Forward Guidance with Limited Commitment and Reputation *

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Abstract

We study monetary policy in an environment where 1) the central bank does not have access to a commitment technology, and 2) it has private information about its discount factor giving rise to type-based reputation. We study to what extent the central bank can engage in forward guidance to stabilize the economy at the Zero Lower Bound, and we find strong mitigation compared to the standard New Keynesian model. Limited commitment imposes an upper bound on the length of sustainable promises which implies that the model does not feature the forward guidance puzzle. We find that there is a non-monotonic relationship between the potency of forward guidance and the persistence of the ZLB when the central bank cannot commit, while there is a monotonically decreasing relationship with full commitment.

Keywords: The forward guidance puzzle, limited commitment, reputation
JEL classification: E58, E52

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1 Introduction

The welfare cost of the Zero Lower Bound (ZLB) depends on the effectiveness of the unconventional monetary policy tools that the central bank can resort to, among which forward guidance. In the baseline New Keynesian model, forward guidance is excessively expansionary: the effect on current output grows without bounds in the promised length of a zero-interest rate path – the so called "forward guidance puzzle" (Del Negro et al., 2012). Hence, any recession can be abated by a sufficiently generous promise about future interest rates (McKay et al., 2016b). In this paper, we take seriously one important, although overlooked, aspect of forward guidance: time inconsistency. We abandon the strong assumption that central banks can commit to a time-inconsistent policy, and we assume that households, looking infinitely into the future when making consumption-savings decisions, understand the incentives of the central bank to keep a promise. We study if this is sufficient to rationalize the forward guidance puzzle.

We make two departures from the baseline New Keynesian model. 1) We add limited commitment to capture the idea that central banks have an incentive to promise an expansionary future path of interest rates at the ZLB, but as soon as the economy leaves the ZLB, they would like to fulfill their actual mandate instead of sticking to the promised path of interest rates. 2) We add type-based reputation that we model by introducing private information of the central banker about her preferences. We assume that a central banker is only in charge of the central bank for a finite period, while the central bank is infinitely-lived. The central banker can be of two types: she may either be: 1) a benevolent one implying that she wants to maximize the lifetime utility of infinitely-lived households, or 2) a selfish central banker that only cares about the utility of the households when she is charge of the central bank. These types can be respectively seen as forward-looking and myopic. Modelling type-based reputation allows us to study how the effectiveness of forward guidance evolves over time as a function of the central banker's past actions. To the best of our knowledge, we are the first paper to explicitly model type-based reputation in a New Keynesian model and use it to study forward guidance.

Our results show that forward guidance is less potent once agents are allowed to understand the incentives of central banks to keep promises. We find that the effect of promising 2 quarters of forward guidance increases output gap with 0.35 percentage points under full commitment while the effect is 0.18 percentage points under limited commitment. This mitigation is increasing in the promised duration of keeping the interest rate at 0. For instance, the effect of promising forward guidance for 1 year is 1.12 percentage points under full commitment and 0.43 percentage points under limited commitment. There is a maximum level of forward guidance that is sustainable, and any promise being more generous than that will be futile, and it can even be harmful in the sense that a less generous promise, expected to be kept with positive probability, will be more expansionary than an overly generous one. For our chosen parameterization we find the maximum sustainable length of forward guidance to be 6 quarters. Contrary to the literature giving rise to a "discounted Euler equation," we show that the marginal effect of increasing the promised duration of 0-interest rates can be negative.

With full commitment, there is a monotonic relationship between the persistence of the ZLB and the effect of forward guidance. The higher the persistence, the lower the effect of forward guidance. We show that this monotonic relationship does not hold in a model with limited commitment. The intuition is that with full commitment, the sole effect of a more persistent ZLB is to decrease the probability of exiting to a more expansionary state where interest rates are still kept at zero. This reduces the effect of forward guidance. The persistence of the ZLB under limited commitment determines not only the probability of transitioning to an expansionary state but also the incentives of the central bank to keep a promise. For a too low persistence of the ZLB, the central bank faces inadequate incentives of keeping a promise rendering forward guidance futile under limited commitment while being very potent with full commitment. As the ZLB gets more persistent, the central bank may face sufficient incentives to keep promises causing the non monotonic relationship.

The effect of forward guidance as well at the severity of the recession at the ZLB both depend on the reputation of the central bank. For instance, a central banker that is known to be myopic will incur a fall in inflation of more than 0.2 % at the ZLB while a central banker, known to be forward-looking offers a fall in inflation half as big.

We find that the central bank is induced to give a more generous promise as limited commitment renders forward guidance less potent at the ZLB, and a promise can be reneged on in states where it is very costly to keep. With our chosen calibration, the optimal length of forward guidance is 18 months under limited commitment, while it is 1 year if the central bank can commit.

The paper is organized as follows. Section 2 provides a literature review. Section 3 introduces forward guidance in a New Keynesian model augmented with limited commitment and reputation. Section 4 presents the main results. Section 5 explores some quantitative features of the model. Section 6 relates our model to the literature on the discounted Euler equation. Section 7 extends the model to study the interaction between forward guidance and government debt. Section 8 concludes. Appendix C contains all the proofs.

2 Related Literature

Our paper is related to four strands of literature: time inconsistency of monetary policy, loose commitment, the forward guidance puzzle, and reputation. Since the seminal contribution of Kydland & Prescott (1977) and Barro & Gordon (1983), it has been known that optimal policies may be time inconsistent which implies that there are gains to commitment. Forward guidance is a perfect example of this. The ability of committing to a low future interest rate at the ZLB can enhance welfare but the policy is time-inconsistent: once the economy exits the zero lower bound, the central bank would be better off by setting a higher interest rate than promised to stabilize current economic outcomes. Related to this observation, Barthélemy & Mengus (2018) argues that central banks should increase inflation prior to the ZLB to make forward guidance more credible while Bhattarai et al. (2023) shows how quantitative easing can serve as a commitment device to keep future interest rates low. Following Stokey (1991) and Chari & Kehoe (1990), we study forward guidance in a model where central banks cannot commit to future policy, and the usefulness of forward guidance therefore depends on the extent to which it constitutes a sustainable plan in the sense that honoring past promises is sequentially optimal. Similar to Walsh (2018) and Nakata (2018), we exploit that the ZLB may be a recurrent event implying that it may constitute a sustainable plan as reputation concerns provide incentives to honor past promises.

Our paper is also related to the literature on loose commitment as developed in Debortoli & Nunes (2010), Debortoli & Lakdawala (2016), and Debortoli et al. (2014). In those papers it is assumed that the policy maker re-optimizes each period with some exogenous probability, while in our model the probability that a promise is kept is endogenous to the promise given. This endogeneity is key to solving the forward guidance puzzle.

Forward guidance has been the subject of a large and growing literature. Eggertsson & Woodford (2003) shows that the possibility of engaging in forward guidance may significantly reduce the welfare cost of the ZLB which is consistent with the view expressed in Bernanke (2020), while Bilbiie (2019) solves for the optimal level of forward guidance. Nevertheless, the large impact of forward guidance predicted by the plain vanilla NK model has been questioned by Del Negro et al. (2012), which has given rise to the well-known "forward guidance puzzle." This puzzle has later spurred a literature trying to solve it where a common approach has been to mute the importance of expected future output on current output. This can be achieved by assuming some kind of bounded rationality as in Farhi & Werning (2019), Gabaix (2020) and García-Schmidt & Woodford (2019) or lack of common knowledge as in Angeletos & Lian (2018). Another approach is to assume incomplete markets as McKay et al. (2016b), yet the degree to which market incompleteness provides a solution to the forward guidance puzzle depends on the cyclicality of risk as remarked by Werning (2015), and elaborated further by Acharya & Dogra (2020). One common implication of these papers is that they give rise to a "discounted" Euler equation which attenuates the effect of higher expected future output. Nakata et al. (2019) show that this implies the prediction that to obtain the same effect of forward guidance as in the standard model, interest rates should be kept low for longer time.

A final strand of literature that is relevant to our paper is reputation as pioneered by Milgrom & Roberts (1982) and Kreps & Wilson (1982). King & Lu (2022) estimates how the reputation of the FED has evolved assuming that the FED has private information about its discount factor. Dovis & Kirpalani (2020) studies a model where the government can be of a commitment type or an optimizing type. The optimizing type chooses policy sequentially which also happens to be the case in our framework for the selfish type. Contrary to them, we don't assume that some type can perfectly commit, and the subject of study is different as they study how the effect of fiscal rules depend on the reputation of the government. Two closely related papers are Loisel (2008) and Nakata (2018). Loisel (2008) shows how the inflation bias and the stabilization bias can be overcome by a reputation-concerned central bank if the punishment length is at least a few years. This can induce the central bank to implement otherwise time-inconsistent optimal monetary policy in the NK model. Nakata (2018) shows how the central bank can credibly commit to keeping the nominal interest rate low for an extended period of time after the ZLB if the ZLB is likely enough to be binding in the future. Contrary to Nakata (2018) and Loisel (2008), we directly model type-based reputation.

3 A Model of Forward Guidance

We consider a plain vanilla New Keynesian model where we assume that demand shocks can bring the economy to the ZLB. Following Eggertsson & Woodford (2003) we allow the central bank to engage in forward guidance once the nominal interest rate reaches the lower bound of zero. Deviating from most of the literature studying this policy, we don't assume that the central bank can commit to it. Instead, we think of forward guidance as cheap talk, and we model promise keeping as a strategic decision of the central banker who can be of two types: myopic or forward-looking. Uncertainty about the discount factor captures the idea that forward guidance is a policy entailing a dynamic trade off and hence valued differently by the two types of central bankers.

3.1 New Keynesian model

We consider a standard New Keynesian model summarized by the New Keynesian Phillips curve and the IS curve following the text book treatment by Woodford (2003) and Galí (2015).

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t \tag{PC}$$

$$y_t = \mathbb{E}_t[y_{t+1}] - \frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \tag{IS}$$

where κ is the slope of the New Keynesian Phillips curve, i.e. it measures the sensitivity of inflation to the output gap, and ρ is the coefficient of relative risk aversion.

The natural rate of interest (r_t^n) follows a persistent stochastic process, and we interpret changes to this rate as a demand shock which is common in the NK literature. For a sufficiently low realization of r_t^n, the economy hits the ZLB.

The IS curve illustrates the root of the "forward guidance puzzle." Future increases in expected output increase one-to-one output today. Hence, any change in future interest rates that are believed with probability 1 can have arbitrarily large effects on current output.

As is standard in the New Keynesian literature, we need to close the model by making an assumption about how the interest rate is determined. We consider two different options: 1) a Taylor rule which appears the standard assumption in the literature, and 2) optimal discretionary monetary policy. In practice, we find that our results are not sensitive to this choice, and we therefore choose to only report the results assuming discretionary policy[1] such that

$$i_t = \arg\min\{\ell(y_t, \pi_t) \text{ s.t. } (PC),\ (IS),\ i_t \geq 0\}$$

where we have imposed that the nominal interest rate cannot be negative, and ℓ is the instantaneous loss function to be defined later[2].

3.2 Forward Guidance

When the economy hits the ZLB, the central banker engages in forward guidance to stimulate economic activity, i.e. she promises to keep the interest rate at zero for τ periods after exiting the ZLB. We thus interpret forward guidance as follows.

Definition 3.1. *Forward guidance is an integer $\tau \in \mathbb{N}$ that prescribes the duration of a zero nominal interest rate path after exiting the ZLB.*

[1] The results assuming a Taylor rule are available upon request.

[2] See appendix A for a derivation of the chosen interest rate.

In words, we consider Odyssean forward guidance where the central bank gives a promise about future policy rates[3].

If the economy hits the ZLB again in a period shorter than τ, i.e. before the promise has expired, we assume that the promise is renewed with the same length τ. We do not take the duration of the promise as a strategic variable, but rather as exogenous.

We can now formally define the effect of forward guidance:

Definition 3.2. *Let $IR_X(\tau) = X(\tau) - X(0)$ be the impact response of forward guidance where X denotes any variable at the ZLB as a function of the promised length τ. Forward guidance is called effective if $IR_X(\tau) > 0$ for some variable X.*

Hence, the impact response is the change in a given variable at the ZLB when the central bank gives a promise of keeping the interest at 0 for τ periods. As emphasised by Farhi & Werning (2019), the response of the output gap grows without bounds in the length of a promise in the standard New Keynesian model if the central bank has access to a commitment technology. In this paper we show how the effect of forward guidance remains bounded for any τ.

3.3 Types, Promise Keeping and Reputation

Monetary policy is conducted by a central banker who can be of two types: 1) she can be a benevolent central banker who cares about consumer welfare even when she is not in charge, or 2) she can be a selfish one who only cares about consumer welfare during her mandate. We assume that a mandate lasts for 1 period causing the benevolent central banker to discount the future as private agents, while the selfish one will have a discount factor of zero. We can thus think of these as being forward-looking and myopic, respectively. We assume that δ, the discount factor of the central banker, is private information to her. We suppose that the central banker is in charge of suggesting her successor to a committee, and she proposes someone with the same type as herself. However, with some small probability γ, the committee chooses someone with a different type giving rise to stochastic and persistent types.

[3] An alternative is Delphic forward guidance where the central gives a promise directly on future output and inflation.

Each central banker minimizes a loss function that is quadratic in inflation and the output gap.

$$L\big(\{y_t\}_{t=0}^{\infty}, \{\pi_t\}_{t=0}^{\infty}\big) = \sum_{t=0}^{\infty} \delta^t \Big[\underbrace{\lambda \cdot (y_t - \bar{y})^2 + \pi_t^2}_{\ell(y_t,\pi_t)} \Big], \quad \lambda = \frac{\kappa}{\theta} \quad \text{and} \quad \bar{y} = \frac{\Phi_y}{\omega + \rho^{-1}}$$

where the notation follows Woodford (2003). λ is the relative weight on output stabilization and $\bar{y} > 0$ is the targeted output gap. The parameter $\bar{y}$ captures that there may be inefficiencies in the economy such that the central seeks to stabilize output at a level which is higher than the natural one and thereby target a positive output gap. We include a positive output gap target to counteract the deflationary forces caused by the ZLB. θ is the elasticity of substitution between intermediate goods, Φ_y is a measure of the distortions in the economy[4], and ω depends on the disutility of labor and the returns to scale to production. For an appropriate choice of the weight on output gap (λ) and the targeted deviation in output from the natural level ($\bar{y}$), it can be shown that this loss function represents a quadratic approximation to the level of expected utility of the representative household in the rational-expectations equilibrium under some regularity conditions[5] (Woodford, 2003).

The above intertemporal loss function plays an important role once the economy exits the ZLB, and the central banker decides whether to honor a promise or to break it and conduct discretionary monetary policy. The trade off she faces is the following: on the one hand, by keeping the promise, inflation and the output gap are too expansionary compared to what could have been achieved by setting the interest rate according to optimal discretion. All else equal, this provides the central banker with an incentive to renege on her promise. On the other hand, by keeping the promise the central banker may gain reputation as captured by the private sector's assessment about her type. This evolves over time as a function of the behavior of the central banker and the state of the economy. A state has three dimensions:

Definition 3.3. *A state is a triplet* (r_t^n, μ_t, P_t) *such that:*

[4] This depends on the monopoly power as well as the degree of taxation

[5] The conditions are that the disturbances are small enough, the outcomes are close enough to the allocation around which the expansion is taken, and finally that distortions are small enough

- $r_t^n \in \mathbb{R}$ *is the natural interest rate at time* t
- $\mu_t \in \Delta\big(\{0, \beta\}\big)$ *is the private agents' belief distribution over types at time* t
- $P_t \in \mathcal{P} = \{0, 1, \ldots, \tau\}$ *is the current period of the promise at calendar time* t*.*

The first dimension is the natural interest rate r_t^n. This determines whether the economy hits the ZLB or not in a given period[6]. The second dimension is the private sector assessment about the distribution of types, μ_t, namely reputation. The third dimension is the period P_t of the promise in place. The state $P_t = 0$ means that there is no promise in place at time t, while for instance $P_t = 3$ means that at time t, the central bank has so far kept a promise for 3 periods. The state contains all the payoff relevant variables as inflation and output gap are pinned down by the natural interest rate, reputation and monetary policy.

For each state of the economy, the central banker needs to decide whether or not to honor a past promise. For this decision, we restrict attention to stationary Markov strategies for the rest of this paper following Maskin & Tirole (2001).

Definition 3.4. *Fix a promise* τ*. For each calendar period* t *and type* δ*, a strategy* σ *prescribes the probability of transitioning from the promise period* $P_{t-1} \in \mathcal{P}$ *to the promise period* $P_t = P_{t-1} + 1 \in \mathcal{P}$ *with natural real interest rate* r_t^n *and reputation* μ_{t-1}*:*

$$\begin{aligned} \sigma : \{0, \beta\} \times \mathbb{R} \times \Delta\big(\{0, \beta\}\big) \times \mathcal{P} &\longrightarrow [0, 1] \\ (\delta, r_t^n, \mu_{t-1}, P_{t-1}) \qquad &\longmapsto \qquad \mathbb{P}(P_t | P_t - 1, r_t^n, \mu_{t-1}, \delta) \end{aligned}$$

We focus on pure strategies, so the possible transition probabilities are either 0 or 1. All actions are publicly observed, so the private sector will learn about the likelihood of each possible type, and the central bank perfectly tracks the evolution of agent's beliefs. The prior distribution $\mu_0 \in \Delta\big(\{0, \beta\}\big)$ is common knowledge. The private sector is composed of Bayesian agents, such that beliefs after promise keeping evolve according to:

$$\mu_t(\delta; r_t^n, \mu_{t-1}, P_t) = \sum_{\hat{\delta}} \frac{\sigma(\hat{\delta}, r_t^n, \mu_{t-1}, P_t) \cdot \mu_{t-1}(\hat{\delta})}{\sum_{\tilde{\delta}} \sigma(\tilde{\delta}, r_t^n, \mu_{t-1}, P_t) \cdot \mu_{t-1}(\tilde{\delta})} \cdot \Gamma(\hat{\delta}, \delta)$$

[6] Whether the ZLB binds or not is endogenous. If forward guidance is very effective and promises are kept more often, the ZLB is less likely to bind for the same realization of the demand shock.

where $\Gamma(\tilde{\delta}, \delta)$ is the transition probability from type $\tilde{\delta}$ to type δ and for any type δ it satisfies $\Gamma(\delta, \delta) = 1 - \gamma$.

If central bankers play pooling strategies or there is no promise in place, then no learning occurs, while the reputation μ_t moves towards the stationary belief distribution due to the transition probability. If, instead, the central bankers play separating strategies, the decision to keep a promise or not is informative about the type of the central banker.

Summarizing, the economy can be in three different scenarios. It can be at the *ZLB* in which case the central banker sets the interest rate at 0, and she engages in forward guidance. When the economy exits the ZLB, the economy can then be in two different scenarios. The central banker can choose to keep her promise in which case the economy transitions to *promise keeping*. Alternatively, the central banker can choose to break her promise causing the economy to transition to optimal discretion, which we dub *normal times*. In Figure 1 below, we illustrate graphically the transitions starting at the ZLB.

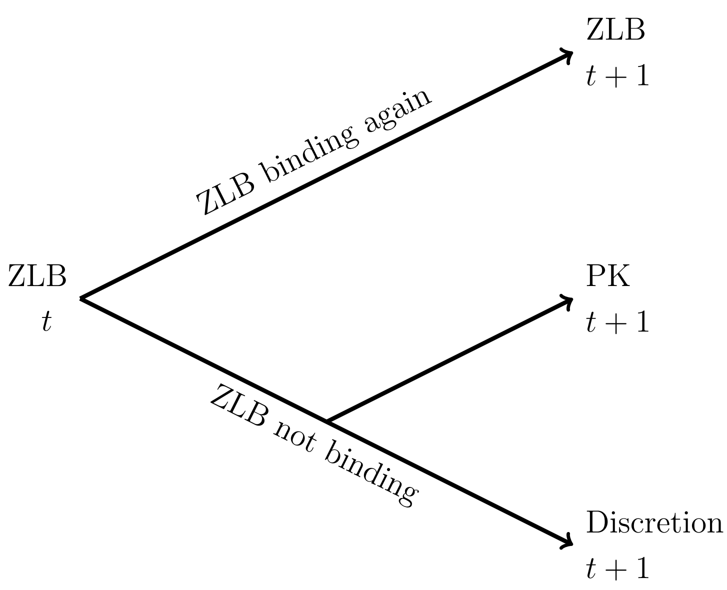

Figure 1: Transitions starting at the ZLB

The strategic of the central bank is whether to transition to promise keeping or discretion once a promise is in place. There is no strategic decision if the ZLB is binding or we are in normal times.

3.4 Equilibrium

An equilibrium in our limited commitment-reputation model has to constitute both a macroeconomic equilibrium, in the sense that the macroeconomic variables are given by the New Keynesian model, and a strategic equilibrium such that the strategies of the central bankers together with the private sector's beliefs constitute a Perfect Bayesian Equilibrium.

Definition 3.5. *An equilibrium is a tuple (σ, μ, y, π, i) such that:*

- *Given strategies and beliefs (σ, μ), output gap y, inflation π, and nominal interest rates i are consistent with the New Keynesian model.*
- *Given (y, π, i), the pair (σ, μ) is a Perfect Bayesian Equilibrium:*
  - *Beliefs μ are consistent with the strategies σ of the central bankers on the equilibrium path.*
  - *Strategies σ of the central bankers are optimal given the beliefs μ.*

To solve for the macroeconomic equilibrium, we first notice that for each pair (σ, μ) there is a subset $Z_{\sigma,\mu} \subset \mathbb{R}$ such that the ZLB binds if and only if $r_t^n \in Z_{\sigma,\mu}$. At the ZLB, output gap and inflation are given by the New Keynesian Phillips curve and the IS curve, imposing that the nominal interest rate is zero. We report the full set of equations characterizing the macroeconomic equilibrium in Appendix B. Below we report the expression for the IS curve at the ZLB.

$$y_t^Z = \mathbb{E}[y_{t+1}] + \frac{1}{\rho}(\mathbb{E}[\pi_{t+1}] + r_t^n)$$

where expected output gap is given by[7]

$$\mathbb{E}[y_{t+1}] = \mathbb{E}\left[\underbrace{y_{t+1}^Z \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\}}_{(a)} + \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\}\left(\underbrace{y_{t+1}^P \mathbb{E}_\delta^{\mu_t}[\sigma(\delta)]}_{(b)} + \underbrace{y_{t+1}^D \mathbb{E}_\delta^{\mu_t}[1-\sigma(\delta)]}_{(c)}\right)\right]$$

[7]Expected inflation can be obtained similarly.

The operator $\mathbb{E}$ denotes the expectation over r_{t+1}^n given r_t^n, $\mathbb{E}_\delta^{\mu_t}$ denotes the expectation with respect to δ using distribution μ_t. Z denotes *zlb*, P denotes *promise keeping* and D denotes *discretion*. We emphasize that output gap and inflation depend on the state of the economy but we have suppressed it for the sake of exposition.

To better describe how to calculate the expectation, we decompose expected output gap into three terms. The term (a) corresponds to output gap whenever the ZLB is binding again. Terms (b) and (c) refer to a realization of the natural rate of interest such that the ZLB is not binding. Term (b) is output gap under promise keeping, i.e. the output gap that prevails if the central bank chooses $i_{t+1} = 0$, multiplied by the probability that the central bank keeps its promise, $\mathbb{E}_\delta^{\mu_t}[\sigma(\delta)]$. For notational convenience, we have suppressed the dependence of this probability on the state of the economy. Finally, term (c) is output gap in normal times, which is multiplied by the probability that the central bank breaks the promise, $\mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta)]$.

Having explained how to solve for the macroeconomic equilibrium at the ZLB, we now explain how to solve for the macroeconomic equilibrium in normal times, i.e. for any shock $r_t^n \notin Z_{\sigma,\mu}$ and $P_t = 0$. As the central bank follows optimal discretion, monetary policy is set to minimize the instantaneous loss function such that the first order condition reads:

$$-\kappa\pi_t = \lambda\left(y_t - \bar{y}\right)$$

where we have imposed that the Lagrange multiplier is zero as the economy is away from the ZLB. Output gap and inflation are then given by the New Keynesian Phillips curve and the IS curve. The full system of equations is reported in Appendix B.

Finally, the economy can also be in promise keeping, i.e. it is away from the ZLB ($r_t^n \notin Z_{\sigma,\mu}$) and there is a promise in place ($P_t > 0$). The central banker then decides whether to keep the promise or not for another period as captured by the strategy σ. Finally, the economy can be in the last period of promise keeping out of the ZLB, in which $P_t = \tau$ and no strategic choice is made. This follows as the economy simply transitions to either discretion or the ZLB depending on the realization of the natural interest rate.

The problem of the central bank given the state of the economy at the ZLB is given by

$$L_t^Z(\delta) = \ell(y_t^Z, \pi_t^Z) + \delta \mathbb{E} \Bigg[\underbrace{L_{t+1}^Z(\delta) \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\}}_{(a)} + \min_{x \in \{0,1\}} \Big\{ \underbrace{x L_{t+1}^P(\delta)}_{(b)} + \underbrace{(1-x) L_{t+1}^D(\delta)}_{(c)} \Big\} \mathbb{1}\{r_{t+1} \notin Z_{\sigma,\mu}\} \Bigg]$$

The loss at the ZLB depends on the current realization of output and inflation and the discounted expected future loss. The future expected loss can be decomposed into 3 terms. Term (a) reflects the loss of being at the ZLB, while term (b) reflects the loss of promise keeping. Finally, term (c) is the loss from discretion. The problems in normal times and in promise keeping are carefully described in Appendix B.

We notice that the myopic central banker faces very poor incentives to keep her promise as she does not value any reputation gains to the central bank in the future. Surprisingly, she may be persuaded to keep her promise as improving reputation today is useful for current economic outcomes. *A priori*, it is not clear if optimal discretion with a low reputation is preferable to promise keeping (i.e. a sub optimal interest rate) with a higher reputation but we find this to be true for all reasonable parameterizations of our model. Yet, there are parameterizations such that this is not the case. To rule out these possibilities, we simply assume that the myopic central banker never keeps her promise.

Assumption 3.1. *For any promise τ, the myopic central banker never keeps her promise, i.e, in any equilibrium we have $\sigma(\delta = 0, r_t^n, \mu_t, P_t) = 0$ for any r_t^n, μ_t and P_t.*

This assumption allows us to simply focus on pooling equilibria where no type keeps her promise, separating equilibria where only the forward-looking type keeps the promise, and semi-separating equilibria where the forward-looking type keeps her promise in some states of the world. We then verify that with the chosen parameterization indeed the myopic type never wants to keep her promise.

4 Solution to the Forward Guidance Puzzle

A New Keynesian model augmented with limited commitment and reputation does not suffer from the forward guidance puzzle. While some level of forward guidance at the ZLB remains useful, we find that an unreasonably expansionary promise cannot be sustained as an equilibrium strategy.

When the central banker decides whether to keep the promise or conduct optimal discretionary policy, she faces the following trade off: on the one hand, keeping a promise causes an excess expansion today. On the other hand, keeping the promise helps the central bank gain reputation which is useful to stabilize output gap and inflation in future ZLB events. We find that there are states of the world where the reputation gains are not important enough to outweigh the cost of an excess expansion:

Proposition 4.1. *There exists* $\hat{R} < \infty$ *such that in any equilibrium, for all* $r_t^n > \hat{R}$, μ_t, P_t *and* δ *we have* $\sigma(\delta, r_t^n, \mu_t, P_t) = 0$.

The intuition is that for a very positive demand shock, the expansion that the economy would undergo with a zero interest rate would be too costly. It would therefore be preferable to deviate to discretion to mitigate the boom in output gap and inflation. In fact, while the loss of the central banker is always bounded under optimal discretion for any realization of the demand shock, the loss goes to infinity as the realization increases if the interest rate is kept at zero.

We further find that the existence of even a minimal cost of keeping the promise is enough to ensure the existence of a pooling equilibrium where neither type keeps the promise:

Proposition 4.2. *For any promise* τ*, there exist equilibrium strategies* σ *such that for any* δ, r_t^n, μ_t *and* P_t*, we have that* $\sigma(\delta, r_t^n, \mu_t, P_t) = 0$.

This is due to the fact that pessimistic off-path beliefs render deviations to promise keeping futile as the reputation channel is muted. Combined with Assumption 3.1, we can therefore restrict attention to semi separating equilibria where the forward-looking central banker keeps her promise in some states of the world only, and the myopic central banker never keeps her promise. There can be no other equilibrium in which forward guidance is effective.

While an excess expansion provides the central banker with a reason to renege on past promises, the reputation channel can be strong enough to provide her with an adequate incentive to honor them. To better understand how this channel works, suppose that the forward-looking central banker keeps her promise in some states of the world. In that case, the effect of forward guidance increases in the probability that agents assign to the central banker being of the forward-looking type. This, in turn, provides the central banker with a reason to indeed keep the promise. For a strong enough reputation channel, there exists a semi separating equilibrium where the forward-looking central banker keeps her promise in some states of the world. We verify that, with our preferred parameterization of the model, such an equilibrium indeed exists, i.e. promises are kept in states of the world where the realization of the demand is sufficiently moderate. This then leads of the main result of the paper: for a too generous promise, the effect of forward guidance is zero.

Proposition 4.3. *There exists $\bar{\tau} \in \mathbb{N}$ such that for any variable X, for all $\tau > \bar{\tau}$, $IR_X(\tau) = 0$.*

We thus find that there is a maximum level of forward guidance that is sustainable in the sense that promises are kept in some states of the world. Any promise with a longer duration will not be believed and hence it will fail to stabilize economic outcomes at the ZLB.

Intuitively, the cost of keeping a promise is increasing in its length. A generous promise, if believed, implies a significant expansion once the economy exists the ZLB. Hence, for a too generous promise agents understand that it cannot be kept, and therefore keeping it does not constitute a strategic equilibrium. The only equilibrium is therefore pooling where promises are never kept rendering forward guidance ineffective. From Proposition 4.3, we immediately have the following corollary:

Corollary 4.1. *The marginal effect of forward guidance is negative for $\tau = \bar{\tau}$:*

$$IR_y(\bar{\tau} + 1) - IR_y(\bar{\tau}) < 0$$

Once a promise goes from being sustainable to unsustainable, the effect of forward guidance drops to zero, and hence the marginal effect of increasing the duration is negative.

This corollary is an example of how our model is different from the literature giving rise to a "discounted Euler equation" where the marginal effect of forwards guidance is always weakly positive.

We provide a graphical representation of Proposition 4.3 and Corollary 4.1 in Figure 2. We find that there exist semi separating equilibria where the forward-looking central banker keeps her promise for moderate demand shocks for a duration of the promise being no higher than 3 periods corresponding to 18 months[8].

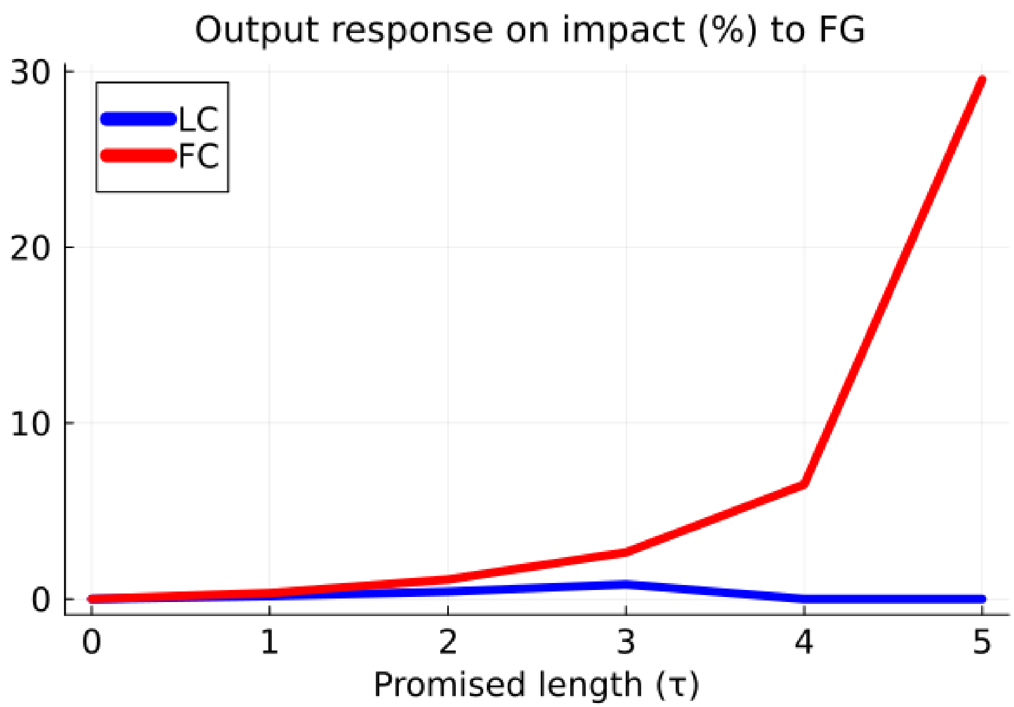

Figure 2: Mitigation of forward guidance effect for $\mu_0(\beta) = 0.5$.

Despite forward guidance being a useful policy tool in the semi separating equilibrium, the effect is much mitigated compared to the full commitment case. This happens for two reasons. First, households realize that the myopic central banker will never keep her promise. Second, they realize that there are states of the world where it is too costly for any central banker to keep her promise regardless of her type.

[8]We assumed that the myopic central banker would never keep her promise, and we verify that indeed this is true.

5 The Effect of Forward Guidance

We choose to parameterize the model such that one period corresponds to two quarters. We calibrate the persistence of the demand shock (ρ_r), the variance of the demand shock (σ_r^2), and the targeted output gap, while we choose values that we consider standard in the New Keynesian literature for the remaining parameters. We display all of the values in Table 1 below.

Parameter	Value	Description
β	0.99	Discount factor of private agents
κ	0.02	Slope of Phillips Curve
$\bar{y}$	0.5%	Output gap target
ρ	2.0	Coefficient of relative risk aversion
λ	0.11	Weight on output gap
$\mu_0(\beta)$	0.5	Ex-ante probability of high type
γ	0.02	Transition probability of types
σ_r	0.0185	Standard deviation of ξ_t
ρ_r	0.56	Auto correlation of r_t^n
$\bar{r}$	1%	Average natural real interest rate

Table 1: Parameter values

We assume that the natural rate of interest follows an AR(1) process:

$$r_t^n = (1-\rho_r)\bar{r} + \rho_r r_t^n + \sigma_r \xi_t, \quad \xi_t \sim \mathcal{N}(0,1)$$

We discretize the AR(1) process by the Tauchen (1986) method where we choose three values for the shock to the natural interest rate. We choose three to have a realization such that the ZLB binds, a realization where the forward-looking central banker keeps the promise, as well as a realization where it is excessively costly for any central banker to keep the promise. This is sufficient to illustrate the main points of the paper.

We calibrate the persistence of the demand shock such that the model with no forward guidance matches the auto correlation in US output gap observed over the period 1982-2020. For the standard deviation of the innovation (σ_r) to the natural interest rate, we choose a value to match the average output gap observed at the height of the

Great Financial Crisis and at the height of the COVID Crisis in the US.

We choose to calibrate the output target to match the average output gap in the US observed in the period 1982-2020. If we did not include a positive target for the output gap, the model would predict a too low average output gap due to the deflationary bias introduced by the ZLB as explained by Eggertsson (2006).

We retrieve data from FRED on real GDP in the US over the period 1947-2022, and we then apply the HP filter (Hodrick & Prescott, 1997) to detrend this data where we choose a smoothing parameter of 400[9]. We obtain an auto correlation of output gap of 0.54 at a semi annual level in the period 1982-2020. We find that the output gap was -3.1 % at the height of the Financial Crisis while it was -9.5 % at the height of the COVID crisis. The implied values for the persistence of the demand shock (ρ_r) and the standard deviation for the innovation (σ_r) are 0.56 and 0.0185, respectively. The average output gap over this period was -0.2 % implying a value of the targeted output gap of 0.5 %.

For the remaining parameters, we choose values that we consider standard in the literature. First, as we consider a semi annual model, we choose β, the discount factor of private agents, to be 0.99, and hence the value of $\bar{r}$, i.e. the natural rate of interest is equal to $-\log(\beta)$, which corresponds to a steady state interest rate of around 2 % a year. We choose a value of the slope of the Phillips curve (κ) to be 0.02. For the coefficient of relative risk aversion (ρ), we choose a value of 2.

5.1 Reputation

The existence of a semi-separating equilibrium implies that the effect of forward guidance depends on reputation. The higher the probability agents assign to the central banker being the forward-looking type, the larger the effect of forward guidance on output as shown in Figure 3. We also report the effect of forward guidance under full commitment in red for comparison.

[9] We explain this procedure more carefully in appendix D

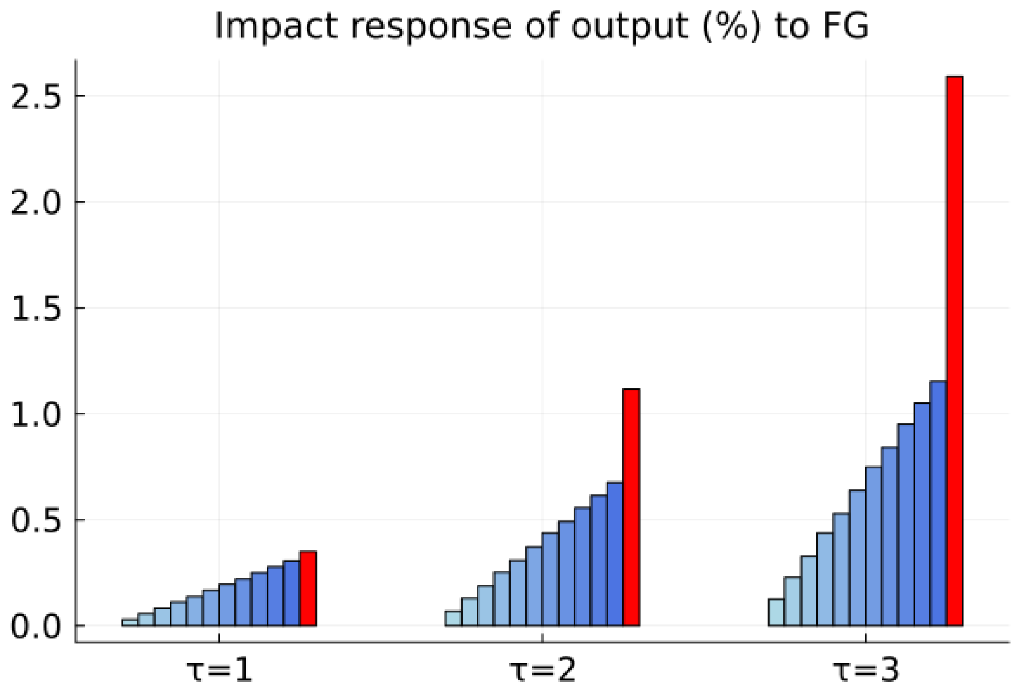

Figure 3: Reputation and forward guidance. In red, the effect of forward guidance under full commitment; in blue, the effect of forward guidance under limited commitment. The darker the blue, the higher the belief $\mu_0(\beta)$, ranging from 0 to 1, with increments of 0.1.

From Figure 3 we see that for $\mu_0(\beta) = 0$, forward guidance is a futile policy tool as the households expect the central banker to not keep the promise. It is then monotonically increasing in the belief that the central banker is the forward-looking type. The effect of forward guidance with limited commitment is mitigated compared to full commitment, even if households believe that the central banker is the forward-looking type (i.e. $\mu_0(\beta) = 1$). This occurs as households realize that there are realizations of the natural interest rate such that it is too costly to keep the promise for the forward-looking type as previously stated. This mitigation effect is not very strong for $\tau = 1$. The intuition is that given that the economy is at the ZLB, it is considered very likely that the economy will not be in an excessive boom in the following period. That is, the realization of the demand shock is likely to be only moderate once the economy exits the ZLB. We see a stronger mitigation effect for larger values of τ when we compare limited commitment to full commitment. The reason is that for higher values of τ, it is more likely that the economy will be hit by a very positive demand shock in which case even the forward-looking central banker will have to deviate from her promise rendering forward guidance less credible.

To illustrate the dynamics of the model, we consider a situation where households initially hold a belief of 0.5 that the central banker is forward-looking, and the economy is in normal times for the first 4 periods while in period 5 the ZLB hits and the central banker engages in forward guidance with $\tau = 1$. For the purpose of illustration, we assume that the ZLB only binds in periods 5. In Figure 4, we show how beliefs evolve depending on whether the central banker keeps her promise or not.

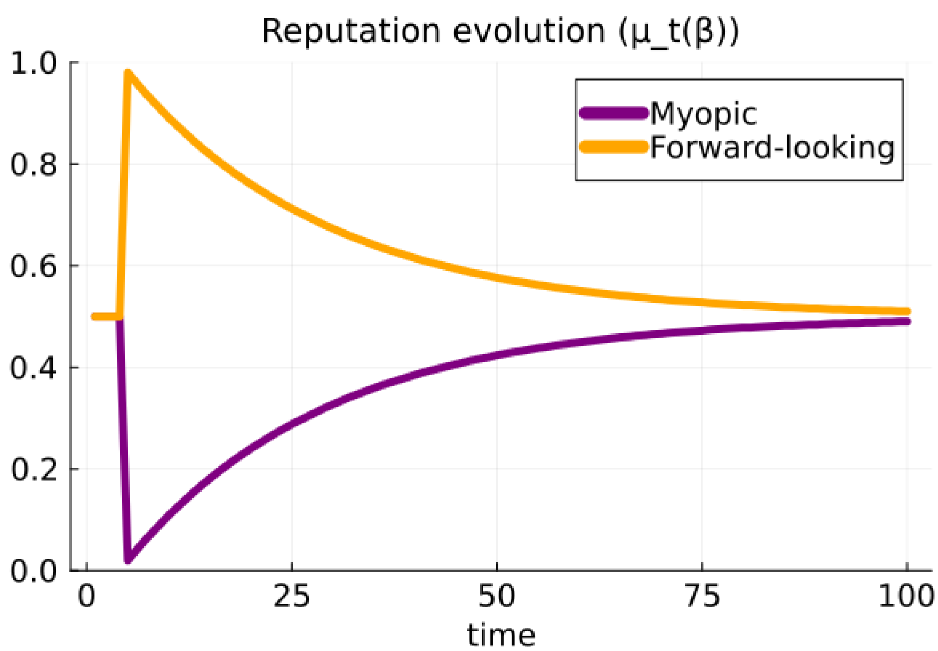

Figure 4: Reputation convergence

We see that immediately after the promise keeping decision of the central banker, there is a large change in beliefs. Following this change, beliefs then converge back to the stationary distribution in the absence of further ZLB periods.

We simulate our model for 100 periods in a path where the type of the central bank happens to not change with the aim of showing how reputation evolves depending on the type of the central banker, and how the reputation of the central banker influences economic outcomes. In particular, we plot the value of inflation and how it evolves together with reputation in a model with $\tau = 3$.

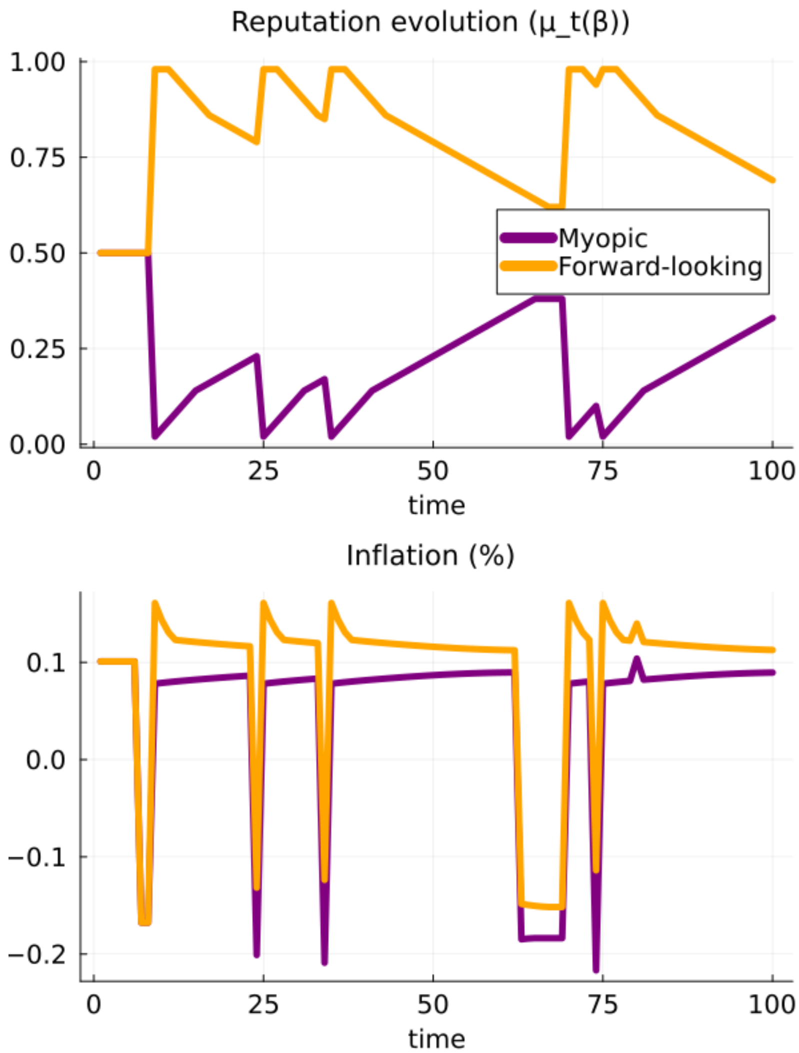

Figure 5: A simulation of reputation and inflation with $\tau = 3$.

We see that having a high reputation is associated to a smaller drop in inflation at the ZLB. Interestingly, if we consider inflation at the ZLB that hits the economy in periods 60 and 75, we notice that the drop is mitigated in period 75 for the forward-looking type while it is exacerbated for the myopic type. This happens as the occurrence of a ZLB in the recent past has given agents a good idea of the type of the central banker.

5.2 Forward guidance and the persistence of the ZLB

We find that the persistence of the demand shock plays an important role in determining the sustainability of forward guidance as well as its effect on the output gap at the ZLB. We vary the autoregressive coefficient (ρ_r) while at the same time adjusting the variance of the innovation σ_r^2 such that the variance of the natural interest rate is kept fixed.

In Figure 6 we show the effect on the output gap at the ZLB of promising a zero interest rate for a time horizon of τ periods as a function of the persistence of the demand shock. We also show the effect of forward guidance with full commitment in red for comparison.

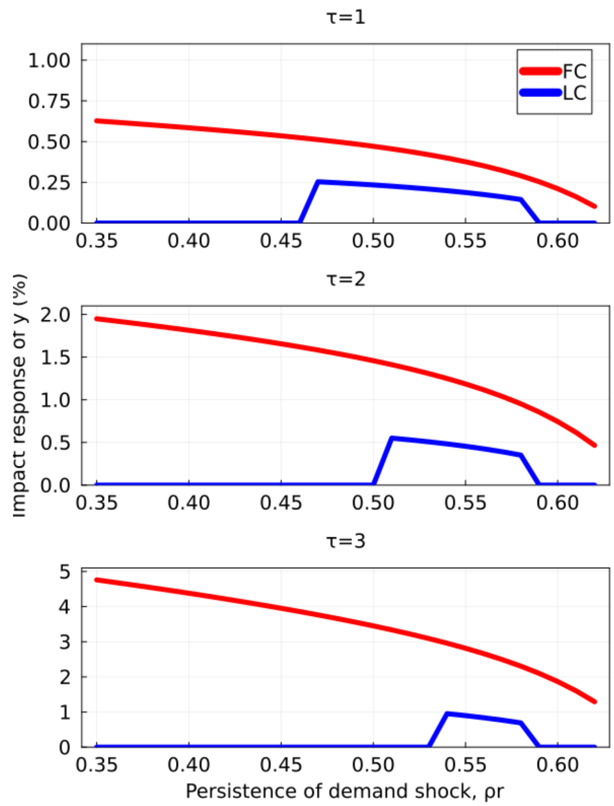

Figure 6: Forward guidance and the persistence of the natural rate of interest

We see that there is a non-monotonic relation between the effect of forward guidance and the persistence of the demand shock when the central bank cannot commit, while the effect of forward guidance is smoothly decreasing in the persistence of the demand shock in a model with full commitment.

Intuitively, a higher persistence of the demand shock implies that the economy is less likely to exit the ZLB in the following period which in turn means that the economy is less likely to transition to the promise keeping state. This explains why the effect of forward guidance is decreasing in the persistence of the demand shock with full commitment.

With limited commitment, there is no longer a monotonic relationship between the persistence of the demand shock and the effect of forward guidance. For a very low persistence of the shock, we find that a promise of keeping the interest rate at 0 for 6 months after the economy exits the ZLB has a zero impact on the output gap at the ZLB. This happens as a low persistence of the demand shock renders the ZLB less severe due to the fact that the economy is likely to exit in the following period, causing expectations about future output gap and inflation to be rather expansionary. Due to the fact that the ZLB is not very severe, the value of having a high reputation, allowing the central banker to stabilize economic outcomes at the ZLB, is modest. Hence, the central banker prefers stabilizing economic outcomes once out of the ZLB by deviating to discretion instead of building reputation.

We see that for a persistence of around 0.47, the effect of forward guidance jumps from zero to 0.25 % when $\tau = 1$. The reason is that as the persistence increases, the ZLB gets more severe implying that the central banker faces stronger incentives to keep her promise to gain reputation. After this jump, the effect of forward guidance then smoothly declines in the persistence of the demand shock for the same reasons as under full commitment: the higher the persistence, the less likely that the economy will transition to the promise keeping state in the following period. We see that eventually the effect of forward guidance drops to zero. This is due to the fact that for a too high persistence, the promise keeping state gets too expansionary implying that the central banker will have to deviate from her promise as it is too costly to keep it.

We find that the higher the value of τ, the smaller the set of values of the persistence such that forward guidance can be sustained as an equilibrium strategy. The intuition is that longer promises are associated to larger expansions implying that the central banker needs stronger incentives to keep the promise, i.e. a very bad ZLB while at the same the persistence cannot be too high as the promise keeping state would then be too expansionary.

5.3 Welfare loss

The presence of a ZLB causes a welfare loss as the central bank is unable to stabilize inflation and output gap through adjustments to the nominal interest rate. Instead, the central bank is forced to rely on unconventional policy measures such as forward guidance to stabilize the economy (Eggertsson & Woodford, 2003).

We compute the loss associated to the ZLB by applying the loss function of the forward-looking central banker which approximates household welfare. Following Galí (2015), we can interpret this loss as the fraction of steady state consumption up to an additive constant that the household is willing to give up to avoid fluctuations in the economy.

We report the welfare loss of the ZLB when the central bank has only limited commitment and we compare this loss to two benchmark cases: 1) a central bank that is unable to conduct forward guidance, and 2) a central bank with full commitment. All of these losses are reported in Table 2.

Scenario	$\tau = 0$	$\tau = 1$	$\tau = 2$	$\tau = 3$
ZLB with no forward guidance	0.0514 %	-	-	-
Loss with full commitment	-	0.0043 %	0.0036 %	0.0040 %
Loss with limited commitment	-	0.0044 %	0.0040 %	0.0037 %

Table 2: Welfare losses. Initial belief distribution is $\mu_0(\beta) = 1/2$.

In general, we find very small welfare differences comparing the alternative scenarios, which is a direct consequence of the fact that we follow the approach by Lucas (1987) and thus the estimated welfare losses of business cycle fluctuations are very small. It is thus not surprising that we do not find important quantitative differences between the welfare loss of a central bank that can engage in forward guidance compared to one that cannot.

However, we find that a central bank which has access to a commitment technology should optimally choose a length of $\tau = 2$. On the contrary, a central bank that does not have access to a commitment technology should optimally choose a length of $\tau = 3$ corresponding to 18 months.

We therefore find that not having access to a commitment technology causes the central bank to give a longer promise. The intuition is two-fold. First, as agents realize that the central bank may deviate from its promise, the expansionary effect of a promise at the

ZLB is reduced under limited commitment. Second, as the central has not committed, it can choose to deviate from the given promise in a state where it is very costly to keep it.

We find that the welfare loss under limited commitment is very close to the loss under full commitment. The intuition is that under full commitment, promises are more effective and hence the drop in output gap and inflation at the ZLB is less severe but on the other hand promises are kept in some states of the world where it is very expensive to do so.

6 Relation to Discounted Euler Equation

McKay et al. (2016a) emphasizes that the highly forward looking nature of the Euler equation is at the core of the forward guidance puzzle. A large literature solving the this puzzle can be summarized by the so-called "discounted Euler equation" which takes the following form:

$$y_t = \Omega \, \mathbb{E}_t[y_{t+1}] - \frac{\chi}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)$$

and can be iterated forward to obtain:

$$y_t = -\frac{\chi}{\rho} \sum_{j=0}^{\infty} \Omega^j \, \mathbb{E}_t \left[i_{t+j} - \pi_{t+j+1} - r_{t+j}^n \right]$$

In the standard New Keynesian model $\Omega = \chi = 1$. The condition $\Omega < 1$ dampens the effect of future expected interest rates on current economic outcomes which explains why forward guidance about interest rates in the distant future is mitigated with a discounted Euler equation compared to the standard model.

However, we find it paradoxical that agents make consumption-savings decisions looking infinitely into the future while failing to understand the incentives of the central bank to keep a time-inconsistent promise. The ability of agents to understand the incentives of the central bank is what we add to the standard model.

The difference between our limited commitment-reputation model and this literature is that we don't obtain attenuation of forward guidance through discounting of the Euler equation, i.e. we have $\Omega = 1$. Instead, we obtain attenuation of forward guidance by assuming that agents understand the incentives of the central bank to keep or renege on a promise. Hence, our attenuation works through the lack of effect that a promise can have on expectations regarding the future interest rate path.

In Table 3, we compare the output gap and inflation that pertain at the ZLB in the standard model, the discounted Euler equation model, and our model with limited commitment and reputation.

Model		Output gap	Inflation
Standard NK	($\Omega = 1.00, \chi = 1.00$ and FC)	-6.61%	-0.32%
Discounted Euler	($\Omega = 0.97, \chi = 0.75$ and FC)	-4.44%	0.01%
LC and reputation	($\Omega = 1.00, \chi = 1.00$ and LC)	-6.61%	-0.32%

Table 3: Recession at the ZLB with no forward guidance

A discounted Euler equation attenuates the fall in output and inflation at the ZLB compared to the standard model as stressed by McKay et al. (2016a). Instead, our model with limited commitment and reputation does not impact economic outcomes at the ZLB in the absence of forward guidance compared to the standard NK model. This is due to the fact that the discounted Euler literature obtains mitigation of a ZLB by reducing the front-loading effect of future drops in inflation and output gap.

We show how the economy responds to a promise of keeping the interest rate at 0 for τ periods in Figure 7 in the three different models.
Both a model with a discounted Euler equation and our limited commitment-reputation model provide attenuation of the effect of forward guidance, but an important difference between the two models is that with the former, the marginal effect of giving a more generous promise is always weakly positive while in the latter we obtain that the marginal effect may be negative if the promise induces agents to loose faith in the ability of the central bank to keep its promise. Hence, central bank communication may be more expansionary by giving a humble promise that can be kept compared to a more expansionary one, which agents expect central banks to renege on.

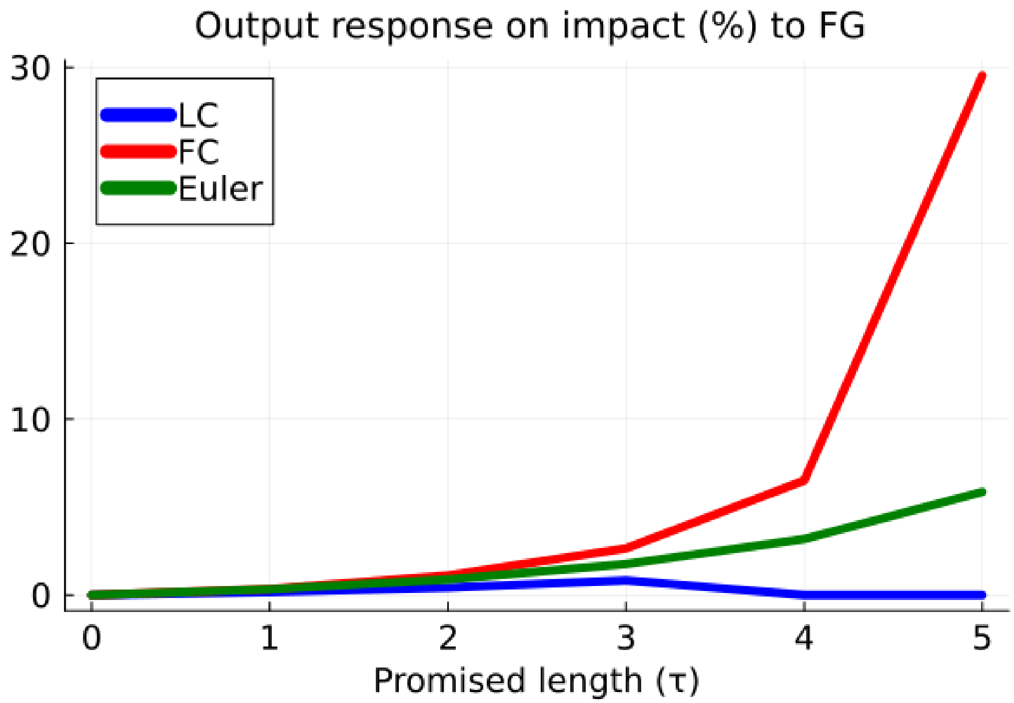

Figure 7: Discounted Euler equation model

7 Extension: Forward Guidance and Government Debt

Bhattarai et al. (2023) show that quantitative easing (an increase in short run government debt) stimulates the economy by inducing the central bank to choose a lower interest rate in the future. They informally discuss how quantitative easing therefore can serve as a commitment device to forward guidance and hence work to strengthen the effect of it. We now extend our quantitative model to study the interaction between forward guidance and government debt in our limited commitment-reputation model.

We assume that there is a level of government spending at the ZLB that is pure waste in the sense that it does not affect aggregate demand nor consumer welfare. Whatever is spent at the ZLB is financed by short term debt that has to be paid back in the following period.

Bhattarai et al. (2023) microfounds a loss function that is increasing in the level of taxation as tax payments are associated to a decrease in welfare due to their distortionary nature. We then follow their approach and assume a loss function of the following form:

$$L\big(\{y_t\}_{t=0}^{\infty}, \{\pi_t\}_{t=0}^{\infty}, \{T_t\}_{t=0}^{\infty}\big) = \sum_{t=0}^{\infty} \delta^t \Big[\ \lambda \cdot (y_t - \bar{y})^2 + \pi_t^2 + \lambda_T \cdot T_t^2 \Big]$$

The term T denotes taxes that are raised to repay government debt raised in the previous period. In particular, $T_t = (1 + i_{t-1}) \cdot G_{t-1}$. The model is otherwise identical to the one described in Section 3. We choose the same calibration as in the baseline model, and we put $\lambda_T = 0.1$.

In Table 4 we report the interest rate chosen by the central bank at the median realization of the demand shock in the model with and without government debt but with no forward guidance. We find that having higher government debt reduces the interest rate chosen by the central bank when they conduct discretionary policy. This result replicates the finding of Bhattarai et al. (2023).

Debt level		i_t
$G_t = 0$	(baseline)	0.39 %
$G_t = 0.035$	(model with debt)	0.06 %
$G_t = 0$	(model with debt)	0.55 %

Table 4: Interest rates under discretionary policy with no forward guidance.

We find that the interest rate in discretion is 0.39 % in the baseline model where government debt is always zero. We then compute the interest rate in a model where the government issues debt at the ZLB. The interest rate is lower in discretion (0.06%) when the government has a positive level of debt. The intuition is that the central bank tries to minimize the tax payments of the private agents while still taking into account how the interest rate influences output and inflation. As a consequence of this, the zero lower bound is less severe in such a model which causes the interest rate to be higher in discretionary periods with no debt as expectations about future outcomes are more expansionary.

We then study the consequences of forward guidance in the model with government debt. We display in Figure 8 the increase in output at the ZLB of promising an interest rate of zero in the period where the economy exists the ZLB.

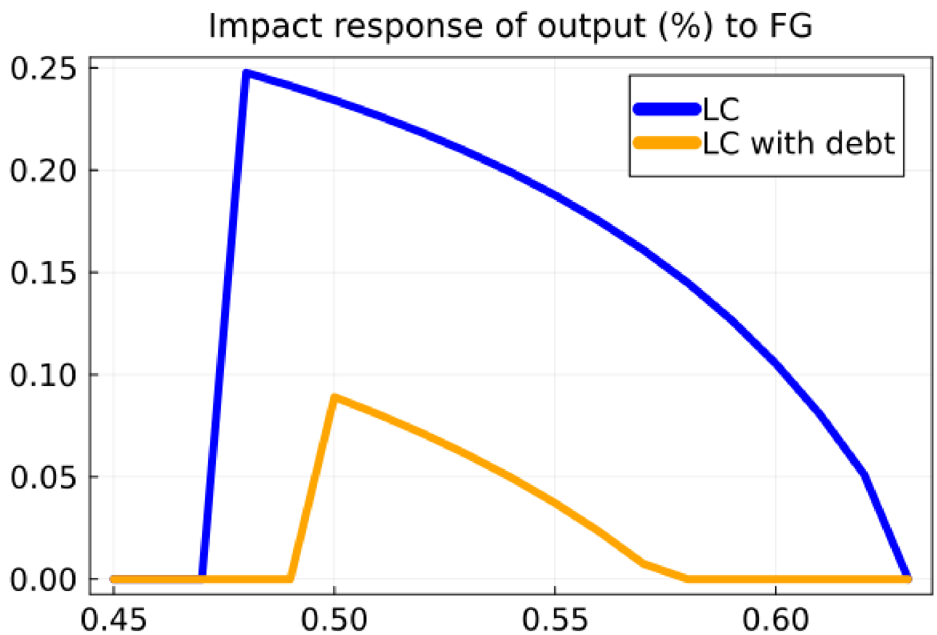

Figure 8: Forward guidance with government debt

From the graph, it follows that the effect of forward guidance is muted once there is government debt. The reason for this is that the presence of government debt reduces the optimal interest rate of the central bank in discretion which renders a promise of low interest rates in the future superfluous as private agents already expect the central bank to keep rates low. We therefore obtain that short term government debt and forward guidance are substitute policies whereas Bhattarai et al. (2023) argue that they are complements.

8 Conclusion

We showed how a standard New Keynesian model augmented with limited commitment and type-based reputation bounds the effect of forward guidance. An important implication of the model is that the marginal effect of the promised horizon becomes negative when the promise is judged unsustainable by private agents. We demonstrated a strong interaction between the persistence of the ZLB, and the sustainability and effect of forward guidance. We conjecture that our model could be useful for studying other time inconsistent policies.

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A Interest Rate under Optimal Discretion

When the central bank is not honoring a promise[10], the interest rate is set to minimize the instantaneous loss function by taking expectations as given. We impose the constraint on the minimization problem that the interest rate cannot be negative. The interest rate chosen by the central banker will therefore be

$$i_t = \arg\min \ell(y_t, \pi_t)$$

subject to

$$\begin{aligned}
\pi_t &= \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t \\
y_t &= \mathbb{E}_t[y_{t+1}] - \frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \\
i_t &\geq 0
\end{aligned}$$

Using the IS curve and the Phillips curve, we can rewrite the loss function of the central bank in terms of the nominal interest rate rather than inflation and output gap:

$$\ell(y_t, \pi_t) = \ell\left(\mathbb{E}_t[y_{t+1}] - \frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)\ ,\ \beta\mathbb{E}_t[\pi_{t+1}] + \kappa\mathbb{E}_t[y_{t+1}] - \kappa\frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)\right)$$

We denote by $\zeta_t \leq 0$ the multiplier associated to the constraint $i_t \geq 0$. The Lagrangian associated to the minimization problem of the central bank writes:

$$\begin{aligned}
\mathcal{L}(i_t, \zeta_t) = &\left(\beta\mathbb{E}_t[\pi_{t+1}] + \kappa\mathbb{E}_t[y_{t+1}] - \kappa\frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)\right)^2 \\
&+ \lambda\left(\mathbb{E}_t[y_{t+1}] - \frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) - \bar{y}\right)^2 \\
&+ \zeta_t i_t
\end{aligned}$$

The first order condition is:

$$2\frac{\kappa}{\rho}\underbrace{\left(\beta\mathbb{E}_t[\pi_{t+1}] + \kappa\mathbb{E}_t[y_{t+1}] - \kappa\frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)\right)}_{=\pi_t} + 2\frac{\lambda}{\rho}\underbrace{\left(\mathbb{E}_t[y_{t+1}] - \frac{1}{\rho}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n)\right)}_{=(y_t - \bar{y})} = \zeta_t$$

[10]Either because no promise has been given or because the central bank decides to break it.

$$\iff 2\frac{\kappa}{\rho}\pi_t + 2\frac{\lambda}{\rho}(y_t - \bar{y}) = \zeta_t$$

So that substituting back π_t and y_t we obtain

$$\kappa\pi_t + \lambda\left(y_t - \bar{y}\right) = \frac{\rho}{2}\,\zeta_t \leq 0$$

Whenever the ZLB is binding, the multiplier ζ_t will generically be strictly negative. On the contrary, whenever the ZLB is slack, the multiplier ζ_t is exactly zero, and the FOC writes:

$$\lambda\left(y_t - \bar{y}\right) + \kappa\pi_t = 0$$

The second order condition is:

$$2\frac{\kappa^2}{\rho^2} + 2\frac{\lambda}{\rho^2} > 0$$

So that the FOC characterizes the unique global minimum.

B Equilibrium Description

We describe here in detail how the equilibrium is computed. For fixed strategies σ and beliefs μ, output gap, inflation and interest rates must constitute a solution to the New Keynesian model. The key differences are that interest rates are set to zero whenever a promise is kept, and that the expectations about future outcomes depend on the private sector assessment about the type of the central bank and how they behave.

B.1 At the ZLB

Whenever the economy is at the ZLB at time t, the nominal interest rate is set to zero, $i_t^Z = 0$. An Odyssean promise is given, and therefore the private sector forms an assessment about how likely it is that the promise will be kept. We have to consider three possibilities:

1. If the ZLB is binding again in the next period, then the promise is renewed. This corresponds to the event $\{r_{t+1}^n \in Z_{\sigma,\mu}\}$.

2. If the ZLB is not binding at time $t+1$, then each type δ of the central bank will keep the promise with probability $\sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)$. As the type of the central bank is private information, the private sector forms an expectation with respect to δ using the probability measure μ_t. Thus conditional on exiting the ZLB, i.e. $r_{t+1}^n \notin Z_{\sigma,\mu}$, the expected probability of transitioning to the first promise-keeping period is $\mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)]$.

3. Finally, if the ZLB is not binding at time $t+1$, the conditional probability of transitioning to normal times is $\mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)]$.

Output gap, inflation and interest rates therefore satisfy the following system:

$$
\begin{cases}
\pi_t^Z = \beta\, \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + \pi_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \\ \quad + \pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \end{bmatrix} + \kappa y_t^Z \\
y_t^Z = \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} y_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + y_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \\ \quad + y_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \end{bmatrix} \\
\qquad - \frac{1}{\rho} \left(0 - \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + \pi_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \\ \quad + \pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t = 1)] \end{bmatrix} - r_t^n \right) \\
i_t^Z = 0
\end{cases}
$$

For the ease of exposition we have suppressed the dependence on the current shock realization. A similar omission applies to the remaining cases.

B.2 Promise-keeping period with $P_t < \tau$

If the economy is away from the ZLB and a promise is to be kept at time t, the interest rate is set to zero, $i_t^P = 0$. In the next period, the ZLB could be binding, so a promise is renewed and the process starts over. If the economy stays away from the ZLB for one more period, a transition to the next period of promise keeping occurs with conditional probability $\mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)]$. The promise is broken in next period with conditional probability $\mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)]$.

Output gap, inflation and interest rates therefore satisfy the following system:

$$\begin{cases} \pi_t^P = \beta \, \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + \pi_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \\ \quad + \pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \end{bmatrix} + \kappa y_t^P \\ \\ y_t^P = \ \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} y_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + y_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \\ \quad + y_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \end{bmatrix} \\ \qquad -\frac{1}{\rho} \left(0 - \mathbb{E}_{r_{t+1}^n} \begin{bmatrix} \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \\ \quad + \pi_{t+1}^P \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[\sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \\ \quad + \pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} \cdot \mathbb{E}_\delta^{\mu_t}[1 - \sigma(\delta, r_{t+1}^n, \mu_t, P_t + 1)] \end{bmatrix} - r_t^n \right) \\ \\ i_t^P = 0 \end{cases}$$

B.3 Last promise-keeping period ($P_t = \tau$)

If the economy is in the last promise-keeping period the interest rate is $i_t^P = 0$. There are only two possible cases in the following period: either the ZLB binds, or the ZLB does not bind and the central bank conducts discretionary policy. Output gap, inflation and interest rates therefore satisfy the following system:

$$\begin{cases} \pi_t^P = \beta \, \mathbb{E}_{r_{t+1}^n} \Big[\pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \Big] + \kappa y_t^P \\ \\ y_t^P = \ \mathbb{E}_{r_{t+1}^n} \Big[y_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + y_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \Big] \\ \qquad -\frac{1}{\rho} \Big(0 - \mathbb{E}_{r_{t+1}^n} \Big[\pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \Big] - r_t^n \Big) \\ \\ i_t^P = 0 \end{cases}$$

B.4 Normal times

Away from the ZLB and with no promise to be kept, the central bank conducts discretionary policy. Thus, the interest rate is set to satisfy the derived FOC:

$$-\kappa\, \pi_t^D = \lambda\, (y_t^D - \bar{y})$$

In the following period, the economy may either hit the ZLB or stay in discretion. Output gap, inflation and interest rates therefore satisfy the following system:

$$\begin{cases} \pi_t^D = \beta\, \mathbb{E}_{r_{t+1}^n}\Big[\pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z(\sigma)\} + \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z(\sigma)\}\Big] + \kappa y_t^D \\ \\ y_t^D = \; \mathbb{E}_{r_{t+1}^n}\Big[y_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z(\sigma)\} + y_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z(\sigma)\}\Big] \\ \qquad -\dfrac{1}{\rho}\Big(i_t^D - \mathbb{E}_{r_{t+1}^n}\Big[\pi_{t+1}^D \cdot \mathbb{1}\{r_{t+1}^n \notin Z(\sigma)\} + \pi_{t+1}^Z \cdot \mathbb{1}\{r_{t+1}^n \in Z(\sigma)\}\Big] - r_t^n\Big) \\ \\ -\kappa\, \pi_t^D = \lambda\, (y_t^D - \bar{y}) \end{cases}$$

B.5 Decisions of the central bank

Once we have computed output gap, inflation and interest rates for fixed (σ, μ) we need to verify that the strategies σ are optimal.
When the economy is at the ZLB, the strategies σ are optimal if they solve the following Bellman equation:

$$\begin{aligned} L_t^Z(\delta, r_t^n, \mu_t, 0) = \ell\Big(y_t^Z(r_t^n, \mu_t, 0), \pi_t^Z(r_t^n, \mu_t, 0)\Big) \\ + \delta\, \mathbb{E}_{r_{t+1}^n}\left[\begin{array}{c} \min\limits_{x \in \{0,1\}} \left\{\begin{array}{l} x\, L_{t+1}^P(\delta, r_{t+1^n}, \mu_{t+1}, 1) \\ \quad + (1-x)\, L_{t+1}^D(\delta, r_{t+1}^n, \mu_{t+1}, 0) \end{array}\right\} \cdot \mathbb{1}\{r_{t+1} \notin Z_{\sigma,\mu}\} \\ + L_{t+1}^Z(\delta, r_{t+1}^n, \mu_{t+1}, 0) \cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\} \end{array}\right] \end{aligned}$$

That is, the optimal action prescribes whether to transit to either discretion (break the promise) or promise-keeping. If the economy is in a promise keeping period with

$P_t < \tau$, optimal strategies solve:

$$L_t^P(\delta, r_t^n, \mu_t, P_t) = \ell\Big(y_t^P(r_t^n, \mu_t, P_t), \pi_t^P(r_t^n, \mu_t, P_t)\Big) + \delta\, \mathbb{E}_{r_{t+1}^n}\left[\min_{x\in\{0,1\}}\left\{\begin{aligned} & x\, L_{t+1}^P(\delta, r_{t+1}^n, \mu_{t+1}, P_t+1) \\ & \quad + (1-x)\, L_{t+1}^D(\delta, r_{t+1}^n, \mu_{t+1}, 0)\end{aligned}\right\} \cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + L_{t+1}^Z(\delta, r_{t+1}^n, \mu_{t+1}, 0)\cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\}\right]$$

When the economy is in the last promise-keeping period, no decision is taken taken with respect to the transition, so the Bellman equation writes:

$$L_t^P(\delta, r_t^n, \mu_t, \tau) = \ell\Big(y_t^P(r_t^n, \mu_t, \tau), \pi_t^P(r_t^n, \mu_t, \tau)\Big) + \delta \mathbb{E}_{r_{t+1}^n}\left[L_{t+1}^D(\delta, r_{t+1}^n, \mu_{t+1}, 0)\cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + L_{t+1}^Z(\delta, r_{t+1}^n, \mu_{t+1}, 0)\cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\}\right]$$

Finally, if the economy is in discretion, no strategic decision is made, so the Bellman equation is:

$$L_t^D(\delta, r_t^n, \mu_t, 0) = \ell\Big(y_t^D(r_t^n, \mu_t, 0), \pi_t^D(r_t^n, \mu_t, 0)\Big) + \delta \mathbb{E}_{r_{t+1}^n}\left[L_{t+1}^D(\delta, r_{t+1}^n, \mu_{t+1}, 0)\cdot \mathbb{1}\{r_{t+1}^n \notin Z_{\sigma,\mu}\} + L_{t+1}^Z(\delta, r_{t+1}^n, \mu_{t+1}, 0)\cdot \mathbb{1}\{r_{t+1}^n \in Z_{\sigma,\mu}\}\right]$$

C Proofs

Proof of Proposition 4.1.
A central banker with type δ will not keep her promise if

$$\ell(y_t^D, \pi_t^D) + \delta \mathbb{E}_t^{\chi}\left[L\big(\{y_i\}_{i=t+1}^{\infty}, \{\pi_i\}_{i=t+1}^{\infty}\big)\right] < \ell(y_t, \pi_t) + \delta \mathbb{E}_t^{\tilde{\chi}}\left[L\big(\{y_i\}_{i=t+1}^{\infty}, \{\pi_i\}_{i=t+1}^{\infty}\big)\right]$$

where χ denotes the probability distribution if the promise is not kept, while $\tilde{\chi}$ denotes the probability distribution when the promise is kept.

It is easy to verify that the left-hand side is bounded while the right-hand side grows unboundedly in r_t^n for $i_t = 0$. Hence, there is a value of r_t^n such that even the patient central banker will have to deviate from her promise. □

Proof of Proposition 4.2.
Given the pooling strategies and the off-path beliefs, the private sector assigns probability zero to promises being kept. Thus, promise-keeping induces no change in expected output gap and inflation, implying that there are no gains from deviating. □

To prove Proposition 4.1, we use the following lemma:

Lemma C.1. *The loss is bounded if the central bank conducts optimal discretionary policy.*

Proof of Lemma C.1.
Losses are bounded for negative shock realizations, as they are assumed to be bounded (Assumption **??**). Moreover, the central bank can always set $i_t = r_t^n$ whenever the natural rate of interest is positive. This bounds from above the values of output gap and inflation for fixed expectations. Furthermore, this policy being infinitely repeated whenever possible bounds expectations from above as well. The instantaneous loss is thus:

$$\ell\left(\mathbb{E}_t[y_{t+1}] + \frac{1}{\rho}\mathbb{E}_t[\pi_{t+1}], \left(\beta + \frac{\kappa}{\rho}\right)\mathbb{E}_t[\pi_{t+1}] + \kappa \mathbb{E}_t[y_{t+1}]\right)$$

Which is bounded as all the terms are bounded. □

Proof of Proposition **??**.
It follows from the fact that in any semi separating equilibrium, only the benevolent type keeps her promises. □

Proof of Proposition 4.3.
A benevolent central banker ($\delta = \beta$) will not keep her promise if

$$\ell(y_t^D, \pi_t^D) + \beta \mathbb{E}_t^{\chi} \left[L\big(\{y_i\}_{i=t+1}^{\infty}, \{\pi_i\}_{i=t+1}^{\infty} \big) \right] < \ell(y_t, \pi_t) + \beta \mathbb{E}_t^{\tilde{\chi}} \left[L\big(\{y_i\}_{i=t+1}^{\infty}, \{\pi_i\}_{i=t+1}^{\infty} \big) \right]$$

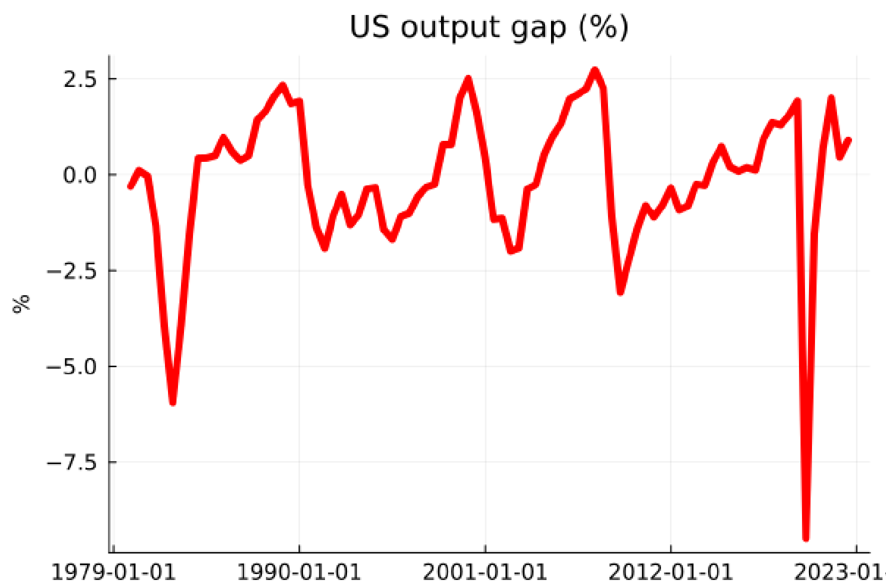
We immediately notice that the LHS is independent of the promised length τ.

It is easy to show that the RHS increases without bounds in the promised length of 0 interest rates. □

Proof of Corollary 4.1.
Follows from Proposition 4.3. □

D Data Appendix

To calibrate the model we used data on US GDP over the period 1947 to 2022 retrieved from the FRED database at the following link FRED data. We then detrend the data using the HP-filter to the period 1947 - 2022 where we choose a smoothing parameter of 400. We then use the estimated output gap for the period 1982-2020 to calibrate the model. In Figure D, we plot our estimated output gap of the US economy.



We report the relevant statistics for the output gap that we use to calibrate our model in Table 5. These statistics are calculated using the years 1982 to 2022.

Moment	US 1982-2020	Model
Average output gap	-0.1%	-0.014%
Fall at the ZLB	-6.3%	-6.61%
Auto correlation	0.57	0.55

Table 5: Targeted moments

We compute the average output gap by using the stationary transition probabilities of the natural real interest rate.

To compute the fall at the ZLB in the data, we consider the average output gap at the height of the Financial crisis and the Covid crisis.

The auto correlation in the model is computed based on a simulation with 1,000 time periods that we repeat 10,000 times and we then compute the average auto correlation across the simulations.