Nonparametric Estimation of Sponsored Search Auctions and Impact of Ad Quality on Search Revenue

Dongwoo Kim (Simon Fraser University)
Pallavi Pal (Stevens Institute of Technology)

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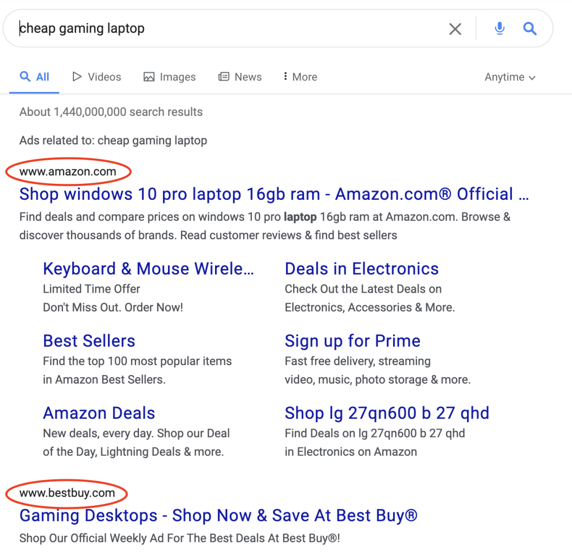
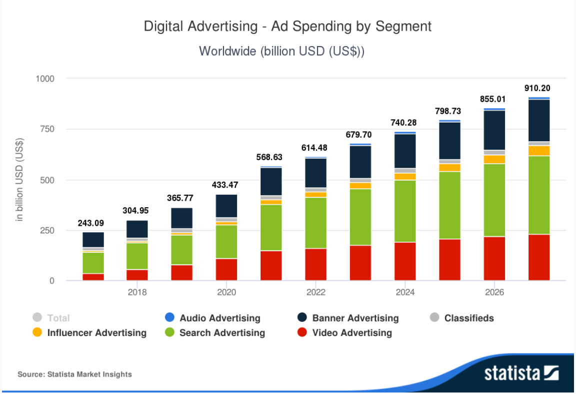
Introduction

Sponsored Search: example



Sponsored Search: important?

- Online advertising is extremely big business
- Sponsored search ad is largest segment (2023: $252 bil USD)



Sponsored Search Auction: GSP

- Search engines used to sell sponsored search ads through a **generalized second price auction (GSP)**
- Advertisers bid for their ads with specific *keywords*
- Consumer submits search query, then auction is held for ads with relevant keywords
- **(untruthfulness)** Bidders don't reveal their value – bid shading

Sponsored Search Auction: weighted bids

- Search ads are contingent objects (price only paid when ads get clicked)
- Therefore, in order to maximize revenue, not only bid but also *probability of being clicked* should be considered

Modified scheme: weighted GSP (GSPw)

- Bids are multiplied by **quality score** (measuring advertiser's clickability)
- Ad positions are allocated in decending order of weighted bids
- Advertisers who won k-th ad position pay $(k+1)$-*th highest* bid divided by their own quality score when the ad gets clicked

Sponsored Search Auction: Example

Table 1: Example of a GSP^w auction

Advertiser	Bid	Quality Score	Weighted Bid	Position	Price per click
A	2	0.8	1.6	2nd	1.50
B	3	0.6	1.8	1st	2.67
C	1	0.9	0.9	X	X
D	4	0.3	1.2	3rd	3.00

This paper's contribution

- Equilibrium analysis of weighted GSP auction under weaker assumptions
  - incomplete information (IPV)
  - general quality scoring rule
  - correlation between value and quality
- Novel nonparametric identification and estimation of valuations and its distribution
  - only given observables and auction characteristics
  - no tuning parameter, no density estimation of bids
- Empirically assess biding behaviours and auction heterogeneity (Yahoo! data)
- Counterfactual: optimal score squashing

Related Literature

- Identification of valuation in search ad auctions
  - Case of complete information : Varian (2007), Edelman, Ostrovsky, and Schwarz (2007)
  - Case of uncertainty: Athey & Nekipelov (2011)
  - Case of Incomplete information: Gomes & Sweeney (2012)
- Methodology
  - Nonparametric estimation in first-price auctions: Guerre, Perrigne, and Vuong (2000), Li and Perrigne (2003), Hendricks, Pinkse and Porter (2003), Flambard and Perrigne (2006), Campo, Perrigne and Vuong (2003)
- Impact of ad quality on auctioneer's revenue
  - Score squashing: S. Lahaie & D. Pennock. (2007), Charles, Devanur & Sivan. (2016), Athey & Nekipelov (2011)

Auction Model

Consumer side

- Consumer i has unit demand for product and puts search query
- Once search results displayed, consumer decides whether or not to click on some ads
- U_{ij}: Expected utility of consumer i from clicking on ad j
- Click decision

$$y^*_{i,j} = \begin{cases} 1 & \text{consumer } i \text{ clicks on ad } j \text{ if } U_{i,j} > 0 \\ 0 & \text{if } U_{i,j} \leq 0 \end{cases}$$

Consumer side: click probability

- Each result page displays K number of ads
- Click probability is computed using the consumer side information
- We assume click probability is product of advertiser effect s_j and position effect c_k

$$\text{probability of click} = s_j \times c_k$$

- Advertiser's optimal response takes s_j and c_k as given

Advertiser Side

- Advertiser $j \in \{1, \cdots, N\}$ puts ad on search engine
- v_j: j's value per click, $v_j \sim_{iid} F_v[\underline{v}, \bar{v}]$
- s_j: j's clickability, $s_j \sim_{iid} F_s[\underline{s}, \bar{s}]$
- q_j: j's quality score, generally a function of s_j i.e. $q_j = q(s_j)$
- $w_j \equiv v_j \times q_j$: j's weighted value, $w_j \sim_{iid} F_w$
- j's type is defined by (v_j, s_j)

Auction setup

- Single auction (weighted GSP) for each search query to sell K ad positions
- Bidding function: $b_j = b(v_j, s_j)$ (in symmetric equilibrium)
- Ad positions are allocated in descending order of weighted bid

$$b_{j,w} = b_j \times q_j$$

- $b_w^{[k]}$: order statistic, k-th highest weighted bid
  - j wins k-th position if $b_{j,w} = b_w^{[k]}$
  - j pays $p_{j,k} = b_w^{[k+1]}/q_j$ if ad gets a click

Assumptions

Assumption 1. (Incomplete information) (Incomplete information) Each advertiser knows their type, (v, s), and the scoring rule, q, but does not know the opponents' bids, quality scores, and values. They only know the weighted value distribution, F_w. The number of advertisers (N), click rates across ad positions ($\mathcal{C} = (c_1, \cdots, c_K)$) and the number of ads per page (K) are common knowledge.

Assumption 2. (monotonic bidding) The advertiser's weighted bid in the GSP^w auction is strictly increasing in his weighted value.

Profit Maximization

- Profit from k-th ad-position:

$$\pi_{k,j} = \underbrace{(c_k \times s_j)}_{\substack{\text{Prob. of click} \\ \text{at position } k}} \times \underbrace{(v_j - p_{j,k})}_{\substack{\text{Per click profit} \\ \text{at position } k}} \tag{1}$$

- Expected profit from auction:

$$\Pi(b_j; v_j, s_j) = \sum_{k=1}^{K} \underbrace{Prob(b_{j,w} = b_w^{[k]})}_{\text{Prob. of winning ad-position } k} \times \underbrace{\mathbb{E}(\pi_{k,j} | b_{j,w}, s_j)}_{\text{Profit from ad-position } k}$$

Equilibrium Analysis

- Equilibrium bid $b(v_j, s_j) = \hat{b}$ maximizes

$$\sum_{k=1}^{K} Prob(\hat{b} \cdot q_j = b_{j,w}^{[k+1]})(c_k \times s_j)\left[v_j - \mathbb{E}\left(\frac{b_w^{[k+1]}}{q_j}\middle| b_w^{[k]} = \hat{b} \cdot s_j\right)\right]$$

- 2D optimization, hard to deal with
- Symmetric equilibrium bid exists in GSP (no weight) auction

  $$b^{GSP}(v_j)$$

  shown by Gomes and Sweeney (2014)
- Solution: solve everything with weighted bid and weighted value (dimensional reduction)

Theorem 1. Under any scoring rule, every equilibrium weighted bid in GSP^w auction is typewise outcome equivalent to an equilibrium bid function in a GSP auction, where the value is replaced by the weighted value.

$$b_w^{GSP^w}(v_j, s_j) = b^{GSP}(\omega_j), \quad \forall j \in \mathcal{J}$$

Corollary 1. The GSP^w has a unique symmetric Bayesian Nash equilibrium, which is efficient if the quality score is equivalent to the advertiser-specific click rate.

Unique Symmetric Bayes-Nash Equilibrium (BNE)

Corollary 2. The unique symmetric Bayesian Nash equilibrium of the weighted GSP auction is given by

$$b_w(\omega) = \omega - \Gamma(\omega) - \sum_{n=1}^{\infty} \int_0^{\omega} M_n(\omega, t)\Gamma(t)dt, \quad \forall \omega \sim F_w(.), \qquad (2)$$

where

$$\Gamma(\omega) = \frac{\sum_{k=1}^{K} c_k \binom{N-2}{k-1}(k-1)(1-F_w(\omega))^{k-2} \int_0^{\omega} F_w^{N-k}(x)dx}{\sum_{k=1}^{K} c_k \binom{N-2}{k-1}(1-F_w(\omega))^{k-1} F_w^{N-k-1}(\omega)},$$

$$M_1(\omega, t) = \frac{\sum_{k=1}^{K} c_k \binom{N-2}{k-1}(k-1)(1-F_w(\omega))^{k-2} F_w^{N-k-1}(t) f(t)}{\sum_{k=1}^{K} c_k \binom{N-2}{k-1}(1-F_w(\omega))^{k-1} F_w^{N-k-1}(\omega)},$$

$$M_n(\omega, t) = \int_0^{\omega} M_1(\omega, \varepsilon) M_{n-1}(\varepsilon, t) d\varepsilon, \text{ for } n \geq 2.$$

Identifying valuation

- Analytically inverting the BNE bidding function is impossible because:
  - Distribution and density of w_j never known to econometrician
  - multiple layers of summations and integrals (no analytic representation)
- We instead derive alternative derivation using creative algebraic manipulation

Theorem 2. Under incomplete information assumption, advertiser j's value is identified by:

$$v_j = b_j + \Phi(G_w, b_{w,j}, q_j | \mathcal{C}, K, N) \tag{3}$$

where

$$\Phi(G_w, b, q | \mathcal{C}, K, N) = \frac{\sum_{k=1}^{K} c_k \binom{N-1}{k-1} (k-1)(1 - G_w(b_w))^{k-2} \int_0^{b_w} G_w(u)^{N-k} du}{q \sum_{k=1}^{K} c_k \binom{N-1}{k-1} G_w(b_w)^{N-k-1} (1 - G_w(b_w))^{k-2} \Big[(N-k)(1 - G_w(b_w)) - (k-1) G_w(b_w) \Big]}$$

Data

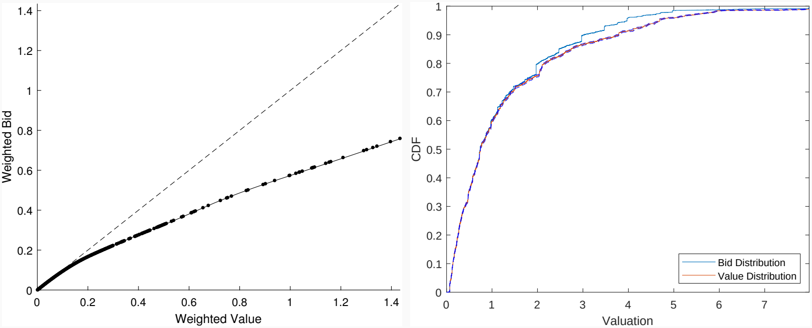
Yahoo! Sponsored Search Auction data

- All search queries in Jan-Apr 2008
- 5 product categories: *Cruise*, *Car Insurance*, *Laptop*, *Cable TV*, *Collectible Coins*
- 30GB data set, 78 million observations, data aggregated at category-advertiser-day level
- Consumer side variables: # of clicks, # of ad displays for each position-advertiser-keyword combination
- Advertiser side: bid, position won for each advertiser-keyword combination
- Keywords and advertiser ids are anonymized
- We focus on the top 10% most popular keywords to ensure common value assumption

Estimation

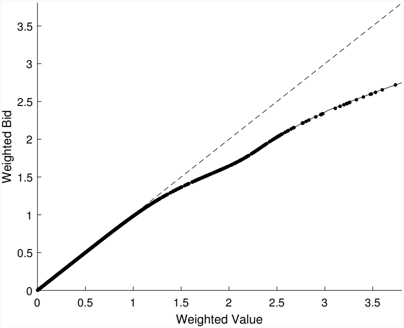
Equilibrium biding function and value distribution

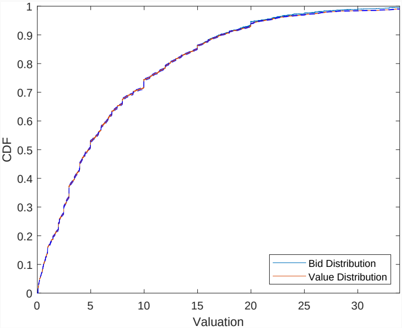
Cruise (N=45)



Equilibrium biding function and value distribution

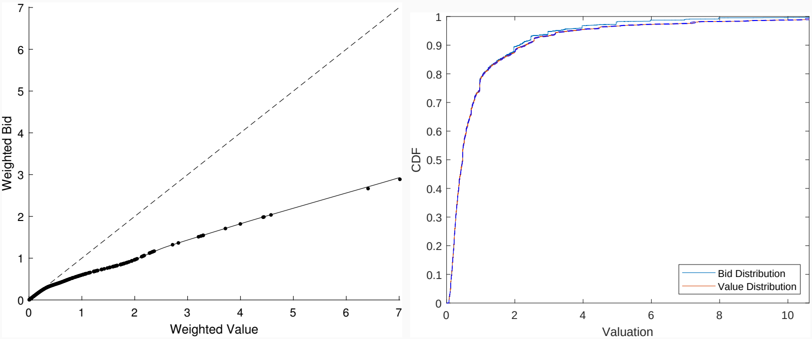
Car insurance (N=403)





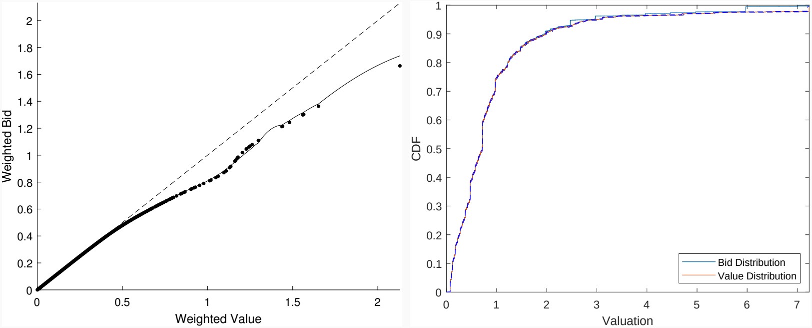
Equilibrium biding function and value distribution

Laptop (N=124)



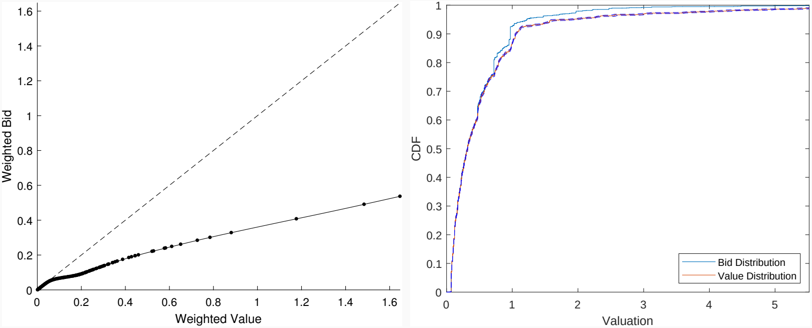
Equilibrium biding function and value distribution

Cable TV (N=142)



Equilibrium biding function and value distribution

Coins (N=55)



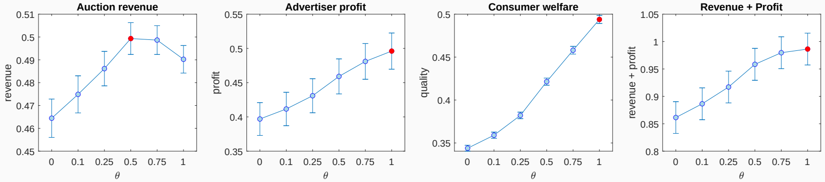
Counterfactual Analysis

Score squashing

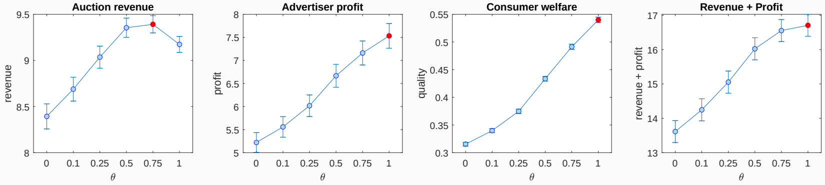
- Why weighted bid?
  - To accommodate the impact of ad quality.
  - Alternate practice: Score Squashing
- Score squashing is a way to change the relative importance of the quality weights by raising the quality score by a parameter $\theta \in [0, 1]$.
  - **Scores:** $q_j = s_j^{\theta}$

Counterfactual: results

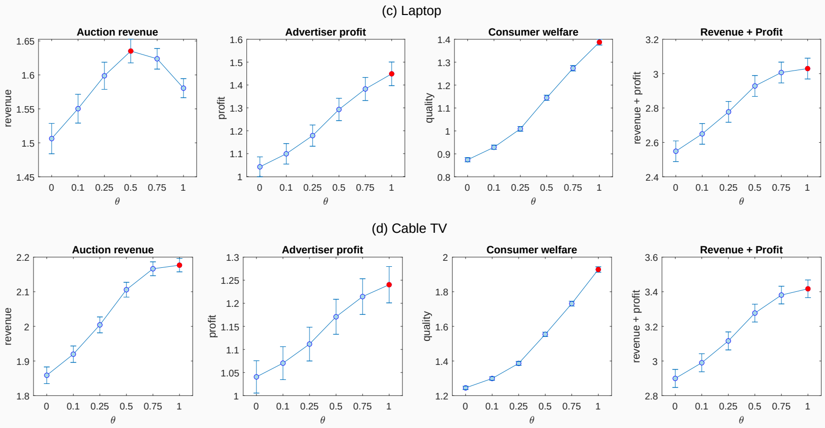
(a) Cruise



(b) Car Insurance

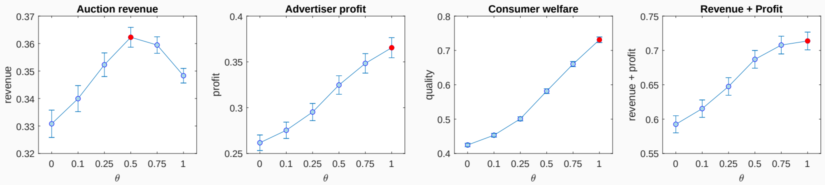


Counterfactual: results



Counterfactual: results

(e) Coins



Conclusion

Conclusion

- Our nonparametric estimator for valuation works well and easy-to-use
- More competition in the market means less bid shading/more revenue
- Score squashing can enhance revenue at the cost of advertiser profit and consumer welfare
- Further extensions in the paper: limited bid data, reserve price