A Continuous Time Experiment on Linking Formation

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  - strategies and computations are complex
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  - multiple equilibria in many models
  - selection based on efficiency and dynamics: sharp predictions
- Goal: test these predictions.

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- Maintaining connections is costly.
- The value flow can be either one-way or two-way.
  - one-way: the link that agent i forms with agent j yields benefits solely to agent i.
  - two-way: the benefits accrue to both agents.

Theory (Bala and Goyal (2000))

- $N = \{1, 2, \ldots, n\}$ with $n \geq 3$
- Each player $i \in N$ chooses a set of links g_i with others, $g_i = (g_{i1}, \ldots, g_{ii-1}, g_{ii+1}, \ldots, g_{in})$, and $g_{ij} \in \{0, 1\}$ for any $j \in N \backslash \{i\}$.
- Thus links are unilateral in this game.
- A strategy profile $g = (g_1, g_2, .., g_n)$ specifies the links made by every player and induces a directed graph, g.

Payoff function

- one-way model: $\Pi_i(g) = V + \sum_{j \in C_i(g)} \delta^{d(i,j;g)} V - \eta_i(g) k$
- two-way model: $\Pi_i(g) = V + \sum_{j \in C_i(\bar{g})} \delta^{d(i,j;\bar{g})} V - \eta_i(g) k$
  - V represents the value of benefit from a connection.
  - $C_i(g)$ is the set of agents that i is path-connected to.
  - $\delta \in (0, 1]$ is the decay factor of value
  - $\bar{g}$ is the closure of g: $\bar{g}_{ij} = \max(g_{ij}, g_{ji})$ for every $i, j \in N$.
  - $d(i, j; g)$ is the length of the shortest path between i and j.
  - $\eta_i(g) = |\{j \in N : g_{ij} = 1\}|$ is the number of links i formed.
  - k is the cost of a link.

Treatments and theoretical predictions

- value of an agent: $V = 10$
- four treatments:
  - two-way, $n = 10$ ($\delta = 0.9,\ k = 20$)
  - two-way, $n = 50$ ($\delta = 0.9,\ k = 100$)
  - one-way, $n = 10$ ($\delta = 1,\ k = 20$)
  - one-way, $n = 50$ ($\delta = 1,\ k = 100$)
- δ: decay factor of value; k: cost per link
- $\frac{k}{n}$ kept constant across treatments

Efficient and Nash networks

For both $n = 10$ and $n = 50$:

- one-way: cycle network
- two-way: star network

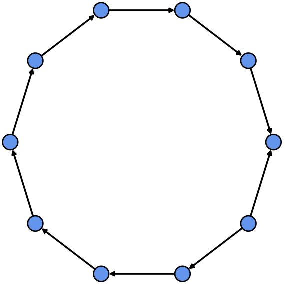

Figure: one-way

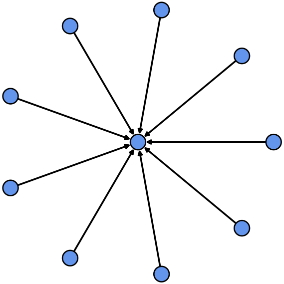

Figure: two-way

General Considerations

- Individuals face a complex decision.
  - compare costs and benefits of linking
  - challenging to compute the value of a link
  - when to make a change
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  - large evolving network
- very unclear what sorts of networks will actually emerge
- How does bounded rational decision-making at an individual level generate aggregate outcomes?

Related work

- existing work: small groups (4–8)
- A: simultaneous choice
  - Goeree et al. (2009): reject the two-way prediction
  - Falk and Kosfeld (2012): match in one-way but reject two-way model
  - Caria and Fafchamps (2020); Callander and Plott (2005): reject the one-way prediction

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- B: asynchronous choice:
  - Berninghaus et al. (2006): match prediction in two-way
  - Friedman and Oprea (2012): continuous time leads to high cooperation rate in repeated prisoner's dilemma game.

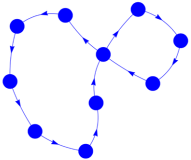
Our work

- unclear if these findings scale with size
- novelty of our work:
  - large and small groups
  - asynchronous decision in continuous time
  - one-way and two-way flow

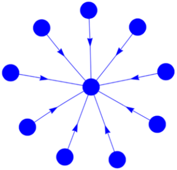
Experiment Design

- continuous time and asynchronuous decision
- 6 minutes a round
- At any instant in the 6-minute game, a subject can form/delete a link with any other subject.
- At any moment, each subject is informed about the network structure and about their own payoff.
- The first minute is a trial period and a time moment is randomly chosen from the last 5 minutes for payment.
- 4 groups per treatment and 6 rounds per group

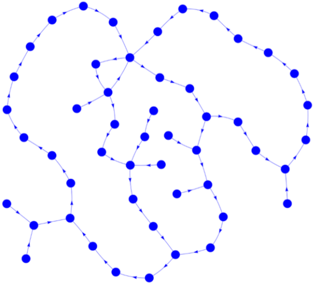
Experimental results — snapshots



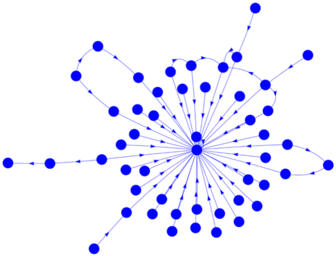
(a) $N = 10$ (one-way): minute 6



(b) $N = 10$ (two-way): minute 6



(c) $N = 50$ (one-way): minute 6



(d) $N = 50$ (two-way): minute 6

Social efficiency

$$SE(g(t)) = \frac{\sum_i \Pi_i(g(t))}{\sum_i \Pi_i(g^*)}$$

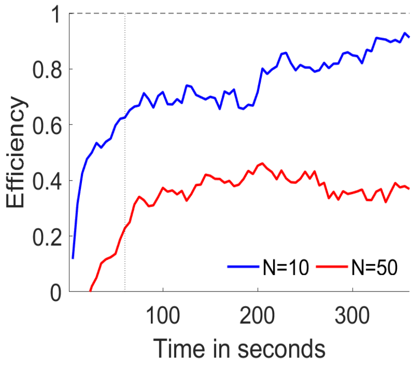


Figure: one-way flow

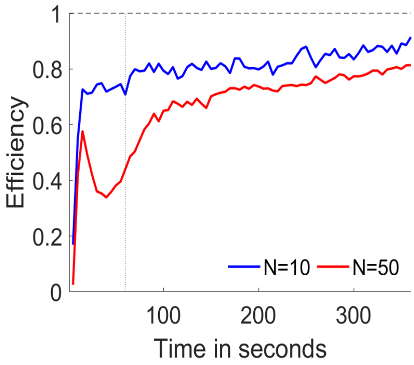
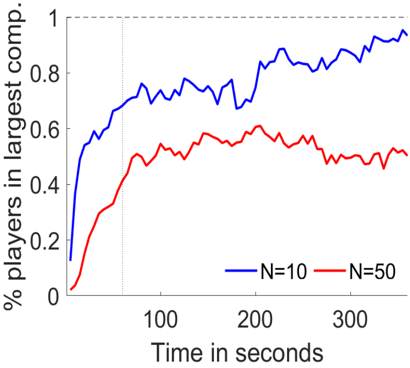


Figure: two-way flow

Size of the largest component (normalised)

- two-way: fract. in largest component of undirected network
- one-way: fract. in largest (strongly connected) component of directed network



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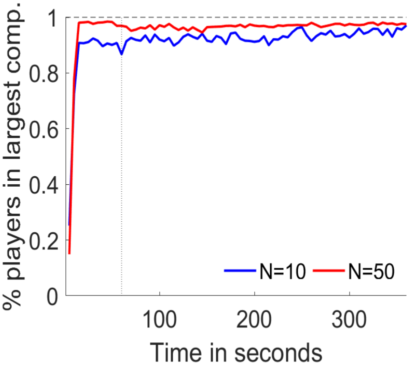


Figure: two-way flow

Summary: aggregate statistics

	one-way ($n = 10$)	one-way ($n = 50$)	two-way ($n = 10$)	two-way ($n = 50$)
social efficiency	77.2% (100%)	37.6% (100%)	82.8% (100%)	71.9% (100%)
average distance	3.59 (5)	6.13 (25)	2.11 (1.8)	2.81 (1.98)
med/max degree	0.578 (1)	0.145 (1)	0.086 (0.11)	0.031 (0.02)
% largest comp.	80.2% (100%)	52.8% (100%)	93.2% (100%)	96.8% (100%)
mean outdegree	1.10 (1)	1.28 (1)	0.93 (0.9)	1.31 (0.98)

equilibrium prediction in parenthesis.

Questions

- How do subjects do so well in the two way model?
- Why is there a breakdown of connectedness and efficiency loss in the one-way model (especially for $n = 50$)?

Summary: individual behavior

	one-way ($n = 10$)	one-way ($n = 50$)	two-way ($n = 10$)	two-way ($n = 50$)
active rate (AR)	9.46%	8.61%	10.3%	9.90%
AR given max pay ≤ 0	3.57%	3.37%	4.91%	4.60%
AR given **max pay > 0**	27.4%	16.1%	20.9%	16.2%
best response rate (BRR)	**76.0%**	**59.4%**	**66.2%**	**54.7%**
BRR given active	**36.6%**	**28.1%**	**31.6%**	**29.0%**

Individual-level performance in one-way is no worse than that in two-way

Noisy myopic best response simulations

- For $t \leq 60$, each player randomly makes action
- For $t > 60$, myopic best response with probability $1 - \epsilon$, random with probability ϵ

Figure: Efficiency for different error rate ϵ

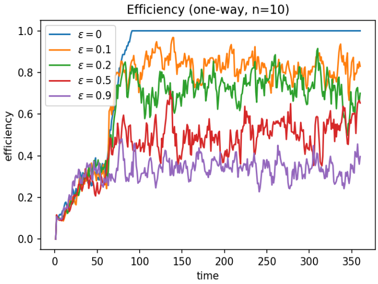
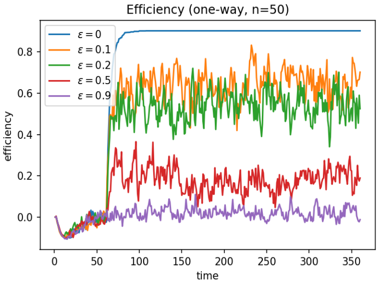


Figure: one-way ($n = 10$)



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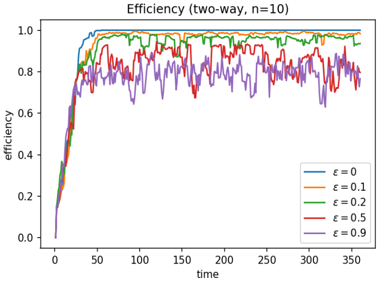

Figure: two-way ($n = 10$)

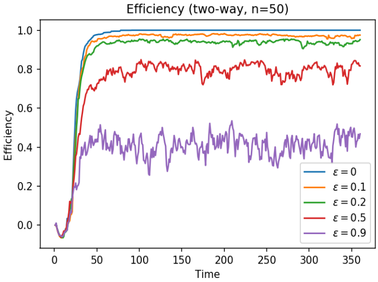

Figure: two-way ($n = 50$)

Individual behavior and aggregate outcome

- one-way model is sensitive to decision noises
- two-way model is robust to decision noises
- more difficult to achieve high social efficiency and be close to theoretical prediction in the one-way model than in the two-way model

Conclusion

- Theory predicts radically different structures in the two models.
- conducted continuous time network formation experiment
  - small groups — close to predictions: different from existing research
  - large groups — breakdown of connectedness and efficiency in the one way model, high efficiency and connectedness in the two way model
- Small noises in decision create great disruption in the one-way model.

Thank you!

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