

- monetary policy can prevent loop
  - hawkish focus on rigid-price goods (core inflation)
  - take into account flexible-price energy (headline inflation, input prices)

- if one price is directly demand-relevant, choice of price index matters
Appendix
Related literature & our contribution

- **input factor scarcity** [e.g. Balleer and Noeller, 2023; Boehm and Pandalai-Nayar, 2022; Comin et al., 2023; Kuhn and George, 2019; Lorenzoni and Werning, 2023; Lucas and Prescott, 1974 and many others]
  - **contribution**: non-fundamental fluctuations & distributional effects of scarcity

- **energy and the macroeconomy** [e.g. Auclert et al., 2023; Blanchard and Galí, 2009; Datta et al., 2021; Känzig, 2021; Nakov and Pescatori, 2009; Olivi et al., 2022; Pieroni, 2023 and many others]
  - **contribution**: scarce energy supply can generate self-fulfilling loops

- **failure of Taylor principle** [e.g. Ascari and Ropele, 2009; Bilbiie, 2008; Branch and McGough, 2009; Galí et al., 2004; Holden, 2022; Ilabaca and Milani, 2021 and many others]
  - **contribution**: novel mechanism through imported energy shortages

- **best monetary policy** [e.g. Airaudo and Zanna, 2012; Aoki, 2001; Bodenstein et al., 2008; Carlstrom et al., 2006; Eusepi et al., 2011; Rubbo, 2022 and many others]
  - **contribution** (i): choice of price index matters for determinacy
  - **contribution** (ii): better not “see through shocks”
the following slides contain all details of the model, in particular:

- households
- firms
- fiscal policy & monetary policy
- foreign economy
- markets
Appendix: Model: Households – decision problem

• maximize lifetime utility $\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ C_{i,t}^{1-\sigma} - \sigma N_{i,t}^{1+\varphi} \right] \right\}$

• by choosing
  • energy and goods consumption, $C_{i,E,t}$ and $C_{i,G,t}$,
  • hours worked, $N_{i,t}$,
  • savers: risk-free nominal domestic-currency bond holdings, $B_{i,t}$,

• subject to
  • period budget constraint,
  • consumption aggregator $C_{i,t} = \left[ \gamma \frac{1}{\eta} \left( C_{i,E,t} - \bar{e} \right) \frac{n-1}{\eta} + (1 - \gamma) \frac{1}{\eta} C_{i,G,t} \right] \frac{n}{n-1}$
    • $\gamma$: share of energy in consumption
    • $\eta$: willingness to substitute goods and energy
    • $\bar{e}$: subsistence level of energy
Appendix: Model: Households – period budget constraints

• spenders’, $H$, and savers’, $S$, budgets:

\[
P_{c,E,t} C_{H,E,t} + P_{G,t} C_{H,G,t} = W_t N_{H,t} + P_t T_{H,t}
\]

\[
\frac{B_t}{1 - \lambda} + P_{c,E,t} C_{S,E,t} + P_{G,t} C_{S,G,t} = W_t N_{S,t} + P_t T_{S,t} + \frac{R_{t-1} B_{t-1}}{1 - \lambda}
\]

• energy consumption, $C_{i,E,t}$, at price $P_{c,E,t}$ (potentially subsidized, see below)
• goods consumption, $C_{i,G,t}$, at price $P_{G,t}$
• hours worked, $N_{i,t}$, at nominal wage $W_t$
• lump-sum net transfers, $T_{i,t}$, see below
• **savers**: risk-free nominal bond holdings, $B_{i,t}$, at nominal return $R_t$
• consumption allocation:

\[ C_{i,E,t} - \bar{e} = \gamma \left( \frac{P_{E,t}^c}{P_t} \right)^{-\eta} C_{i,t} \quad \text{and} \quad C_{i,G,t} = (1 - \gamma) \left( \frac{P_{G,t}}{P_t} \right)^{-\eta} C_{i,t} \]

with marginal price index \( P_t = \left[ \gamma(P_{E,t}^c)^{1-\eta} + (1 - \gamma)(P_{G,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \)

• labor supply decision: \( W_t / P_t = \chi C_{i,t}^\sigma N_{i,t}^{i^\sigma} \)

• savers’ intertemporal consumption decision: \( C_{S,t}^{\neg\sigma} = \mathbb{E}_t \left[ \beta C_{S,t+1}^{\neg\sigma} R_t / \Pi_{t+1} \right] \)
Appendix: Model: Firms – aggregate production function

- typical New Keynesian structure with energy and labor as inputs
  - differentiated goods, demand elasticity $\varepsilon > 1$, Rotemberg adjustment costs

- aggregate production function: $Y_{G,t} = \left[ \alpha E_t^{\frac{\theta - 1}{\theta}} + (1 - \alpha) N_t^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}$
  - $\alpha$: share of energy
  - $\theta$: elasticity of substitution between energy and labor

- firms’ real profits, with sales subsidy & potentially subsidized energy price:

$$D_t = (1 + \tau^y) \frac{P_{G,t}}{P_t} Y_{G,t} - \frac{W_t}{P_t} N_t - \frac{P_{E, t}^f}{P_t} E_t - \frac{\psi}{2} \frac{P_{G, t}}{P_t} Y_{G, t} (\Pi_{G, t} - 1)^2$$
• non-linear PPI Phillips curve with savers’ stochastic discount factor for profits

\[ \psi \Pi_G(t)(\Pi_G, t - 1) = (1 + \tau^y)(1 - \varepsilon) + \varepsilon \Lambda_t \left( \frac{P_G, t}{P_t} \right)^{-1} \]

\[ + \psi \mathbb{E}_t \left[ \beta \left( \frac{C_{S,t+1}}{C_{S,t}} \right)^{-\sigma} \Pi_G, t+1(\Pi_G, t+1 - 1) \frac{Y_G, t+1}{Y_G, t} \frac{P_E, t+1/P_{t+1}}{P_G, t/P_t} \right] \]

• optimal factor input shares: \( W_t/P_E^f, t = \frac{1-\alpha}{\alpha} (E_t/N_t)^{1/\theta} \)

• real marginal costs: \( \Lambda_t = \left[ \alpha^\theta \left( P_E^f, t/P_t \right)^{1-\theta} + (1 - \alpha)^\theta (W_t/P_t)^{1-\theta} \right]^{1/1-\theta} \)
• energy-price subsidies for households and firms, $k \in \{c, f\}$:

$$\log(P_{E,t}^k / P_t) - \log(P_E / P) = (1 - \tau_E^k) \left[ \log(P_{E,t} / P_t) - \log(P_E / P) \right]$$

where $P_{E,t}$ denotes the wholesale energy price and $\tau_E^k$ is the subsidy

• government budget constraint:

$$P_tD_t = (P_{E,t} - P_{E,t}^c)C_{E,t} + (P_{E,t} - P_{E,t}^f)E_t + \lambda P_t T_{H,t} + (1 - \lambda) P_t T_{S,t} + \tau^Y P_{G,t} Y_{G,t}$$

revenues: firms’ profits; expenditures: energy-price & sales subsidies, net transfers

  • transfers to hand-to-mouth households: $P_t T_{H,t} = \nu \left( P_tD_t - \tau^Y P_{G,t} Y_{G,t} \right)$
  • transfers to savers, $T_{S,t}$, balance the budget
monetary policy controls the gross nominal interest rate \( R_t \)

- baseline: Taylor rule responds to core inflation, i.e., \( R_t/R = (\Pi_{G,t})^{\phi_\Pi} \), with \( \phi_\Pi = 1.5 \)
- later: respond to other concepts of “inflation” and/or output etc.

"Taylor principle", extension for multi-sector models

\( \phi_\Pi > 1 \) ensure a unique bounded equilibrium, irrespective of what inflation index the central bank responds to (Carlstrom et al., 2006).

Note: household heterogeneity may shift the cutoff away from unity (Bilbiie, 2021).
Appendix: Model: Energy supply and international trade – scarce energy

- energy is supplied and owned by Foreign
  - quantity of energy, $\xi_E$, is fixed
  - quantity is sold in Home at the currently-prevailing, wholesale price of energy, $P_{E,t}$
  - energy price is flexible and endogenous to demand conditions in Home
Appendix: Model: Energy supply and international trade – scarce energy

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  - quantity of energy, $\xi_E$, is fixed
  - quantity is sold in Home at the currently-prevailing, wholesale price of energy, $P_{E,t}$
  - energy price is flexible and endogenous to demand conditions in Home

- Foreign can accumulate net foreign assets out of energy revenues
  - Foreign’s budget: $P_{G,t}X_{G,t} - [B_t - R_{t-1}B_{t-1}] = P_{E,t}\xi_E$
  - Foreign’s energy revenues, in real terms: $Y^*_t = P_{E,t}/P_{G,t} \times \xi_E$
  - Foreign’s export demand:
    \[
    \log\left(\frac{X_{G,t}}{X_G}\right) = \mu_{F,1} \log\left(\frac{Y^*_t}{Y^*}\right) - \mu_{F,2} \frac{B_{t-1}/P_{t-1}}{Y^*}
    \]
    - $\mu_{F,1}$: Foreign’s marginal propensity to demand exports out of energy revenues
    - $\mu_{F,2}$: Foreign’s marginal propensity to consume out of savings
Appendix: Model: Market clearing – four markets

- bond market: domestic savings equal foreign debt
- labor market: firms’ labor demand equals households’ labor supply
- energy market: $\xi_E = (1 - \lambda)C_{S,E,t} + \lambda C_{H,E,t} + E_t$
- goods market: $Y_{G,t} = (1 - \lambda)C_{S,G,t} + \lambda C_{H,G,t} + X_{G,t}$
Appendix: Model: Market clearing – four markets

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- labor market: firms’ labor demand equals households’ labor supply
- energy market: \( \xi_E = (1 - \lambda)C_{S,E,t} + \lambda C_{H,E,t} + E_t \)
- goods market: \( Y_{G,t} = (1 - \lambda)C_{S,G,t} + \lambda C_{H,G,t} + X_{G,t} \)

- GDP definition: \( P_t GDP_t = P_{G,t}C_{G,t} + P_{E,t}C_{E,t} + P_{G,t}X_{G,t} - P_{E,t}\xi_E \)
  - equivalent to value-added definition: \( P_t GDP_t = P_{G,t}Y_{G,t} - P_{E,t}E_t \)
Appendix: Model: Firms – retailer’s problem

- unit mass of producers of differentiated goods, indexed by $j \in [0, 1]$
- retailer assembles differentiated goods into consumption good
- retailer’s production function: $Y_{G,t} = \left[ \int_0^1 y_{G,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^\frac{\varepsilon}{\varepsilon-1}$
- retailer’s demand function: $y_{G,t}(j) = \left( \frac{P_{G,t}(j)}{P_{G,t}} \right)^{-\varepsilon} Y_{G,t}$
- producer-price index: $P_{G,t} = \left[ \int_0^1 P_{G,t}(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$
Appendix: Model: Firms – intermediate firms’ problem

• differentiated good, $y_{G,t}(j)$ is produced using labor, $N_t(j)$, and energy, $E_t(j)$:

$$y_{G,t}(j) = \left[ \alpha E_t(j)^{\frac{\theta-1}{\theta}} + (1 - \alpha) N_t(j)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

• each firm sets its price subject to retailer’s demand, its production function, and price adjustment costs, by maximizing profits:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{S,t+k}}{C_{S,t}} \right)^{-\sigma} \frac{1}{P_{t+k}} \left[ P_{G,t+k}(j)(1 + \tau^y)y_{G,t+k}(j) - W_{t+k} N_{t+k}(j) ight] ight. \\
-\left. P_{E,t+k} E_{t+k}(j) - \psi \frac{P_{G,t+k}}{2} Y_{G,t+k} \left( \frac{P_{G,t+k}(j)}{P_{G,t+k-1}(j)} - 1 \right)^2 \right\}$$
Appendix: Model: Firms – symmetric equilibrium

• firms’ real profits, with sales subsidy & potentially subsidized energy price:

\[
D_t = (1 + \tau^y) \frac{P_{G,t}}{P_t} Y_{G,t} - \frac{W_t}{P_t} N_t - \frac{P_{E,t}^f}{P_t} E_t - \frac{\psi}{2} \frac{P_{G,t}}{P_t} Y_{G,t} (\Pi_{G,t} - 1)^2
\]

• non-linear PPI Phillips curve with savers’ stochastic discount factor for profits

\[
\psi \Pi_{G,t} (\Pi_{G,t} - 1) = (1 + \tau^y)(1 - \varepsilon) + \varepsilon \Lambda_t \left( \frac{P_{G,t}}{P_t} \right)^{-1}
\]

\[
+ \psi \mathbb{E}_t \left[ \beta \left( \frac{C_{S,t+1}}{C_{S,t}} \right)^{-\sigma} \Pi_{G,t+1} (\Pi_{G,t+1} - 1) \frac{Y_{G,t+1}}{Y_{G,t}} \frac{P_{G,t+1}/P_{t+1}}{P_{G,t}/P_t} \right]
\]

• optimal factor input shares: \( W_t/P_{E,t}^f = \frac{1-\alpha}{\alpha} (E_t/N_t)^{1/\theta} \)

• real marginal costs: \( \Lambda_t = \left[ \alpha^\theta \left( P_{E,t}^f/P_t \right)^{1-\theta} + (1 - \alpha)^\theta (W_t/P_t)^{1-\theta} \right]^{1/\theta} \)
Appendix: Model: Monetary and fiscal policy – inflation indices

Input-price inflation:

\[ \Pi_{nmc,t} = \frac{\left[ \alpha^\theta (P_{E,t}^f)^{1-\theta} + (1 - \alpha)^\theta (W_t)^{1-\theta} \right]^{1/(1-\theta)}}{\left[ \alpha^\theta (P_{E,t-1}^f)^{1-\theta} + (1 - \alpha)^\theta (W_{t-1})^{1-\theta} \right]^{1/(1-\theta)}} \]
the following slides contain all details of the paper-and-pencil intuition
Appendix: Paper-and-pencil – simplifying assumptions

• for the sake of tractability, allowing to derive a 3-equation representation:
  • energy is used in production only
  • balanced trade, i.e., no international financial trade
  • no energy price subsidies

• two versions:
  • representative-household version (RA)
  • heterogeneous-household version (HA)

• focus on representative-household version here
Appendix: Paper-and-pencil – RA three-equation representation

• Dynamic IS curve:

\[ \hat{Y}_{G,t} = E_t \hat{Y}_{G,t+1} - \frac{1}{\tilde{\sigma}} \left[ \hat{R}_t - E_t \hat{\Pi}_{G,t+1} \right] \quad \text{with} \quad \tilde{\sigma} := \frac{\sigma}{1-\alpha} \frac{1 - \alpha \left[ 1 + \varphi + \frac{1}{\theta} \right]}{1 - \alpha + \alpha \sigma} \]

• New Keynesian Phillips curve:

\[ \hat{\Pi}_{G,t} = \beta E_t \hat{\Pi}_{G,t+1} + \tilde{\kappa} \hat{Y}_{G,t} \quad \text{with} \quad \tilde{\kappa} := \frac{\epsilon}{\psi} \frac{\sigma + \varphi + \frac{\alpha}{\theta} (1 - \sigma)}{1 - \alpha + \sigma \alpha} \]

• Taylor rule:

\[ \hat{R}_t = \phi_n \hat{\Pi}_{G,t} \quad \text{with} \quad \phi_n \geq 0 \]
Proposition: Determinacy

The following two cases summarize the conditions for determinacy.

1. “Conventional.” If \( \tilde{\sigma} \) and \( \tilde{\kappa} \) have the same sign, there is local determinacy iff \( \phi_{\Pi} > 1 \).
2. “Unconventional.” If \( \tilde{\sigma} < 0 \) and \( \tilde{\kappa} > 0 \), there is local determinacy iff 

\[
\phi_{\Pi} > \max \left( 1, -4 \frac{\tilde{\sigma}}{\tilde{\kappa}} - 1 \right).
\]

• inverted IS curve behind indeterminacy, as in Bilbiie (2021)’s closed economy
Appendix: Paper-and-pencil – interpretation for RA model

- room for self-fulfilling energy-price-activity feedback loop if
  - $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$, and
  - $|\tilde{\sigma}/\tilde{\kappa}|$ is sufficiently large

- this is true if sufficiently
  - unwilling to substitute intertemporally (high $\sigma$)
  - inelastic labor supply (high $\varphi$)
  - flat Phillips curve (low $\epsilon/\psi$)
  - important role of energy in costs (high $\alpha$)
  - hard to substitute energy with labor in production (low $\theta$)
Appendix: Paper-and-pencil – the role of heterogeneity

- HA version can be represented by the same three equations as the RA version, only mapping from structural parameters to reduced-form parameters $\tilde{\sigma}$ and $\tilde{\kappa}$ changes.

- Consider, for simplicity, $\sigma = 1$: $\tilde{\kappa} > 0$, indeterminacy can occur only if $\tilde{\sigma} < 0$.

- Risk of indeterminacy:
  - $\partial \tilde{\sigma} / \partial \alpha < 0$: a higher share of energy ($\alpha$) raises the risk of indeterminacy (as in RA version).
  - $\partial^2 \tilde{\sigma} / (\partial \alpha \partial \lambda) < 0$: the larger the share of hand-to-mouth households ($\lambda$), the more does the share of energy ($\alpha$) raise the risk of indeterminacy.
Appendix: Paper-and-pencil – sign of $\tilde{\kappa}$ (RA and HA)

$$\text{sgn}\tilde{\kappa} = \text{sgn} \frac{\epsilon}{\psi} \frac{\sigma + \varphi + \frac{\alpha}{\theta}(1 - \sigma)}{1 - \alpha + \sigma\alpha} = \text{sgn} \left( \sigma + \varphi + \frac{\alpha}{\theta}(1 - \sigma) \right)$$

- $\frac{\sigma + \varphi + \frac{\alpha}{\theta}(1 - \sigma)}{1 - \alpha + \sigma\alpha}$ is the elasticity of marginal costs with respect to output
- $\sigma + \varphi$ is standard wealth effect and effect of compensation for disutility of work on wages, would capture entire effect if wages and energy prices move in lock-step
- $\frac{\alpha}{\theta}(1 - \sigma)$ captures the excess effect of energy prices on marginal costs, matters if (i) large energy share in production, $\alpha$, or (ii) little substitutability, $1/\theta$
- $1 - \sigma$ captures two countervailing effects of excess sensitivity:
  - direct effect: higher output comes with higher marginal costs
  - indirect effect: given output, a rise in energy prices reduces households’ consumption (a larger share of output is consumed by foreign), wealth effect reduces wages and thus marginal costs (wealth effect increases in $\sigma$
- if $\frac{\alpha}{\theta}(1 - \sigma)$ is negative and large in absolute value, $\tilde{\kappa}$ inverts
Appendix: Paper-and-pencil – sign of $\tilde{\sigma}$ (RA)

$$\text{sgn} \tilde{\sigma} = \text{sgn} \frac{\sigma}{1 - \alpha} \frac{1 - \alpha [1 + \varphi + \frac{1}{\theta}]}{1 - \alpha + \alpha \sigma} = \text{sgn} \left( 1 - \alpha \left[ 1 + \varphi + \frac{1}{\theta} \right] \right)$$

- $1 - \alpha [1 + \varphi + \frac{1}{\theta}]$ reflects the comovement of aggregate consumption with output
- $\alpha$ measures the share of energy in production and thus the share of output exported
- with constant energy prices and linear production, $1 - \alpha$ would capture all effects
- $[1 + \varphi + \frac{1}{\theta}]$ captures disproportionate movements with output in input prices, if energy prices would move one-to-one with wages, $1 + \varphi$ would capture all effects; $1/\theta$ measures (again) the excess sensitivity of energy prices to output
- $\tilde{\sigma}$ inverts if energy is important ($\alpha$), labor supply is inelastic ($\varphi$), or energy is hard to substitute ($1/\theta$)
Appendix: Paper-and-pencil – signs of $\tilde{\sigma}$ and $\tilde{\kappa}$ (RA)

\[
\begin{align*}
\text{sgn} \tilde{\sigma} &= \text{sgn} \left( 1 - \alpha \left[ 1 + \varphi + \frac{1}{\theta} \right] \right) \implies \tilde{\sigma} > 0 \iff 1 - \frac{\alpha}{\theta} > \alpha (1 + \varphi) \\
\text{sgn} \tilde{\kappa} &= \text{sgn} \left( \sigma + \varphi + \frac{\alpha}{\theta} (1 - \sigma) \right) \implies \tilde{\kappa} > 0 \iff 1 - \frac{\alpha}{\theta} > -\frac{1}{\sigma} (\varphi + \frac{\alpha}{\theta})
\end{align*}
\]

- $\alpha(1 + \varphi) > 0$ and $-\frac{1}{\sigma} (\varphi + \frac{\alpha}{\theta}) < 0$
- hence, whenever $\tilde{\sigma} > 0$, also $\tilde{\kappa} > 0$
- for $\tilde{\sigma} < 0$, we can still have either $\tilde{\kappa} > 0$ or $\tilde{\kappa} < 0$
Corollary: Insufficiency of Taylor principle

Consider the same conditions as above. In addition, suppose that $\alpha = \theta$, meaning the weight of energy in production equals the elasticity of substitution between energy and labor. This implies that case 2) of the proposition is the relevant case (that is, $\tilde{\sigma} < 0$ and $\tilde{\kappa} > 0$).

We have the following result: An arbitrary response $\phi_\Pi > 1$ ensures determinacy if and only if the following inequality holds:

$$\frac{1}{2} \frac{\varepsilon/\psi}{\sigma} \frac{1 - \alpha}{\alpha} \geq 1.$$

If the above inequality is violated, determinacy requires a stronger response to inflation than suggested by the Taylor principle.
### Households

- **$\beta$** discount factor; 2% annualized real rate of interest
- **$\sigma$** inverse of IES; realistic IES of consumption of $\frac{1}{3}$
- **$\chi$** disutility of labour supply; normalize labor supply to unity; implies 0.713
- **$\varphi$** Frisch elasticity of labor supply of $\frac{1}{3}$; in line with range in literature
- **$\lambda$** share of hand-to-mouth households of 0.22; estimates by Slacalek et al. (2020)
- **$\eta$** elasticity of substitution energy/goods of 0.1; Bachmann et al. (2022)
- **$\gamma$** share of energy in consumption; 5% of GDP, see above and BDEW (2023)
- **$\bar{e}$** subsistence consumption; 25% of HH energy cons., Fried et al. (2022)

### Firms

- **$\varepsilon$** elasticity of substitution varieties; conventional 10% markup
- **$\psi$** price adjustment costs; match 0.1 slope of NKPC, implies 389
- **$\theta$** elasticity of substitution energy/labor of 0.1; Bachmann et al. (2022)
- **$\alpha$** production share of energy: 10% of GDP, see above and BDEW (2023)
Appendix: Calibration – full parameters (II)

Energy supply

$\mu_{F,1}$  MPC out of energy rev. of 0.25, similar as in Home

$\mu_{F,2}$  Foreign’s MPC out of savings of 0.02, stabilize net foreign assets

Government

$\tau^y$  production subsidy; no markup in steady state

$\nu$  no profit redistribution; savers receive profits and pay all taxes

$\phi_\Pi$  response to inflation of 1.5; standard value

$\tau^h_\bar{E}$  energy price subsidy of 33% for firms and households, in range of literature
### Appendix: Calibration – symmetric steady state

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td></td>
<td></td>
<td>Prices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>1.192</td>
<td>Consumption</td>
<td>$\Pi_G = \Pi$</td>
<td>1</td>
<td>Inflation</td>
</tr>
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<td>$C_E$</td>
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<td>Energy cons.</td>
<td>$P_E/P$</td>
<td>0.121</td>
<td>Real energy price</td>
</tr>
<tr>
<td>$C_G$</td>
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<td>Goods cons.</td>
<td>$P_G/P$</td>
<td>1.328</td>
<td>Real goods price</td>
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<tr>
<td>$N$</td>
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<td>Labor supply</td>
<td>$W/P$</td>
<td>1.207</td>
<td>Real wage</td>
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<tr>
<td>Production</td>
<td></td>
<td></td>
<td>$R$</td>
<td>1.005</td>
<td>Gross nom. rate</td>
</tr>
</tbody>
</table>

- symmetric steady state for both types of households
- energy shares of GDP as targeted
Appendix: Calibration – how to think about Foreign’s MPC?

- energy exporter is an emerging-market economy w/o sovereign wealth fund
- facts to keep in mind:
  - emerging-market economies have higher MPCs
  - financial trade with Russia, but also other energy exporters, is limited due to sanctions, implying a relatively higher MPC (closer to instant settlement)
  - current situation: are Russians in the middle of a severe crisis likely to save or spend?
  - MPC also governs the behavior of debt relative to the trade volume, how much would a country borrow to another country (in percent of trade volume)?
  - MPC out of energy revenues is likely to be higher than “normal” MPC (e.g. due to pro-cyclicality of government spending in energy exporting countries)!
Suppose non-fundamental beliefs of high prices for the scarce factor (energy)

- high marginal costs
- costs not fully passed on (nominal rigidities)
- depresses markups → redistribute to high MPC households
- aggregate demand rises unless monetary policy curbs domestic demand enough
- production rises and this requires energy
- energy price responds to demand conditions

⇒ validated
Suppose non-fundamental beliefs of high prices for the scarce factor (energy)

- high marginal costs
- costs not fully passed on (nominal rigidities)
- depresses markups $\rightarrow$ redistribute to high MPC households
- and redistribute to Foreign (lower markups and high energy price)
- external demand linked to terms of trade: higher external demand (MPC of Foreign)
- aggregate demand rises unless monetary policy curbs domestic demand enough
- production rises and this requires energy
- energy price responds to demand conditions

$\Rightarrow$ validated
· Bianchi and Nicolò (2021): approach to deal with indeterminacy in LRE models
  · augment original state space with a set of auxiliary exogenous equations to achieve the adequate number of explosive roots
  · the solution in the expanded state space is always determinate and identical to the indeterminate solution in the original state space
  · selection of equilibrium based on zero restriction: set correlation of the fundamental disturbances with the sunspot shocks to zero

· other approaches select other equilibrium but span the same set of equilibria
  · e.g. Lubik and Schorfheide (2003) minimize distance between IRFs of indeterminate and determinate solution at boundary of determinacy region
  · irrelevant for determinacy threshold, only matters for the precise shape of the IRFs which we just use to illustrate the mechanism that causes the indeterminacy
our case: up to one degree of indeterminacy \(\rightarrow\) add one auxiliary equation and sunspot shock \(\varepsilon_{\omega,t}\), linking (any) forecast error to the sunspot shock, for instance,

\[
\log \omega_t = \rho_\omega \log \omega_{t-1} + \varepsilon_{\omega,t} + \left( \log C_t - \mathbb{E}_{t-1}[\log C_t] \right)
\]

- determinate model: choose \(\rho_\omega < 1\), equation is irrelevant
  - sunspot does not affect equilibrium, just drives \(\omega_t\) which does not enter the economy
- indeterminate model: \(\rho_\omega > 1\), one additional explosive root
  - sunspot affects equilibrium, \(\omega_t\) must be zero, hence, sunspot shifts forecast error thereby affecting agents decisions, for instance, consumption is non-fundamentally higher than expected (variable does not matter here)
Appendix: Further policy alternatives

• core inflation plus **economic activity**: depends on the measure of activity
  • intuition: energy price feedback loop increases output but decreases GDP
  • for output, determinacy if $\phi_Y > 0.61$
  • targeting GDP exacerbates the feedback loop

• core inflation and **real rate** as intercept: determinacy if $\phi_\Pi > 1$
  • intuition: savers’ income drop puts upward pressure on $r_t$
  • real rate rule as in Holden (2022)
Appendix: Feedback loop in closed economy

- redistribution from low-MPC savers to higher-MPC foreigners & hand-to-mouth
- do we need the open economy dimension?

- consider: closed economy, hand-to-mouth $H$ and savers $S$

- Taylor principle more likely to break if $H$ consumption sufficiently procyclical
  - $H$’s consumption equals their income:
    - labor income + share in energy revenues + share in firms’ profits
  - energy income makes $H$’s consumption more procyclical, profit income less

- fundamentally, redistribution from low- to high-MPC households matters
References


References


