# Sticky Information and the Taylor Principle

Optimal monetary policy in the Sticky Information model

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# Motivation

# This paper presents determinacy bounds on monetary policy

**Q**: How can we use the Sticky Information model (Mankiw and Reis, 2002) to study optimal monetary policy?

**A**: Analytically derive determinacy bounds on monetary policy

→ previously unobtainable

→ Existing analysis relies on time series methods:

- only approximates the solution
- misses important insights in model dynamics

**Sticky information**: agents update their information occasionally rather than instantaneously through a Calvo (1983)-mechanism

# Contribution

We derive

- a fully recursive expression of the sticky information model, SI-Phillips Curve
- implement the model in frequency domain
- determinacy bounds on monetary authority's policy rule

Woodford (2003, pp. 254-255), "... indeed, a large enough [response to] *either* [the output gap or inflation] suffices to guarantee determinacy."

→ This does not hold true for the Sticky Information model!

# Literature

# Sticky Information and monetary policy

Our paper brings together two literatures:

- **Sticky information**:
  - **Information frictions**: Mankiw and Reis (2007), Branch (2007), Coibion and Gorodnichenko (2015), Nason and Smith (2021), Cornand and Hubert (2022), Link et al. (2023), Andrade and Le Bihan (2013)
  - **Solution**: Trabandt (2007), Kiley (2007), Meyer-Gohde (2010), Huo and Takayama (2023), Kasa (2000), Jurado (2023)
- **Monetary policy**:
  - **In non-FIRE**: Ball, Mankiw, and Reis (2005), Roth and Wohlfart (2020) Iovino, La'O, and Mascarenhas (2022), Chou, Easaw, and Minford (2023), An, Abo-Zaid, and Sheng (2023), Angeletos and La'O (2020), Bernstein and Kamdar (2023), Chou, Easaw, and Minford (2023)

# The Sticky Information model

## The Sticky Information Phillips Curve (SIPC):

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})] \quad (1)$$

where $1-\lambda$ is the fraction of firms that obtains new information about the state of the economy and computes new path of optimal prices in period $t$; $\lambda$ is fraction of firms that uses old information and prices

## The IS equation:

$$y_t = E_t y_{t+1} - \sigma R_t + \sigma E_t \pi_{t+1} \quad (2)$$

## Interest rate rule:

$$R_t = \phi_\pi \pi_t + \phi_y y_t \quad (3)$$

$\rightarrow$The infinite regress of lagged expectations precludes a recursive representation

# A frequency domain approach

# The frequency domain

**What we do**:

- Express the model entirely in the frequency domain by applying the *z-transform* following Whiteman (1983)
- Solve the system of equations using Cauchy's residue theorem
- Determine boundary conditions on monetary policy

**Some Intuition**:

- Time domain: analyze mathematical functions of signals w.r.t. time
- Frequency domain: analyze mathematical functions or signal w.r.t. frequency
  - a **frequency** is *"the number of repetitions of a periodic process in a unit of time"*
  - at every point in time we get eventually different heights of frequencies

# A legitimate approach?

Following the **Riesz-Fischer Theorem** (see Sargent, 1987):

Let $\{c_n\}_{n=0}^{\infty}$ be a sequence of complex numbers for which $\sum_{n=0}^{\infty} c_n^2 < \infty$. Then, there exists a complex-valued function $f(\omega)$ defined for real $\omega$'s belonging to the interval $[-\pi, \pi]$ such that:

$$f(\omega) = \sum_{j=0}^{\infty} c_j e^{i\omega j}$$

where $f(\omega)$ is called the Fourier transform of the series $c_n$.

- Every object that exists in the time domain also has a representation in the frequency domain $\rightarrow$ We get two representations of the same information.
- Formulating the problem in the frequency domain is equivalent to formulating the problem in the time domain $\rightarrow$ It doesn't alter the model in itself!

# The z-transform

Consider the unilateral square-summable sequence of real numbers

$$\{c_n\}_{n=0}^{\infty}$$

where $c_n \in \mathbb{R}$ such that $\sum_{n=0}^{\infty} c_n^2 < \infty$. Following from the Wold's decomposition theorem, the stationary random variable always has an $MA(\infty)$-representation:

$$x_t = \sum_{n=0}^{\infty} c_n \epsilon_{t-n} = \sum_{n=0}^{\infty} c_n L^n \epsilon_t = c(L)\epsilon_t, \qquad \epsilon_t \sim WN(0, \sigma_\epsilon^2)$$

The z-transform of the variable is then given by the following function

$$x(z) = \sum_{j=0}^{\infty} c_j z^j$$

where $z \in \mathbb{C}$ and $z = e^{i\omega}$ for angular frequency $\omega \in [-\pi, \pi]$.

# Scaling in the z-domain

Scaling in the z-domain

$$x(\lambda z) = \sum_{j=0}^{\infty} \lambda^j c_j z^j,$$

connects initial conditions

$$x(0) = c_0$$

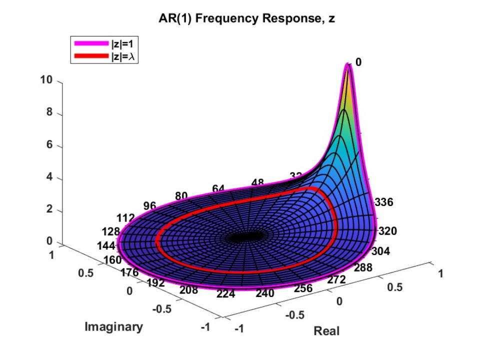
and unconditional moments

$$IFFT(|x(z)|^2 \sigma_\epsilon^2)_{|z|=1} = \sigma_x^2,$$

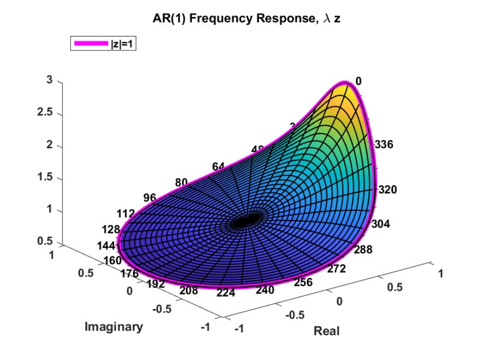
where $\sigma_\epsilon^2$ is the variance of $\epsilon_t$.

For $0 < \lambda < 1$, more distant (lower frequency) movements' contribution to moments reduced.

# Scaling in the z-domain: AR(1)




**(a)** $|H(z)|$ - $y(z) = \rho z y(z) + \epsilon(z)$,
*where* $|H(z)| = |\frac{1}{1-\rho z}|$




**(b)** $|H(z)|$ - $y(z) = \lambda \rho z y(z) + \epsilon(z)$,
*where* $|H(z)| = |\frac{1}{1-\lambda\rho z}|$

# A Comparison of the Phillips Curves: Sticky Price and Sticky Information

# The Sticky Price Phillips Curve

The Sticky Price Phillips Curve in the time domain:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

To get the frequency representation we use the Wiener-Kolmogorov prediction formula:

$$\mathcal{Z}\{E_t(x_{t+1})\} = \left[\frac{X(z)}{z}\right]_+ = \frac{1}{z}(X(z) - X(0))$$

Then, the Sticky Price Phillips Curve in the frequency domain:

$$\pi(z) = \beta \frac{1}{z}(\pi(z) - \pi_0) + \kappa y(z)$$

- Output gap at some frequency depends on inflation at the same frequencies
- no stickiness and dampening

# The Sticky Information Phillips Curve in the frequency domain

Sticky Information Phillips Curve in the time domain

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})]$$

→ Inflation depends on output, past expectations of current inflation and past expectations of current output growth.

Sticky Information Phillips Curve in the frequency domain

$$\pi(\lambda z) = \frac{1-\lambda}{\lambda}\xi y(z) + \xi(1-\lambda z)y(\lambda z)$$

→ Inflation in the dampened frequency depends on output in the regular frequency and output in the dampened frequency

# The Sticky Information Phillips Curve in the frequency domain

The SIPC in the frequency domain gives a recursive representation:

$$y(z) = \frac{\lambda}{\xi}\left(\frac{1}{1-\lambda z}\right)\pi(\lambda z) + \lambda y(\lambda z) \tag{4}$$

which holds for all frequencies $\omega$ resp. $z$ $(= e^{i\omega})$.

- Output gap at some frequency depends on output gap and inflation at dampened frequencies
- $\lambda$ introduces stickiness in the frequency domain: If the fraction of firms with an info update is low (high), output gap is driven more strongly by inflation at low (high) frequencies
- Output gap depends only on inflation at higher frequencies: vertical SIPC in the long run

# Monetary policy implications

# Determinacy in the frequency domain

Following Whiteman (1983)

- Impact response of forward looking variables will be pinned down
- by setting the residue of their z-transform to zero at removable singularities inside the unit circle

For example the solution for $y_t$ is a function $y(z)$ analytic on the unit disk:

$$y_t = \alpha E_t y_{t+1} + \epsilon_t \quad \overset{z}{\to} \quad y(z) = \alpha \frac{1}{z}(y(z) - y_0) + 1 \quad \Rightarrow \quad y(z) \quad = \left(1 - \frac{1}{\alpha} z\right)^{-1} \left(y_0 - \frac{z}{\alpha}\right)$$

If $|\alpha| < 1$, then for $z = \alpha$ there is a removable singularity inside the unit circle and we can solve for a boundary condition on $y_0$

$$\lim_{z \to \alpha} \left(1 - z\frac{1}{\alpha}\right) y(z) = 0 \quad \to \quad y_0 = 1$$

# Determinacy of the Sticky Price Model

$$\left(\begin{bmatrix}-\beta & 0\\ \sigma & 1\end{bmatrix} - z\begin{bmatrix}-1 & \kappa\\ \sigma\phi_\pi & 1+\sigma\phi_y\end{bmatrix}\right)\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} = \begin{bmatrix}-\beta & 0\\ \sigma & 1\end{bmatrix}\begin{bmatrix}\pi_0\\ y_0\end{bmatrix}$$

Decoupling as follows gives two singularities $\lim_{z\to 1/\gamma_i}(1-z\gamma_i)w_i(z)=0,\quad i=1,2$ (Cauchy's residue theorem)

$$\begin{bmatrix}w_1(z) & w_2(z)\end{bmatrix}' = V^{-1}\begin{bmatrix}\pi(z) & y(z)\end{bmatrix}'$$

which pins down initial conditions $\pi_0$ and $y_0$.

- Singularities inside the unit circle if $1-\frac{1-\beta}{\kappa}\phi_y < \phi_\pi$
  - $\Rightarrow$ Standard SP result, output gap targeting can substitute for inflation
- Taylor principle sufficient but not necessary for determinacy

# Determinacy of the Sticky Information Model

$$\begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} = \begin{bmatrix}\phi_\pi & \frac{1+\sigma\phi_y-\lambda}{\sigma}\\ 0 & \lambda\end{bmatrix} z \begin{bmatrix}\pi(z)\\ y(z)\end{bmatrix} + \begin{bmatrix}\frac{1-\lambda}{\lambda}\xi + \frac{1}{\sigma}\\ 0\end{bmatrix} y_0 + \begin{bmatrix}-\frac{\lambda}{\sigma\xi} & -\frac{\lambda}{\sigma}(1-\lambda z)\\ \frac{\lambda}{\xi} & \lambda(1-\lambda z)\end{bmatrix}\begin{bmatrix}\pi(\lambda z)\\ y(\lambda z)\end{bmatrix}$$

$x(\lambda z)$: Irrelevant for determinacy (a property of $x(z)|_{|z|=1}$)

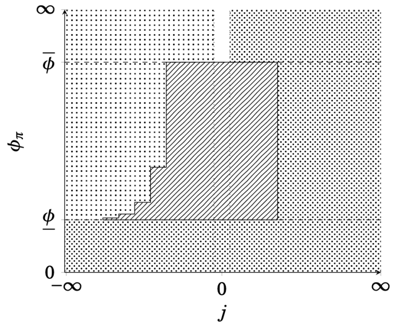
- Backward looking (recursive in frequency) SIPC gives one initial condition $\xi\frac{1-\lambda}{\lambda}y_0 = \pi_0$
- Of the two eigenvalues ($\lambda$ and $\phi_\pi$), one and only one must provide a removable singularity to pin down remaining condition
  - As the probability of no information update, $0 < \lambda < 1$, inside the unit circle
- Hence it must hold that $1 < \phi_\pi$
  - $\Rightarrow$ A stricter bound, output gap targeting cannot substitute for inflation
- Taylor principle sufficient and necessary for determinacy

# Extension: Extending to arbitrary horizons

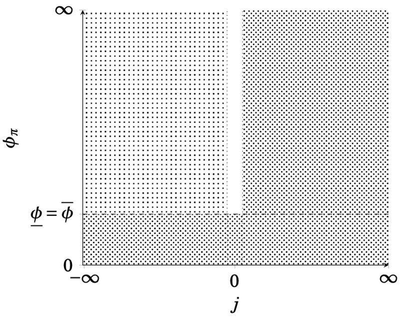
Consider the following general Taylor rule:

$$R_t = \phi_\pi E_t \pi_{t+j} + \phi_y(\alpha_y E_t y_{t+m}(1-\alpha_y) E_t \Delta y_{t+m})$$

Then the determinacy regions are given by:




(A) Sticky Price, $\underline{\phi} = 1$, $\overline{\phi} = 1 + 2\frac{1+\beta}{\kappa\sigma}$ (Loisel, 2022)




(B) Sticky Information, $\underline{\phi} = \overline{\phi} = 1$

# Extension: Interest rate smoothing

Consider the more general Taylor rule

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[\phi_\pi(\alpha_\pi \pi_t + (1-\alpha_\pi)E_t\pi_{t+1}) + \phi_y(\alpha_y y_t + (1-\alpha_y)\Delta y_t)\right]$$

- $0 \leq \rho_R < 1$ allows for interest rate smoothing
- $\alpha_\pi$: both contemporaneous ($\alpha_\pi = 1$) as well as future ($\alpha_\pi = 0$) inflation targeting
- $\alpha_y$: output gap level ($\alpha_y = 1$) as well as output gap growth ($\alpha_y = 0$) targeting

The IS and Taylor rule in frequency representation

$$\begin{aligned}
&[1-(1-\rho_R)\phi_\pi(1-\alpha_\pi) - z(\rho_R(1-\phi_\pi\alpha_\pi) + \phi_\pi\alpha_\pi)]\,\pi(z) \\
&= [1-(1-\rho_R)\phi_\pi(1-\alpha_\pi) - z\rho_R]\,\pi(0) - \frac{1-z\rho_R}{\sigma}y(0) \\
&+ \left[\frac{1}{\sigma} + \phi_y(1-\rho_R) - z\left(\frac{1+\rho_R}{\sigma} + \phi_y(1-\rho_R)(1-\alpha_y)\right) + z^2\frac{\rho}{\sigma}\right] y(z)
\end{aligned}$$

$\rightarrow$ This posits a relation between $\pi(z)$ and $y(z)$

# Extension: Determinacy Sticky Price Model

Sticky Price Determinacy Sticky Price Phillips curve

$$\left(1 - \beta\frac{1}{z}\right)\pi(z) = -\beta\frac{1}{z}\pi_0 + \kappa y(z)$$

- posits a long-run ($|z| = 1$) tradeoff between inflation and the output gap
- fragile, specification specific determinacy bounds
  $\rightarrow$ difficult to derive determinacy bounds with the general Taylor rule here
- Taylor principle not directly relevant

# Extension: Determinacy Sticky Information Model

Sticky Information Determinacy Sticky Information Phillips curve

$$y(z) = \frac{\lambda}{\xi}\frac{1}{1-\lambda z}\pi(\lambda z) + \lambda y(\lambda z) = \frac{1}{\xi}\sum_{j=1}^{\infty}\frac{\lambda^j}{1-\lambda^j z}\pi(\lambda^j z)$$

- long-run ($|z|= 1$) tradeoff between inflation and the output gap
- Recursive in frequency SIPC gives one initial condition $\xi\frac{1-\lambda}{\lambda}y_0 = \pi_0$
- Removable singularity must be in the IS + Taylor rule w.r.t. inflation

$$\lim_{z\to\gamma}\left[1-(1-\rho_R)\phi_\pi(1-\alpha_\pi) - z(\rho_R(1-\phi_\pi\alpha_\pi)+\phi_\pi\alpha_\pi)\right]\pi(z) = 0$$

- Is relevant for determinacy if and only if

$$|\gamma| = \left|\frac{1-(1-\rho_R)\phi_\pi(1-\alpha_\pi)}{\rho_R(1-\phi_\pi\alpha_\pi)+\phi_\pi\alpha_\pi}\right| < 1$$

- *No amount or type of output targeting can replace necessary focus on inflation!*

# Conclusion

- How can we analyze the SI model?
  $\rightarrow$ Derive a recursive representation in the frequency domain by applying the z-transform
- Benefits of analyzing models with lagged expectations in the frequency domain:
  - No need to extend the model's state-space
  - No need to solve for an infinite sequence of undetermined $MA(\infty)$ coefficients
  - Allows for a closed form results
  - Possible to obtain conditions on monetary policy to ensure determinacy
    $\rightarrow$ implications for stabilization of an economy
- Implications for Monetary policy:
  - Cannot substitute a reaction to real conditions for a reaction to inflation as in SP model!

**Thank you!**

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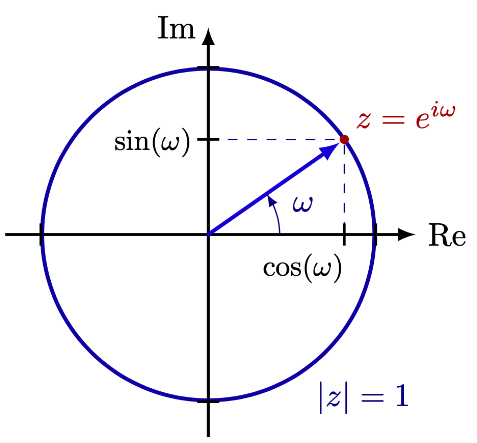
# Appendix

# Unit circle frequency domain



# Wiener-Kolmogorov for lagged expectations

The Wiener-Kolmogorov prediction formula for lagged expectations provides the following representation:

$$\mathcal{Z}\{E_{t-i}[x_t]\} = z^i \left[\frac{X(z)}{z^i}\right]_+ = X(z) - \sum_{j=0}^{i} X^j(0)z^j \tag{5}$$

where $X^j(0)$ is the $j'th$ derivative of $X(z)$ evaluated at the origin and $_+$ is the annihilation operator.

# Phillips Curve in the z-domain

The Sticky Information Phillips Curve in the time domain is given by:

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty} \lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})] \tag{6}$$

Using the Wiener-Kolmogorov prediction formula for lagged expectations, the sticky information Phillips curve can be expressed in the frequency domain as:

$$\pi(z) = \frac{1-\lambda}{\lambda}\xi y(z) + (1-\lambda)\sum_{i=0}^{\infty} \lambda^i \left[\pi(z) - \sum_{j=0}^{i} \pi^j(0) z^j + \xi(1-z)\left(y(z) - \sum_{j=0}^{i} y^j(0) z^j\right)\right] \tag{7}$$

# Phillips Curve in the z-domain

The infinite sums in (7) can be resolved by noting that:

$$\sum_{i=0}^{\infty}\lambda^i\left[x(z)-\sum_{j=0}^{i}x_jz^j\right]=\frac{1}{1-\lambda}x(z)-\sum_{i=0}^{\infty}\lambda^i\sum_{j=0}^{i}x_jz^j \tag{8}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\sum_{i=j}^{\infty}x_jz^j\lambda^i \tag{9}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}\lambda^ix_jz^j\lambda^j \tag{10}$$

$$=\frac{1}{1-\lambda}x(z)-\sum_{j=0}^{\infty}\frac{1}{1-\lambda}\lambda^ix_jz^j\lambda^j \tag{11}$$

$$=\frac{1}{1-\lambda}\left(x(z)-x(\lambda z)\right) \tag{12}$$

# Phillips Curve in the z-domain

In the time domain, a recursive representation of the lagged expectations of the endogenous variables is given by:

$$(1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[x_t], \quad x_t = \left(\sum_{j=0}^{\infty} x_j z^j\right)\epsilon_t \tag{13}$$

$$= (1-\lambda)\left(E_{t-1}[x_t] + \lambda E_{t-2}[x_t] + \lambda^2 E_{t-3}[x_t] + \ldots\right) \tag{14}$$

Applying the Wiener-Kolmogorov prediction formula to the lagged expectations, we get the frequency domain representation as:

$$\begin{aligned} & (1-\lambda)\left(x(z) - x_0 + \lambda(x(z) - x_0 - zx_1) + \lambda^2(x(z) - x_0 - zx_1 - z^2x_2) + \ldots\right) \\ = \quad & x(z) - \sum_{j=0}^{\infty}\lambda^i z^j x_j = x(z) - x(\lambda z) \end{aligned}$$

# Phillips Curve in the z-domain

Hence, the lagged expectations in (14) can be transformed from the time into the frequency domain as:

$$(1-\lambda)\sum_{j=0}^{\infty}\lambda^i E_{t-i-1}[x_{t-1}] = (1-\lambda)\left(\frac{z}{1-\lambda}x(z) - \frac{\lambda z}{1-\lambda}x_0 - \frac{(\lambda z)^2}{1-\lambda}x_1 - \ldots\right)$$
$$= zx(z) - \lambda z x(\lambda z)$$

Applying the z-transform we get the following representation of the Phillips curve:

$$\pi(z) = \xi\left(\frac{1-\lambda}{\lambda}\right)y(z) + \pi(z) - \pi(\lambda z) + \xi(1-z)y(z) - \xi(1-\lambda z)y(\lambda z)$$

such that

$$\xi\left(\frac{1}{\lambda} - z\right)y(z) = \pi(\lambda z) + \xi(1-\lambda z)y(\lambda z) \tag{15}$$

# Analytic function and singularity

**Definition: Analytic function**:

A function $f(z)$ is said to be analytic in a region $\mathcal{R}$ of the complex plane if $f(z)$ has a derivative at each point of $\mathcal{R}$ and if $f(z)$ is single valued.

**Theorem**:

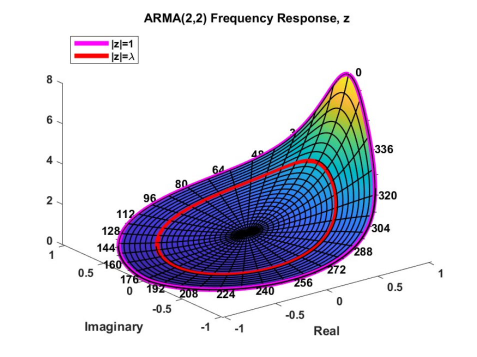
If $f(z)$ is analytic at a point $z$, then the derivative $f(z)$ is continuous at $z$.

**Definition: Singularity**

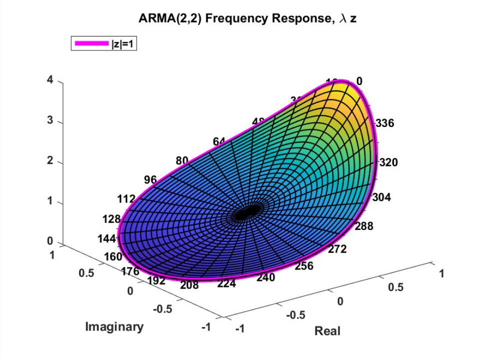
A singularity is a point at which a given mathematical object is not defined, or a point where the mathematical object ceases to be well-behaved in some particular way, such as by lacking differentiability or analyticity.

For example, the function $f(x) = \frac{1}{x}$ has a singularity at $x = 0$, where the value of the function is not defined, as involving a division by zero.

# Scaling in the z-domain: ARMA(2,2)




**(e)** $|H(z)|$ - $(1+\rho_1 z+\rho_2 z^2)y(z)$
$= (1+\theta_1 z+\theta_2 z^2)\epsilon(z)$,
*where* $|H(z)| \models |\frac{1}{1-\rho_1 z-\rho_2 z^2}|$




**(f)** $|H(z)|$ - $(1+\lambda\rho_1 z+\lambda^2\rho_2 z^2)y(z)$
$= (1+\lambda\theta_1 z+\lambda\theta_2 z^2)\epsilon(z)$
*where* $|H(z)| \models |\frac{1}{1-\lambda\rho_1 z-\lambda\rho_2 z^2}|$