

Detecting Grouped Local Average Treatment Effects and Selecting True Instruments

With an Application to the Effect of Imprisonment on Recidivism

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① Introduction

② Introduction

③ Assumptions & Method

④ Application

⑤ Conclusion

Motivation: The effect of imprisonment on recidivism

- Does incarceration prevent future crime (*recidivism*)?
- The results in the literature so far are mixed: crime-reducing, no effect or even a crime-inducing effect of prison
- Maybe there is heterogeneity of effects, maybe even inside states?
 - In some counties prisons are good at rehabilitating, in others they have reverse effect

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This paper in a nutshell

- 1 Estimate the effect of incarceration on recidivism in the US based on judge instruments when allowing for effect heterogeneity

Single estimate might mask coexistence of null, positive and negative effects

- 2 Proposition of a two-step method to allow for selection of valid IVs in presence of LATEs
 - 1 Find clubs of propensity scores (i.e. imprisonment rates) and apply clustering
 - 2 Inside each club find largest group of same reduced form estimates (i.e. recidivism rate)
 - 3 Estimate grouped heterogeneous treatment effects for all pairs of clubs using only the largest group of (valid) instruments (i.e. judges)

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LATE Assumptions: For two distinct IV values $z \neq z'$:

- **Assumption 1: Validity (Exogeneity and Exclusion restriction)**

- (i) Random assignment: $Z \perp (D(z), Y(z', d))$ and

- (ii) Exclusion: $Y(z, d) = Y(z', d) = Y(d)$

- **Assumption 2: Monotonicity** $\Pr(D(z) \geq D(z')) = 1$

- **Assumption 3: First Stage:** $E(D|Z = z) - E(D|Z = z') \neq 0$

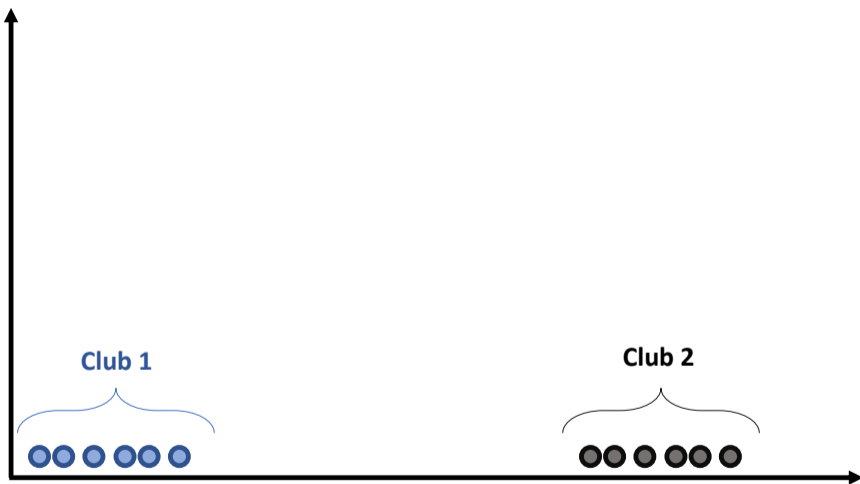
- *Clubs:* Sets of judges with comparable probability to incarcerate

- *Group:* Set of judges inside club with same judge-specific mean of the outcome

- **Assumption 4: Plurality:** Largest group of judges in each club fulfils LATE assumptions

Method: Visualization of Clubs and Groups

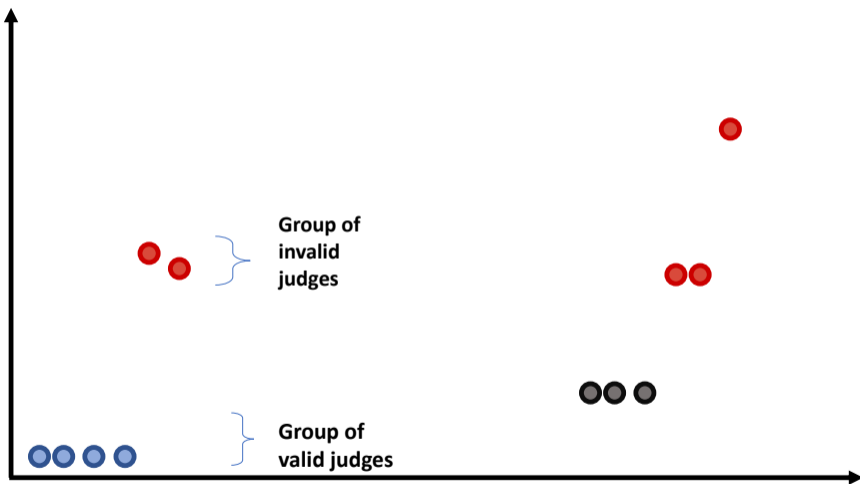
Mean recidivism
per judge



Mean incarceration per judge

Method: Visualization of Clubs and Groups

Mean recidivism
per judge



Group of
invalid
judges

Group of
valid
judges

Mean incarceration per judge

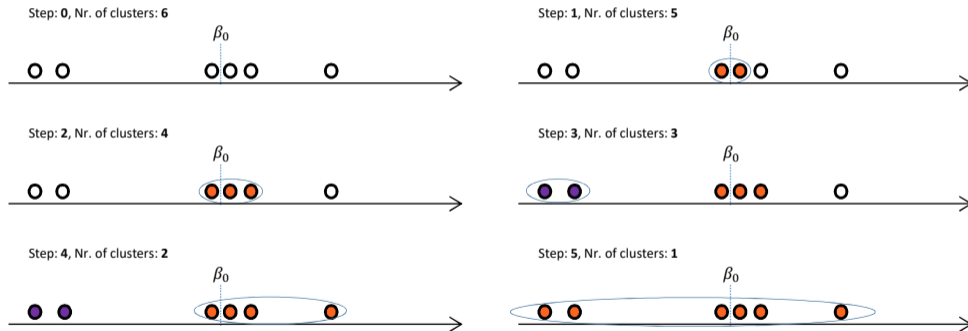
Method: Group-pair LATEs

$$LATE = \frac{R_{C1} - R_{C2}}{P_{C1} - P_{C2}}$$

- R_{C1} , R_{C2} : Recidivism rate for Group 1 and 2
- P_{C1} , P_{C2} : Imprisonment rate for Group 1 and 2
- LATE is the effect of imprisonment on recidivism for the sub-population of people who are sentenced to jail by a **judge in Group 1**, but **would not have been** sentenced by a **judge in Group 2**
- If we group judges into **4** clubs, we get only **6** estimates (instead of several thousands from a judge-wise comparison)

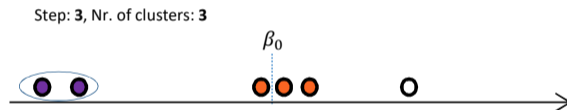
Method: AHC Visualization

Classification of mean imprisonment and recidivism via Agglomerative Hierarchical Clustering (AHC) (Ward, 1963; Apfel and Liang, 2021) [Details](#)



Method: AHC Visualization

Select the number of clusters via a **stopping rule**: Stop when the F-test for equality of all first-stage parameters in the cluster does not reject any more



- **Idea:** Club pairs that fulfil LATE assumptions produce same LATE
- **Step I:** Classify Judges into Clubs
 - Find clubs of IVs with the same propensity score P_z
- **Step II:** Find largest groups of judges with same reduced form inside each club
- Estimate LATEs for group-pairs

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Application: Incarceration and future crime

$$\text{Recidivism}_{ojt} = \beta_0 + \text{Prison}_{ojt}\beta + u_{ojt}$$

Equation of Interest

$$\text{Prison}_{ojt} = \gamma_0 + \mathbf{Z}_{ojt}\gamma + \varepsilon_{ojt}$$

First Stage

- o : Offender ID, j : Judge ID, t : Time period (2009-2014)
- *Recidivism*-Dummy: 1 if offender has reoffended within 3 years
- *Prison*-Dummy: 1 if offender has been convicted
- Adult offenders in US state Minnesota: Data obtained by linking Minnesota Judicial Branch case database with Minnesota Sentencing Guidelines dataset
- Judges assigned randomly according to the Minnesota Order for Assignment of Cases
- Controls: race-, gender-, offense-type-, year-dummies, severity of crime, age, the squares of the latter two and race-gender interaction dummies

Application: Incarceration and future crime

- Number of judges: 78
- Cases with presumptive sentences of up to three years (minor crimes), e.g. robbery, assault, theft, stalking, fleeing from the police, lottery fraud
- Number of cases per judge:
 - Min: 202
 - Mean: 307
 - Max: 935

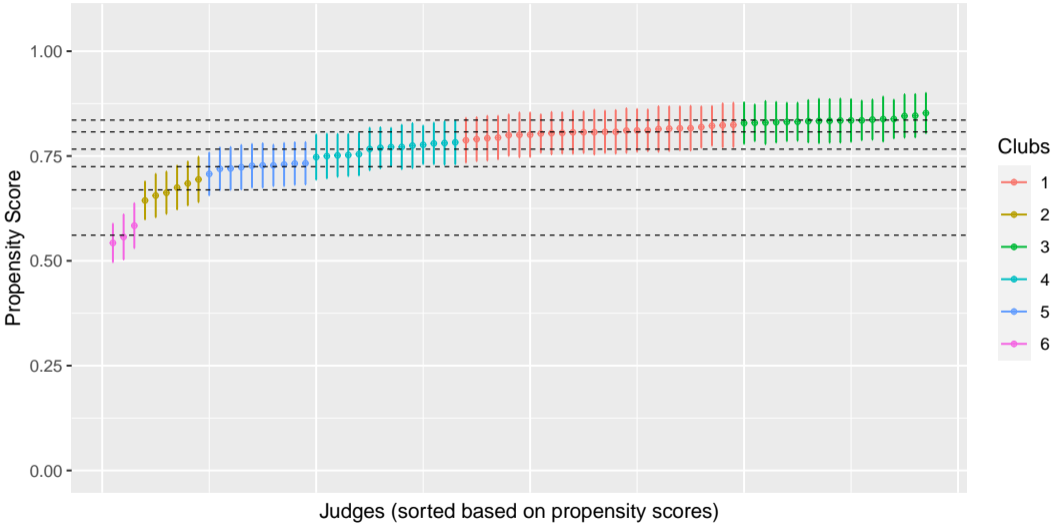
Descriptives

Application: Clusters based on imprisonment rate

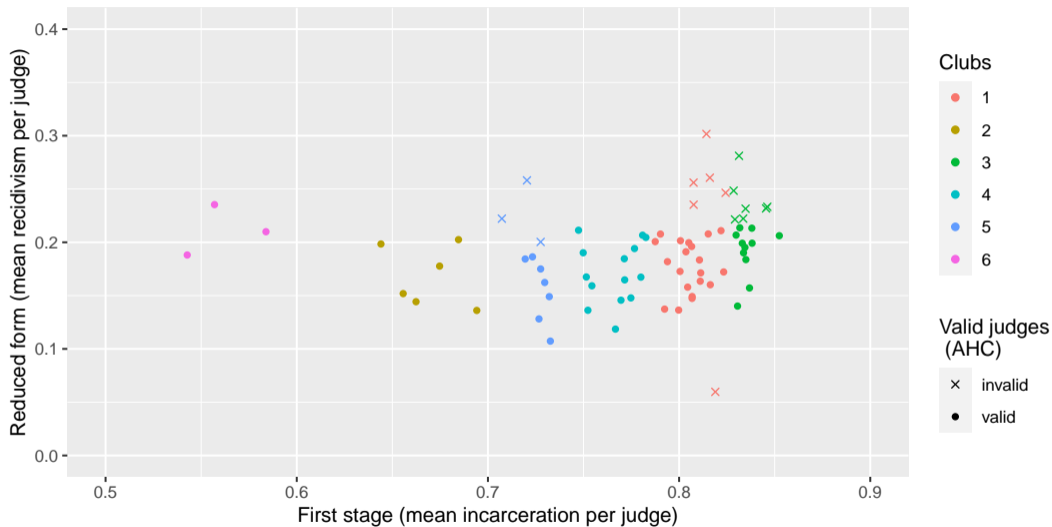
Club	Mean	Nr
1	0.81	26
2	0.67	6
3	0.84	18
4	0.77	14
5	0.72	10
6	0.56	3
7	0.15	1

Table: First step: Club allocation

Application: Judges sorted by imprisonment rate



Application: Imprisonment & Recidivism rate



Application: Group-pair LATEs

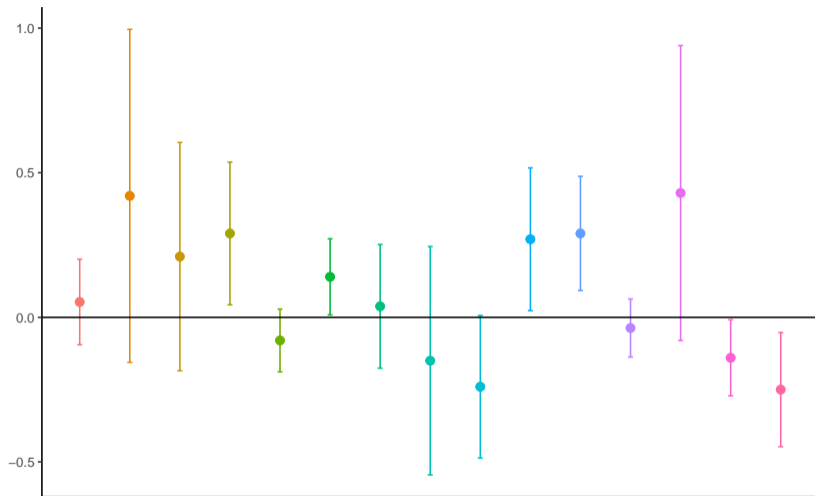


Figure: Effect of Imprisonment on Recidivism - AHC (group-pair LATEs)

Application: Group-pair LATEs (significant only)

Table: Effect of Imprisonment on Recidivism - AHC

	OLS	2SLS	1-5	2-3	3-5	5-6
Prison	0.061 (0.0095)	0.095 (0.022)	0.294 (0.154)	0.135 (0.078)	0.286 (0.121)	-0.248 (0.114)
J			27	17	18	10
N	23958	23958	7156	5787	5758	2703
Diff			-0.083	0.166	-0.111	-0.163

Cluster-robust standard errors in parentheses.

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Conclusion

This paper...

- suggests a cluster method to detect grouped LATEs
- suggests a cluster method to select valid instruments
- applies the method to estimate heterogeneous effects on incarceration on recidivism

Thank you for your attention!
Comments & Questions?
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References

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Appendix: Simulation: Setting w/o Invalidity

- $V \sim Unif(0, 1)$
- $U = 0.5V + Unif(0, 1)$
- $D = 1(\mathbf{Z}\mathbf{p} > V)$
- \mathbf{Z} matrix of judge dummy
- Three clubs: $\mathbf{p} = (0.9\iota_4, 0.7\iota_4, 0.5\iota_2)$
- $Y = (D \cdot 0.5 + D \cdot U + U)^4$
- $J = 10$
- Number of cases: $Unif(3, 5)$ multiplied by 10 (few cases setting) or by 100 (many cases setting).
- 1000 repetitions

Simulation: Setting with Invalidity

- $Y = (D \cdot 0.5 + D \cdot U + \mathbf{Z}\gamma + U)^4$
 - $\gamma_{inv} = \begin{matrix} 0.1, 0\iota_3 \\ 0\iota_2, -0.1, -0.2 \\ 0\iota_2 \end{matrix}$
- Club 1: Three judges are valid and one is invalid (majority and plurality holds)
Club 2: Two judges are valid and two are invalid but invalidity has different magnitudes (that is, majority is violated but plurality holds)
Club 3: All judges are valid

Simulation: Extent of effect heterogeneity

Setting	Invalidity	OLS	Overall	Oracle Clubs 2SLS		
			2SLS	1-2	1-3	2-3
few cases	no	18.94	20.39	30.78	26.47	22.08
	yes	22.75	24.52	29.13	32.02	43.59
many cases	no	18.90	20.39	30.29	26.19	20.10
	yes	22.60	24.40	28.67	31.62	36.26

- Overall 2SLS using judge dummies as IVs would ignore effect heterogeneity.
- Hansen test rejects due to effect heterogeneity and invalidity.

Simulation: 1st clustering based on first stage parameters

- #clubs: Mean number of clusters selected by AHC
- #corr: fraction of times the correct number of clubs has been selected

Setting	Invalidity	Method	#clubs	#corr	Hansen p
few cases	no	Oracle	3	1	0.51
		AHC	2.36	0.36	0.48
	yes	Oracle	3	1	0.00
		AHC	2.35	0.35	0.15
many cases	no	Oracle	3	1	0.51
		AHC	3.02	0.98	0.49
	yes	Oracle	3	1	0.00
		AHC	3.02	0.99	0.00

Simulation: 1st clustering based on first stage parameters

- #clubs: Mean number of clusters selected by AHC
- #corr: fraction of times the correct number of clubs has been selected

Setting	Invalidity	Method	#clubs	#corr	Hansen p
few cases	no	Oracle	3	1	0.51
		AHC	2.36	0.36	0.48
	yes	Oracle	3	1	0.00
		AHC	2.35	0.35	0.15
many cases	no	Oracle	3	1	0.51
		AHC	3.02	0.98	0.49
	yes	Oracle	3	1	0.00
		AHC	3.02	0.99	0.00

- If number of cases is sufficiently large, clustering works well (even if some judge violate LATE assumptions)
- Invalid IV setting: Hansen test still rejects due to invalidity

Simulation: Estimation after 1st clustering

Setting	Invalidity	Method	2SLS		
			1-2	1-3	2-3
many cases	no	Oracle	30.23	26.15	20.01
		AHC Clubs	30.29	26.19	20.09
	yes	Oracle	30.16	26.08	19.94
		AHC Clubs	28.67	31.62	36.25

- Invalid IV setting: LATEs are biased

Simulation: 2nd clustering based on reduced form parameters

- Separately for each club (i.e. constant imprisonment rate) we perform a 2nd clustering based on RF parameters (Hansen test as stopping rule)
- If plurality holds (i.e. largest group of judges are valid), we can identify the valid judges.

Setting	Method	ValDet	InvDet	Hansen p
few cases	Oracle	1	1	0.50
	AHC	0.96	0.33	0.34
many cases	Oracle	1	1	0.51
	AHC	0.99	0.86	0.44

- Again clustering works well if number of cases is large enough
- Hansen test does not reject any longer.

Simulation: Estimation after 1st and 2nd clustering

- Average LATE estimates for the three possible comparisons

Method	1-2	1-3	2-3
Oracle	30.28	26.13	20.12
AHC Clubs & Groups	28.74	26.12	22.37

- Estimation of the LATEs work very well if we drop the invalid judges

Appendix: LATE assumptions

There exist three compliance types under Assumption 3:

Type	Potential treatment variable	Interpretation in application
Compliers (C)	$D^1 = 1, D^0 = 0$	> 2 children only if same sex
Defiers (F)	$D^1 = 0, D^0 = 1$	> 2 children only if opposite sex
Always-takers (A)	$D^1 = 1, D^0 = 1$	> 2 children in any case
Never-takers (N)	$D^1 = 0, D^0 = 0$	2 children in any case

Which are distributed in the data as

	Z=1	Z=0
D=1	C,A	A
D=0	N	C,N

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Potential outcomes model (Rubin, 1974)

$$D = 1(\psi(Z) > V)$$

$$Y = \eta(D, U)$$

First Stage relationship

Outcome

- V and U are unobservable
- $\psi(\cdot)$ is a nonparametric function of Z
- Binary instruments Z
- Potential treatment: $D(z)$
- Potential outcome: $Y(d, z)$
- Propensity score: $Pr(D = 1|Z = z) = P_z$

Model II: Key implication

Under the LATE Assumptions (Exogeneity and Exclusion, Monotonicity and First Stage), the Wald estimator identifies the LATE $\Delta_{z,z'}$

Central implication

Two IV pairs with values (z, z') and (z'', z^*) identify the same LATE *iff* their propensity scores are equal $p(z) = p(z'')$ and $p(z') = p(z^*)$:

$$\Delta_{z,z'} = E[Y(1) - Y(0) | p(z) > V \geq p(z')] = \Delta_{z'',z^*}$$

Intuition: We are looking at the same section of the population in terms of V

$$\begin{aligned}\Delta_{z,z'} &= \frac{E(Y|Z = z) - E(Y|Z = z')}{\Pr(D = 1|Z = z) - \Pr(D = 1|Z = z')} \\ &= E[Y(1) - Y(0)|D(z) = 1, D(z') = 0] \\ &= E[Y(1) - Y(0)|p(z) > V \geq p(z')]\end{aligned}$$

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Method IV: First-step selection of clubs

- 1 Cluster the mean incarceration per judge via Agglomerative Hierarchical Clustering (AHC) (Ward, 1963; Apfel and Liang, 2021) [Details](#)
- 2 Select the number of clusters via a stopping rule: Stop when the F-test for equality of all first-stage parameters in the cluster does not reject any more

Method V: Dealing with invalidity

- ① Create a set of instrumental variables from two clubs
 - ② Test overidentifying restrictions (Hansen-Sargan test)
 - ③ Select valid IVs
 - Apply Agglomerative Hierarchical Clustering on the judge-specific means of recidivism
- [Details](#)
- Search for largest cluster (group) inside each club found in the preceding step

Method VI: Agglomerative Hierarchical Clustering

- 1 **Input:** Calculate all propensity scores \hat{p}_j with a first-stage regression
- 2 **Initialization:** Each \hat{p}_j has its own cluster. The total number of clusters in the beginning hence is J .
- 3 **Joining:** The two clusters k and l which are closest in terms of their weighted Euclidean distance $\frac{J_k \cdot J_l}{J_k + J_l} \|\bar{\mathbf{p}}_k - \bar{\mathbf{p}}_l\|^2$ are joined to a new cluster.
- 4 **Merging:** Recalculate the cluster means. Recalculate the pair-wise Euclidean distances with the new cluster.
- 5 **Iteration:** The joining and merging steps are repeated until all just-identified point-estimates are in one cluster. For each joining step, the number of clusters decreases by 1.

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Method VII: Consistent classification

Consistent classification

As $(J, T) \rightarrow \infty$, classification is individually consistent if the probability of wrongly assigning judges goes to zero for all judges, for all clubs.

Possible wrong assignments:

- **Not** assigning a judge from a certain club C_k^0 to an estimated club \hat{C}_k
- Assign a judge from club C_k^0 to an estimated club \hat{C}_k to which it **doesn't** belong

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Descriptives

Variable	Overall	$D = 1$	$D = 0$	Diff	pval
Recidivism	0.292	0.3	0.219	0.08	4.4e-37
Female	0.183	0.173	0.276	-0.1	4.3e-53
Age at Sentence	33.04	33.02	33.2	-0.18	0.28
Race					
White	0.59	0.585	0.638	-0.05	6.9e-13
Black	0.261	0.267	0.204	0.06	6.3e-24
Amerindian	0.073	0.074	0.065	0.01	0.021
Hispanic	0.05	0.047	0.074	-0.03	5.6e-12
Asian	0.026	0.027	0.018	0.01	5e-05
Unknown	0	0	0	0	0.73
Crime Type					
Property Crime	0.332	0.324	0.412	-0.09	9.7e-32
Crime against a Person	0.299	0.306	0.235	0.07	8e-28
Drug Crime	0.238	0.242	0.2	0.04	7.5e-12
Sex Offenses	0.056	0.056	0.054	0	0.49
Weapons Offense	0.007	0.007	0.009	0	0.19
Other	0.068	0.065	0.091	-0.03	1.3e-09
Severity	3.44	3.46	3.25	0.21	6.1e-09

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