Racing with a rearview mirror: innovation lag and investment dynamics

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Introduction

Situations where:

- Agents are racing for an innovation under uncertainty, i.e.,
  - they want to be the first to achieve breakthrough with a risky technology;
  - they are uncertain about the feasibility of the breakthrough.
- Outcomes of experimentation effort occur with delay.

Typical example: patent races for new drugs/ vaccines.

→ Strategic experimentation with positive informational externality, negative payoff externality and outcome lag.

The model

- time is continuous, no discounting;
- continuum of "short-lived" players: player t only plays at time t;
- each player t chooses $k_t \in [0, 1]$ to invest in a risky technology at unit cost α;
- good news model of experimentation with delayed outcomes: the technology can be good ($\theta = 1$) or bad ($\theta = 0$):
  - if $\theta = 0$, the technology never yields any success;
  - if $\theta = 1$, the technology yields a success at every jump of a time-inhomogeneous Poisson process with rate $\lambda k_{t-\Delta} \mathbb{1}_{t \geq \Delta}$, with $0 < \alpha < \lambda$;

    Probability of a breakthrough before t: $\begin{cases} 0 & \text{if } t \leq \Delta \\ 1 - e^{-\lambda \int_0^{t-\Delta} k_s ds} & \text{if } t > \Delta \end{cases}$

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- efforts and outcomes are public;
- p_t: common belief at time t that $\theta = 1$ (p_0: a priori belief):

$$\dot{p}_t = -p_t(1-p_t)\lambda k_{t-\Delta}\mathbb{1}_{t\geq\Delta}\ \forall\, t$$

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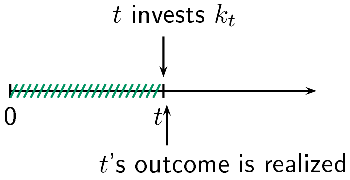

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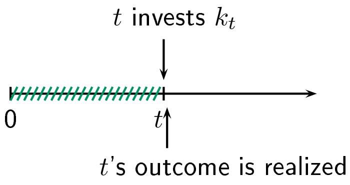

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Investment

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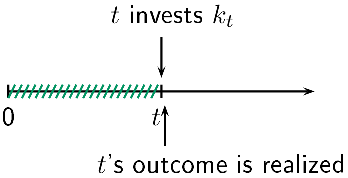

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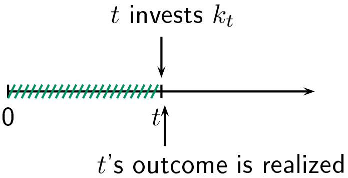

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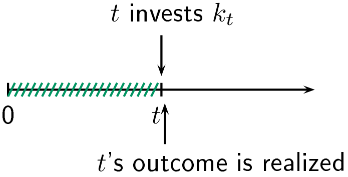

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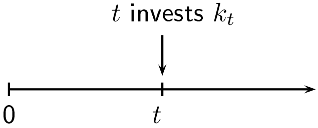

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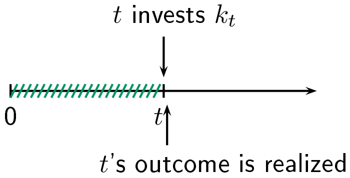

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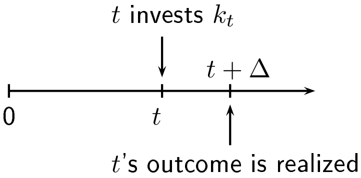

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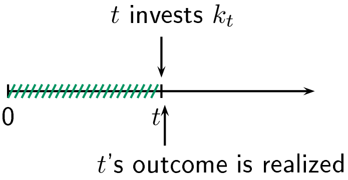

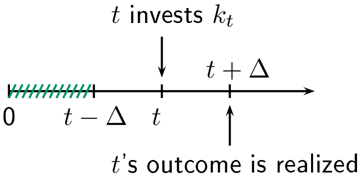

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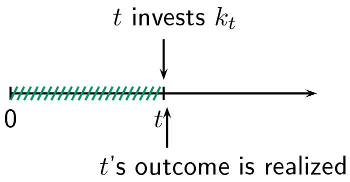

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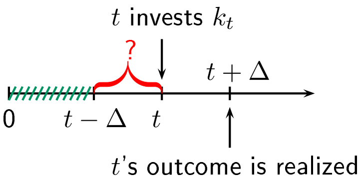

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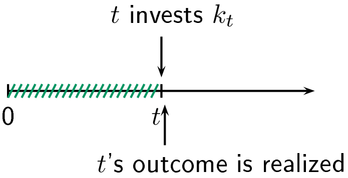

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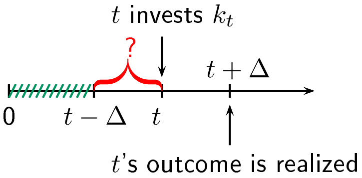

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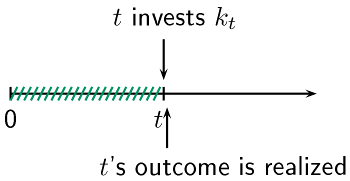

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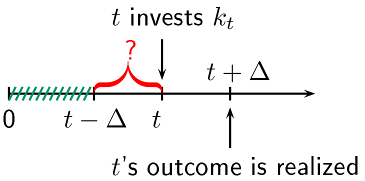

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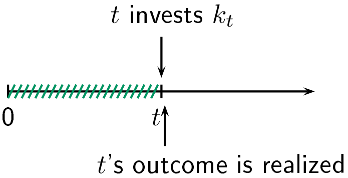

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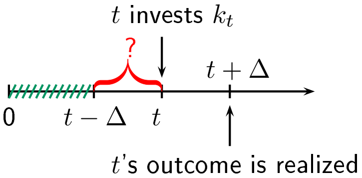

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Equilibrium analysis

Recall that player t's expected payoff is:

$$u(k_t; k_{-t}) = k_t \left(-\alpha + \lambda \underbrace{p_t e^{-\lambda \int_{(t-\Delta)\mathbb{1}_{t \geq \Delta}}^{t} k_s ds}}_{:= \mu_t} \right)$$

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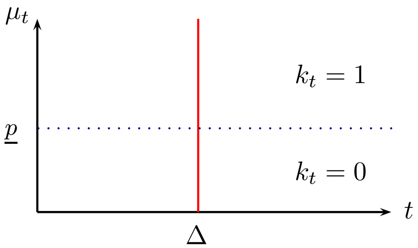
Player t's best response to k_{-t}:

$$\Rightarrow k_t \begin{cases} = 1 & \text{if} \quad \mu_t > \underline{p} \\ \in [0,1] & \text{if} \quad \mu_t = \underline{p} \\ = 0 & \text{if} \quad \mu_t < \underline{p} \end{cases}$$

where $\underline{p} = \frac{\alpha}{\lambda}$.



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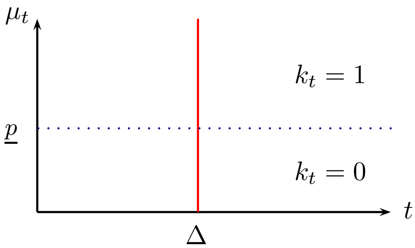

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$$\dot{\mu}_t = -\mu_t \lambda (k_t - p_t k_{t-\Delta} \mathbb{1}_{t \geq \Delta})$$
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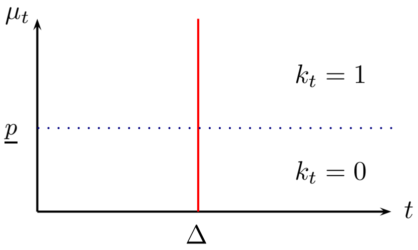
$\Rightarrow$ μ_t weakly decreases when $t \leq \Delta$ or $k_{t-\Delta} = 0$.



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$\rightarrow$ investment is less and less attractive on $[0, \Delta]$ and during periods of no (past) competition.

Initial pessimism: $p_0 < \underline{p}$

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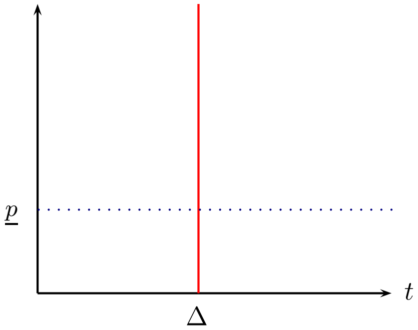

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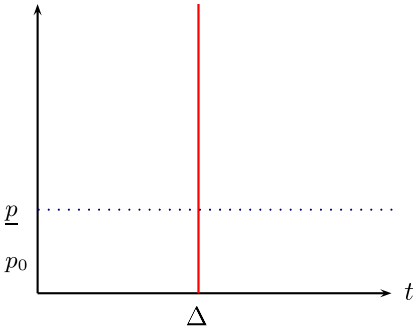

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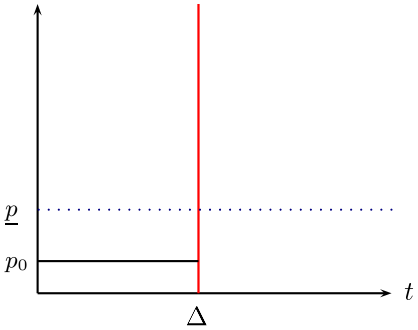

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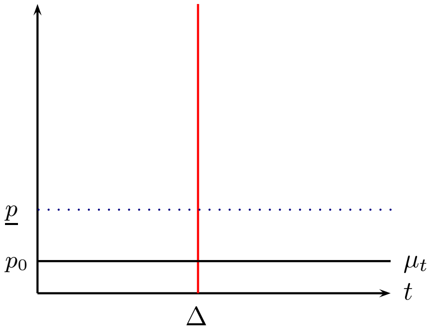

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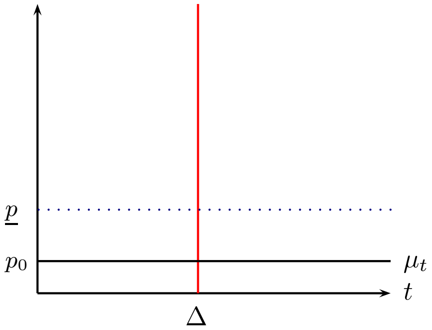

If $p_0 \leq \underline{p}$, then $k_t = 0$ for all t.

Initial optimism: $p_0 > \underline{p}$

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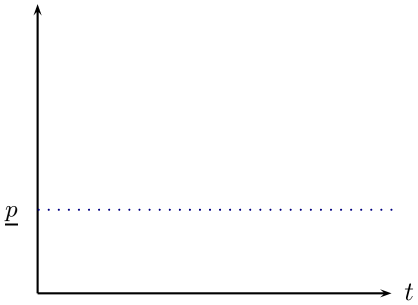

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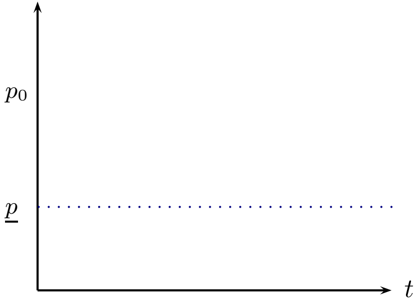

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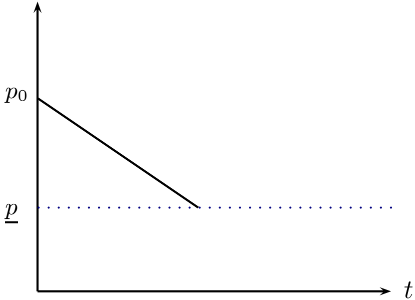

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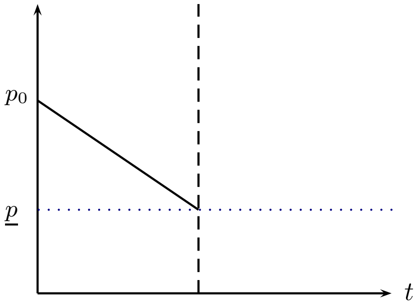

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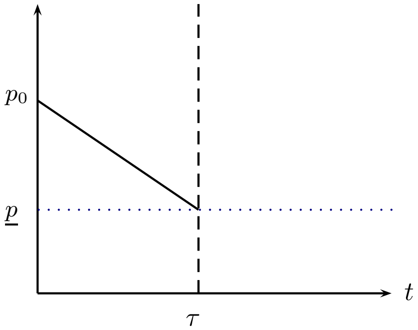

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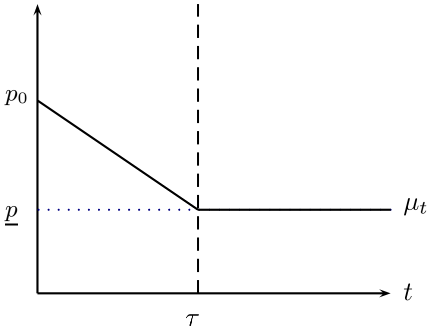

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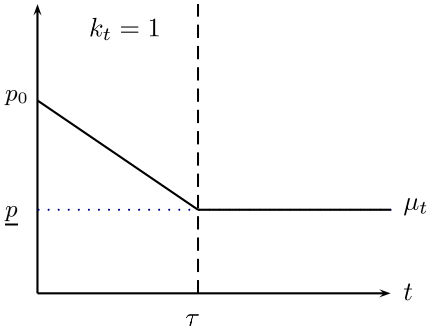

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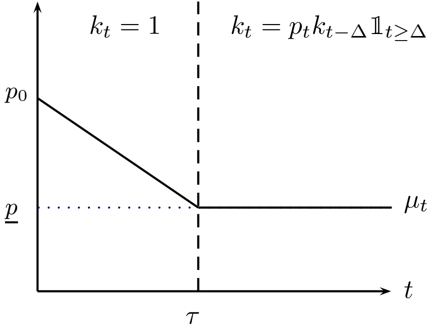

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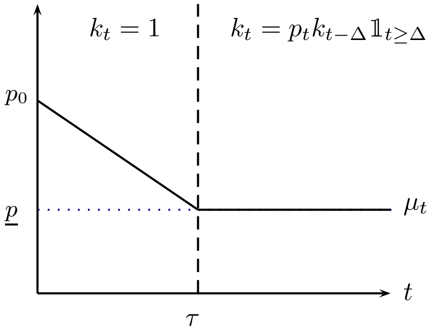
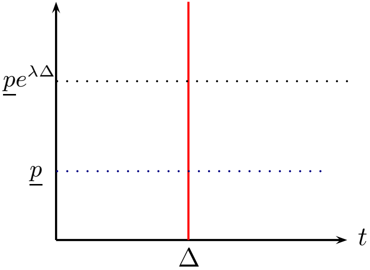
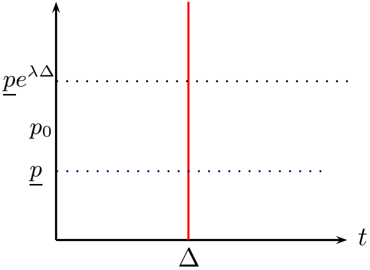


If $p_0 > \underline{p}$, there is $\tau > 0$ such that $k_t = \begin{cases} 1 & \text{for } t < \tau \\ p_t k_{t-\Delta} \mathbb{1}_{t \geq \Delta} & \text{for } t \geq \tau \end{cases}$


$\underline{p}e^{\lambda\Delta}$
$\underline{p}$
t
Δ



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p_0
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t
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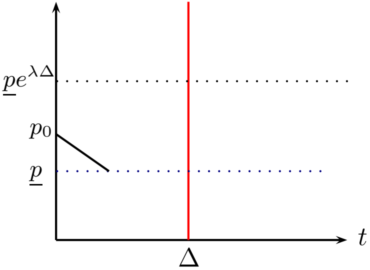

$\underline{p}e^{\lambda\Delta}$
p_0
$\underline{p}$
t
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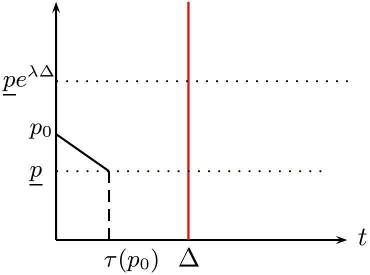
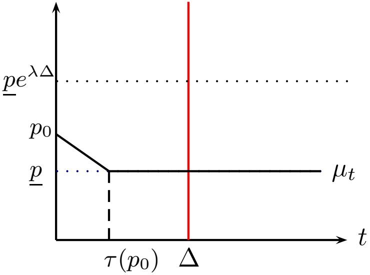

$\underline{p}e^{\lambda\Delta}$
p_0
$\underline{p}$
t
$\tau(p_0)$
Δ


$\underline{p}e^{\lambda\Delta}$
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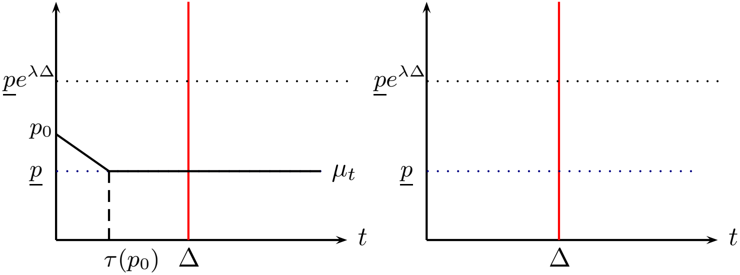
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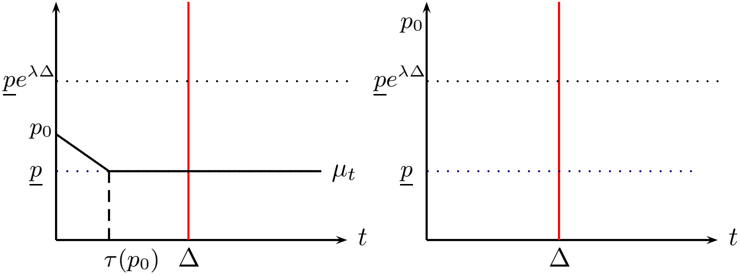
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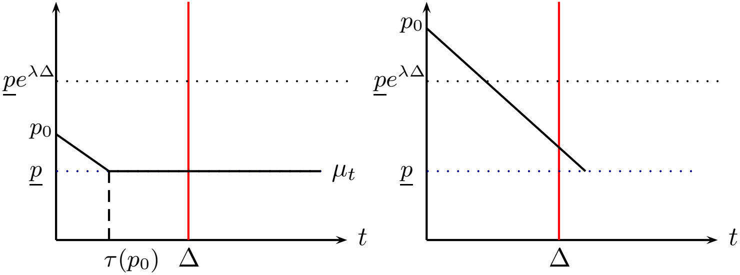

$\underline{p}e^{\lambda\Delta}$
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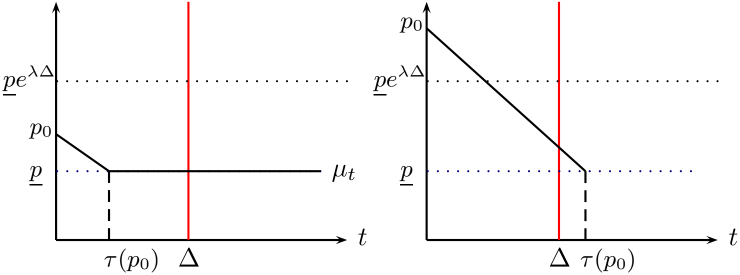
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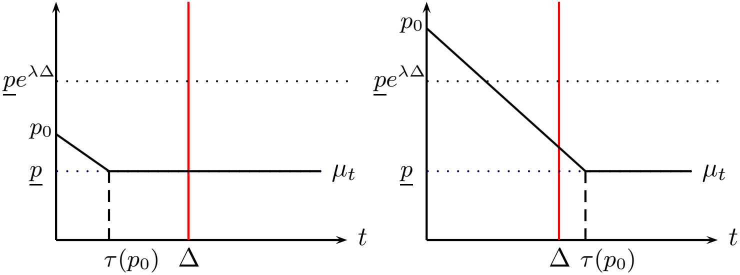
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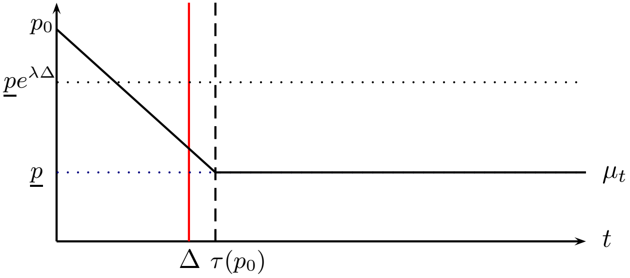
$$p_0 > \underline{p}e^{\Delta}$$



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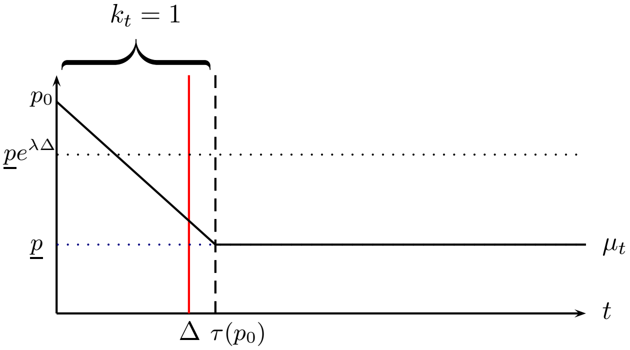

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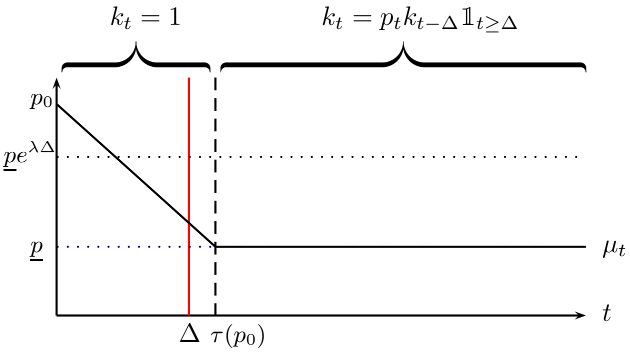

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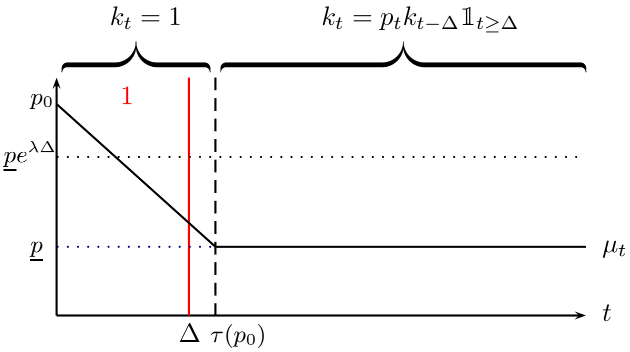

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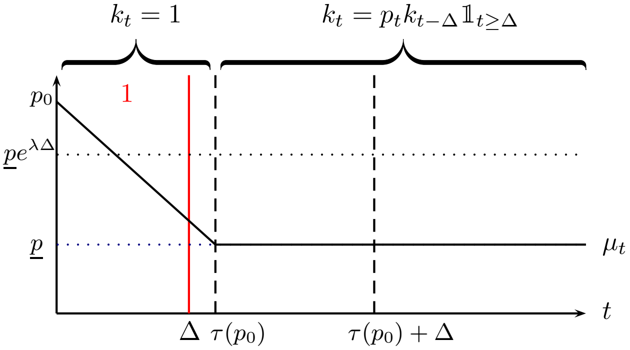

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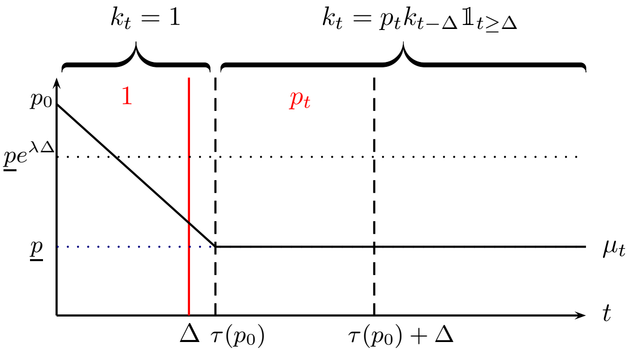

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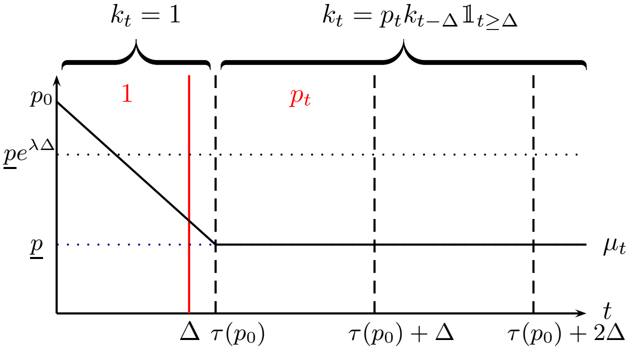

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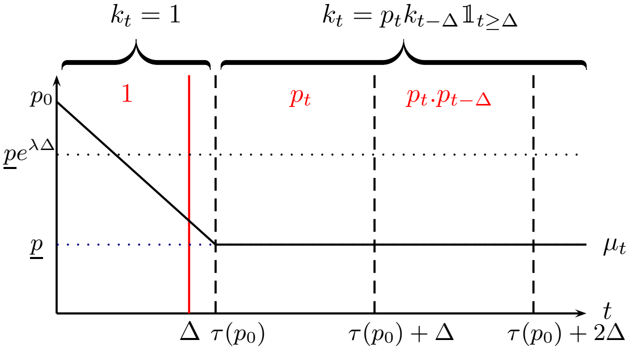

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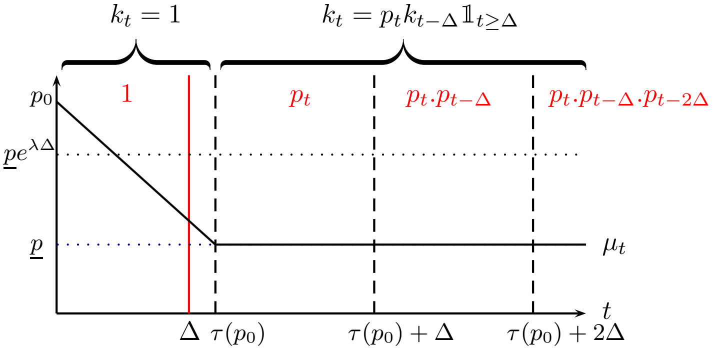
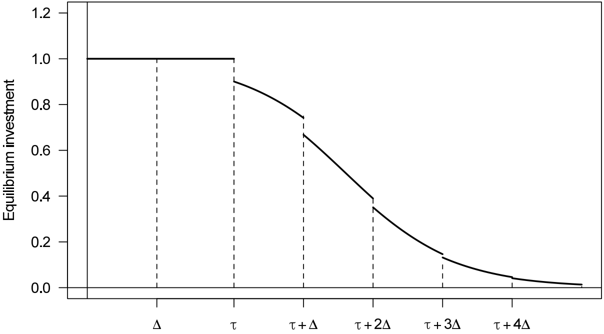


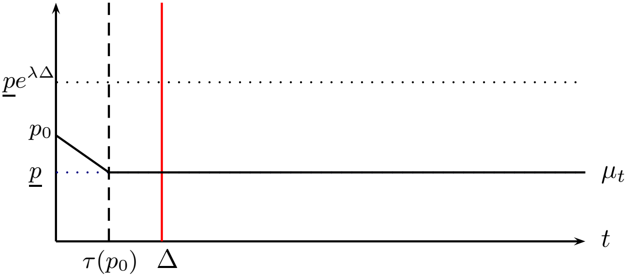
Proposition At the unique Nash equilibrium, if $p_0 > \underline{p}e^{\Delta}$, then

$$k_t^* = \begin{cases} 1 \text{ for } t < \tau \text{ and, } \forall\, n \in \mathbb{N}, \\ \prod_{m=0}^{n} p_{t-m\Delta} \text{ for } t \in [\tau + n\Delta, \tau + (n+1)\Delta) \end{cases}$$

$\rightarrow$ investment is monotonically decreasing, with downward jumps at τ, $\tau + \Delta$, $\tau + 2\Delta$, ...



$$\underline{p} < p_0 < \underline{p}e^{\Delta}$$



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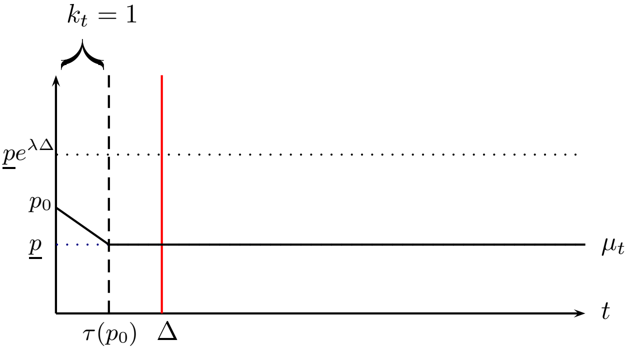

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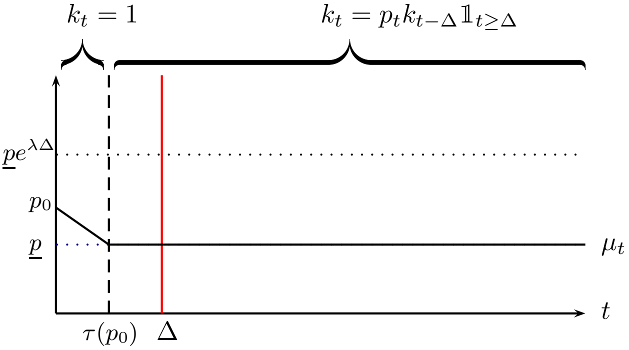

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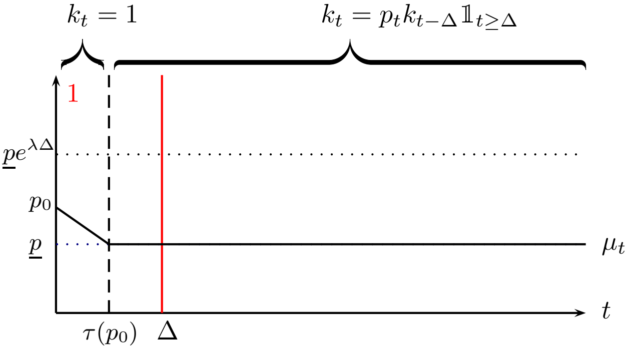

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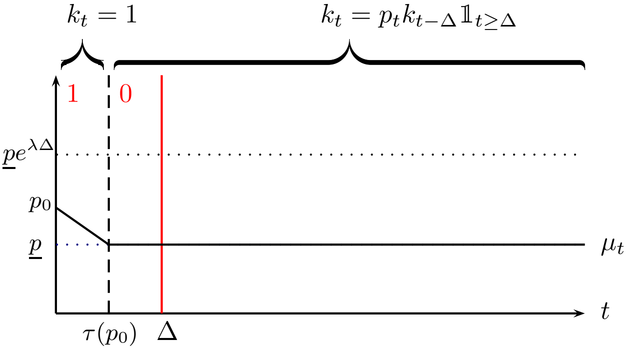

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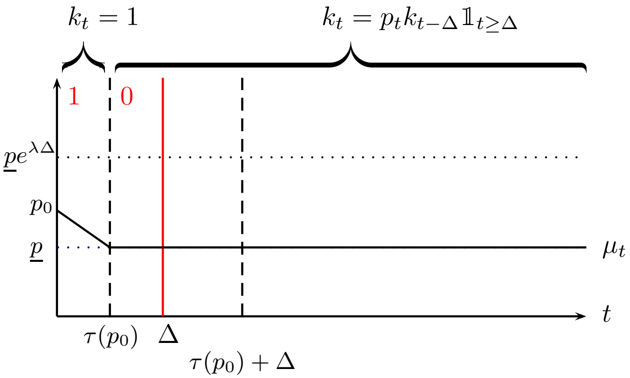

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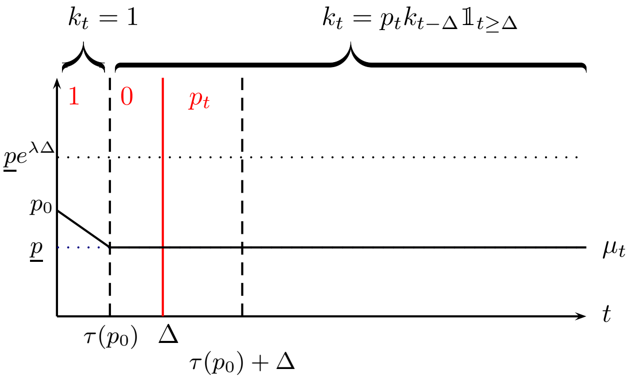

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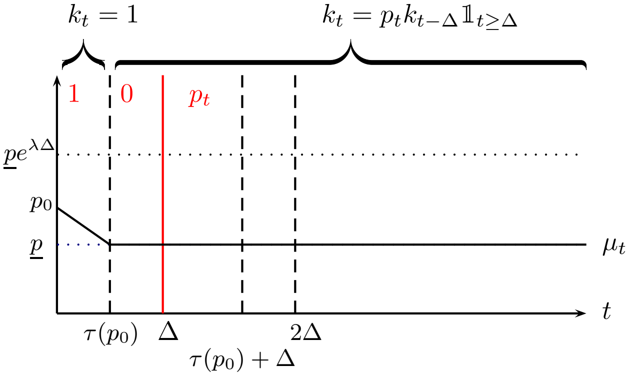

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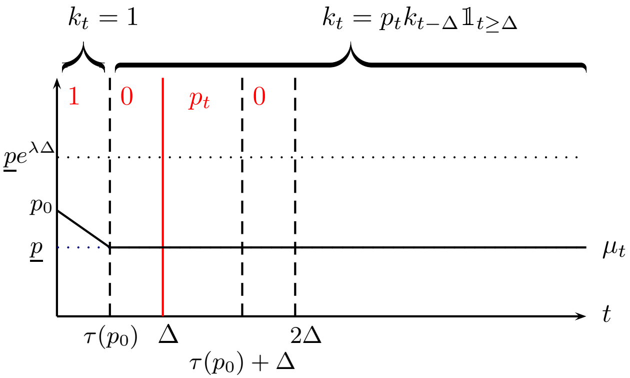

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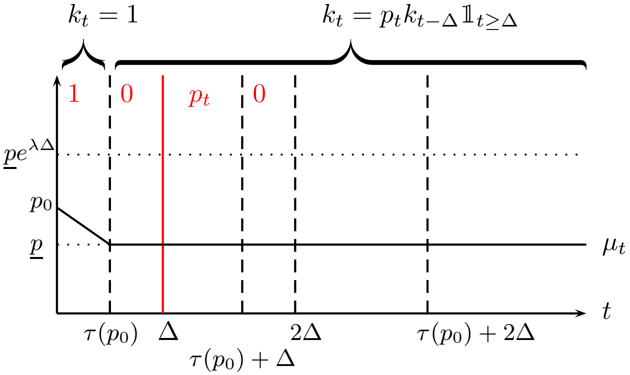

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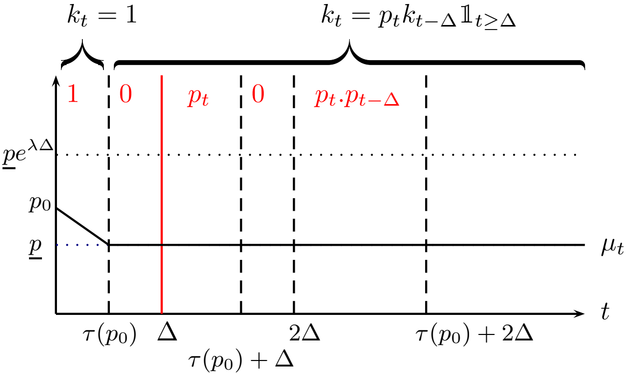

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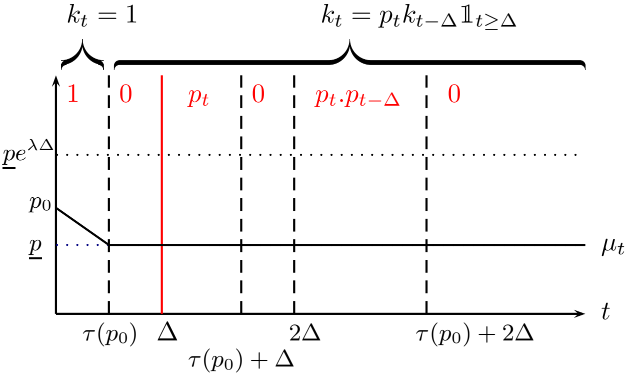

Proposition At the unique Nash equilibrium, if $\underline{p} < p_0 < \underline{p}e^{\Delta}$, then

$$k_t^* = \begin{cases} 1 & \text{for } t < \tau \text{ and, } \forall n \in \mathbb{N}, \\ 0 & \text{for } t \in [\tau + n\Delta, (n+1)\Delta) \\ \prod_{m=0}^{n} p_{t-m\Delta} & \text{for } t \in [(n+1)\Delta, \tau + (n+1)\Delta) \end{cases}$$

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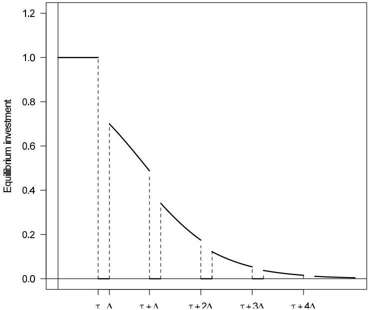
$\rightarrow$ Investment is non-monotonic: jumps down at times $\tau + n\Delta$, jumps up at times $n\Delta$.

$\Rightarrow$ more pessimistic generations may experiment more, because they fear less to be preempted



Asymptotics

- Investment converges to 0: $\lim_{t\to\infty} k_t^* = 0$ for any $p_0 < 1$;
- Same amount of experimentation as cooperative players:

$$\int_0^\infty k_t^* dt = \hat{K} \text{ for any } p_0 > \underline{p};$$

- Common belief converges to $\underline{p}$ if $p_0 > \underline{p}$.

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Welfare analysis

The aggregate payoff in equilibrium is

$$W(k^*) = \begin{cases} 0 & \text{if } p_0 \le \underline{p} \\ p_0 - \underline{p} + \underline{p} \ln\left(\frac{\underline{p}}{p_0}\right) & \text{if } p_0 \in \left[\underline{p}, \underline{p}e^{\lambda\Delta}\right] \\ -\alpha\Delta + p_0 - \underline{p} + \underline{p}(1-p_0)\ln\left(\frac{\Omega(\underline{p}e^{\lambda\Delta})}{\Omega(p_0)}\right) & \text{if } p_0 \ge \underline{p}e^{\lambda\Delta} \end{cases}$$

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$\rightarrow$ increases with p_0; either does not depend on, or decreases with Δ.

Proposition The equilibrium is inefficient, i.e., $W(k^*) < W(\hat{k})$ if $p_0 > \underline{p}$.

Argument: the cutoff strategy $\tilde{k}_t = \mathbb{1}_{t \leq \tau}$ replicates the equilibrium payoff. Yet for any cutoff strategy, the social planner can improve the total payoff by postponing the last "period" of experimentation after the cutoff.

Source of inefficiency: intermediate investment.

Thank you!

Concluding remarks

The outcome lag is a source of inefficiency because players are afraid to be preempted, thus do not fully experiment.

$\Rightarrow$ is it possible to improve the aggregate payoff with another mechanism/reward scheme?

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The outcome lag is a source of inefficiency because players are afraid to be preempted, thus do not fully experiment.

⇒ is it possible to improve the aggregate payoff with another mechanism/reward scheme?

The family of *Hidden outcomes mechanisms* work as follows:

- Principal observes the outcomes but keeps them secret until some deadline T.
- If there has been at least one success between 0 and T, then the payoff 1 is shared among all those players who obtained a success according to **some reward scheme** (equal sharing, first takes all, etc...)

Aggregate payoff under a hidden mechanism: If $\int_0^\infty k_t dt < +\infty$,

$$W^H(k) =$$

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This expression is maximal when $\int_0^\infty k_t dt = \frac{1}{\lambda} \ln\left(\frac{p_0}{\underline{p}}\right) < \hat{K}$.

Aggregate payoff under a hidden mechanism: If $\int_0^\infty k_t dt < +\infty$,

$$W^H(k) = -\alpha \int_0^\infty k_t dt + p_0(1 - e^{-\lambda \int_0^{+\infty} k_t dt})$$

This expression is maximal when $\int_0^\infty k_t dt = \frac{1}{\lambda} \ln\left(\frac{p_0}{\underline{p}}\right) < \hat{K}$.

Interpretation

- at the best hidden outcomes mechanism, there is under experimentation;

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Interpretation

- at the best hidden outcomes mechanism, there is under experimentation;
- $\max_k W^H(k) \begin{cases} = W(k^*) \text{ if } p_0 \in [\underline{p}, \underline{p}e^\Delta] \\ < W(k^*) \text{ if } p_0 \geq \underline{p}e^\Delta \end{cases}$

A *hidden outcomes mechanism* cannot improve the aggregate payoff.

Related literature

- Strategic experimentation
  - Bolton & Harris (1999) Keller, Rady, Cripps (2005), Keller & Rady (2010), Klein & Rady (2011), Keller & Rady (2015),...
  - Bonatti & Hörner (2011, 2017), Heidhues, Rady, Strack (2015), Marlats & Ménager (2021),...
  - Rosenberg, Solan, Vieille (2007), Murto & Välimäki (2011), Rosenberg, Salomon, Vieille (2013), Renault, Solan, Vieille (2022),...
- Experimentation with a competition component (payoff externality)
  - Moscarini & Squintani (2010), Das & Klein (2022),...
- Contest Design
  - Halac, Kartik, Liu (2017), Bimpikis, Ehsani, Mostagir (2019),...
- Observation lags
  - Gordon, Marlats, Ménager (2021)