Household Portfolios, Monetary Policy and Asset Prices*

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Abstract

We study the role of the household portfolio rebalancing channel for the effects of monetary policy on redistribution and risk premia. Beyond income and substitution effects, monetary policy transmits through wealth effects from changes in equilibrium asset prices that are induced by endogenous portfolio rebalancing. We introduce a heterogeneous household life-cycle model, disciplined by Euro Area survey data, with multiple assets and combine it with an incomplete markets asset pricing framework. We find that, absent wealth effects, older cohorts reduce consumption as they face lower expected asset returns, while younger cohorts raise consumption as they can borrow more cheaply. Introducing wealth effects preserves the heterogeneity. We also find that asset risk premia rise because the risk compensation effect (need for more returns to hold more risk) dominates the risk tolerance effect (positive wealth effect on risky asset holdings), but flips sign without heterogeneity.

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I. Introduction

A household's portfolio generally consists of tradable (equity, housing, bonds) and non-tradable (labour income) assets used to make consumption plans. In making these plans, households face both aggregate risk, such as tradable asset returns, and uninsurable idiosyncratic risk like labour income shocks. Given these risks, variation in age and asset holdings, amongst other factors, can potentially generate differences in responses across households to common aggregate shocks.

In this paper, we analyse monetary policy shocks and their effect on household asset allocations and consumption plans. Household heterogeneity, in terms of age, wealth and *ex-ante* asset allocation, will notably affect the pass-through of monetary policy into consumption. In the paper we combine a life-cycle model with an asset pricing framework with heterogeneous agents and incomplete markets. This allows us to study the effect of monetary policy not only through the usual income and substitution effects, but also through an endogenous portfolio rebalancing effect and the subsequent wealth effect that results from the change in equilibrium prices of equity and housing.

We calibrate the model to Euro area countries, using microeconomic data from the Household Finance and Consumption Survey (HFCS) in order to shed light on how monetary policy shocks affect different countries in different ways. Empirical evidence shows significant variation across Euro area countries in terms of income profile, distribution of *ex-ante* asset allocations as well as demographics. These factors - as we are going to show in the paper - notably interact with the pass-through of monetary policy, leading to substantial heterogeneity in the aggregate response of different countries to the same monetary policy shocks.

We first develop a life-cycle model to compute the household's optimal portfolios. The model is a sequence of 4-year trading periods. In the model, the income process combines a deterministic age profile with aggregate shocks and a combination of idiosyncratic permanent and transitory shocks. Households of different ages enter the period with different holdings of nominal bonds, equity and housing. They observe their income realization, consume and trade in the asset market. Households can invest in bonds, equity or housing and can borrow subject to a collateral constraint, where the borrowing limit is proportional to the value of their housing. All assets are risky (in real terms), and we model their expected return and variance based on historical moments. Assets can be bought and sold without any frictions or transaction costs.

We then derive equilibrium prices for equity and housing as in Piazzesi and Schneider (2009) and Landvoigt et al. (2015). Based on the optimal policy of households, we can compute aggregate demand and finally compute the equilibrium prices for equity and housing that equate aggregate supply and hence clear the markets in the current period. We compute the initial price level for the assets. Expectations of future prices are exogenous (consistent with the returns data generating process) and therefore market clearing prices establish a 'temporary equilibrium' for the current period.

Household-level endowments and the quantity of aggregate supply are derived from the data, using the Household Finance and Consumption Survey (HFCS) and the Euro Area Economic Accounts (EEA). The HFCS is available for 2010 and 2014 for Euro area countries. We derive household asset allocations in 2010, using directly observed figures from the HFCS and indirect exposures measured through aggregate financial data from the EEA using the methodology as in Doepke and Schneider (2006) and Adam and Zhu (2015).

Combining our model with the data, we derive, for each cohort, the asset allocation and consumption decision. We will then compare our model-implied allocation with empirical moments from the 2014 survey to assess the ability of the model to match actual household decisions. Finally, we will evaluate the impact of monetary policy shocks. Within our framework, this is modeled as a reduction in the supply of bonds. In order to clear the bond market, this will, all else equal, necessitate a reduction in the expected return on bonds, consequently changing the demand for risky assets and so their market-clearing prices. We consider a 21bps shock reduction in the expected return on long-term bonds, modelled to capture the effects of QE on term premia. The consequent adjustment in market-implied asset prices will induce wealth effects as well as changes in expected returns on risky assets.

The overall effect on consumption depends on a combination of a) the positive wealth effect caused by movements in asset prices and b) changes in the savings margin induced by a reduction in expected returns and borrowing rates. The strength, and sometimes direction, of these two forces notably varies depending on a household's age.. We find that, absent wealth effects, older cohorts reduce consumption while younger cohorts increase their consumption slightly. Cohorts experience a negative income effect from declining expected returns, and so consume less. This can be offset by the benefits of cheaper borrowing rates, which only young cohorts benefit from in equilibrium.

Once we incorporate the positive wealth effects, consumption responses turn positive for all cohorts. In response to the monetary policy shock, there is a reallocation away from bonds and towards risky assets. Young cohorts raise borrowing in order to increase leveraged investments in housing, while older rich cohorts reallocate away from bond savings. This increases the demand for risky assets, causing market-clearing asset prices to rise. As older cohorts hold a disproportionate share of risky assets ex ante, they experience a stronger positive wealth effect on consumption. This wealth effect dominates such that, overall, consumption rises for all cohorts. We hence conclude that considering wealth effects is very important for both the sign and scale of responses. Nevertheless, the heterogeneity in responses is unaffected by the wealth effect.

Interestingly, the model conveys implications of monetary policy shocks on asset risk premia. We find that, in response to a 21bps reduction in expected returns on bonds, the risk premia on housing and equity rise by 7.5bps and 5bps, respectively. Intuitively, in response to a lower expected real return on bonds, households rebalance toward the risky assets, equity and housing, and away from bonds. Consequently, they now hold more risk in their portfolio. However, households need to be compensated for bearing more risk in equilibrium. This is called the *risk compensation* effect. On the other hand, as asset prices rise, households have a higher tolerance for holding more risky assets, termed the *risk tolerance* effect. As the risk premium rises, here the risk compensation effect dominates the risk tolerance effect.

A. Literature review

Our paper relates to the literature along a variety of dimensions. Firstly, there has been a large body of work establishing theoretical channels of quantitative easing (QE) on the real economy, for example: the bank lending channel (Gertler and Karadi (2013)), the collateral channel (De Fiore et al. (2021)) and the liquidity premium channel (Bigio and Sannikov (2023)). This paper instead focuses on the households portfolio rebalancing channel of QE that induces heterogeneous changes in asset prices, wealth and consumption.

Secondly, we relate to the literature on the interaction between monetary policy (and in particular Quantitative Easing) and asset prices. Krishnamurthy and Vissing-Jorgensen (2011) and Gagnon et al. (2011) estimate the effect of quantitative easing on asset prices. A large number of papers also estimate the effect of monetary policy on asset prices, starting from the seminal paper by Bernanke and Kuttner (2005), to Rigobon and Sack (2004) and Gurkaynak et al. (2004). Our model is closely related to Piazzesi and Schneider (2009). Piazzesi and Schneider (2009) study the effect of inflation expectations on household asset allocations and asset prices. Koijen et al. (2021) take a demand-system approach to understand the effect of the ECB's (Public Sector Purchase Program) on government bond yields by estimating demand functions of different investors. This paper instead focuses on the demand of the household sector, where asset demand functions are instead derived from a calibrated household life cycle model.

Thirdly, we relate to the literature on monetary policy with heterogeneous agents and the redistributive effect of monetary policy. Lenza and Slacalek (2023) are closely related to our work from an empirical standpoint. They evaluate the impact of quantitative easing on the income and wealth of individual Euro Area households. Cloyne et al. (2018) show the importance of homeowners with mortgages for the aggregate consumption response to monetary policy shocks. From a theoretical point of view, Auclert (2017), Kaplan et al. (2018) and McKay et al. (2016) all assess the role of household heterogeneity in the transmission of monetary policy. In our paper we develop a life-cycle model highlighting the role of demographics, as in Wong et al. (2016). Wong et al. (2016), as well as Greenwald (2017), focus on the role of the mortgage market and housing in generating heterogeneous responses to monetary policy shocks.

The paper is also related to the literature that empirically estimates household-level exposures to shocks, as in Doepke and Schneider (2006), Adam and Tzamourani (2016) and Adam and Zhu (2015). Carroll et al. (2014) estimate marginal propensities to consume for households, taking into account the distribution of wealth, also using the HFCS survey.

Our model, generating asset pricing appreciations following a monetary policy shock, is able to assess in a coherent framework the role of wealth effects on consumption. A paper by Berger et al. (2017) analyzes the effect of house prices on consumption in a heterogenous agents framework. The wealth effects on consumption have also been largely explored in previous work, such as Case et al. (2005) and Campbell and Cocco (2007).

Finally, one of the main focuses of the paper is studying the interaction between monetary policy and asset allocation. Related papers are Adrian and Shin (2010), Borio and Zhu (2012), and Hau and Lai (2016). Moreover, a strand of literature analyzes the effect of monetary policy on leverage and risk taking. Examples are Gambacorta (2009) and Altunbas et al. (2010).

II. Data

We estimate how households of different ages and wealth levels allocate their portfolios across different asset classes. For this purpose, we use microeconomic information from the Household Finance and Consumption Survey (HFCS) in combination with aggregate data from the Euro Area Economic Accounts (EEA).

When estimating the portfolio allocation, we consider three asset classes: equity, housing and bonds. We define the investment in bonds as the total investment in fixed income assets (i.e. deposits and bond securities) minus debt (i.e. mortgage and non-mortgage debt). Therefore, according to our definition, a household which is a net borrower has a negative share in the bond's asset class.

A. Euro Area Accounts

The European sector accounts provide a comprehensive and comparable overview of the European economy. They record transactions between economic agents grouped by sector. Stocks of assets and liabilities are recorded in balance sheets. The institutional sectors include, amongst others, households, non-financial corporations, financial institutions, insurance corporations and pension funds.

B. HFCS

The HFCS collects household-level data on a wide range of variables, with a particular focus on the composition of household savings/borrowings. It contains 20 EU member states with a sample of more than 84,000 households. The primary advantage of this survey is that data collection is harmonized across countries, allowing direct comparison of results across countries. In our paper, we use the first two waves, in 2010 and 2014, which are repeated cross-sections of similar size.[1]

The HFCS provides us with data on household-level consumption and income as well as portfolio allocations. We extract data on holdings of equity, housing and net bonds, as well as investments in pension and mutual funds.

Equity investment includes the value of self-employment businesses, shares of private companies as well as publicly traded shares. We define bond investment as the net exposure to fixed income assets: deposit and bond securities minus mortgage and non-mortgage debt. Housing investment includes the value of a household's main residence as well as other real estate properties. Investment in intermediaries is the sum of investment in pension funds and mutual funds.[2] Appendix E details the variables we use to construct these series.

Figure 1 shows the portfolio allocation for Euro area households. We divide households by age into 4-year cohorts. The youngest cohort is composed of households ranging from 20 to 24 years old. The oldest cohort we consider includes households aged 84 to 88 years old. Investment in equity, housing and intermediaries are hump-shaped. Investment in equity ranges from around 10,000 euros for the younger and older cohorts to 40,000 for the middle aged cohorts. Housing investment is clearly larger and ranges from 40,000 to 200,000 euros. Net Bond is defined as fixed income investment minus debt. We notice that younger cohorts tend to be net borrowers (borrowing up to 30,000 euros) while older cohorts tend to be net investors, investing up to 20,000 euros. In nominal terms, households of age 44 - 48 hold the largest amount of debt.

Although the survey includes data on household-level investment in intermediaries, it does not obviously provide any information on how these funds are invested in different asset classes. For this reason, we combine microeconomic data from the survey with

[1] Many countries also have panel data available, allowing for longitudinal studies.

[2] We only consider investment in mutual funds and non-defined benefits pension funds. We exclude defined benefit pension funds and insurance funds because the pay-out from these investments do not depend on the portfolio allocation, given that they are fixed payments.

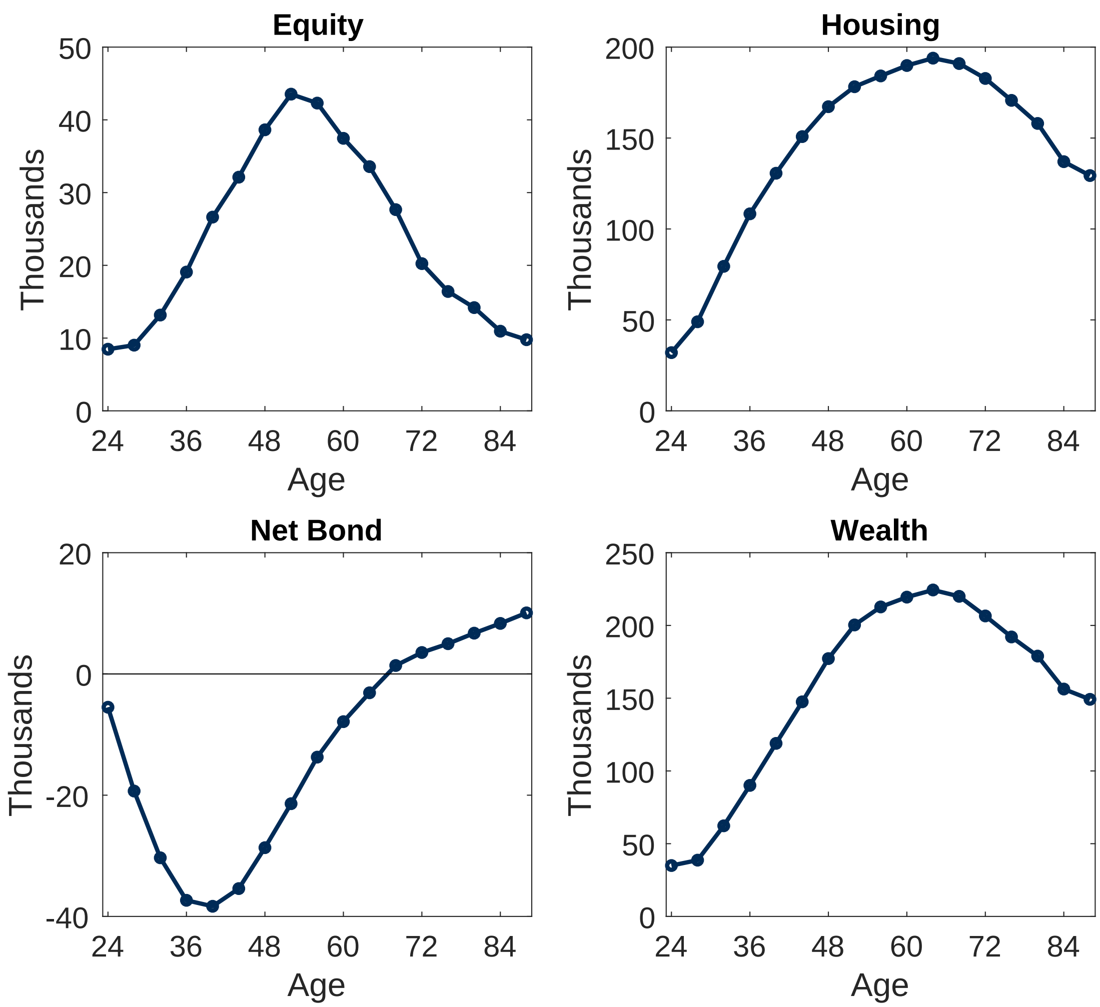

Figure 1. Portfolio allocation for Euro Area households in 2014 across equity, housing and net bond, in thousands of Euros, along with total wealth in the bottom right quadrant. Households are divided by age into 4-year cohorts, where we plot the average holdings at the cohort level.

aggregate data from the EEA. In this way we are able to *look-through* the investment in intermediaries and have a complete pictures of households' exposure to different assets.

Before detailing the *look-through* exercise, we provide information on the comparability of the HCFS and EEA data by matching household data from these two data sources. In this way, we are able to shed light on the differences and similarities between the two data sets.

C. *Aggregate: EEA vs HFCS*

We expect aggregate information provided by the EEA and HFCS to be comparable. However, we notice large discrepancies once we aggregate HFCS and compare them to the EEA figures. Table I compares the HFCS and EEA aggregates. The table shows that the numbers reported by the HFCS are significantly smaller than the EEA's. Kavonius et al. (2013) already pointed out that there are major differences between the HFCS aggregated data and the EEA data, and discuss possible explanations for this gap. Our numbers are broadly in line with what they found in their aggregation exercise. While we acknowledge discrepancies are not negligible, we believe the information from the EEA is valuable in the assessment of indirect portfolios of households. We therefore proceed with the consolidation exercise, where we combine the two data sets and compute a comprehensive measure of household portfolios.

Table I. Comparison of Aggregates: HFCS vs. EEA

	HFCS	EEA	HFCS/EEA
Deposits	2928.81	7074.23	0.41
Bonds	304.42	1070.82	0.28
Listed Equity	462.22	871.90	0.53
Unlisted Equity	4061.42	3134.61	1.30
Loans	4232.04	6115.71	0.69

Notes: Comparisons of aggregate data from HFCS and EEA for the household sector as of 2014. Data are in EURbn for columns 1 and 2.

D. *Consolidation*

As discussed, the HFCS provides information on household-level investment in pension funds and mutual funds. We then use data from the EEA to understand how these funds allocate their portfolio. The EEA does not provide any information on investment in housing and therefore we only consider investments in equity and bonds.

We firstly calculate portfolio allocations for the mutual funds (defined in EEA as Non-MMF Investment Funds). Portfolio shares are reported in Table II. Data from the EEA shows that mutual funds invest 57% of their net wealth in bonds and 44% in equity. They borrow only 1% of their net wealth.

Moving to pension funds (defined in EEA as Insurance Corporations and Pension Funds), we notice that they not only invest in equity and bonds but also in Investment Fund Shares. The Investment Fund Shares entails an indirect exposure whose allocation we assume to be equal to the portfolio shares estimated for the mutual funds. We then also consider the indirect exposure. Pension funds invest 76% of their net wealth in equity and 30% in bonds. They borrow 7% of their net wealth.

Table II. Portfolio Allocation of Intermediaries

	Mutual Funds	Pension
Equity	0.44	0.31
Bond	0.57	0.77
Debt	0.01	0.07

Notes: Portfolio allocation by intermediaries across Equities, Bonds and Debt. The table conveys portfolio shares for Mutual Funds and Pension Funds.

With this data to hand, we are able to compute total household exposures.

E. Household Portfolio Allocation

We then consolidate the HFCS data with the EEA data. As mentioned earlier, we observe from the HFCS the investment in pension funds and mutual funds. We assign that investment pro-rata to the three asset classes of interest according to the share we calculated from the EEA. For example, if an Italian household in 2014 holds 10,000 euros in mutual funds, we assume that his indirect exposure to equity is of 4,400 euros (44% of 10,000) and we add this amount to the direct investment in equity we observe from the HFCS.

We have now calculated the total exposure for each household in our survey. We then calculate the portfolio shares allocated to the three asset classes. Figure 2 shows the average shares of household portfolio invested in each asset by households in different cohorts. For equity, housing and net bonds we show the share as a percentage of net wealth (housing+equity+net bonds). The bottom-right panel shows the evolution of net-wealth across cohort ages.

Net wealth has the well known hump shape, increasing until the age of 52-60 and decreasing thereafter. Housing investment notably dominates all other asset classes. Equity investment ranges from 20% for the younger cohorts to 10% for the older cohorts. Housing investment ranges from 80% for young households, to peak of 110% for households of age 36. Finally, we notice a large difference in the net bond shares. Young households tend to be net borrowers, borrowing up to 30% of their net wealth. Older households instead tend to decrease the riskiness of their portfolio, holding a positive portfolio share in bonds and a lower equity investment.

We observe that portfolio allocations not only change across cohorts but also across different wealth levels. For this reason we split the cohort into three sub-cohorts according

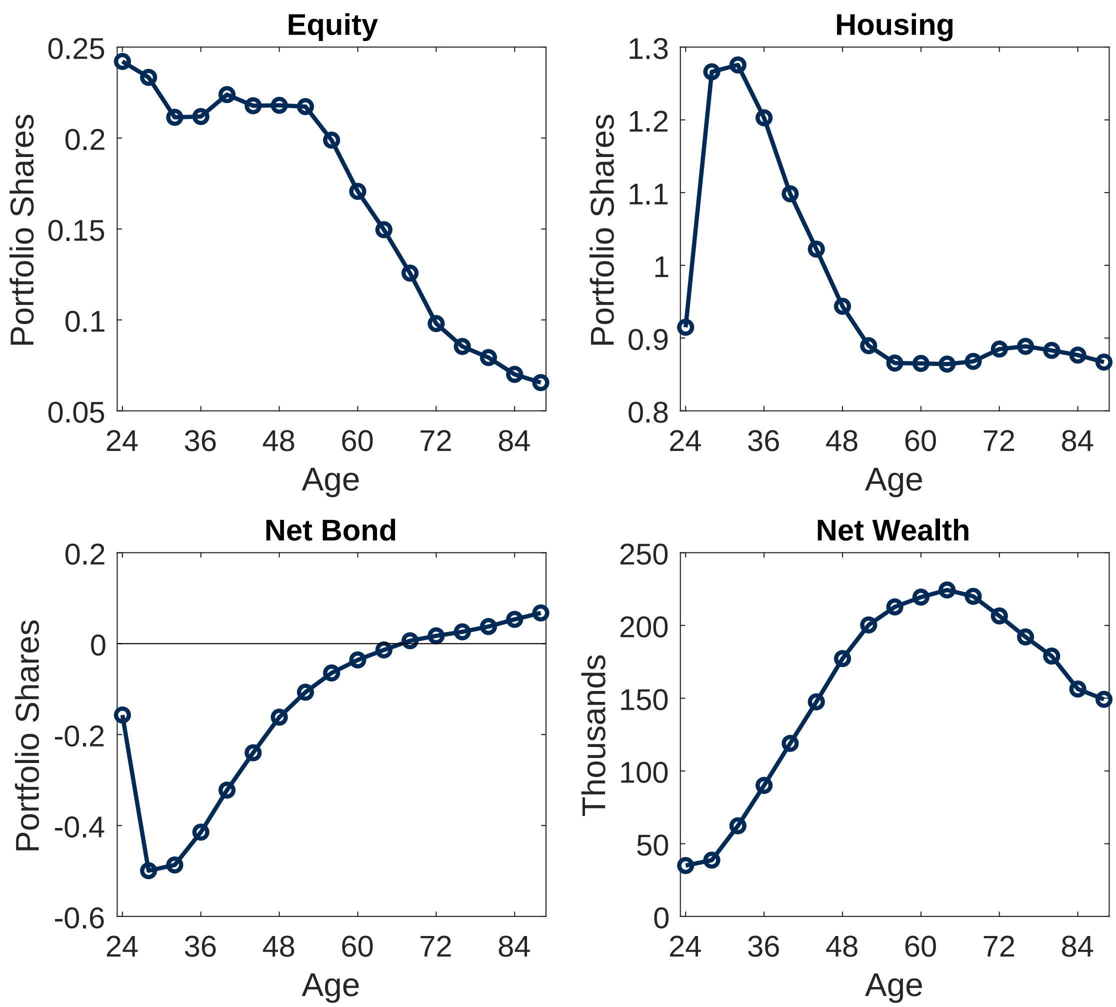

Figure 2. Portfolio shares for Euro Area households in 2014 across equity, housing and net bond, along with total wealth in thousands of Euros in the bottom right quadrant. Households are divided by age into 4-year cohorts, where we plot the average holdings at the cohort level.

to their wealth level. We subdivide cohorts into the bottom 40, the medium 40 and the top 20 percentile. For each of these age-wealth level combinations we calculate portfolio shares. These are shown in Appendix A in Figures 13 - 15.

F. Rest of the Economy

As we will discuss in the model section, we also have a Rest of the Economy (RoE) sector. We use the EEA and national tables data in order to estimate the supply of assets provided by the RoE.

III. Model

A. Environment

A.1. Preferences

Time t is discrete, where each period corresponds to 4 years. Households begin at the age of 24, and live up to a maximum of 18 (T) periods. The probability that a household aged a is alive at time $(t+1)$ conditional on being alive at time t is p_a.

Households have Epstein-Zin utility functions defined over the single homogeneous consumption good, C_t. Let W_t denote wealth at time t. Household preferences can then be written as:

$$V_{a,t} = \left[C_t^{1-\frac{1}{\sigma}} + \beta E_t \left[p_a V_{a+1,t+1}^{1-\gamma} + (1-p_a)\phi_B^{1-\gamma} W_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \tag{1}$$

where γ is the coefficient of relative risk aversion, σ is the elasticity of intertemporal substitution, and ϕ_B controls the strength of the bequest motive. Given the presence of a bequest motive, the terminal condition is:

$$V_{T,t} \equiv \phi_B W_t. \tag{2}$$

A.2. Labour Income Process

Following what is standard in the literature, the labour income process for household i is given by:

$$Y_{i.t} = G_t P_{i,t} U_{i,t}, \tag{3}$$

$$P_{i.t} = \exp(f(a)) P_{i,t-1} N_{i,t}, \tag{4}$$

$$G_t = \exp(\mu^G)\exp(\epsilon_t^G) G_{t-1}, \tag{5}$$

where $f(a)$ is a deterministic function of household age a, G_t is GDP per capita, $P_{i,t}$ is a permanent component with innovation $N_{i,t}$, and $U_{i,t}$ is a transitory component. We assume that $\ln U_{i,t}$ and $\ln N_{i,t}$ are iid with mean $\{-0.5 * \sigma_u^2, -0.5 * \sigma_n^2\}$ and variances σ_u^2 and σ_n^2, respectively. ϵ_t^G is an iid normal variable with mean $-0.5 * \sigma_G^2$ and variance σ_G^2. Thus, G_t is a random walk with deterministic drift $\exp(\mu^G)$.

A.3. Financial Assets

There exists three financial assets each period: bonds, equity and housing. Bonds pay a real return:

$$R^B_{t+1} = \frac{1 + i_t}{\Pi_{t+1}}, \tag{6}$$

where i_t is the nominal return and Π_{t+1} is the inflation rate. Households can short bonds (i.e. get a mortgage) by paying a spread ι in addition to the real return on bonds.

Equity and housing are single-period lived trees with aggregate dividends at time $t+1$ of D^E_{t+1} and D^H_{t+1} and have prices at time t of p^E_t and p^H_t. The gross returns on these assets are therefore defined as:

$$R^E_{t+1} = D^E_{t+1}/p^E_t, \tag{7}$$

$$R^H_{t+1} = D^H_{t+1}/p^H_t. \tag{8}$$

A.4. Wealth Accumulation

We define wealth (W) as the resources that are available for consumption and saving. Household i's next period wealth is given by:

$$W_{i,t+1} = \theta^E_{i,t} D^E_{t+1} + \theta^H_{i,t} D^H_{i,t+1} + B^+_{i,t} R^B_{t+1} - B^-_{i,t}\left(R^B_{t+1} + \iota\right) + Y_{i,t+1}, \tag{9}$$

where $\theta^E_{i,t}$ and $\theta^H_{i,t}$ denote equity and housing shares, respectively, at time t, $B^+_{i,t}$ is the holdings of one period bonds, and $B^-_{i,t}$ is holdings of one-period debt. ι is the spread faced by households who want to borrow.

Since the households must allocate wealth between consumption and savings, the budget constraint is:

$$W_{i,t} = C_{i,t} + p^E_t \theta^E_{i,t} + p^H_t \theta^H_{i,t} + B^+_{i,t} - B^-_{i,t}. \tag{10}$$

Holdings of assets are restricted to being non-negative:

$$\theta^{E}_{i,t} \geq 0, \tag{11}$$

$$\theta^{E}_{i,t} \geq 0, \tag{12}$$

$$B^{+}_{i,t} \geq 0, \tag{13}$$

$$B^{-}_{i,t} \geq 0. \tag{14}$$

Finally, household-level borrowing is subject to a collateral constraint:

$$B^{-}_{i,t} \leq \phi p^{H}_{t} \theta^{H}_{i,t}, \tag{15}$$

where housing holdings is the sole source of collateral, and ϕ is the associated Loan-to-Value (LTV) ratio limit.

A.5. Optimization Problem

The complete optimization problem is then:

$$\max_{\{\theta^{E}_{i,t}, \theta^{H}_{i,t}, B^{+}_{i,t}, B^{-}_{i,t}, C_{i,t}\}_{t=0}^{T-a}} E\left[V_{a,0}\right],$$

where a_i is the age of household i and $V_{a,0}$ is given by Equations 1 - 2 and is subject to the constraints given by Equations 9 - 15 and to the stochastic labour income process given by Equations 3 - 5.

A.6. Central Bank

The central bank chooses the nominal interest rate i_t. The central bank provides B^{CB}_{t} units of bonds to accommodate household sector demand, under the set nominal rate i_t:

$$B^{CB}_{t} = \sum_{i} B^{+}_{i,t} - \sum_{i} B^{-}_{i,t}. \tag{16}$$

This is consistent with current ECB monetary policy operations whereby the central bank fixes the policy rate and provides liquidity through tender operations based on the principle of full allotment.

A.7. Rest of Economy (RoE)

We introduce another sector, RoE, that exogenously supplies assets to the household sector. The RoE is not explicitly modelled, but instead exogenously supplies new assets and consumes or provides goods to households. At time t, the RoE supplies one unit of housing and one unit of equity. The RoE can consume or provide goods to the household sector. We define the net consumption as C^{RoE}; its value is negative in case the RoE is a net supplier of goods. In order to determine C_t^{RoE}, we assume that the RoE consumes or provides goods to the household sector in order to accommodate any discrepancy between households' aggregate income and aggregate consumption. This will allow the household sector to increase (or decrease) their savings in a way that is usually ignored in an endowment economy.

B. *Temporary Equilibrium at Time 0*

The economy starts at time 0. Under our equilibrium concept, asset prices in period 0 are set such that asset markets clear in period 0, taking as given household expectations on future income and future asset dividends $\{D_{t+1}^E, D_{t+1}^H\}_{t=0}^{\infty}$ and prices $\{p_{t+1}^E, p_{t+1}^H\}_{t=0}^{\infty}$ as well as future nominal interest rates and inflation: $\{i_{t+1}, \Pi_{t+1}\}_{t=0}^{\infty}$.

An **equilibrium** for time 0 is a vector of prices $p_0 \equiv (p_0^H, p_0^E)$, a supply of bonds B_0^{CB}, and period 0 choices $A_{i,0} = \{\theta_{i,0}^E, \theta_{i,0}^H, B_{i,0}^+, B_{i.0}^-, C_{i,0}\}_i$ s.t.

1. $A_{i,0}$ is part of their optimal plan given age, endowment, beliefs and the nominal rate i_0;

2. All markets clear i.e.

$$\sum_i p_0^E \theta_{i,0}^E(a_0, W_{i,0}, P_{i,0}, p_0, i_0) = p_0^E, \quad \textbf{(equity)} \quad (17)$$

$$\sum_i p_0^H \theta_{i,0}^H(a_0, W_{i,0}, P_{i,0}, p_0, i_0) = p_0^H, \quad \textbf{(housing)} \quad (18)$$

$$\sum_i B_{i,0}^+(a_0, W_{i,0}, P_{i,0}, p_0, i_0) - \sum_i B_{i,0}^-(a_0, W_{i,0}, P_{i,0}, p_0, i_0) = B_0^{CB}, \quad \textbf{(bonds)} \quad (19)$$

$$\sum_i C_{i,0}(a_0, W_{i,0}, P_{i,0}, p_0, i_0) + C_0^{ROE} = \sum_i Y_{i,0}. \quad \textbf{(goods)} \quad (20)$$

C. *Equivalent Formulation*

Lets define $E_{i,t} = p_t^E \theta_{i,t}^E$ and $H_{i,t} = p_t^H \theta_{i,t}^H$ as the amount of wealth household i invested in equity and housing, respectively. We can re-define Equation 9 as:

$$W_{i,t+1} = E_{i,t} R_{t+1}^E + H_{i,t} R_{t+1}^H + B_{i,t}^+ R_{t+1}^B - B_{i,t}^- \left(R_{t+1}^B + \iota\right) + Y_{i,t+1}, \quad (21)$$

where $R^B_{t+1}, R^E_{t+1}, R^H_{t+1}$ are the gross returns on the three assets. The budget constraint is then:

$$W_{i,t} = C_{i,t} + E_{i,t} + H_{i,t} + B^+_{i,t} - B^-_{i,t}. \tag{22}$$

Non-negativity constraints on holdings are now defined as:

$$E_{i,t} \geq 0, \tag{23}$$

$$H_{i,t} \geq 0, \tag{24}$$

$$B^+_{i,t} \geq 0, \tag{25}$$

$$B^-_{i,t} \geq 0. \tag{26}$$

Finally, the collateral constraint is now defined as:

$$B^-_{i,t} \leq \phi H_{i,t}. \tag{27}$$

Moreover, as households take initial prices as given, we can solve the equivalent problem:

$$\max_{\{E_{i,t}, H_{i,t}, B^+_{i,t}, B^-_{i,t}, C_{i,t}\}_{t=0}^{T-a}} E\left[V_{a,0}\right],$$

where $V_{a,0}$ is given by Equations 1 and 2 and is subject to the constraints given by Equations 21 - 27 and to the stochastic labour income process given by 3 - 5. The new formulation is consistent with usual portfolio problems where households take into account the distribution of expected returns.

D. Beliefs

We assume that households have expectations about future returns for housing and equity R^E_{t+1}, R^H_{t+1} as given by:

$$\log R^E_{t+1} = \mu^E_t + \epsilon^E_{t+1}, \tag{28}$$

$$\log R^H_{t+1} = \mu^H_t + \epsilon^H_{t+1}, \tag{29}$$

where $\epsilon^E_{t+1}, \epsilon^H_{t+1}$ are iid normal variables with mean zero and variances σ^2_E, σ^2_H, respectively. Inflation expectations are given by

$$\log \Pi_{t+1} = \mu_t^{\Pi} + \epsilon_{t+1}^{\Pi}. \tag{30}$$

Finally, using Equation 6, bonds return expectations are then given by:

$$\log R_{t+1}^B = \log(1+i_t) - \log(\Pi_{t+1}) = \mu_t^B + \epsilon_{t+1}^B, \tag{31}$$

where $\mu_t^B = \log(1+i_t) - \mu_t^{\Pi}$ and $\epsilon_{t+1}^B = -\epsilon_{t+1}^{\Pi}$ are iid normal variables with mean zero and variances $\sigma_B^2 = \sigma_{\Pi}^2$. The central bank can therefore directly affect the expected return on bonds by choosing i_t.

E. Monetary Policy Shock

At time 0^+, after households have chosen their allocation and made their consumption decision, the central bank can decide to change the nominal interest rate i_0 to a new rate:

$$\bar{i}_{0^+} = i_0 + \bar{v}_{0^+}, \tag{32}$$

where $\bar{v}_{0^+}$ is the monetary policy shock.

The shock is fully transitory and we assume that i) expected labor income is unchanged, ii) expectations about future dividends $\{D_{t+1}^E, D_{t+1}^H\}_{t=0}^{\infty}$, prices $\{p_{t+1}^E, p_{t+1}^H\}_{t=0}^{\infty}$ as well as future nominal interest rates and inflation $\{i_{t+1}, \Pi_{t+1}\}_{t=0}^{\infty}$ are unchanged. This set of assumptions makes sure that the problem from period $t=1$ onward is unchanged and hence the value functions at time $t=1$, $V_{a,1}$, for each cohort a are still valid.

After the shocks, households update their optimal plan $\bar{A}_{i,0^+} = \{\bar{\theta}_{i,0^+}^E, \bar{\theta}_{i,0^+}^H, \bar{B}_{i,0^+}^+, \bar{B}_{i,0^+}^-, \bar{C}_{i,0^+}\}_i$ given the new interest rate $\bar{i}_{0^+}$. The asset market opens and households can trade housing, equity and bonds. The new equilibrium after the shocks is a new vector of prices $(\bar{p}_{0^+}^H, \bar{p}_{0^+}^E)$ given the new optimal plans $\bar{A}_{i,0^+}$ and the condition that all markets clear. As expectations about future dividends are unchanged, new equilibrium prices at time $t=0$ imply new expected returns on risky assets, inducing portfolio rebalancing that is needed to clear asset markets after the shocks.

F. Wealth-Effect

Note that, after the monetary policy shock - and the subsequent asset price change - households experience a revaluation of their wealth. Household i at time 0 holds θ_0^E shares of equity and θ_0^H shares of housing. After the shock, at time 0^+, equilibrium prices of equity and housing have changed from p_0^E to $\bar{p}_0^E$ and from p_0^H to $\bar{p}_0^H$, respectively. This implies a change in wealth of:

$$\Delta \bar{W}_{i,0^+} = \left(\bar{p}_{0^+}^E - p_0^E\right)\theta_0^E + \left(\bar{p}_{0^+}^H - p_0^H\right)\theta_0^H. \tag{33}$$

We define that as the *wealth effect* of monetary policy. This also means that household i's new plan $\bar{A}_{i,0^+}$ has to be optimal given his new wealth $\bar{W}_{i,0^+}$.

G. Solving the Problem

Analytical solutions to this problem do not exist. Thus, we use a numerical solution method. The details are given in Appendix C, but here we provide the main idea. Firstly, we exploit how the value function can be normalized by $P_{i,t}$, which transforms the value function to one containing just a single state variable $w_{i,t} \equiv \frac{W_{i,t}}{P_{i,t}}$. We then solve for optimal policies, starting with the terminal period T, and then iterating backwards. Computational speed for deriving optimal policy is aided by evaluating first-order conditions at each candidate policy. This will then generate the following policy functions:

$$E_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right), \tag{34}$$

$$H_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right), \tag{35}$$

$$B^{+}_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right), \tag{36}$$

$$B^{-}_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right), \tag{37}$$

$$C_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right), \tag{38}$$

where a_i is the age of household i. Finally, we then define the level of savings, $S_{i,t}$, as:

$$S_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right) = E_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right) + H_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right) + B^{+}_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right) - B^{-}_{i,t}\left(a_i, W_{i,t}, P_{i,t}\right). \tag{39}$$

IV. Calibration

A. Asset Returns and Covariances

Our baseline calibration assumes that the mean and variance of returns are constant across time. We calibrate the mean (μ) and standard deviation σ in Equations 28 - 29 to match empirical moments estimated on historical real returns for bonds, equity and housing. Appendix D details the series we used for the estimation. The resulting estimates are detailed in Table III below.

Our choice on i_0 is the historical mean nominal return on bonds.

We include an idiosyncratic shock to the housing return, represented as a scaling factor, σ^{H}_{idio}, to aggregate housing variance, as is done in Piazzesi et al. (2007). We also scale equity return variances by σ^{E}_{idio} to reflect idiosyncratic variation in household-level equity returns.

Table III. Mean and standard deviation of real annualized returns

	μ	σ
Bond	-0.04	0.89
Housing	5.08	1.88
Equity	4.89	21.39

Notes: Mean (μ) and standard deviation (σ) of real annualized returns for bonds, housing and equity. Calculated using a sample from Q1 1996 to Q4 2018. See Appendix D for details.

B. *Labour Income Process*

The deterministic labour income profile $f(a)$ exhibits the familiar hump-shape of earnings over the life cycle. This is estimated from the HFCS by averaging household-level income at the cohort level.[3] Idiosyncratic income shock variances (σ_u^2, σ_n^2) are chosen to be $(7\%, 2\%)$, as estimated in Krueger et al. (2010)

C. *Initial Wealth*

We take initial wealth directly from the data. To evaluate initial wealth levels we aggregate households at the cohort level, where each cohort is of a 4-year range. The representative household of each cohort holds wealth equalling the average of what is observed in the data in the survey in 2014, equalling asset holdings plus consumption. This is in accordance with Equation 10.

We then assume that each household i in cohort a holds shares equal to the representative household of that cohort.

D. *Other Non-Calibrated Parameters*

Death probabilites are extracted from the European population survey data. We decide on an LTV ratio limit of $\phi = 0.9$ as there exists a very distinct bunching of households with a portfolio at this ratio. Finally, we calibrated $\beta = \exp(-0.01 * 4)$ i.e. a 1% annual discount rate.

E. *Calibrated Parameters*

We calibrate the remaining model parameters: $(\gamma, \sigma, \phi_B, \iota, \sigma^H_{\text{idio}}, \sigma^E_{\text{idio}})$. Recall that initial wealth is drawn from the 2014 wave of the HFCS. Given a parameter selection, it will

[3] For further details, see Appendix B

generate a variety of model-implied outcomes for asset holdings that can be compared to the 2014 wave of the HFCS. We decide to calibrate the parameters to match the following aggregate outcomes:

1. Wealth/GDP
2. Housing/GDP
3. Equity/GDP
4. Gross Lending/GDP
5. Gross Borrowing/GDP

Firstly, we want to ensure wealth levels in the model are consistent with the data. Secondly, we want to match aggregate portfolio shares, and the composition of net lending between gross lending and gross borrowing. The final two are important because a key form of heterogeneity across households relevant for responses to MPSs is being a borrower vs. a saver.

Note that, given exogenous expected returns, once we have market-clearing (p_0^E, p_0^H) (equivalent to aggregate value of equity and housing), we then back out implied dividend expectations. These dividend expectations will then be held constant in our comparative statics exercises.

The empirical outcomes are shown in the table below:

Table IV. Empirical Outcomes

	Wealth/GDP	Housing/GDP	Equity/GDP	Lending/GDP	Borrowing/GDP
data	1.76	1.11	0.20	0.02	−0.12

Each parameter plays a distinct role in matching these targets. The degree of risk aversion, γ, controls the split between bonds and the relatively riskier housing and equity assets. σ controls aggregate wealth via intertemporal substitution. $(\sigma^H_{\text{idio}}, \sigma^E_{\text{idio}})$ manage the level and portfolio split between housing and equity. sp helps target the split between savings and borrowings on aggregate. Finally, ϕ_B, helps to match the portfolio decision of older cohorts.

We constructed a grid of possible values for the parameters listed above. We then ran thousands of simulations (one for each combination of parameters) and selected the combination that best matched the targeted values. The calibrated parameters values are given in Table V below.

Table V. Calibrated Parameters

	γ	σ	σ^H_{idio}	σ^E_{idio}	sp	ϕ_{beq}
parameters	9.00	0.50	6.20	2.50	0.03	1.00

Table VI. Aggregate Outcomes: Model vs. Data

	Wealth/GDP	Housing/GDP	Equity/GDP	Lending/GDP	Borrowing/GDP
data	1.76	1.11	0.20	0.02	−0.12
model	1.76	1.11	0.18	0.03	−0.12

Table VI compares model-implied aggregates to the data.[4]

V. Baseline Results

With the calibrated parameters at hand, we firstly present and discuss the policy functions of households. Secondly, we evaluate the success of the model by comparing model-implied wealth and portfolio shares across cohorts to the data.

A. Policy Functions

We now lay out the policy functions of each household conditional on their age and wealth. Specifically, we analyze both their portfolio choice and consumption levels relative to the state variable P_{it}. In other words, Figure 3 graphs optimal choices $\left(\frac{B}{P}, \frac{H}{P}, \frac{E}{P}, \frac{C}{P}\right) \equiv (b, h, e, c)$ for households aged 28, 52 and 76 as a function of the state variable $w_{i,t}$ i.e. normalized initial wealth.

[4] In future work, as we have more parameters than targets, we wish to target an additional moment, in particular borrowing of the younger cohorts.

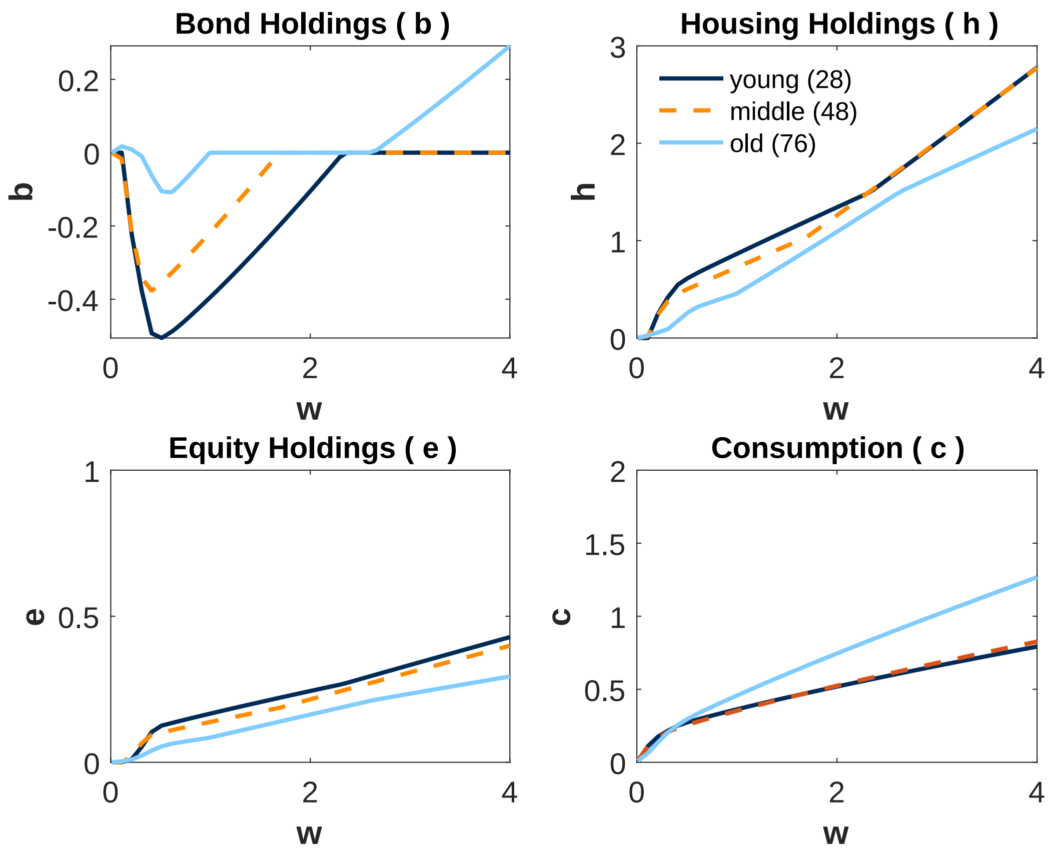

Figure 3. Policy functions at the baseline calibration for bond (b), housing (h) and equity (e) holdings and consumption (c) as a function of wealth. All variables are relative to the state variable of the permanent component of income, P_{it}. Policy functions conveyed for households aged 28 (young), 48 (middle) and old (76).

We begin with asset holdings. Even at very low wealth levels, agents prefer not to consume all of their wealth but instead decide to borrow in order to take on leveraged risky asset investments. Despite high risk aversion and marginal propensities to consume, lucrative expected returns induce early investment.

Beyond this observation, bond holdings are notably heterogeneous across cohorts. For the young with low wealth, most of their future wealth is in the form of labour income, which is uncorrelated with asset returns. Consequently, they exhibit high risk tolerance and so allocate savings to leveraged housing (i.e. they borrow to finance investment in the housing asset), the riskiest type of investment. Then, as wealth rises, risk tolerance declines. Current asset holdings become increasingly important for future wealth, causing them to deleverage and diversify their portfolio.

Middle-aged cohorts follow a similar path to the young, but start deleveraging at lower wealth levels. Such lower risk tolerance occurs because i) future income is less important simply due to lower remaining life expectancy, and ii) they anticipate a declining age profile of income.

For the old, risk tolerance is even lower, for the same reasons as middle-aged cohorts. Moreover, as conditional life expectancy is short, they no longer have long investment horizons that can smooth out negative asset return shocks. Thus, bond holdings are non-negative for almost all wealth levels.

Notice also that, for each age, there exists a region of wealth in which bond holdings are zero. This is generated by the borrowing spread (ι) as then it is feasible to strictly prefer to neither save in bonds nor borrow. The larger is ι, the larger is this region of wealth.

Finally, we turn to consumption choices. Consumption policy functions are similar for the young and middle-aged households. Older households tend to have higher consumption levels as well as higher propensities to consume (given the greater slope of the policy function).

B. Comparing Model to the Data

By combining market-clearing asset prices with household-level policy functions, we then derive period $t = 0$ model-implied portfolio shares and wealth across cohorts. To evaluate the success of the model, we compare this to the data. Figure 4 below shows the portfolio shares, $\left(\frac{E_{i,t}}{S_{i,t}}, \frac{H_{i,t}}{S_{i,t}}, \frac{B_{i,t}}{S_{i,t}}\right)$, for different cohorts together with the wealth levels.

The model matches wealth dynamics and bond shares very well. Moreover, it gets the level and trend housing and equity shares reasonably right. The only real shortfall is that the youngest agents borrow a bit too much. However, as they are a small fraction of overall households, it matters little for responses of asset prices to shocks.

VI. Monetary Policy Shock

A. Assumptions

After we calibrated the model, we estimate the effects of a monetary policy shock. Within our framework, a monetary policy shock is a 21bps change in the nominal interest rate, as described in Equation 32. As explained in Section III , expectations about future dividends, $\{D^E_{t+1}, D^H_{t+1}\}^{\infty}_{t=0}$, prices $\{p^E_{t+1}, p^H_{t+1}\}^{\infty}_{t=0}$, nominal interest rates $\{i_{t+1}\}^{\infty}_{t=0}$ and inflation $\{\Pi_{t+1}\}^{\infty}_{t=0}$ are unchanged. Given the equity and housing prices (p^E_0, p^H_0), we are then able to derive new returns expectations $E_0\left[\log R^E_1\right]$, $E_0\left[\log R^H_1\right]$ that matter for portfolio allocation choice of households.

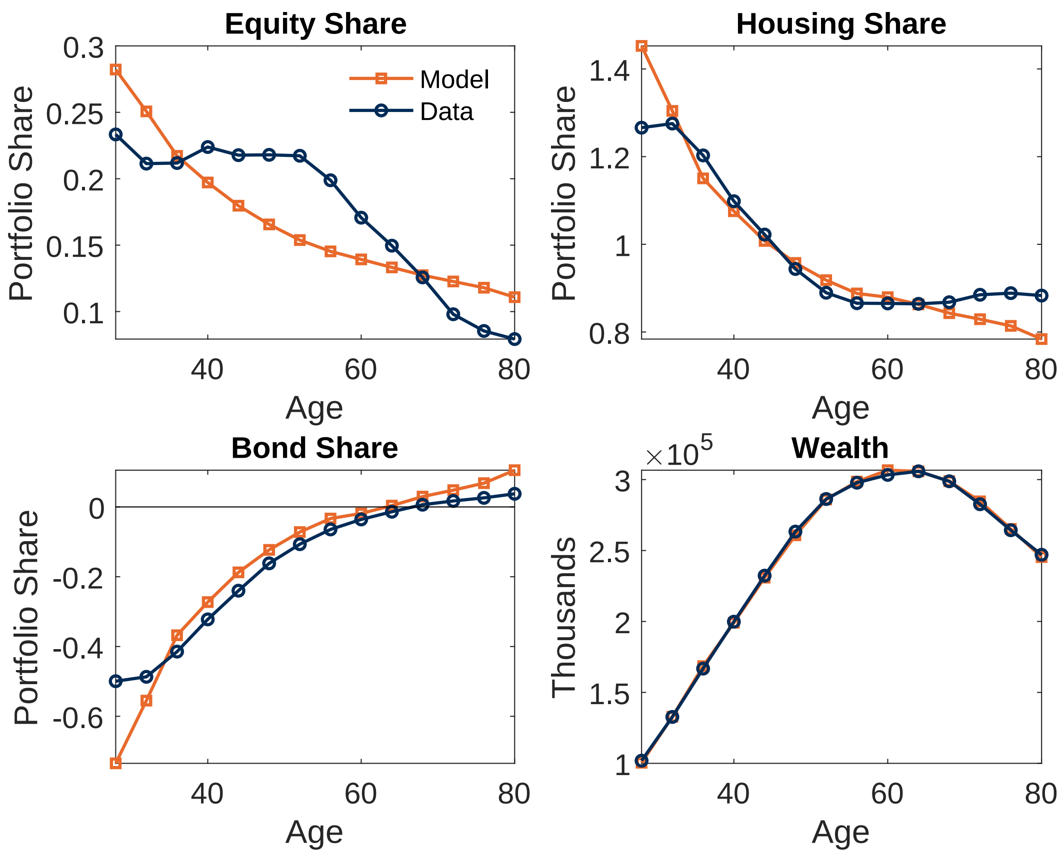

Figure 4. Compares the data (blue) and model-implied (orange) portfolio shares in equity, housing and bonds across different household cohort ages, along with wealth levels.

B. Portfolio Rebalancing

The change in the nominal interest rate, and the consequent shift in expectations about time $t = 1$ bond returns, will prompt households to rebalance their portfolio. In this section, we detail the portfolio rebalancing of different households following the change in the interest rate, holding constant the price of other assets (hence keeping expected returns on housing and equity unchanged). Figure 5 shows the change in portfolio shares for each asset class together with the change in consumption. The reduction in the nominal interest rate leads younger households to increase their leveraged investment in riskier assets. For very young households, we notice the largest change in debt, increasing their shares in debt (with a peak of 6% for households of age 28) and use the borrowing to buy more housing shares and equity shares. The eldest households, who tend to be net investors in bonds, also reduce their bond shares and rebalance toward riskier assets. Finally a 21 bps change in the nominal rate induces a consumption response of 0.2% for younger households. This response decreases in age, to the point where older households barely adjust consumption.

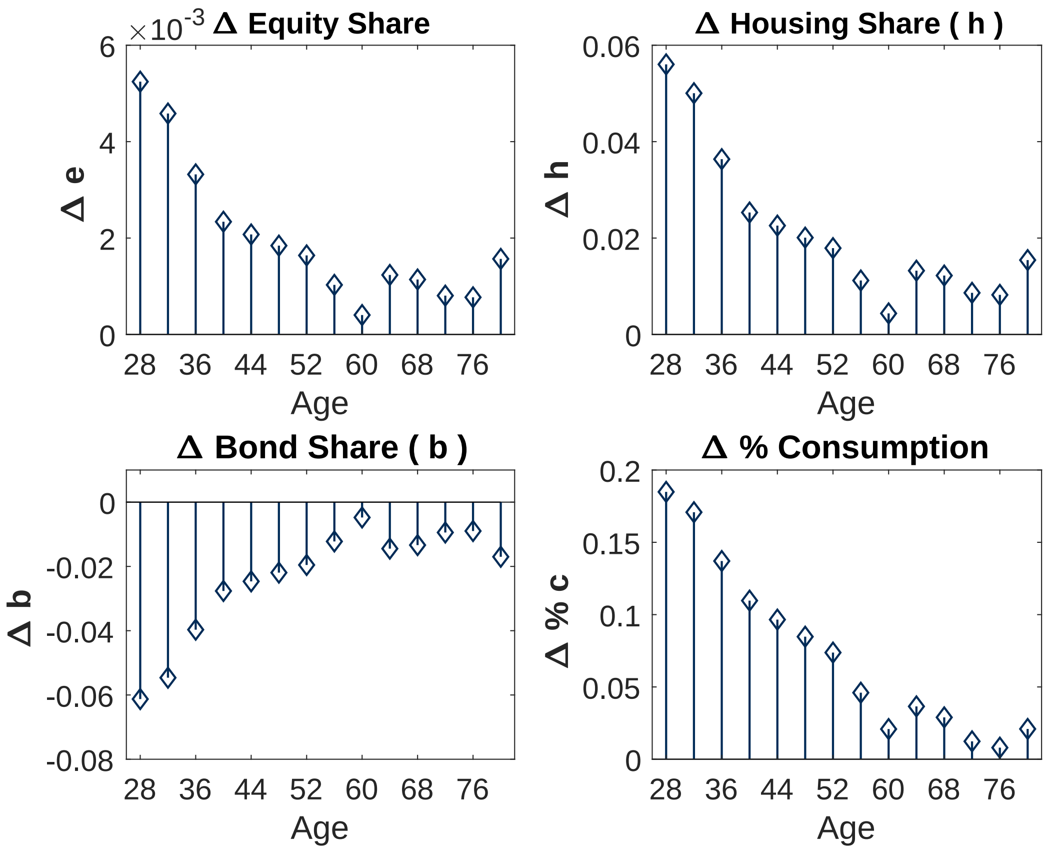

Figure 5. Change in the portfolio shares for equity, housing and bonds for households across 4-year cohorts after a one-time 21bps reduction in the expected annual return on bonds. Also conveys the percentage change in consumption in the bottom right quadrant. This comparative static is holding wealth and the expected return on equity and housing constant.

C. *Asset Prices*

The portfolio rebalancing described above increases aggregate demand for riskier assets. In order to clear markets, asset prices need to adjust.

Figure 6 plots the demand and supply for risky assets. The demand curve is always downward sloping. Intuitively, a higher asset price lowers the expected return on the asset, reducing demand as households rebalance their portfolio in response. As the quantity supplied is fixed at 1, the value of supply is its asset price, hence the 45-degree line for the supply curve.

The Baseline demand curve crosses the supply at the pre-shock equilibrium price. The monetary policy shock prompts an outward shift in the demand curve, holding wealth constant, from Baseline to Constant Wealth as households increase demand for riskier assets.[5]

[5]Note that each curve in Figure 6 takes into account the ultimate equilibrium effect on the expected return

This generates an increase in asset prices and so wealth. As a result, it induces a positive wealth effect on demand, which is reflected in the outward shift in the demand curve from Constant Wealth to Post Shock in Figure 6. The intersection of the supply and Post Shock demand curve determines the new equilibrium asset prices.

D. Overall Effect on Consumption and Portfolio Choice

Now that we have understood the effect on asset prices, we finally turn to the general equilibrium effects on portfolio choice and consumption. Results are shown in Figure 7.

Given the change in asset prices induced by the monetary policy shock, we find that, in equilibrium, the risk premia on risky assets rise. In particular, for a 21bp shock to the return on bonds, equity and housing risk premia increase by 5bps and 7.5bps, respectively.

Intuitively, in response to a lower targeted real return on bonds, households rebalance toward the risky assets, equity and housing, and away from bonds. Consequently, they now hold more risk in their portfolio. However, households need to be compensated for bearing more risk in equilibrium. This is called the *risk compensation* effect. On the other hand, as asset prices rise, households have a higher tolerance for holding more risky assets, termed the *risk tolerance* effect. As the risk premium rises, here the risk compensation effect dominates the risk tolerance effect.

The top two quadrants of Figure 7 demonstrate the percentage change in risky asset holdings holding wealth constant (in light blue) and incorporating wealth effects (in dark blue). Because of the increase in the risk premia on risky assets, households rebalance in favour of housing and equity across all cohorts. This effect is somewhat more pronounced for the young as their willingness to take risk is higher. However, once we incorporate the wealth effects induced by higher asset prices, reflected in the bottom right quadrant, holdings rise significantly for all cohorts, although the heterogeneity in asset holding responses remains.

Finally, and most importantly, we turn to consumption responses demonstrated in the bottom left quadrant of Figure 7. Absent wealth effects (in light blue), older cohorts reduce consumption while younger cohorts raise consumption slightly. Intuitively, all cohorts experience a negative income effect from declining expected returns, and so consume less. This can be offset by the benefits of cheaper borrowing rates, which only young cohorts benefit from in equilibrium. But once we incorporate positive wealth effects, consumption rises for all cohorts (in dark blue), while the heterogeneity remains.

It is quite clear, then, that the percentage change in consumption is higher for the younger cohorts partly because they benefit relatively more from cheaper borrowing rates.

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of the other asset.

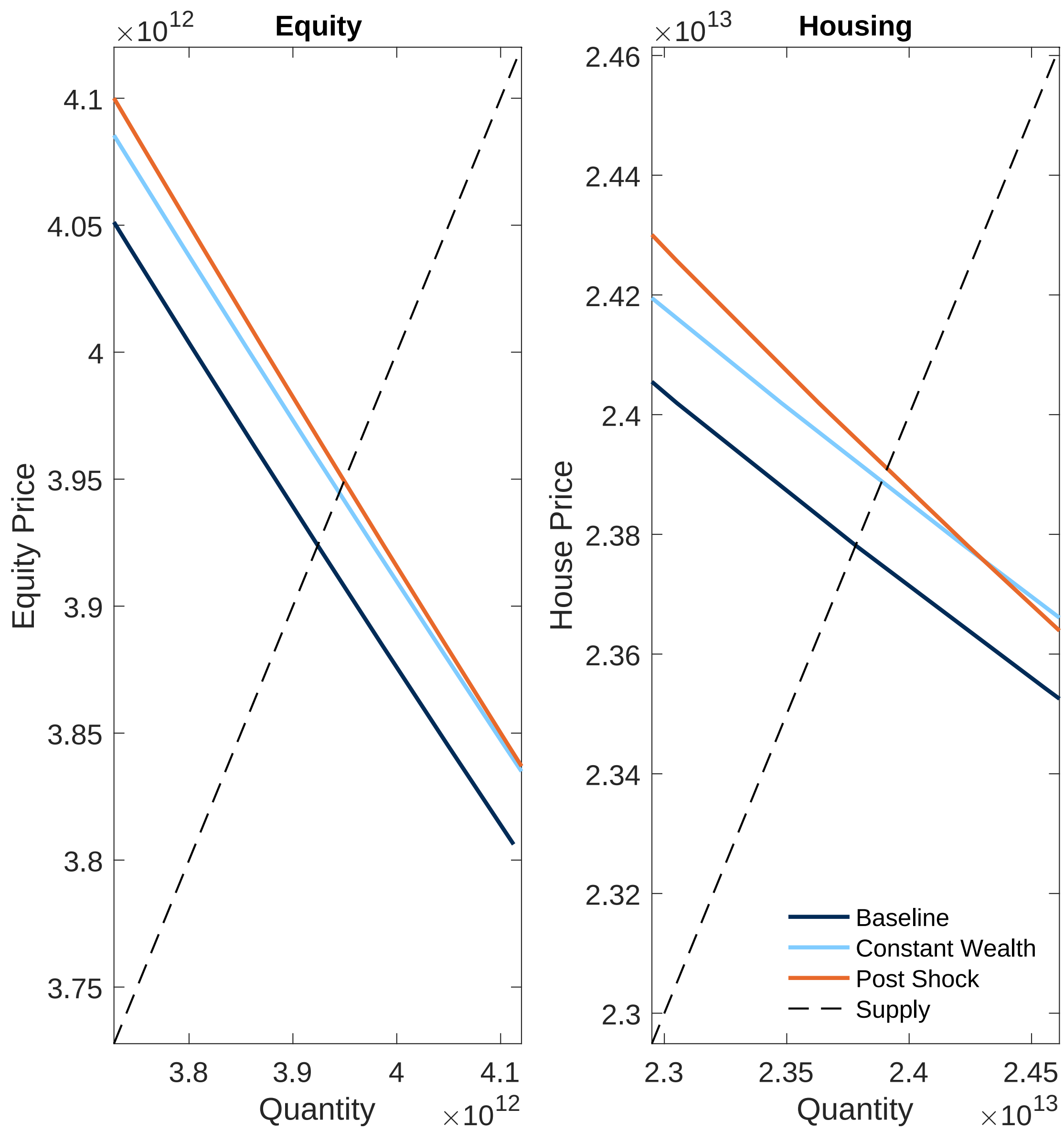

Figure 6. Demand and supply curves for equity and housing, respectively. Conveys demand curve under Baseline calibration (dark blue), after the 21bps shock reduction in expected return on bonds but holding wealth constant (light blue), and post-shock but accounting for general equilibrium effects on asset prices (red). Each demand curve takes into account the change in the expected return of the other asset in equilibrium. Supply curve is a 45-degree line as there is one constant unit in supply. Price/quantity measured in Euros.

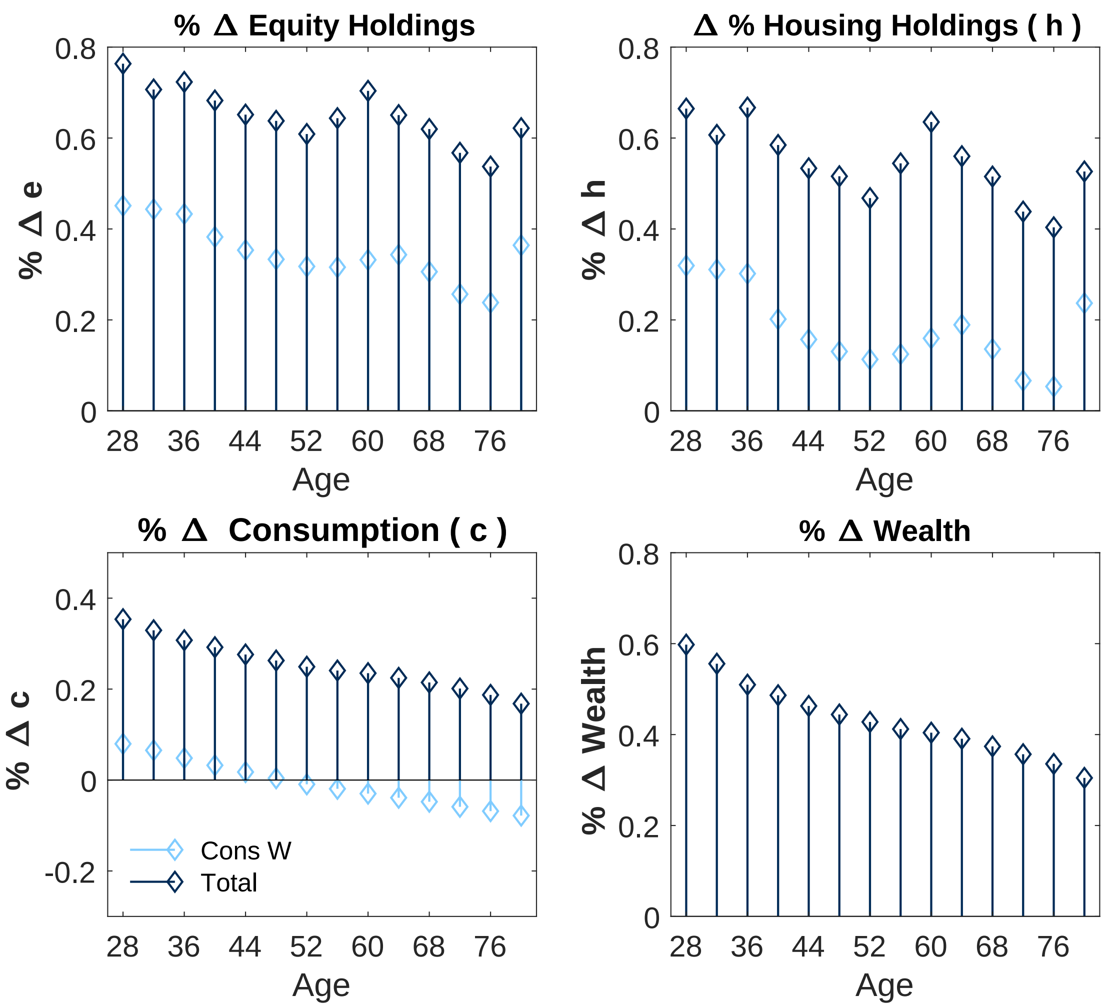

Figure 7. Change in the portfolio shares for equity and housing for each of the 4-year cohort households after a one-time 21bps reduction in the expected annual return on bonds. Also conveys the percentage change in consumption and wealth in the bottom quadrants. We consider effects holding wealth constant (light blue) and also allowing for general equilibrium wealth effects (dark blue). The comparative statics account for the change in the expected return on equity and housing.

But does this heterogeneity also play a role for the effect on risk premium? More generally, is the size/directional effect on risk premia dependent on the assumptions we make about the monetary policy shock? This model is a tractable setup that can help us understand the key drivers, which we turn to in the next section.

VII. Monetary Policy and Risk Premia

The key advantage of this modelling framework is that it is tractable for understanding endogenous effects on asset risk premia. For our results in Section VI, we imposed a certain set of assumption for the monetary policy shock, for example on labor income and dividends expectations. To further the intuition of risk premium effects, we relax them in turn, and also consider the role of heterogeneity.

A. Labor Income

For our baseline experiment, we assume that current labor income is unaffected by the monetary policy shock. We now relax this assumption. and garner estimates of the effect on labor income using a VAR approach. We construct a VAR using 4 Euro-Area variables: inflation, unemployment rate, GDP and the 3-month interbank rate. We take quarterly time series over the time horizon of 1999-2014. We then consider the impulse responses to a 1% shock to the 3-month rate. Results are shown in Figure 8.

Results are shown in Figure 8. Taking the average fall in the 3-month rate over the 16 quarters (4 years) after the shock, comparing it to the effect on GDP over the same horizon, and using GDP as an initial proxy for labor income, we garner an estimate of the mapping of a shock to the 4-year rate on labor income. In this setting, we find the 3-month rate falls 0.3% over the 4-year horizon while GDP rises 1% on average over the same horizon. This implies then that, for our 21bps shock to the policy rate, we assume current labor income rises by 0.7%.

Figure 9 then shows the impact on risk premia depending on our assumption on labor income. Dark blue is our baseline result, while in light blue is when current labour income rises 0.7% for all cohorts. As we can see, the risk premium falls, and in fact turns negative for equity. Intuitively, higher labor income is equivalent to a positive wealth income, inducing a *risk tolerance* effect on risky asset demand. If households expect the labor income change to be permanent, this strengthens the impact on future household wealth, inducing a large *risk tolerance* effect.

(Include a commentary on why the effect on equity risk premia is larger)

B. Asset Dividends Expectations

In our baseline experiment, we also assume that expectations about future dividends, $\{D^E_{t+1}, D^H_{t+1}\}^{\infty}_{t=0}$, are unchanged. What happens if we instead assume expected dividends next period for housing or equity rise 1% in response to monetary policy shocks - what happens to risk premia?

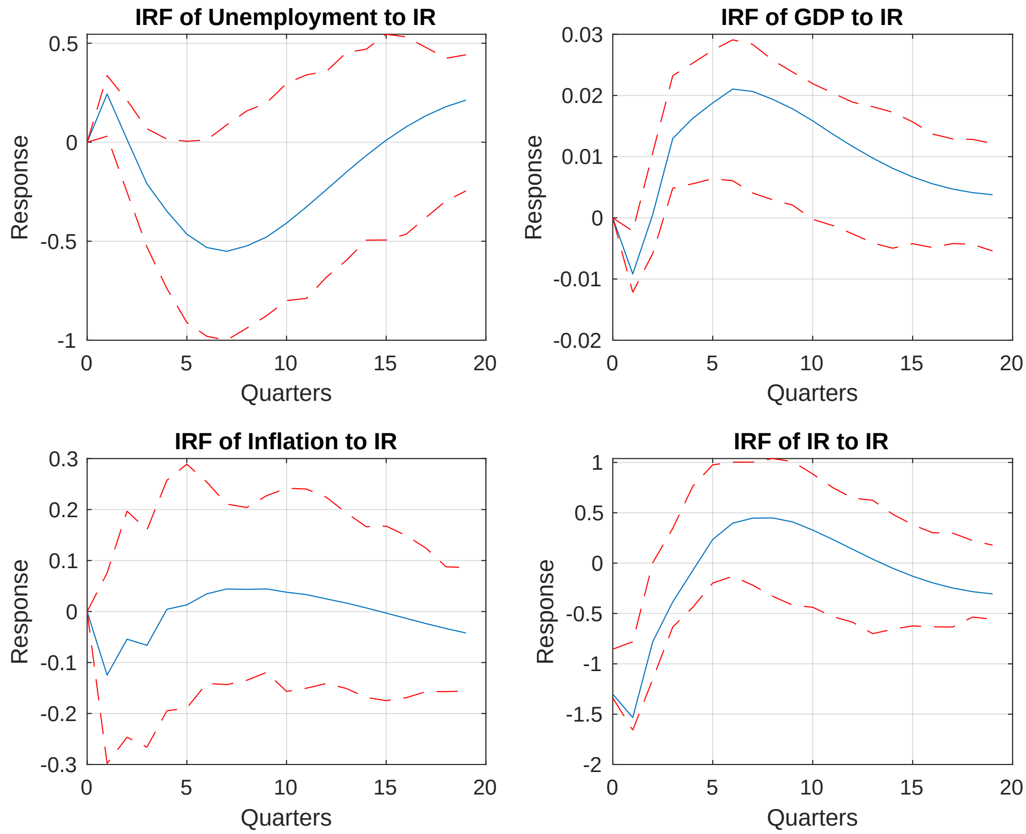

Figure 8. Impulse Responses from an Estimated VAR

Results are demonstrated in Figure 10. Let's firstly consider the housing dividend shock, shown in light blue. There are two forces at play. On the one hand, receiving more housing dividends in expectation next period generates a positive wealth effect on asset demand. This *risk tolerance* effect lowers risk premia for both assets. On the other hand, for a constant risk premium, 1% higher housing dividends requires, in equilibrium, 1% more investment in housing today, generating a *risk compensation* effect on housing to induce these higher holdings. The latter effect causes the housing risk premium to rise, while equity risk premia fall.

C. *Role of Heterogeneity*

Another question to ask is the role played by household heterogeneity for the effect on risk premium. Appendix F outlines how we calibrate the same model but under the case of a representative agent.

We then simulate a monetary policy shock under the same assumptions detailed in Section VI. In this case, the change in the expected return on bonds is again going to spur a portfolio re-balancing. Figure 11 plots the change in the portfolio of the three assets

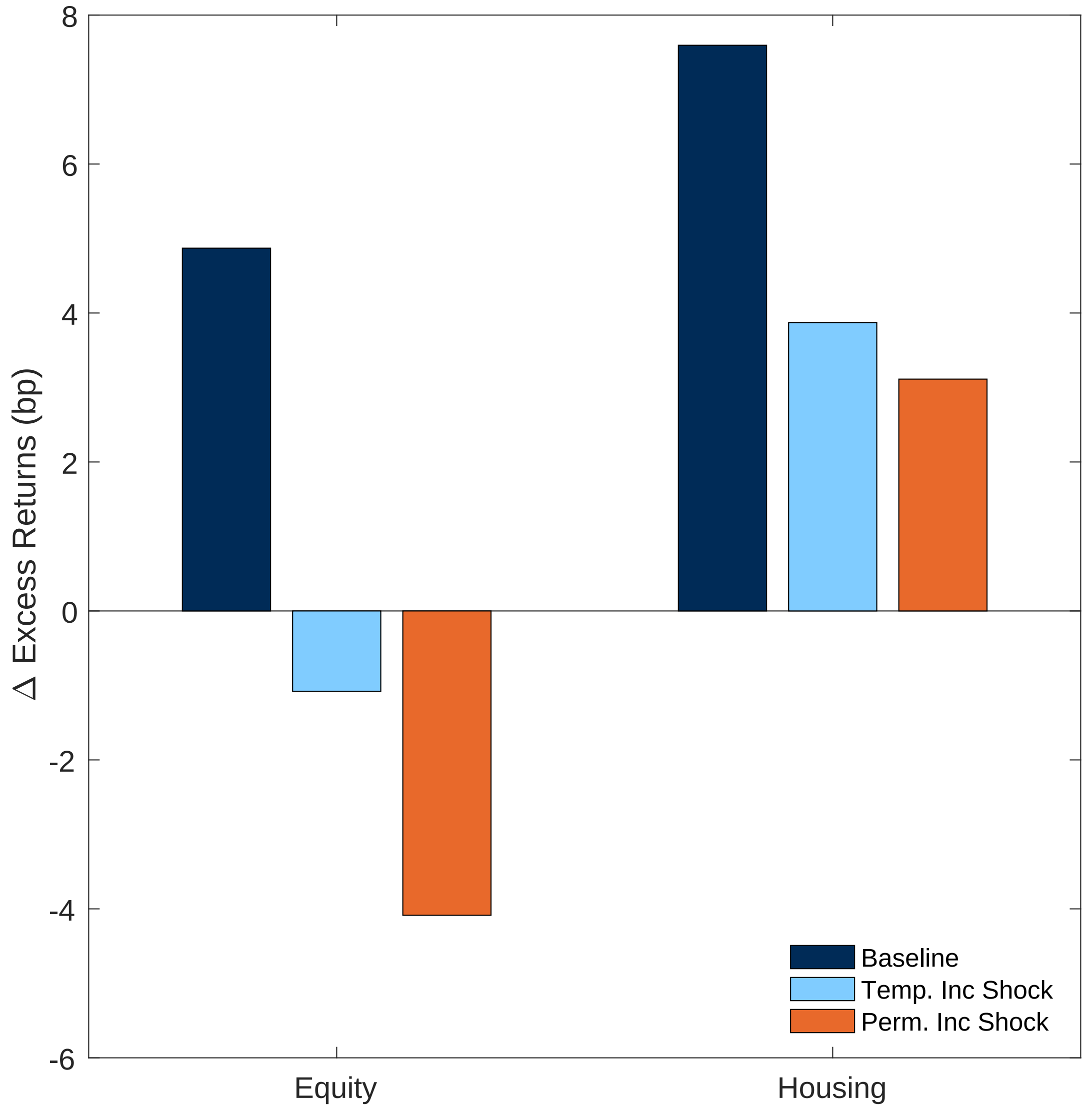

Figure 9. Change in housing and equity risk premia in response to the monetary policy shock under three different assumptions: (i) unchanged labor income, (ii) temporary (current period) increase in labor income, (iii) expected permanent increase in labor income

by the representative agent in comparison to the baseline heterogeneous agent model. In this case, we find that the directional movement is the same in both models, but the size of the movement is larger under the representative agent model. Intuitively, in order to have the representative model match the data, I needed to make the Epstein-Zin preferences less risk-averse (lower γ) relative to the heterogeneous agent benchmark. As a result, households are more sensitive to changes in expected returns, inducing a larger portfolio

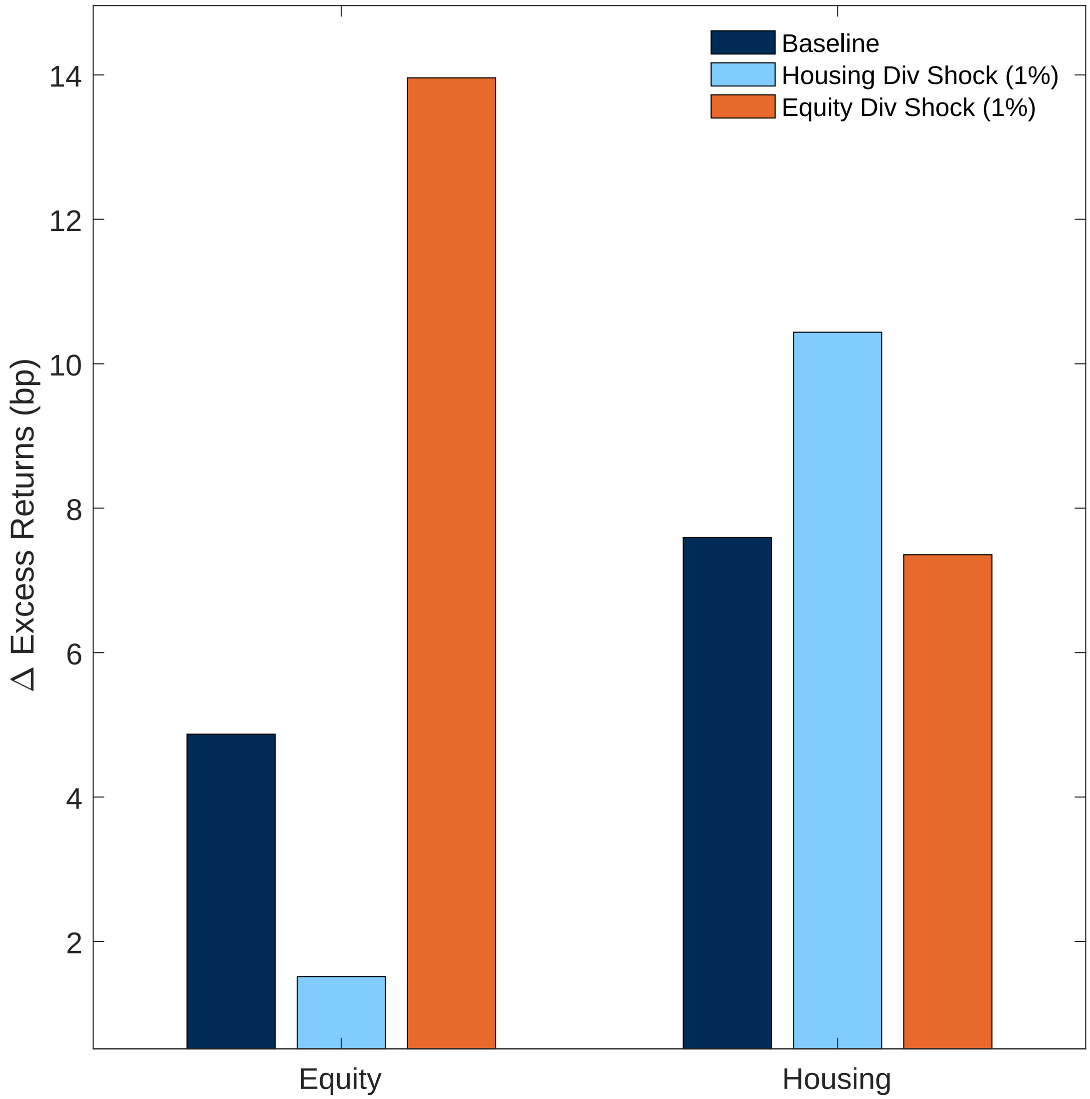

Figure 10. Change in housing and equity risk premia in response to the monetary policy shock under three different assumptions: (i) unchanged dividend expectations, (ii) increase in next period's (temporary) expected housing dividends only, (iii) increase in next period's (temporary) expected equity dividends only

response.

As in the heterogeneous agents case, the shock induces an outward shift in the demand curve for both assets, due to the combination of a portfolio rebalancing effect, and an additional wealth effect caused by rising asset prices. This change in asset prices will determine the equilibrium effect on risk premia, as outlined in Figure 12 below.

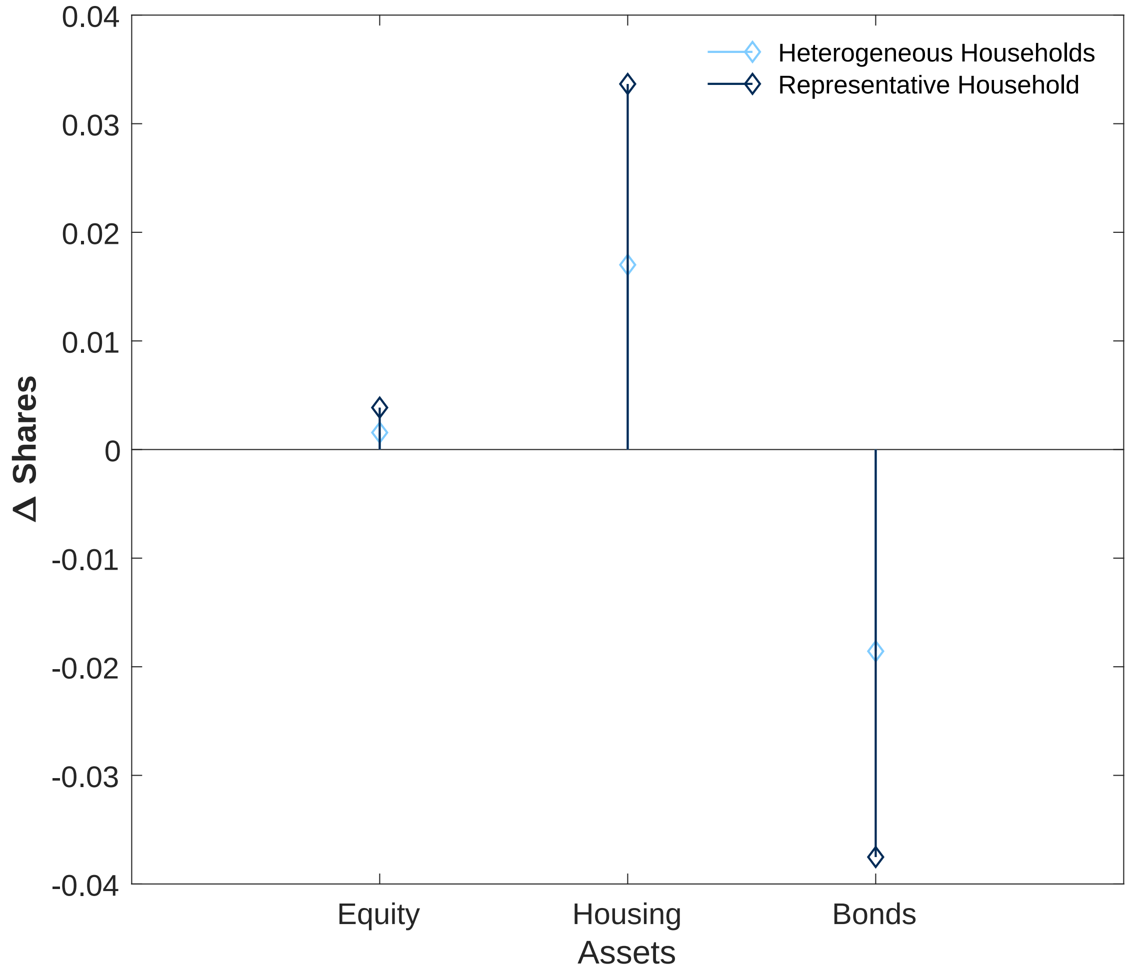

Figure 11. Change in the portfolio shares for equity, housing and bonds for the representative household after a one-time 21bps reduction in the expected annual return on bonds. Also conveys the percentage change in consumption in This comparative static is holding wealth and risky asset prices constant.

Figure 12 demonstrates the change in risk premia, comparing the outcomes with heterogeneous agents (in dark blue) vs. the representative agent (in light blue). What appears to happen is that the risk premia move very similarly, suggesting that even though portfolio rebalancing is stronger, the equilibrium asset price effects are similar. Therefore, the heterogeneity does not appear to drive neither the size nor the direction of the response. Intuitively, for the majority of households, they are on the part of their policy functions for equity and housing demand that are linear and of similar slope across ages. Without additional frictions, many agents would require holding very low wealth for heterogeneity to matter for risk premia.[6]

[6]It of course matters for consumption precisely because the MPCs differ across cohort ages for given

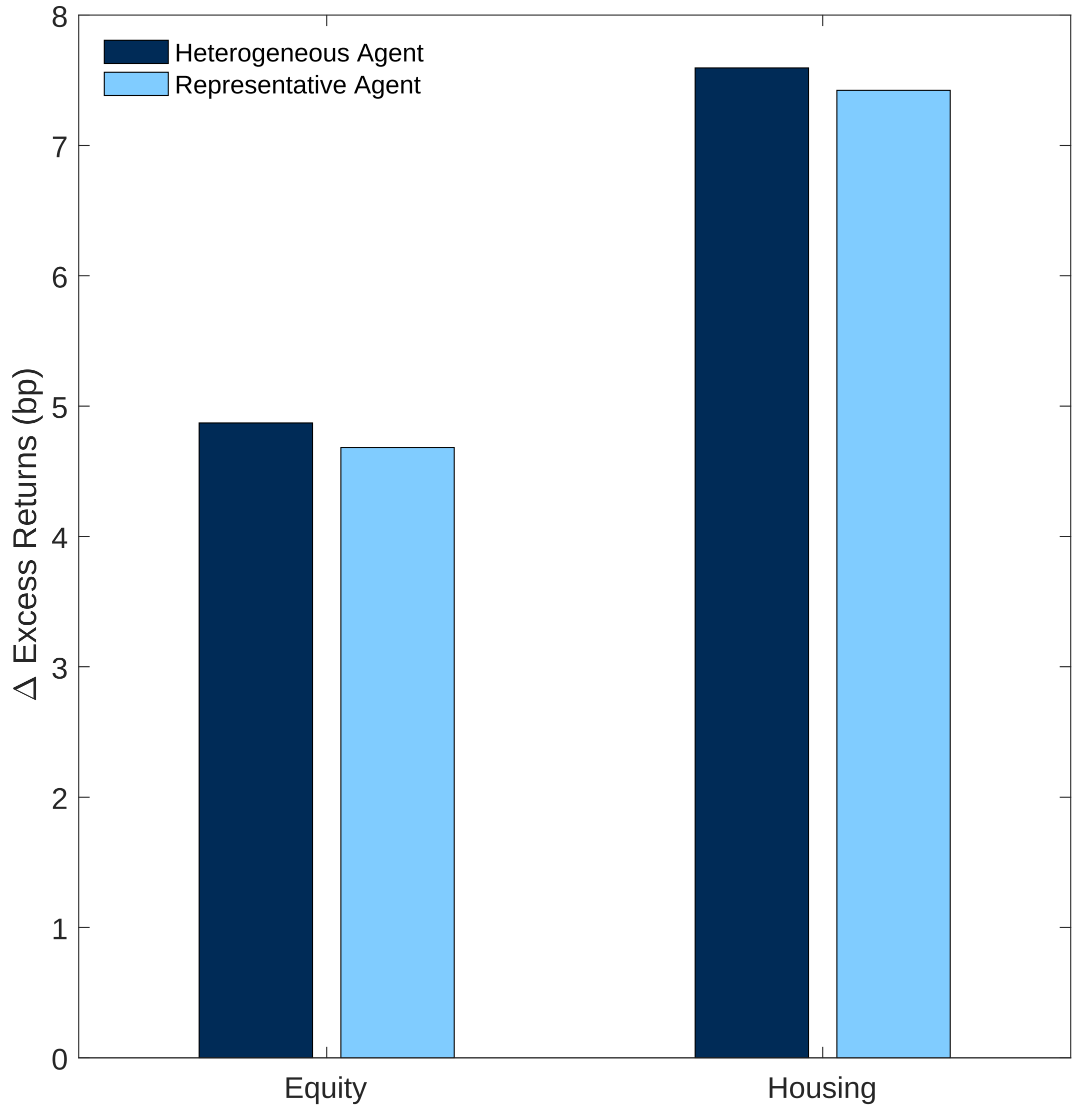

Figure 12. Change in the annual risk premium of equity and housing in general equilibrium after a one-time 21bps reduction in expected return in bonds, for heterogeneous agent (dark blue) vs. representative agent (light blue) model.

VIII. Conclusion

In this paper, we attempt to understand the role of the household portfolio rebalancing channel for the aggregate and redistributive effects of monetary policy shocks. We introduce a heterogeneous household life-cycle model with multiple assets and combine it with

wealth levels.

an incomplete markets asset-pricing framework. After using both micro and macro-level data to derive initial household endowments, we calibrate the model to the data by targeting key empirical aggregates. We then find that, in response to a reduction in safe asset supply, households rebalance portfolios towards riskier assets due to a rise in their risk premia in equilibrium, thereby increasing asset prices and so household wealth.

We find that, absent wealth effects, older cohorts reduce consumption as they face lower expected asset returns, while younger cohorts raise consumption as they can borrow more cheaply. This heterogeneity remains with wealth effects, but responses turn positive for all cohorts. Asset risk premia rise because the risk compensation effect (need for more returns to hold more risk) dominates the risk tolerance effect (positive wealth effect on risky asset holdings). Shutting down household heterogeneity does little for impacting the risk premium responses.

References

Adam, Klaus and Panagiota Tzamourani (2016), "Distributional consequences of asset price inflation in the euro area." *European Economic Review*, 89, 172–192.

Adam, Klaus and Junyi Zhu (2015), "Price-level changes and the redistribution of nominal wealth across the euro area." *Journal of the European Economic Association*, 14, 871–906.

Adrian, Tobias and Hyun Song Shin (2010), "Financial intermediaries and monetary economics." In *Handbook of monetary economics*, volume 3, 601–650, Elsevier.

Altunbas, Yener, Leonardo Gambacorta, and David Marques-Ibanez (2010), "Does monetary policy affect bank risk-taking?"

Alvarez, David Argente, Fernando E. and Francesco Lippi (2020), "A simple planning problem for covid-19 lockdown." *Working Paper 26981, National Bureau of Economic Research*.

Auclert, Adrien (2017), "Monetary policy and the redistribution channel." Technical report, National Bureau of Economic Research.

Berger, David, Veronica Guerrieri, Guido Lorenzoni, and Joseph Vavra (2017), "House prices and consumer spending." *The Review of Economic Studies*, 85, 1502–1542.

Bernanke, Ben S and Kenneth N Kuttner (2005), "What explains the stock market's reaction to federal reserve policy?" *The Journal of finance*, 60, 1221–1257.

Bigio, Saki and Yuliy Sannikov (2023), "A model of credit, money, interest and prices." NBER Working Paper 28540.

Borio, Claudio and Haibin Zhu (2012), "Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism?" *Journal of Financial stability*, 8, 236–251.

Campbell, John Y and Joao F Cocco (2007), "How do house prices affect consumption? evidence from micro data." *Journal of monetary Economics*, 54, 591–621.

Carroll, Christopher D, Jiri Slacalek, and Kiichi Tokuoka (2014), "The distribution of wealth and the mpc: implications of new european data." *American Economic Review*, 104, 107–11.

Case, Karl E, John M Quigley, and Robert J Shiller (2005), "Comparing wealth effects: the stock market versus the housing market." *Advances in macroeconomics*, 5.

Cloyne, James, Clodomiro Ferreira, and Paolo Surico (2018), "Monetary policy when households have debt: new evidence on the transmission mechanism."

De Fiore, Fiorella, Marie Hoerova, and Harald Uhlig (2021), "Money markets, collateral and monetary policy." University of Chicago, Becker Friedman Institute for Economics Working Paper No. 2018-79.

Doepke, Matthias and Martin Schneider (2006), "Inflation and the redistribution of nominal wealth." *Journal of Political Economy*, 114, 1069–1097.

Gagnon, Joseph, Matthew Raskin, Julie Remache, Brian Sack, et al. (2011), "The financial market effects of the federal reserve's large-scale asset purchases." *international Journal of central Banking*, 7, 3–43.

Gambacorta, Leonardo (2009), "Monetary policy and the risk-taking channel."

Gertler, Mark and Peter Karadi (2013), "Qe 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool." *International Journal of Central Banking*, 9, 5–51.

Gourinchas, Pierre-Olivier and Jonathan A Parker (2002), "Consumption over the life cycle." *Econometrica*, 70, 47–89.

Greenwald, Daniel (2017), "The mortgage credit channel of macroeconomic transmission."

Gurkaynak, Refet S, Brian P Sack, and Eric T Swanson (2004), "Do actions speak louder than words? the response of asset prices to monetary policy actions and statements."

Hau, Harald and Sandy Lai (2016), "Asset allocation and monetary policy: Evidence from the eurozone." *Journal of Financial Economics*, 120, 309–329.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018), "Monetary policy according to hank." *American Economic Review*, 108, 697–743.

Kavonius, Ilja Kristian, Juha Honkkila, et al. (2013), "Reconciling micro and macro data on household wealth: A test based on three euro areacountries." *Journal of Economic & Social Policy*, 15, 42.

Koijen, Ralph, Francois Koulischer, Benoit Nguyen, and Motohiro Yogo (2021), "Inspecting the mechanism of quantitatve easing in the euro area." *Journal of Financial Economics*, 140, 1–20.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen (2011), "The effects of quantitative easing on interest rates: channels and implications for policy." Technical report, National Bureau of Economic Research.

Krueger, Dirk, Fabrizio Perri, Luigi Pistaferri, and Giovann Violante (2010), "Cross sectional facts for macroeconomists." *Review of Economic Dynamics*, 13, 1–14.

Landvoigt, Tim, Monika Piazzesi, and Martin Schneider (2015), "The housing market (s) of san diego." *American Economic Review*, 105, 1371–1407.

Lenza, Michele and Jiri Slacalek (2023), "How does monetary policy affect income and wealth inequality? evidence from quantitative easing in the euro area." *Manuscript*.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson (2016), "The power of forward guidance revisited." *American Economic Review*, 106, 3133–58.

Piazzesi, Monika and Martin Schneider (2009), "Inflation and the price of real assets." *Manuscript*.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel (2007), "Housing, consumption and asset pricing." *Journal of Financial Economics*, 83, 531–569.

Rigobon, Roberto and Brian Sack (2004), "The impact of monetary policy on asset prices." *Journal of Monetary Economics*, 51, 1553–1575.

Wong, Arlene et al. (2016), "Population aging and the transmission of monetary policy to consumption." In *2016 Meeting Papers*, volume 716, Society for Economic Dynamics.

A. Portfolio Shares Sub-cohorts

Figures 14 - 15 show the portfolio shares for the bottom 40 percentile, the mid 40 percentile and the top 20.

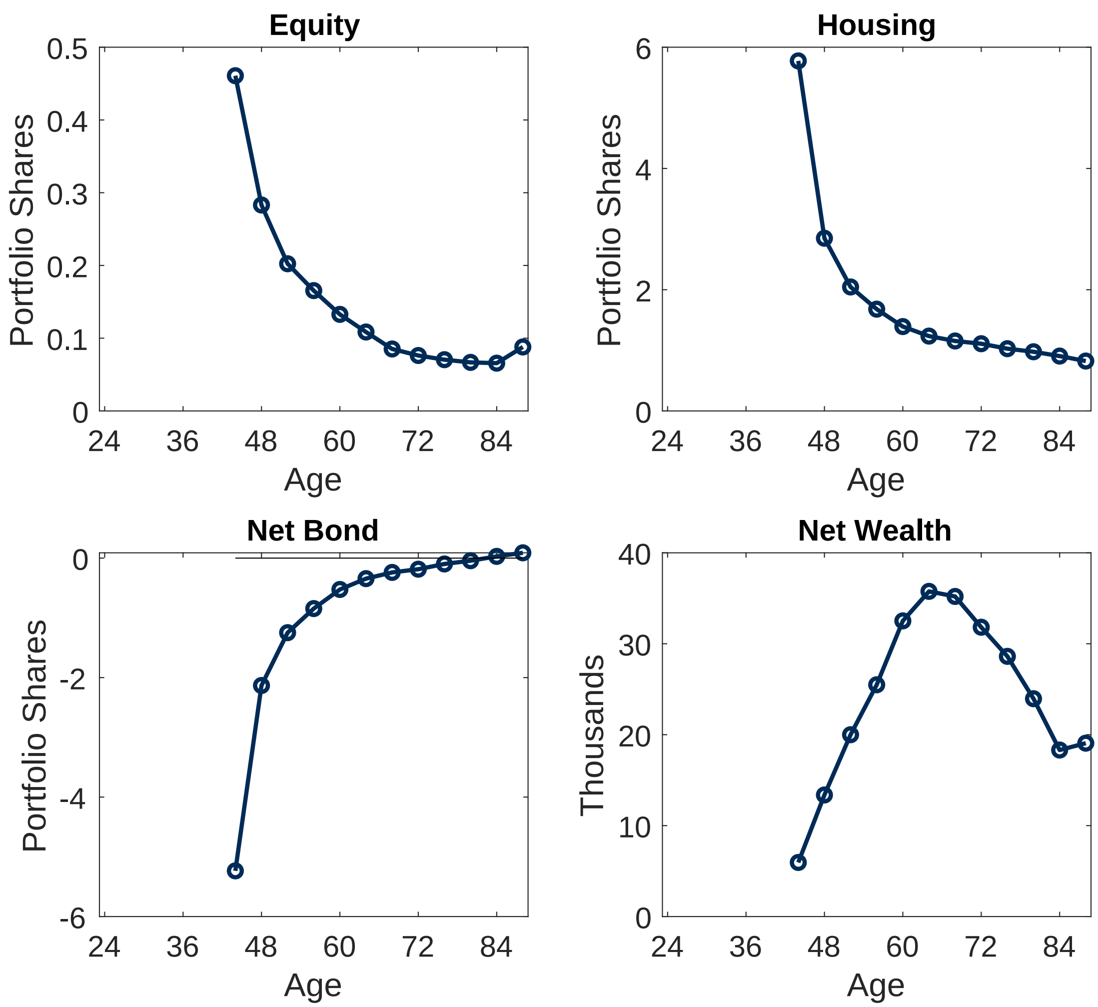

Figure 13. Portfolio shares for Euro Area households within the Bottom 40% of each 4-year cohort in 2014 by wealth. Consider shares of equity, housing and net bond, along with total wealth in thousands of Euros in the bottom right quadrant. Note: Only show results for sub-cohorts with positive net wealth.

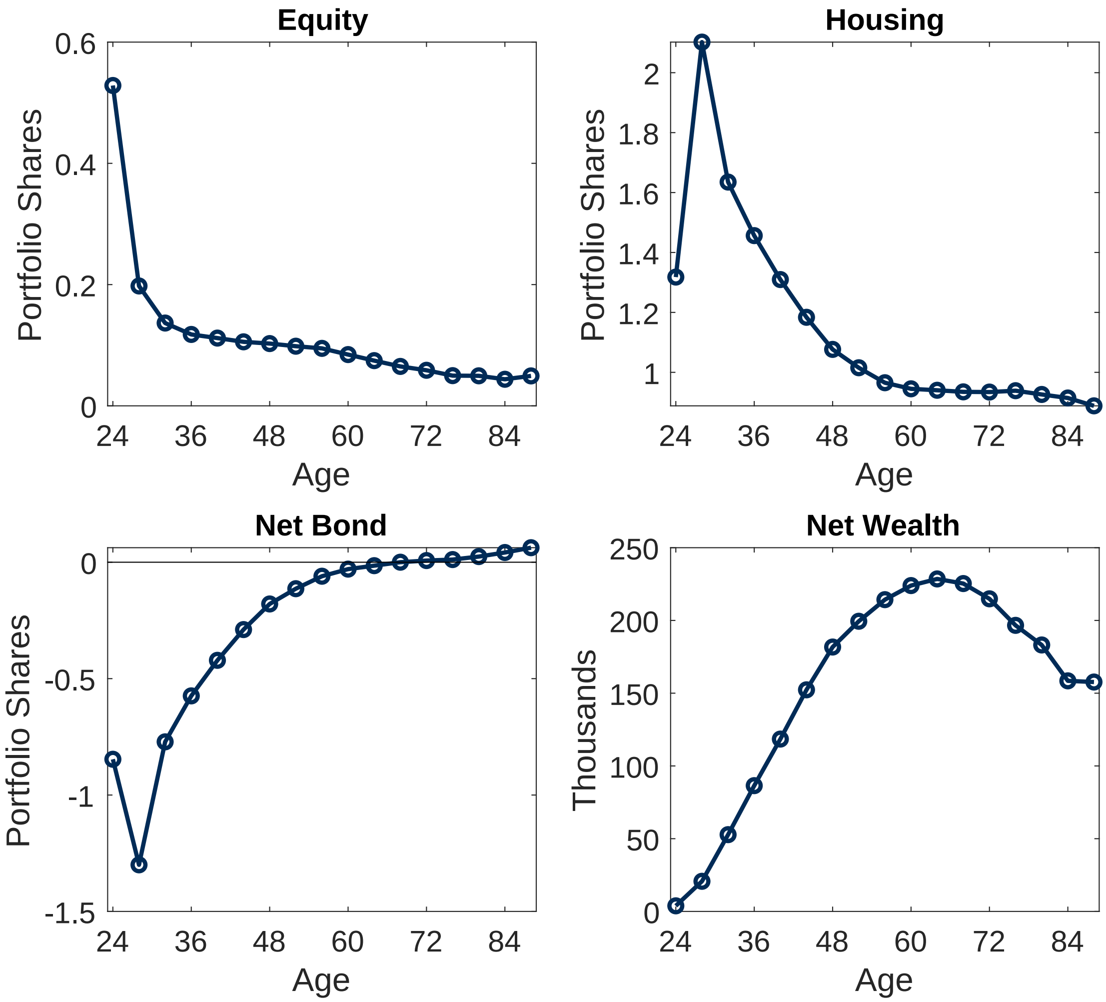

Figure 14. Portfolio shares for Euro Area households within the Middle 40% of each 4-year cohort in 2014 by wealth. Consider shares of equity, housing and net bond, along with total wealth in thousands of Euros in the bottom right quadrant.

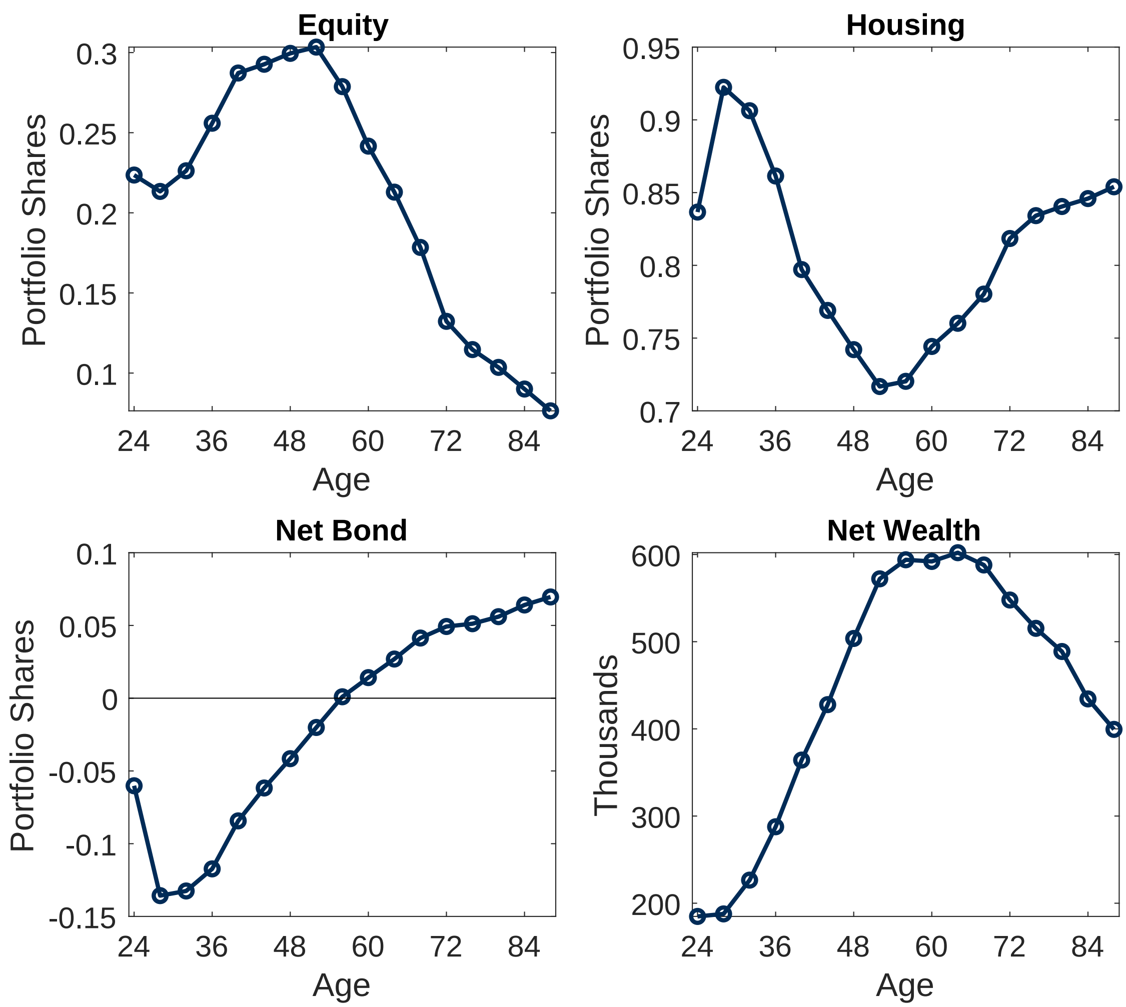

Figure 15. Portfolio shares for Euro Area households within the Top 20% of each 4-year cohort in 2014 by wealth. Consider shares of equity, housing and net bond, along with total wealth in thousands of Euros in the bottom right quadrant.

B. Labour Income Process

Following Gourinchas and Parker (2002), the labour income process for household i is given by

$$Y_{i.t} = G_t P_{i,t} U_{i,t} \tag{40}$$

$$P_{i.t} = \exp(f(a)) P_{i,t-1} N_{i,t} \tag{41}$$

$$G_t = \exp(\mu^G) \exp(\epsilon_t^G) G_{t-1} \tag{42}$$

To estimate $f(a)$, we take advantage of the HFCS household-level income data. Households within cohort a vary in income through their idiosyncratic permanent and temporary components. We calculate the average income within each cohort:

$$\frac{1}{n_a} \sum_{i \in a} Y_{i,t} = G_t \frac{1}{n_a} \sum_{i \in a} P_{i,t} U_{i,t} \tag{43}$$

Assuming we have a large number of households within each cohort and knowing that $\ln U_{i,t}$ and $\ln N_{i,t}$ are independent and iid normal with mean $\{-0.5 * \sigma_u^2, -0.5 * \sigma_n^2\}$ and variances σ_u^2 and σ_n^2, respectively, then by law of large numbers:

$$\text{plim} \frac{1}{n_a} \sum_{i \in a} Y_{i,t} = G_t e^{\sum_{j=1}^{a} f(j)} P_0 \tag{44}$$

If we also estimate the average income for cohort $a - 1$ we obtain:

$$\text{plim} \frac{1}{n_{a-1}} \sum_{i \in a-1} Y_{i,t} = G_t e^{\sum_{j=1}^{a-1} f(j)} P_0 \tag{45}$$

Applying these results, we then know that:

$$\text{plim} \left(\log(\frac{1}{n_a} \sum_{i \in a} Y_{i,t}) - \log(\frac{1}{n_a - 1} \sum_{i \in a-1} Y_{i,t}) \right) = \log \left(\text{plim} \frac{1}{n_a} \sum_{i \in a} Y_{i,t} \right) - \log \left(\text{plim} \frac{1}{n_{a-1}} \sum_{i \in a-1} Y_{i,t} \right) \tag{46}$$

$$= f(a) \tag{47}$$

Consequently, $f(a)$ can be consistently estimated by the log difference in mean income between cohorts a and $a - 1$.

C. Value Function Normalization

$$V_{i,a,t}(W_{i,t}, P_{i,t}) = \left[C_{i,t}^{1-\frac{1}{\sigma}} + \beta E_t \Big[p_a V_{i,a+1,t+1}^{1-\gamma}(W_{i,t+1}, P_{i,t+1}) + (1-p_a)\phi_B^{1-\gamma} W_{i,t+1}^{1-\gamma} \Big]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{48}$$

To show that this can be normalized, we guess and verify that $V_t(W_{i,t}, P_{i,t}) = \hat{V}_t(\omega_{i,t})P_{i,t}$. The verification argument involves conveying that the only state variable necessary is $\omega_{i,t}$. Plugging in the guess, we have that:

$$\hat{V}_{i,a,t}(\omega_{i,t}) = \left[c_{i,t}^{1-\frac{1}{\sigma}} + \beta E_t \left[p_a \hat{V}_{i,a+1,t+1}^{1-\gamma}(\omega_{i,t+1}) \left(\frac{P_{i,t+1}}{P_{i,t}} \right)^{1-\gamma} + (1-p_a)\phi_B^{1-\gamma} \omega_{i,t+1}^{1-\gamma} \left(\frac{P_{i,t+1}}{P_{i,t}} \right)^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}}$$

where

$$\begin{aligned}
\omega_{i,t+1} &= \left(e_{i,t}R_{t+1}^E + h_{i,t}R_{t+1}^H + b_{i,t}^+ R_{t+1}^B - b_{i,t}^-(R_{t+1}^B + \iota) \right) \left(\frac{P_{i,t+1}}{P_{i,t}} \right) + \frac{Y_{i,t+1}}{P_{i,t+1}} \\
\frac{P_{i,t+1}}{P_{i,t}} &= \exp(f(a+1))N_{i,t+1} \\
\frac{Y_{i,t+1}}{P_{i,t+1}} &= G_t \exp(\mu^G)\exp(\epsilon_{t+1}^G)U_{i,t+1}
\end{aligned}$$

where lower-cased policy functions are choices relative to the permanent component of income, $P_{i,t}$. As $\ln N_{i,t+1}, \ln U_{i,t+1}$ and ϵ_{t+1}^G are independent and i.i.d. normal, then conditional on knowing $\omega_{i,t}$, no additional variable is informative.

D. Appendix: Historical Returns

We use historical data to estimate the mean and variance of different assets. Below we detail for each asset class the series we use and the sample.

Bond: We use quarterly data on the Eonia 3m OIS yield downloaded from Bloomberg (ticker: EUSWEC Curncy).

Equity: We use quarterly data on the The MSCI EMU Index (ticker: GDDLEMU Index). The index captures large and mid cap representation across the 10 Developed Markets countries in the EMU. With 246 constituents, the index covers approximately 85% of the free float-adjusted market capitalization of the EMU.

Housing: We use quarterly data on the Residential Property Prices for Euro area. We download the data from Fred (ticker: QXMN628BIS). We use data on price-to-rent index from the OECD to estimate the rental yield evolution. However, in order to transform the index in level terms, we calibrate the time series to match the average rental yield estimated from the HFCS in 2014: 4%. We then use historical price index together with the rental yield series to construct a total return index for housing.

Inflation: We use the quarterly Euro area consumer price index. We download data from Bloomberg (ticker: EUHICPI Index)

We transform the bond, housing and equity series into a real index, using the Euro area price index. Our sample runs from Q1-1996 through Q4-2018 for all the series. The only exception is the bond series, which only starts in Q1-2000.

We use the quarterly log-change for all the series. We then calculate the mean and variance of real returns for bonds, housing and equity.

E. Appendix: Data Collection

Each variable is equal to the sum of the corresponding listed items.

Income:

- DI1200 Self-employment income
  - Sum of PG0210 Gross self employment income (profit/losses of unincorporated enterprises) for household members
- DI1200 Self-employment income
- DI1500 Income from pensions
- DI1600 Regular social transfers (except pensions)

Consumption:

- HI0100 amount spent on food at home
- HI0200 amount spent on food outside home
- HI0210 amount spent on utilities

- HI0220 amount spent on consumer goods and services

Equity:

- DA2104 Value of non self-employment private business
  - HD1010 value of non-selfemployment not publicly traded businesses
- DA2105 Shares, publicly traded
  - HD1510 value of publicly traded share
- DA1140 Value of self-employment businesses

Unlisted Equity:

- DA2104 Value of non self-employment private business
- DA1140 Value of self-employment businesses

Listed Equity:

- DA2105 Shares, publicly traded

Housing:

- DA1110 Value of household's main residence. DA1110=HB0500 $*$ HB0900
  - HB0500 % of ownership of household main residence
  - HB0900 current price of household main residence
- DA1120 Value of other real estate property. DA1120=Sum of (HB270x $*$ HB280x) $+$ HB2900
  - HB270$x other property $x: % of the property belonging to household
  - HB280$x other property $x: current value
  - HB2900 additional properties current value

Bonds:

- DA2103 Bonds
  - HD1420 market value of bond

Deposits:

- DA2101 Deposits. DA2101=HD1110+HD1210

- HD1110 value of sight accounts
- HD1210 value of saving accounts

Debt:

- DL1000 Total outstanding balance of household's liabilities. DL1000=DL1100+DL1200.
  - DL1100 Outstanding balance of mortgage debt. DL1100=DL1110+DL1120
    - DL1110 Outstanding balance of HMR mortgages. DL1110=Sum of (HB170x) + HB2100
      - HB170$x HMR mortgage $x: amount still owed
      - HB2100 money still owed on additional HMR loans
    - DL1120 Outstanding balance of mortgages on other properties. DL1120=Sum of (HB370x) + HB4100
      - HB370$x other property mortgage $x: amount still owed
      - HB4100 money still owed on additional other property loans
  - DL1200 Outstanding balance of other, non-mortgage debt. DL1200=HC0220+ HC0320 + Sum of (HC080x) + HC1100
    - HC0220 amount of outstanding credit line/overdraft balance
    - HC0320 amount of outstanding credit cards balance
    - HC080$x non-collaterised loan $x: outstanding balance of loan
    - HC1100 total amount owed for additional non-collaterised loans

Pensions:

- Occupational pension plan non-defined benefit
  - Sum of PF0710 current value of all occupational pension plans that have an account.
  - Select only non-defined benefit pension (PF0800 = 2). PF0800 occupational plan has regular benefit in retirement (1 means yes, 2 means no).
- Voluntary pension.
  - Sum of PF0920 voluntary pension schemes - value of accounts
- Social security plans
  - Sum of PF0510 current value of all social security plans that have an account

Mutual Funds:

- DA2102 Mutual funds, total. DA2102=HD1320g OR sum of(HD1320a-f)
  - HD1320x market value of mutual funds.
    - a - Funds predominantly investing in equity
    - b - Funds predominantly investing in bonds
    - c - Funds predominantly investing in money market instruments
    - d - Funds predominantly investing in real estate
    - e - Hedge funds
    - f - Other fund types (specify)
    - g - Aggregate amount of all funds together

F. Role of Heterogeneity for Risk Premia

We wish to understand the role of heterogeneity by age for the transmission of monetary policy shocks on asset prices and risk premia. To do so, we compare our results to a representative agent model, featuring an infinitely lived household with a survival probability p who holds as an asset portfolio the holdings of the household sector on aggregate.[7]

Households exhibit Epstein-Zin utility functions defined over the single homogeneous consumption good, C_t. Let W_t denote wealth at time t. Household preferences can then be written as:

$$V_t = \left[C_t^{1-\frac{1}{\sigma}} + \beta E_t \left[p V_{t+1}^{1-\gamma} + (1-p) \phi_B^{1-\gamma} W_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\sigma}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{49}$$

As in the heterogeneous case, we have a bequest motive, whose strength is controlled by ϕ_B. When the agent dies, the flow utility is:

$$V_t \equiv \phi_B W_t \tag{50}$$

The labor income process is described by Equation 3-5, but we now set $f(a) = 1$ (i.e. we drop the deterministic age profile). The representative household still experiences idiosyncratic income shocks.

The complete optimization problem is then:

[7] p is chosen such that life expectancy conditional on being aged 24 is the same as that in the data

$$\max_{\{\theta^E_{i,t},\theta^H_{i,t},B^+_{i,t},B^-_{i,t},C_{i,t}\}_{t=0}^{\infty}} E\left[V_0\right]$$

V_0 is given by Equations 49 and 50 and is subject to the constraints given by Equations 9 - 15, the modified stochastic labour income process given by Equations 3 - 5, and the same structure of beliefs on future asset returns as the heterogeneous case.

A. Calibration

We re-calibrate the representative agent problem to target the same aggregate moments as in the heterogeneous agents case. We use the same non-calibrated parameters, and calibrate the remaining model parameters $(\gamma, \sigma, \phi_B, \iota, \sigma^H_{\text{idio}}, \sigma^E_{\text{idio}})$ to match the following aggregate outcomes:

1. Wealth/GDP
2. Housing/GDP
3. Equity/GDP
4. Net Borrowing/GDP

Notice that, compared to the heterogeneous agents case, we cannot separately target gross lending and gross borrowing, as we only have one agent that either borrows or lends. For this reason, we only target the Net Borrowing/GDP moment. The estimated parameters are shown in Table VII.

Table VII. Calibrated Parameters Rep

	γ	σ	σ^H_{idio}	σ^E_{idio}	sp	ϕ_{beq}
parameters	7.00	0.30	9.20	3.80	0.03	1.00

Table VIII. Aggregate Outcomes: Model vs. Data Rep

	Wealth/GDP	Housing/GDP	Equity/GDP	Bonds/GDP
data	1.76	1.11	0.20	-0.11
model	1.76	1.12	0.19	-0.10

As you can see, we choose the parameters to match close the empirical outcomes closely in Table VIII. The main difference vs. the heterogeneous agent calibration is that risk aversion, γ, declines, coupled with a rise in the idiosyncratic variance of the risky assets.