

Domestic Policies and Sovereign Default*

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Abstract

A model with two essential elements, sovereign default and distortionary fiscal and monetary policies, explains the interaction between sovereign debt, default risk and inflation in emerging countries. We derive conditions under which monetary policy is actively used to support fiscal policy and characterize the intertemporal tradeoffs that determine the choice of debt. We show that in response to adverse shocks to the terms of trade or productivity, governments reduce debt and deficits, and increase inflation and currency depreciation rates, matching the patterns observed in the data for emerging economies.

JEL Classification: E52, E62, F34, F41, G15.

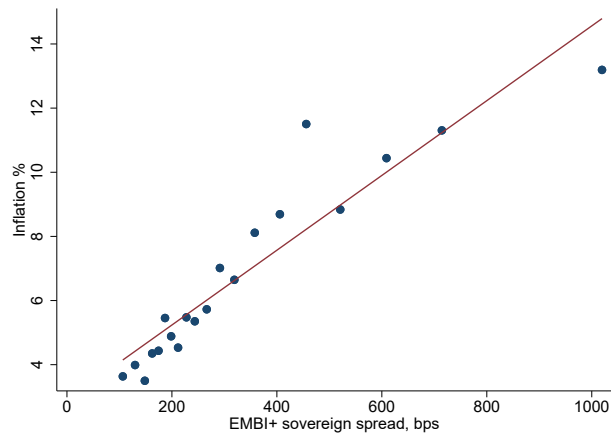
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1 Introduction

A now large literature, spanned by the work of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008), studies recurrent sovereign debt crises by contending that emerging countries underinsure against negative shocks by overborrowing during booms. It is also widely understood, though mostly ignored by this literature, that emerging markets have traditionally experienced high inflation, especially during debt crises.¹ Figure 1 illustrates this fact by showing the correlation between sovereign debt spreads and inflation in emerging markets countries since 1990.

Figure 1: Inflation and Sovereign Spreads in Emerging Markets



Note: This figure is a binscatter. Each dot is the mean for a bin containing the same number of observations. We removed outliers, defined as values below the 3rd percentile or above the 97th percentile, for both series.

In this paper, we argue that extending the standard sovereign default model to incorporate distortionary *domestic policies*, explains the close connection between debt crises and inflation. In our framework, domestic fiscal and monetary policies interact with the availability of external credit and the possibility of default. As in the standard model, debt accumulation is promoted by relative impatience and hindered by default risk. When policy instruments are distortionary and the government cannot commit to future policy choices, the government actively employs monetary policy to support fiscal policy. It also faces an intertemporal tradeoff that may moderate or amplify the incentives to accumulate debt. We also find that, since more significant distortions are necessary to support a larger debt, the money growth rate, taxes, and the exchange rate are increasing in debt.

As in the standard model, the government's inability to commit to debt repayment implies that aggregate shocks affect debt financing costs. In particular, adverse shocks raise the cost of

¹More generally, there is a concern that poorly designed or executed domestic policy frameworks adversely affect the resilience of emerging economies to shocks. For example, see Caballero (2003) and Kehoe, Nicolini, and Sargent (2020).

rolling over the debt. Barring default, higher financing costs induce the government to reduce the level of outstanding debt, which it pays for with a combination of fiscal and monetary actions. On the fiscal side, the government raises taxes and lowers spending. On the monetary side, the government prints money and devalues the currency at faster rates. This policy response leads to a tight connection between distress in sovereign debt markets, procyclical deficits, high inflation and currency depreciation. Importantly, we show that distortionary domestic policies and sovereign default risk are both necessary to understand the real and nominal effects of adverse shocks in emerging economies.

Our framework consists of a tradable-nontradable (TNT) small open economy (as in Uribe and Schmitt-Grohé, 2017, §8), extended to include production, money and sovereign default. Firms produce both non-tradable goods and exported goods; agents consume non-tradable goods and imported goods. Consumers need money to finance their purchases of non-tradable goods, which gives rise to a demand for fiat money.² The government provides a valued public good and makes transfers to individuals; expenditures are financed with labor taxes, money creation and external debt. Government debt is issued in foreign currency to foreign risk-neutral investors. In the event of default, the government enjoys a haircut on its external liabilities but suffers temporary exclusion from financial markets and a productivity loss. We further assume that the government's inability to commit extends to all future policy actions.

We derive necessary first-order conditions to characterize government policy, including a Generalized Euler Equation that characterizes debt choice.³ We first derive conditions under which monetary policy is used to support fiscal policy, thus implementing an inefficient rate of return on money, i.e., creating distortionary inflation. We find that these results depend on agents' preferences and the level of (exogenous) transfers from the government to households. Then, we show that the decision of how much debt to issue depends on three channels. The first involves distortion-smoothing: debt allows the government to trade-off intertemporally how severely the balance of payments restricts its policy. If the domestic economy is impatient relative to the rest of the world, then this factor provides incentives to accumulate debt. The second channel reflects the negative impact of more debt, which leads to a higher default premium and, thus, counters the desire to accumulate debt. The third channel arises because fiscal and monetary policies are distortionary. Higher debt tomorrow leads to larger future distortions when repaying and a larger default probability, both of which affect the demand for money today and, hence, the government's current budget constraint. This factor may be

²Though we model domestic government liabilities as fiat money, one could also interpret them more generally to include debt issued domestically in local currency as long as this debt is liquid to some extent.

³Arellano, Mateos-Planas, and Ríos-Rull (2019) and Karantounias (2019) also derive the Generalized Euler Equation to solve and characterize a sovereign default model.

positive or negative, depending on agents' preferences, and thus may amplify or moderate the incentives to issue debt.

We conduct several quantitative exercises to evaluate the model's ability to capture notable regularities in emerging markets. First, we compare model simulations with data from seven Latin American economies since 1980. In the presence of fluctuations in the terms of trade or productivity, the model replicates standard business cycle statistics and, more importantly, the cyclical properties of fiscal and monetary policies. Second, we conduct an event study to show that inflation and currency depreciation rise during debt crises, a fact that the model is able to replicate. Third, we estimate impulse responses to shocks to the terms-of-trade and productivity. We show that the dynamics of the EMBI spread, inflation, currency depreciation and output have the same signs and similar magnitudes in the data and the model.

Our model has two main elements: distortionary domestic policies and sovereign default. We argue that both are necessary to understand the real and nominal effects of adverse shocks in emerging economies. First, we characterize the role of domestic policies by studying a version of our model in which the government has access to lump-sum taxes. We show that in such an economy, monetary policy is no longer distortionary and the equilibrium allocation coincides with the one that would obtain from a real economy. This version of our model cannot match the empirical levels of nominal variables, such as inflation and currency depreciation.

Second, we analyze the role of default risk by comparing our benchmark calibration with an economy with low sovereign default risk. A key difference between the two economies is that the default premium reacts very differently to adverse shocks. As we discussed above, in the typical emerging economy, as captured in our benchmark calibration, the interest rate on external debt rises significantly after an adverse shock, which induces the government to contract the amount of outstanding debt. This leads to significant responses in fiscal and monetary policies. In contrast, in a low-default economy, the default premium and hence, the interest rate, do not react significantly to an adverse shock, which implies debt, fiscal and monetary policies do not react as much in the event of an adverse shock. Overall, we show that the effect of adverse shocks on inflation and currency depreciation is significantly muted when default risk is low.

Related literature. The literature on sovereign default has evolved from the framework developed by Eaton and Gersovitz (1981) to quantitative models that account for stylized facts about business cycles in emerging countries (Aguiar and Gopinath, 2006; Arellano, 2008). Although recent models have added realistic features, such as long-term debt (Hatchondo and Martinez, 2008; Hatchondo, Martinez, and Sosa-Padilla, 2016) and sovereign-debt restructuring (Yue, 2010; Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021), there are few papers concerned

with the role of domestic policies. We aim to bridge that gap by developing a sovereign default model that incorporates both monetary and fiscal policies and has implications for inflation and currency depreciation.

The literature has argued that the prevalence of sovereign default risk rationalizes the fact that fiscal policy in emerging markets is procyclical, rather than countercyclical as in developed countries. Cuadra, Sánchez, and Sapriza (2010) makes this argument for tax policy in a standard sovereign debt default model. Bianchi, Ottonello, and Presno (2023) shifts attention to government spending and shows that even with strong nominal rigidities and Keynesian stabilizing gains, fiscal policy is procyclical when default risk is taken into consideration. Karantounias (2019) and Pouzo and Presno (2022) also study models in which the government chooses distortionary taxes, government debt and whether to default.

These papers do not take into account how monetary policy affects the cyclical properties of government policy. Introducing money, as we do, extends the scope of the analysis and complicates the environment by adding an intertemporal optimization problem for households, which the government needs to take into account when formulating policy. In this context, we show that a rise in sovereign debt spreads during bad times leads to an increase in tax and inflation rates, even though monetary policy alone could, in principle, absorb the financial burden of the shock and allow the primary deficit to react countercyclically. There is, thus, a similarity between our contribution and that of Bianchi, Ottonello, and Presno (2023): they show that procyclical fiscal policy in emerging market is robust to adding nominal rigidities, while we show that it is also robust to adding a flexible monetary policy.

Some papers are concerned with monetary policy in related environments. Díaz-Giménez, Giovannetti, Marimón, and Teles (2008) and Martin (2009, 2011), among others, study government policy without commitment in monetary economies. Unlike our work, they do not consider the role of sovereign default risk, which is critical for understanding emerging countries. Ottonello and Perez (2019) and Sunder-Plassmann (2020) study how the composition of sovereign debt interacts with default risk and inflation. Importantly, the former assumes that inflation has a utility cost and the government cannot raise seigniorage; the latter abstracts from the distinction between tradable and non-tradable goods, as well as exchange rates. Also related, Galli (2020) studies default risk and inflation when debt is issued in domestic currency and taxes are either lump-sum or exogenous. All these papers stress the role of inflation in diluting the real value of debt issued in domestic currency. This mechanism offers an alternative explanation for the rise in inflation during recessions. However, lenders' anticipation of this strategy makes it difficult for countries with weak central banks to borrow in domestic currency and may explain

why, during periods of high inflation in Latin America (most notably in the 1980s and 1990s), practically all of government debt was denominated in foreign currency.⁴ This fact leads us to abstract from domestic currency debt in our analysis.

More recently, Arellano, Bai, and Mihalache (2020) analyze the interaction of sovereign default risk with a monetary policy rule in a cashless economy. Their work complements ours since they study the case in which central bankers can commit to a Taylor rule, whereas we assume that both fiscal and monetary policies are discretionary and useful to finance government spending.

Another important aspect of our model is the role of nominal exchange rates. In this regard, our work connects with Na, Schmitt-Grohé, Uribe, and Yue (2018), which point to the link between devaluations and default. In a model with downward nominal wage rigidity, they show that an optimal exchange rate devaluation occurs in periods of default, lowering the real value of wages to reduce unemployment. Their paper and ours both show how to recover a “real” economy as in Eaton and Gersovitz (1981). In Na, Schmitt-Grohé, Uribe, and Yue (2018), the key is an optimal devaluation to undo the wage rigidity, while in our model, it is the availability of unconstrained lump-sum taxation.

The paper is structured as follows. Section 2 describes the environment and characterizes the monetary equilibrium. Section 3 formulates the problem of the government. Section 4 characterizes policies and derives theoretical results. Section 5 shows how we take the model to the data. Section 6 studies how the model responds to shocks to the terms of trade and productivity, and how these results compare to the data and help us understand their impact on emerging economies. Section 7 concludes.

2 Model

2.1 Environment

We study a small open economy populated by a large number of identical infinitely-lived agents with measure 1. Time is discrete. Throughout the paper, we make use of recursive notation, denoting next-period variables with a prime.

Preferences, endowments and technology

There are three private goods and one public good in the economy. First, there is a non-tradable good that is consumed and produced domestically, their quantities being denoted c^N and y^N , respectively. Second, there is tradable imported good that is consumed domestically but not produced. Let c^T denote the consumption of this imported good. Third, there is a

⁴For example, Eichengreen, Hausmann, and Panizza (2005) find that, between 1993 and 2001, almost all the domestic debt issued by Mexico, Brazil, and Argentina’s was in foreign currency.

tradable exported good that is not consumed domestically and is only produced to be exported. Let y^T denote the production of this exported good. Finally, the government can transform non-tradable output y^N one-to-one into a public good, g .

The representative household is endowed with one unit of time each period, which can be either consumed as leisure, ℓ , or supplied in the labor market, h . Thus, $\ell + h = 1$.

Preferences are represented by a time-separable, expected discounted utility. Let the period utility be given by $u(c^N, c^T) + v(\ell) + \vartheta(g)$, where u , v and ϑ are strictly increasing, strictly concave, C^2 and satisfy standard boundary conditions. Let $\beta \in (0, 1)$ denote the discount factor. In what follows, u_j denotes the partial derivative of u with respect to the consumption good c^j , with $j = \{N, T\}$, and v_ℓ denotes the derivative of v with respect to $\ell = 1 - h$. We assume that cross derivatives are zero; i.e., $u_{NT} = u_{TN} = 0$.

There is an aggregate production technology that transforms hours worked, h , into non-tradable output, y^N , and exported goods, y^T . This technology is represented by a cost function $F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$, which is strictly increasing, strictly convex and homogeneous of degree 1. Given h , feasible levels of (y^N, y^T) must satisfy

$$F(y^N, y^T) - h \leq 0, \quad (1)$$

where F_j is the partial derivative of F with respect to y^j , $j = \{N, T\}$.

Market structure

Agents can exchange both tradable and non-tradable goods, as well as domestic currency (fiat money), while trading of other financial assets will be restricted to the government. Let M^d denote individual money holdings. Prices are denominated in domestic currency (i.e., pesos) and given by P^X , P^M and P^N for exports, imports and non-tradable goods, respectively. Let W denote the nominal wage in units of domestic currency.

The nominal exchange rate E is defined as the units of domestic currency necessary to purchase one unit of foreign currency (i.e., pesos per dollar). We assume that the law of one price holds for tradable goods and so $P^X = Ep^T$ and $P^M = E$, where p^T is the (potentially time-varying) international price of exported goods, while the international price of imported goods is assumed to be constant and normalized to 1. Given these assumptions, p^T also stands for the terms of trade.

In order to study a stationary environment, we normalize nominal variables by the stock of the money supply, M . Let μ denote the growth rate of the money supply and $M' = (1 + \mu)M$ denote its law of motion. We define the corresponding normalized variables as $p^N = P^N/M$, $w = W/M$, $e = E/M$ and $m = M^d/M$.

To motivate a role for fiat money, we introduce a cash-in-advance constraint. Since a large

majority of international trade is conducted on credit, we impose the constraint only on goods produced and consumed domestically.⁵ Hence,

$$p^N c^N \leq m. \quad (2)$$

That is, (normalized) expenditure on non-tradable goods, $p^N c^N$, cannot exceed (normalized) money balances available at the beginning of the period, m .

Government and the balance of payments

The government provides a public good, g , which is transformed one-to-one from non-tradable output. It may also make lump-sum transfers to households. Let γ be the real value (in units of the non-tradable good) of government transfers. We assume that transfers are exogenous, non-negative and represent a non-discretionary redistributive policy.⁶

To finance its expenditure, the government may tax labor income wh at rate τ , increase the money supply at rate μ , and issue debt in international credit markets. Debt takes the form of one-period discount bonds that pay one unit of foreign currency and trade at the price q , also denominated in foreign currency. Let B denote the value of maturing debt and qB' the funds collected from issuing new debt B' , both expressed in foreign currency units.

We consolidate the fiscal and monetary authority and write the government budget constraint in (normalized) units of domestic currency as follows⁷:

$$p^N(g + \gamma) + eB \leq \tau wh + \mu + eqB'. \quad (3)$$

The balance of payments, expressed in units of foreign currency, implies

$$p^T y^T - c^T = B - qB', \quad (4)$$

where the left-hand side of (4) is the trade balance, while the right-hand side is the change in the country's net asset position plus implicit debt interest payments.

Combining (3) and (4) we can express the government budget constraint as the relationship between the external sector (the trade balance) and the public sector (the primary surplus plus seigniorage):

$$\tau wh - p^N(g + \gamma) + \mu - e(p^T y^T - c^T) \geq 0. \quad (5)$$

⁵For example, see Auboin (2009), Schmidt-Eisenlohr (2013) and Antras and Foley (2015). More recently, Kohn, Luttini, Szkup, and Zhang (2023) uses micro-level export transactions by Chilean firms and finds that 85% of sales are on credit. In Appendix A.5 we consider a general case, when both non-tradable and imported goods are subject to a cash-in-advance constraint. Note that (2) would still be the relevant constraint if imports were settled with end-of-period balances.

⁶For a model of sovereign default and endogenous tax progressivity see Ferriere (2015).

⁷As we argue in Section 4.4.2, when lump-sum taxes are available, the government sets distortionary taxes equal to zero and follows the optimal monetary policy (the Friedman rule). In this case, the model becomes Ricardian: the government budget constraint solves for lump-sum taxes and places no further restrictions on government policy.

2.2 The problem of the representative firm

Local firms produce non-tradable and tradable goods by hiring labor according to the technology represented by F . Constant returns to scale imply that we can assume that the industry behaves as if there were a representative firm that solves the static problem

$$\max_{y^N, y^T, h} \{p^N y^N + ep^T y^T - wh\}$$

subject to (1). The necessary and sufficient first-order conditions imply expressions for the wage and exchange rate as functions of (y^N, y^T, p^N, p^T) as follows:

$$w = \frac{p^N}{F_N} \quad e = \frac{p^N}{p^T} \frac{F_T}{F_N}. \quad (6)$$

2.3 The problem of the representative household

The endogenous aggregate state of the economy consists of the amount of maturing foreign debt, B , and an indicator function \mathcal{I} , which specifies whether the government is in default ($\mathcal{I} = D$) or not ($\mathcal{I} = P$). As we shall explain below, the default state may last several periods while the country is excluded from international credit markets. Agents know the government's default state before making any decisions at the beginning of every period. The exogenous aggregate state of the economy is summarized by s and known at the beginning of each period. The state s may include any variable that evolves stochastically over time, e.g., the terms of trade, p^T . The set of all possible realizations for the stochastic state is S . Note that we are allowing for the possibility that state variables may depend on the default state.

Agents know the laws of motion of all aggregate state variables. All prices and government policies are perceived by agents as being functions of the aggregate state. This dependence is omitted to simplify notation. The period budget constraint of the household is

$$p^N c^N + ec^T + m'(1 + \mu) \leq (1 - \tau)wh + m + p^N \gamma, \quad (7)$$

where, as mentioned above, p^N , w , e , m are all normalized by the aggregate money supply at the beginning of the period. As also mentioned above, purchases of non-tradable goods are subject to the cash-in-advance constraint, (2).

The individual state variable is the household's (normalized) money balances at the beginning of the period, m . Let $V(m, B, \mathcal{I}, s)$ denote the agent's value function as a function of individual and aggregate state variables. Let $\mathbb{E}[V(m', B', \mathcal{I}', s') | B, \mathcal{I}, s]$ be the conditional expected value of the agent's value function in the next period, given current aggregate state (B, \mathcal{I}, s) .

The problem of the representative household is

$$V(m, B, \mathcal{I}, s) = \max_{(c^N, c^T, m', h)} u(c^N, c^T) + v(1 - h) + \vartheta(g) + \beta \mathbb{E} [V(m', B', \mathcal{I}', s') | B, \mathcal{I}, s]$$

subject to (2) and (7). As derived in Appendix A.1, the solution to this problem must necessarily satisfy

$$\frac{(1 - \tau)wu_T}{e} = v_\ell \quad \frac{(1 + \mu)u_T}{e} = \beta \mathbb{E} \left[\frac{u'_N}{p^{N'}} \middle| B, \mathcal{I}, s \right] \quad (8)$$

plus constraints (2) and (7).

Conditions (8) show how policies distort households' choices. The tax rate introduces an intra-temporal wedge between the marginal utilities of consumption of tradable goods and leisure, while the money growth rate introduces an inter-temporal wedge, as it distorts the substitution between current consumption of tradable goods and future consumption of non-tradable goods.

2.4 Monetary equilibrium

Since all agents are identical, c^N , c^T and h should be interpreted as referring to aggregate quantities from now on.

The resource constraint in the non-tradable sector is $c^N + g = y^N$. From (1) and the resource constraint, labor is a function of non-tradable (private) consumption, public expenditures and the production of tradables; i.e., $h = F(c^N + g, y^T)$.

All agents enter the period with the same money balances, m . Market clearing implies that $m = m' = 1$. Without loss of generality, the cash-in-advance constraint (2) is satisfied with equality.⁸ Then, in equilibrium

$$p^N = \frac{1}{c^N}. \quad (9)$$

The equilibrium wage can be derived by combining (6) and (9):

$$w = \frac{1}{c^N F_N}. \quad (10)$$

Similarly, the equilibrium exchange rate follows from (6) and (9):

$$e = \frac{1}{c^N p^T} \frac{F_T}{F_N}. \quad (11)$$

Finally, the Lagrange multiplier associated with the cash-in-advance constraint must be non-negative, which implies the following equilibrium condition:

$$\frac{u_N}{u_T} - \frac{p^T F_N}{F_T} \geq 0. \quad (12)$$

⁸If the cash-in-advance constraint is slack, then the price level p^N is, in general, indeterminate. A standard assumption is to take the limiting case, when the constraint is satisfied with equality.

If (12) is positive, then the cash-in-advance constraint (2) binds; if is equal to zero, then (2) is slack. Condition (12) reflects an inefficiency wedge created by monetary policy, as the marginal rate of substitution between non-tradable and tradable goods is larger than the corresponding marginal rate of transformation. Though monetary policy creates this intra-temporal wedge, similar to a tax on non-tradable consumption, it actually operates through an inter-temporal channel, as reflected by (8).

3 Government policy

3.1 Government budget constraint in a monetary equilibrium

Below, we formulate the problem of the government following the *primal approach*. That is, we solve for allocation and debt choices that are implementable in a monetary equilibrium. In order to proceed, we need to use the equilibrium conditions derived above, to replace prices (p^N, w, e) and policies (μ, τ) in the government budget constraint (5).

To obtain an expression for the tax rate, combine (8), (10) and (11):

$$\tau = 1 - \frac{v_\ell F_T}{u_T p^T}. \quad (13)$$

Similarly, the money growth rate can be written by combining (8), (9) and (11):

$$\mu = \frac{\beta \mathbb{E} [u'_N c^{N'} | B, \mathcal{I}, s]}{u_T c^N p^T (F_N / F_T)} - 1. \quad (14)$$

One important difference in the expressions for the tax and money growth rates is that the latter depends on tomorrow's state of the world. Importantly, among other possible events, whether the country is in repayment or default. This follows from the fact that the marginal value of money tomorrow, which is a function of the aggregate state, affects the demand for money today. This connection will be key for understanding the government's debt choice.

Using (9)–(14) we obtain the government budget constraint in a monetary equilibrium:

$$u_T [c^T - \gamma p^T (F_N / F_T)] - v_\ell F(c^N + g, y^T) + \beta \mathbb{E} [u'_N c^{N'} | B, \mathcal{I}, s] \geq 0, \quad (15)$$

which depends on $(c^N, c^T, y^T, g, c^{N'})$.

3.2 Repayment and default

Suppose the government is currently not excluded from international credit markets. At the beginning of any such period, the government decides between repaying (P) and defaulting (D) on its debt. If it decides to default, then debt is set to $B^D \geq 0$. Define

$$\hat{V}(B, s, \varepsilon^P, \varepsilon^D) = \max\{V^P(B, s) + \varepsilon^P, V^D(s) + \varepsilon^D\},$$

where $V^P(B, s)$ and $V^D(s)$ denote the values of repayment and default, respectively, which are defined in detail below. The decision to repay or default is influenced by random additive

shocks to utility. The expectation of the value function with respect to the utility shocks is $\mathcal{V}(B, s) = E_\varepsilon[\hat{\mathcal{V}}(B, s, \varepsilon^P, \varepsilon^D)]$. Finally, let $\mathcal{P}(B, s)$ be the probability of repayment for any given (B, s) , which can be expressed as $\mathcal{P}(B, s) = \Pr(V^P(B, s) - V^D(s) \geq \varepsilon^D - \varepsilon^P)$. We assume that ε^P and ε^D are independently, identically distributed extreme value (Gumbel or type I extreme value) shocks. In Appendix A.2 we derive simple analytical expressions for $\mathcal{V}(B, s)$ and $\mathcal{P}(B, s)$, and their derivatives with respect to B , all as functions of $V^P(B, s)$ and $V^D(s)$.

3.3 Problem of the government

Every period, the government first decides on whether to repay or default on its debt. After that, it implements policy for the period, taking into account the response of private domestic agents and international lenders and the government policies it expects to be implemented in the future. A period policy consists of choices on the amount of future debt, the money growth rate, the tax rate and government expenditure.

If the government decided to default, then the country is excluded from international credit markets. It regains access at the beginning of the period with probability δ ; hence, $1/\delta$ is the expected duration of exclusion. The country reenters credit markets with a renegotiated level of debt $B^D \geq 0$, which is exogenous. This assumption implies that debt haircuts are increasing in the level of defaulted debt. While in default, $\mathcal{I} = D$, the country experiences a productivity penalty, $\Omega(s)$, which generally depends on the exogenous state of the economy.

If the government is currently repaying, the probability that it will remain in repayment status tomorrow is given by $\mathcal{P}(B', s')$ for any given (B', s') , as derived above. On the other hand, if the government is currently in default, the probability that it will transition to repayment status tomorrow is given by $\delta\mathcal{P}(B^D, s')$ for any given s' . To compute these probabilities we need to know the value functions $V^P(B, s)$ and $V^D(s)$, which we will derive below.

In equilibrium, zero expected profits by risk-neutral international lenders implies that

$$Q(B', s)B' = \frac{\mathbb{E}[\mathcal{P}(B', s')|s]B' + \mathbb{E}[(1 - \mathcal{P}(B', s'))Q^D(s')|s]B^D}{1 + r}, \quad (16)$$

where $Q^D(s') \equiv \delta Q(B^D, s') + (1 - \delta)\mathbb{E}[Q^D(s'')|s']/(1 + r)$ for all s' . The first term in (16) reflects the expected debt repayment, while the second term reflects the expected debt recovery. $Q^D(s')$ stands for the price an investor would pay to earn B^D in the period the defaulting country reenters international credit markets. Note that, $Q^D(s')$ is an endogenous object, as it depends on the equilibrium function \mathcal{P} through $Q(B^D, s')$; however, the government takes it as given. Using (16) and the expression for $Q^D(s')$ we obtain the following recursion for all s :

$$Q^D(s) = \frac{\delta\mathbb{E}[\mathcal{P}(B^D, s')|s] + \mathbb{E}[(1 - \delta\mathcal{P}(B^D, s'))Q^D(s')|s]}{1 + r}$$

As explained above, we follow the primal approach to formulate the government's problem.

Hence, we use equilibrium conditions to express domestic prices, the money growth rate, and the tax rate as functions of current and future allocations. Every period, the government then chooses a debt level (when repaying) and domestic policies that implement the allocation (c^N, c^T, y^T, g) . These choices need to satisfy the balance of payment, (4), the government budget constraint, (15), and the non-negativity constraint, (12).

When the government is in the repayment state, $\mathcal{I} = P$, its policies are a function of the state (B, s) ; let the relevant policy functions be denoted by $\{\mathcal{B}, \mathcal{C}^N, \mathcal{C}^T, \mathcal{Y}^T, \mathcal{G}\}$. While in the default state, $\mathcal{I} = D$, its policies are a function of the state s ; let the relevant policy functions be denoted by $\{\bar{\mathcal{C}}^N, \bar{\mathcal{C}}^T, \bar{\mathcal{Y}}^T, \bar{\mathcal{G}}\}$. Given these policy functions, in Appendix A.2 we define the value functions $V^P(B, s)$, $V^D(s)$ and $\mathcal{V}(B, s)$.

3.3.1 Repayment

The problem of the government in the repayment state is

$$\max_{(B', c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta \mathbb{E}[\mathcal{V}(B', s')|s] \quad (\text{PP})$$

subject to

$$p^T y^T - c^T + Q(B', s)B' - B = 0, \quad (17)$$

$$u_T c^T - \gamma u_T p^T (F_N/F_T) - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}[u'_N c^{N'}|P, s] = 0, \quad (18)$$

$$u_N - u_T p^T (F_N/F_T) \geq 0. \quad (19)$$

The constraints in the government's problem correspond to the balance of payment, (4), the government budget constraint, (15), and the non-negativity constraint, (12). Note that the expectation term in the government budget constraint is conditioned on the current state being repayment ($\mathcal{I} = P$); hence, the relevant transition probabilities are $\mathcal{P}(B', s')$ for repayment and $1 - \mathcal{P}(B', s')$ for default, for all (B', s') .

3.3.2 Default

The problem of the government in the default state is

$$\max_{(c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta \mathbb{E}[\delta \mathcal{V}(B^D, s') + (1 - \delta)V^D(s')|s] \quad (\text{DP})$$

subject to

$$p^T y^T - c^T = 0, \quad (20)$$

$$u_T c^T - \gamma u_T p^T (F_N/F_T) - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}[u'_N c^{N'}|D, s] = 0, \quad (21)$$

$$u_N - u_T p^T (F_N/F_T) \geq 0. \quad (22)$$

and where total factor productivity, embedded in the cost function $F(y^N, y^T)$, is reduced by a default penalty, $\Omega(s)$.

In this case, note that the balance of payments is simply the trade balance, as the government is excluded from international credit markets. The expectation term in the government budget constraint is conditioned on the current state being default ($\mathcal{I} = D$); hence, the relevant transition probabilities are $\delta\mathcal{P}(B^D, s')$ for repayment and $1 - \delta\mathcal{P}(B^D, s')$ for default, for all s' .

4 Characterization

4.1 The non-negativity constraint

The balance of payments and the government budget constraint clearly restrict government actions. However, the non-negativity constraints, (19) and (22), may or may not bind. These constraints follow from (12), which is the requirement that the Lagrange multiplier on the cash-in-advance constraint be non-negative. As we discussed above, when (12) is satisfied with strict inequality, the Lagrange multiplier on the cash-in-advance constraint is positive, which implies households face a positive opportunity cost of holding domestic currency across periods. This acts as a tax on the consumption of non-tradable goods, which require money to be purchased. However, note that the money growth rate μ is not equivalent to a sales tax on c^N since μ distorts the tradeoff between consumption of tradables today versus non-tradables tomorrow—see (14). Furthermore, the government cannot subsidize non-tradable consumption relative to tradable consumption. With the instruments at its disposal, the most it can do is eliminate the wedge between the consumption of the two goods; this is accomplished by making the cash-in-advance constraint slack for agents.

We define the *Friedman rule* as a policy that satisfies the relevant non-negativity constraint, (19) or (22), with equality, so that the cash-in-advance constraint is slack. Note that this policy may arise because the government is at a corner; i.e., it would prefer to implement a policy that involves not satisfying the non-negativity constraint, but is prevented from doing so as such a policy would not be consistent with a monetary equilibrium.⁹

Implementing the Friedman rule typically leads to deflation. As we can see from (14), when (12) is satisfied with equality globally and in the absence of aggregate shocks, we obtain $\mu = \beta - 1 < 0$. In the long run, inflation is equal to the money growth rate and so, we get deflation. This result is not supported by the empirical evidence on emerging countries, which report positive, and oftentimes elevated, rates of inflation. Hence, in the analysis below we will focus on the case when the non-negativity constraints, (19) and (22), do not bind (i.e. the cash-in-advance constraint is binding) and derive conditions under which this assumption holds.

⁹In our environment, non-monetary equilibria lead to infinite misery for households as they would not be able to consume non-tradable goods.

4.2 Intratemporal tradeoffs

We now analyze the intratemporal tradeoffs faced by the government when formulating policy. This involves characterizing the choice for (c^N, c^T, y^T, g) , given state (B, s) , repayment status $\mathcal{I} = \{P, D\}$ and a debt policy $\mathcal{B}(B)$. To simplify some of the notation below, let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T(F_N/F_T)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma/dc^T = \Gamma \times (u_{TT}/u_T) < 0$, while the convexity of F implies that $\Gamma_N = \Gamma_g = \Gamma \times (F_{NN}/F_N - F_{NT}/F_T) > 0$ and $\Gamma_y = \Gamma \times (F_{NT}/F_N - F_{TT}/F_T) < 0$. Also define $\Phi \equiv v_\ell - v_{\ell\ell}F(c^N + g, y^T) > 0$. All proofs of the propositions below are in Appendix A.4.

Since the problems in repayment and default are functionally identical with respect to (c^N, c^T, y^T, g) , we focus on (PP)–(19). Let ξ and λ be the Lagrange multipliers associated with the constraints (17) and (18), respectively. We will assume the non-negative constraint (19) is slack and verify when this is so—the general case is characterized in Appendix A.3. The necessary first-order conditions with respect to (c^N, c^T, y^T, g) are

$$u_N - v_\ell F_N - \lambda(F_N \Phi + \gamma \Gamma_N) = 0, \quad (23)$$

$$u_T - \xi + \lambda(u_T + u_{TT}c^T - \gamma \Gamma_T) = 0, \quad (24)$$

$$-v_\ell F_T + \xi p^T - \lambda(F_T \Phi + \gamma \Gamma_y) = 0, \quad (25)$$

$$-v_\ell F_N + \vartheta_g - \lambda(F_N \Phi + \gamma \Gamma_g) = 0. \quad (26)$$

From (23) and (26) it follows that $u_N = \vartheta_g$. The provision of public goods is optimal, in the sense that the marginal utility of non-tradable goods consumption is equal to the marginal utility of public good consumption. Recall that the public good is transformed on-to-one from the non-tradable good.

We now show under which conditions the non-negativity constraint does not bind and when policy is away from the Friedman rule. As it turns out, these results rely critically on the value of transfers γ and the curvature of the utility for tradable goods.

Proposition 1. (i) Assume that $\gamma = 0$. The non-negativity constraint (19) is slack if and only if $\frac{-u_{TT}c^T}{u_T} \leq 1$. Policy is away from the Friedman rule if and only if $\frac{-u_{TT}c^T}{u_T} < 1$.

(ii) Assume that $\gamma > 0$. There exists a $\hat{\sigma}^T > 1$ such that if $\frac{-u_{TT}c^T}{u_T} \in (0, \hat{\sigma}^T)$ then the non-negativity constraint (19) is satisfied with strict inequality. Policy is away from the Friedman rule if $\frac{-u_{TT}c^T}{u_T} < \hat{\sigma}^T$.

The argument supporting Proposition 1(i), the case with no transfers, is simpler and illustrative. When $\gamma = 0$, we can use (23) and (25) to solve for the Lagrange multipliers and obtain

$\lambda = \frac{u_N - v_\ell F_N}{F_N \Phi}$ and $\xi = \frac{u_N F_T}{p^T F_N}$. Hence, (24) implies

$$F_T \Phi \left(\frac{u_N}{u_T} - \frac{p^T F_N}{F_T} \right) = p^T (u_N - v_\ell F_N) \left(1 + \frac{u_{TT} c^T}{u_T} \right). \quad (27)$$

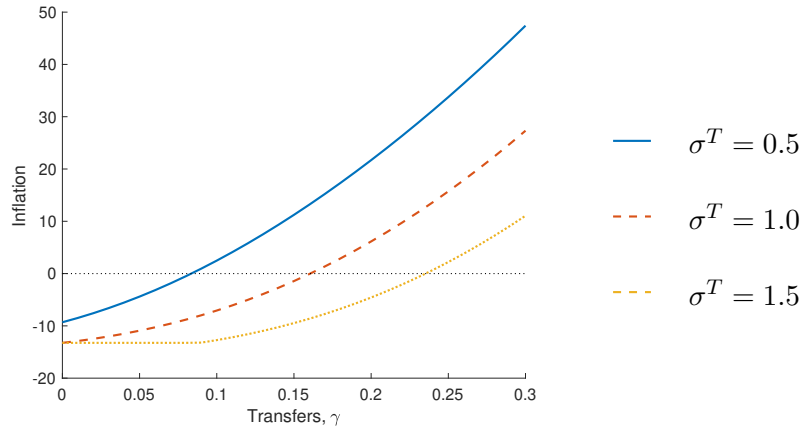
The term in brackets in the left-hand side is non-negative by (19) and so both sides of (27) need to be non-negative. The term $u_N - v_\ell F_N$ in the right-hand side is positive given $\lambda > 0$. Thus, (27) holds if and only if $\frac{-u_{TT} c^T}{u_T} \leq 1$. Furthermore, when $\frac{-u_{TT} c^T}{u_T} < 1$, we obtain $\frac{u_N}{u_T} - \frac{p^T F_N}{F_T} > 0$ and so, (19) is satisfied with strict inequality. That is, the cash-in-advance constraint binds and policy is away from the Friedman rule, which is the empirically relevant case. In contrast, if $\frac{-u_{TT} c^T}{u_T} = 1$ then (19) is satisfied with equality but still slack, and if $\frac{-u_{TT} c^T}{u_T} > 1$ then (19) binds. In these latter two cases, the cash-in-advance is slack and the Friedman rule is implemented.

Asking whether the non-negativity constraint binds or not boils down to asking whether the government wants to tax the consumption of non-tradable goods relative to the consumption of tradable goods.¹⁰ As is standard in the optimal tax literature, this tradeoff is resolved depending on the relative price and income elasticities (Chari and Kehoe, 1999). Roughly speaking, as long as the cash-in-advance constraint binds, two key factors are at play. First, individual policy functions for consumption of tradable goods and leisure are independent of beginning-of-period money holdings, m ; i.e., there is no income effect with respect to money holdings for c^T and ℓ . Second, and as consequence, goods are taxed according to their price-elasticities: goods with relatively lower price-elasticity are taxed relatively more. In this setting, it can be shown that the price-elasticity of tradable goods is the inverse of $\frac{-u_{TT} c^T}{u_T}$, while the price-elasticity for non-tradable goods is equal to 1 due to the cash-in-advance constraint. Money holdings are (implicitly) taxed as long as the demand for non-tradable goods is less elastic than the demand for tradable goods, i.e., when $1 < \left(\frac{-u_{TT} c^T}{u_T} \right)^{-1}$. Hence, whether and by how much the relative consumption of non-tradables and tradables is distorted depends only on the curvature of preferences with respect to the latter. In other words, the sign of $1 + \frac{u_{TT} c^T}{u_T}$ determines whether the government implements policy at or away the Friedman rule.

When transfers are positive, the non-negativity constraint (19) is slack when $\frac{-u_{TT} c^T}{u_T} \leq 1$ and, in contrast to the case with zero transfers, also for some range when $\frac{-u_{TT} c^T}{u_T} > 1$ and this range increases as γ increases. Thus, there exists a $\hat{\sigma}^T > 1$ such that if $\frac{-u_{TT} c^T}{u_T} < \hat{\sigma}^T$ then the non-negativity constraint (19) is satisfied with strict inequality. The intuition for this result goes as follows. Since transfers enter the household's budget constraint in units of the non-tradable good, a change in the price of the tradable good, e , has now an income effect in addition to the substitution effect we described above. As this price increases, the cost of the tradable good

¹⁰As we discussed above, given the available policy instruments the government cannot subsidize non-tradable goods relative to tradables.

Figure 2: Transfers and Inflation



becomes relatively more expensive (the substitution effect) while the value of transfers in terms of tradable goods falls (the income effect). Thus, income and substitution effects go in the same direction and make the demand of tradable more elastic. That is, the price-elasticity of the tradable good is now larger than the inverse of $\frac{-u_{TT}c^T}{u_T}$. As a consequence, positive transfers enlarge the set of primitives consistent with a binding cash-in-advance constraint as taxing the tradable good is relatively more undesirable and hence, the optimal monetary policy calls for staying away from the Friedman rule.

Figure 2 illustrates the impact of these results. The example uses the calibration we discuss in the following section, though these details are not important for our argument here. Importantly, we assume no aggregate shocks (other than the extreme value shocks) and preferences with a constant elasticity-of-substitution; in particular, $\frac{-u_{TT}c^T}{u_T} = \sigma^T$. After solving for the equilibrium, we look at the case when the repayment state lasts indefinitely; then, we draw the equilibrium inflation rate, which in this case is equal to the money growth rate, μ , as a function of transfers, γ . When $\sigma^T < 1$, (19) is satisfied with strict inequality and so, the cash-in-advance constraint binds. Thus, monetary policy is away from the Friedman rule and inflation is above optimal. This inefficiency worsens as transfers γ increase. When $\sigma^T = 1$, (19) is slack and equal to zero for $\gamma = 0$ but is satisfied with strict inequality for $\gamma > 0$; thus, inflation increases in γ , as monetary policy becomes progressively more inefficient. When $\sigma^T > 1$, (19) binds for γ small enough, but eventually becomes slack and then inflation starts increasing with γ .

4.3 Debt policy

We now characterize debt choice in the event the government decides to repay its inherited debt. The necessary first-order condition of problem (PP) with respect to B' is

$$\beta \frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} + \xi \frac{[\partial Q(B', s)B']}{\partial B'} + \lambda \beta \frac{\partial \mathbb{E}[u'_N c^{N'}|P, s]}{\partial B'} = 0. \quad (28)$$

As reflected by the three terms in (28), debt choice affects the continuation value for the

government and how tightly the balance of payment and government budget constraints bind. We can further characterize this equation using some of the expressions derived above.

Proposition 2. *The Generalized Euler Equation, which characterizes the government's debt choice, is*

$$\underbrace{\mathbb{E} \left[\mathcal{P}(B', s') \left(\frac{\xi}{1+r} - \beta \xi' \right) \middle| s \right]}_{\text{distortion-smoothing}} - \underbrace{\frac{\xi}{\kappa(1+r)} \mathbb{E} \left[\mathcal{P}(B', s') (1 - \mathcal{P}(B', s')) (B' - Q^D(s') B^D) \xi' \middle| s \right]}_{\text{default-risk premium}} + \lambda \beta \mathbb{E} \left\{ \underbrace{\mathcal{P}(B', s') \left[(u'_N + u'_{NN} \mathcal{C}^{N'}) \mathcal{C}_B^{N'} - \frac{(u'_N \mathcal{C}^{N'} - \bar{u}'_N \bar{\mathcal{C}}^{N'}) (1 - \mathcal{P}(B', s')) \xi'}{\kappa} \right] \middle| s}_{\text{distortionary policies}} \right\} = 0. \quad (29)$$

The Generalized Euler Equation (29) highlights three channels in the government's debt decision. The first channel corresponds to distortion smoothing: debt allows the government to trade off intertemporally how tightly the balance of payments binds. The distortion-smoothing term has an intrinsic bias since the government is relatively impatient as $\beta(1+r) < 1$. In other words, this term would not be zero if the government kept expected distortions constant over time, i.e., set $\xi = E[\xi' | s]$. This relative impatience motivates debt accumulation in the sovereign default literature.

The second channel captures the default-risk premium: more debt leads to a higher probability of default and, hence, a higher interest rate. The default-premium term reflects the added financial cost due to default risk. This term, as is standard in the literature, moderates debt accumulation. $B^D > 0$ counters this effect, as lenders take into account that they partially recover their loan after a default event.

To further understand the role of default risk, consider the case when debt is always repaid. Since the probability of repayment is now always one, the interest rate on debt is equal to the risk-free rate, i.e., $\mathcal{P}(B, s) = 1$ for all (B, s) , which implies $Q(B', s) = (1+r)^{-1}$ for all (B', s) . The Generalized Euler equation, characterizing debt choice, (29), becomes $\xi/(1+r) - \beta E[\xi' | s] + \lambda \beta E[(u'_N + u'_{NN} \mathcal{C}^{N'}) \mathcal{C}_B^{N'} | s] = 0$. The equations characterizing the choice (c^N, c^T, y^T, g) remain functionally the same as those derived above. Thus, Proposition 1 still applies. In particular, whether policy is away from the Friedman rule depends on the values of $\frac{-u_{TT} c^T}{u_T}$ and γ . In Section 6.5 we explore quantitatively how default-risk affects the policy response to aggregate shocks.

The third channel in (29) is analyzed in the following section.

4.4 The role of distortionary policies

Our model has two main elements: domestic policies and sovereign default risk. This section analyzes the role of domestic policies, while we address the role of sovereign default in the quantitative evaluation of the model. Here, we first focus on how distortionary policies affect the government's borrowing choice. Then, we analyze how monetary and fiscal policies would look like if the government had access to lump-sum taxes and thus, would not need to rely on distortionary instruments.

4.4.1 Effect of distortionary policies on government debt policy

The third term in (29) reflects an intertemporal tradeoff due to the fact that fiscal and monetary policies are distortionary. Note that higher debt leads to a change in policies tomorrow, which affects the demand for money today and, hence, how tightly the current government budget constraint binds. There are two parts in the distortionary-policies term in (29), and we will analyze each in turn.

We follow Martin (2011), which analyzes a closed monetary economy with domestic debt and no default, to interpret the expression $\mathcal{P}(B', s')(u'_N + u'_{NN}\mathcal{C}^{N'})\mathcal{C}_B^{N'}$. The envelope condition from the household's problem implies $V_m = u_N/p^N$ (see derivation in Appendix A.1). When using equilibrium condition (9) we obtain $V_m = u_N\mathcal{C}^N$, which states the equilibrium value of entering the period with an additional unit of domestic currency. We can further establish how this value changes with debt: $dV_m/dB = (u_N + u_{NN}\mathcal{C}^N)\mathcal{C}_B^N$. Hence, the first part of the distortionary-policies term in (29) reflects how a change in debt directly affects the demand for money and, therefore, how tightly the government budget constraint binds. Note that this first part of the channel is multiplied by $\mathcal{P}(B', s')$, implying that it only operates within repayment states today and tomorrow (recall that $\bar{\mathcal{C}}_B^N = 0$).

The level of debt affects the level of implemented distortions since the policy instruments available to the government are distortionary; these expected distortions, in turn, affect the demand for money. Two opposing forces determine how higher debt and distortions affect the demand for money in equilibrium: a substitution effect and an income effect. The substitution effect dictates that larger distortions should lead to lower consumption and lower demand for money to finance non-tradable goods. The income effect induces households to want to mitigate the drop in consumption due to larger distortions; hence, it increases the demand for money. Which effect dominates depends on the curvature of u , more specifically, the sign of $u_N + u_{NN}c^N$. If this term is positive (negative), the substitution (income) effect dominates.

The second part of the term is the expression $(u'_N\mathcal{C}^{N'} - \bar{u}'_N\bar{\mathcal{C}}^{N'})\mathcal{P}'_B$, where $\mathcal{P}'_B = -\mathcal{P}(B', s')(1 - \mathcal{P}(B', s'))(\xi'/\kappa) < 0$. As explained above, $V_m = u_N\mathcal{C}^N$; hence, this term reflects the impact

of debt choice on the current money demand, through the change in the repayment probability. As the government issues more debt, it lowers the probability of repayment, $\mathcal{P}'_B < 0$. This matters to domestic households since the value of an extra unit of money depends on whether the government repays or defaults, as policies (and distortions) are different in each case. Again, the sign of $u'_N \mathcal{C}^{N'} - \bar{u}'_N \bar{\mathcal{C}}^{N'}$ depends on the curvature of u . Assuming that $\mathcal{C}^{N'} > \bar{\mathcal{C}}^{N'}$ (an assumption we verify numerically), the difference is positive (negative) if the substitution (income) effect dominates.¹¹

So, how does the distortionary-policies channel operate? First, issuing more debt today alters future fiscal and monetary policies in the repayment state, as well as the probability of repayment. Second, since domestic policy instruments are distortionary, anticipated changes in these future policies alter the marginal value of money tomorrow and, hence, households' current money-holding decisions. Third, the change in future repayment probability also alters the expected marginal value of money tomorrow, as policies differ if the government repays or defaults. Fourth, these changes in the current demand for money affect the real value of domestic government liabilities and hence, the government's budget constraint. Importantly, this effect is not internalized by the government tomorrow, which results in a time-consistency problem, as the government values current debt issuance differently today and tomorrow.

The distortionary-policies channel alters how the other two components in (29) are traded off when the government decides how much debt to issue. The effect may be positive, zero, or negative, depending on the assumptions on preferences, thus altering the standard tradeoff in sovereign debt choice. For example, if the utility is logarithmic, then $u_N + u_{NN} \mathcal{C}^N = u'_N \mathcal{C}^{N'} - \bar{u}'_N \bar{\mathcal{C}}^{N'} = 0$, and so there is no time-consistency problem due to the interplay between debt policy and the demand for money. In this case, the government would trade off its relative impatience with the default risk premium; i.e., the desire to accumulate debt is moderated by the extra financial cost of supporting it due to the higher default probability. Suppose instead that $u_N + u_{NN} \mathcal{C}^N < 0$, which also implies $u'_N \mathcal{C}^{N'} - \bar{u}'_N \bar{\mathcal{C}}^{N'} < 0$, as argued above. Given that $\mathcal{C}^N_B < 0$ (since higher debt implies larger distortions and, hence, lower consumption) and $\mathcal{P}^N_B < 0$ (as shown above), the distortionary-policies term would now be positive, countering the default premium term and reinforcing the relative impatience term. That is, when the income effect dominates the substitution effect in the preference for the non-tradable good (and the demand for money), the distortionary-policies channel provides additional incentives to

¹¹Note from (14) that how the repayment probability affects the mapping between allocations and the money growth rate also depends on the curvature of u with respect to c^N . Taking the probability of repayment as a parameter, one can show that, when the substitution (income) effect on the money demand dominates, the equilibrium money growth rate that implements a given allocation is higher (lower) when faced with a higher repayment probability tomorrow.

accumulate debt. In the opposite case, $u_N + u_{NN}c^N > 0$, which implies $u'_N c^{N'} - \bar{u}'_N \bar{c}^{N'} > 0$, the distortionary-policies channel mitigates the incentive to accumulate debt.

4.4.2 Effect of distortionary policies on fiscal and monetary policy

We now explain the role of distortionary taxation, in particular, its effects on monetary policy. Suppose that lump sum, unconstrained taxes \mathcal{T} are available.¹² In such a case, the government budget constraint (3) becomes $p^N(g + \gamma) + eB \leq \mathcal{T} + \tau wh + eqB'$. Proceeding as we did to derive (15), the government budget constraint in a monetary equilibrium, when lump sum taxes are available, can be written in terms of allocations as follows:

$$u_T[c^T - \gamma p^T(F_N/F_T)] - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}[u'_N c^{N'} | \mathcal{I}, s] + \mathcal{T} \geq 0 \quad (30)$$

for $\mathcal{I} = \{P, D\}$. The problem of the government in the repayment state is (PP) subject to (17), (19) and (30) for $\mathcal{I} = P$. Similarly, when in default, the problem of the government is (DP) subject to (20), (22) and (30) for $\mathcal{I} = D$.

Consider now the following centralized version of the government's problem, where the only constraint is the balance of payments. When repaying, the problem of the government is

$$\max_{(B', c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta \mathbb{E}[\mathcal{V}(B', s') | s] \quad (\text{PPEP})$$

subject to $p^T y^T - c^T + Q(B', s)B' - B = 0$. The problem of the government in the default state is defined similarly. The solution to problem (PPEP), denoted $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$, will be referred to as the *EG real allocation*, where EG stands for Eaton and Gersovitz (1981). The associated lump-sum taxes necessary to finance this allocation are given by

$$\hat{\mathcal{T}} = -\hat{u}_T \hat{c}^T + \gamma \hat{u}_T p^T (\hat{F}_N / \hat{F}_T) + v_\ell F(\hat{c}^N + \hat{g}, \hat{y}^T) - \beta \mathbb{E}[\hat{u}'_N \hat{c}^{N'} | P, s] \quad (31)$$

We now establish the following equivalence result between the two problems.

Proposition 3. *Given lump sum, unconstrained taxes $\hat{\mathcal{T}}$ given by (31), the EG real allocation $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ solves the problem (PP), with the corresponding value of default, $V^D(s)$, that solves (DP), and the constraints (19), (22) and (30), for $\mathcal{I} = \{P, D\}$, are slack.*

When lump-sum taxes are available, the EG real allocation can be decentralized as a competitive equilibrium as follows. The government finds it optimal to (i) set the distortionary tax rate τ equal to zero and (ii) conduct monetary policy μ so that the cash-in-advance constraint in the household's problem does not bind, i.e., implement the Friedman rule given by

$$\hat{\mu} = \frac{\beta \mathbb{E}[\hat{u}'_N \hat{c}^{N'} | B, \mathcal{I}, s]}{\hat{u}_N \hat{c}^N} - 1. \quad (32)$$

¹²If endogenous lump-sum taxes are restricted to be non-positive (i.e., transfers are allowed but taxes are not) then the government would optimally set them to zero, as they are not valued by private agents and need to be financed with distortionary instruments. This case corresponds to the case $\gamma = 0$ we analyzed above.

Lump-sum taxes adjust so that under these policies the government budget constraint is satisfied with no need for distortions. Hence, in this version of the model, the government budget constraint is no longer a restriction in the government's problem. In effect, the policy regime becomes Ricardian.

We can write the analog of equation (29) for the case with unconstrained lump-sum taxes as: $\mathbb{E}[\mathcal{P}(B', s')(\frac{u_T}{1+r} - \beta u'_T)|s] - \frac{u_T}{\kappa(1+r)}\mathbb{E}[\mathcal{P}(B', s')(1 - \mathcal{P}(B', s'))(B' - Q^D(s')B^D)u'_T|s] = 0$. In this case, the multiplier of the balance of payment constraint, ξ , is equal to u_T .¹³ We can see that, with lump-sum taxes, the government trades off distortion-smoothing (plus relative impatience) and the default premium. The intertemporal tradeoff due to distortionary policies is absent since the government budget constraint is automatically satisfied with lump-sum taxes and thus, is no longer a binding restriction to policy implementation.

Next, we show that the EG real allocation cannot be decentralized in the absence of lump-sum taxes, i.e., cannot be implemented if only distortionary policy instruments are available. We focus on the case when $\hat{\mu} \leq 0$, which is isomorphic to requiring a non-negative implicit real interest rate on a domestic bond denominated in non-tradable goods.

Proposition 4. *Suppose that $\hat{\mu}$, given by (32), is non-positive. If lump-sum, unconstrained taxes are not available, then there are no feasible monetary and fiscal policies that decentralize the EG real allocation.*

To conclude this section, we consider the implications of eliminating monetary policy distortions on their own (i.e., in the absence of lump-sum taxes). In Proposition 1 we stated the cases when monetary policy is conducted away from the Friedman rule and hence, monetary policy is distortionary. To better understand the role played by monetary policy distortions in these cases, we can analyze a situation when the government is forced to run the Friedman rule when implementing policy. This involves imposing that the non-negativity constraint, (19) or (22), be satisfied with equality in the government's problem for all states. This assumption eliminates the monetary policy distortion as it forces the government to always implement a slack cash-in-advance constraint, i.e., to always leave households satiated with money balances.

When monetary policy is unconstrained, the government handles the tradeoff between non-tradable and public goods efficiently, $u_N = \vartheta_g$, but at the cost of distorting the current consumption of non-tradable versus tradable goods—recall from the discussion above, that the government cannot directly affect the latter wedge as μ distorts the tradeoff between consumption

¹³Note that with distortionary taxes, ξ is not equal to u_T in general; from (24), this would require either the government budget constraint to be slack, i.e., $\lambda = 0$, or $\lambda(u_T + u_{TT}c^T) - (\gamma\lambda + \zeta)\Gamma_T = 0$. This last case obtains when $u_T + u_{TT}c^T = \gamma = 0$, while it cannot happen when $u_T + u_{TT}c^T > 0$ and may be possible when $u_T + u_{TT}c^T < 0$.

of tradables today versus non-tradables tomorrow. When the non-negativity constraint binds (either optimally or because it was imposed) the government no longer equates the marginal utilities for non-tradable and public goods. That is, eliminating monetary policy distortions leads to an inefficient provision of public goods: $u_N > \vartheta_g$ (see the derivations in Appendix A.3).

When $u_N > \vartheta_g$, the equation characterizing the static trade-off faced by the government becomes more involved to account for this additional wedge, while the equation characterizing debt choice remains functionally the same as when monetary policy is unconstrained. However, the values of the Lagrange multipliers on the constraints of the government's problem are different, which would have quantitative implications in debt choice and domestic policy. We analyze these implications in Appendix B.6.

5 Quantitative Evaluation

This section describes how we take the model to the data. We describe the functional forms adopted for the quantitative analysis and discuss the sources of the parameters set externally. Then we explain how we set the remaining parameters' values to match some relevant statistics. We proceed in two steps. First, we discipline most parameters by calibrating the model without aggregate shocks, i.e., when $s = \bar{s}$. Second, we consider version of the model with either productivity or terms-of-trade shocks and calibrate the remaining parameters. This two-step process greatly improves the speed of computation needed for the calibration as most of the parameters are determined in an economy in which there are no shocks. Appendix B.4 describes the computational procedure.

5.1 Functional forms

The utility functions are $u(c^N, c^T) = \frac{\alpha^N (c^N)^{1-\sigma^N}}{1-\sigma^N} + \frac{\alpha^T (c^T)^{1-\sigma^T}}{1-\sigma^T}$ and $v(\ell) = \frac{\alpha^H \ell^{1-\varphi}}{1-\varphi}$ for consumption and leisure, respectively. We let $\sigma^N = \sigma^T = \sigma$, which implies that $1/\sigma$ represents both the intratemporal elasticity of substitution between c^N and c^T and the intertemporal elasticity of substitution.

The utility function for the public good is $\vartheta(g) = \alpha^G \ln g$, which is a standard representation in the optimal taxation literature and close to empirical estimates.¹⁴

The function $F(y^N, y^T) = (1/A)[(y^N)^\rho + (y^T)^\rho]^{1/\rho}$ describes the labor requirement for production, where ρ determines how costly it is to change the composition of y^N and y^T that is produced, in terms of labor units, and A is a measure of labor productivity.

Finally, we assume that the economy experiences a drop in productivity when the government is in default. Following Arellano (2008), we allow this penalty to vary with the state of the

¹⁴A more general representation with constant relative risk aversion, $\alpha^G (g^{1-\nu} - 1)/(1-\nu)$, converges to log utility as ν approaches 1. Nieh and Ho (2006) estimate values of ν around 0.8. Azzimonti et al. (2016), among others, use log utility for the public good. See also the discussion in Debortoli and Nunes (2013).

economy. Productivity, while in the default state, takes the following form: $A^{def} = A - \Omega(s)$ with $\Omega(s) = \max \left\{ \omega_1 + \omega_2 \frac{(s-\bar{s})}{\bar{s}}, 0 \right\}$, where $\omega_1 > 0$ is the intercept and the slope parameter ω_2 makes the default cost a function of the stochastic variable s . The parameter \bar{s} represents the value of s in an economy without aggregate shocks. Note that s is a scalar since we will consider one type of shock at a time. For the case without aggregate shocks, which we use below to calibrate the majority of the long-run statistics, the default penalty on TFP is equal to ω_1 .

5.2 Exogenous parameters

Table 1 shows the values of the parameters set externally. The annual risk-free interest rate is 3%, in line with the average real interest rate of the world since 1985 in King and Low (2014). We calibrate the value of φ to 1.50 so that the Frisch elasticity is one-half on average.¹⁵ Considering the duration of a default episode from Das et al. (2012) and the length of exclusion after restructuring from Cruces and Trebesch (2013), we choose an expected period of exclusion after a default of 6 years, which implies $\delta = 1/6$.

Table 1: Exogenous parameters

| Parameter | Description | Value | Basis |
|------------|--|-------|--------------------|
| r | risk-free rate | 0.03 | long-run average |
| φ | curvature of leisure | 1.50 | Frisch elasticity |
| δ | reentry probability | 0.17 | exclusion duration |
| α^T | preference share for c^T | 1.00 | normalization |
| σ^N | curvature of c^N | 0.50 | see appendix B.5 |
| σ^T | curvature of c^T | 0.50 | see appendix B.5 |
| ρ | elasticity of substitution between y^N and y^T | 1.50 | see appendix B.5 |
| p^T | terms of trade | 1.00 | normalization |

We set $\sigma^N = \sigma^T = 0.5$. As shown in the previous section, $\sigma^T < 1$ is sufficient for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers γ are zero). In addition, $\sigma^N < 1$ implies the distortionary-policies channel has a negative sign, mitigating the incentives to accumulate debt. This choice implies that imported goods are gross substitutes for non-tradable goods, as in the estimates of Ostry and Reinhart (1992).¹⁶

The value of ρ , which determines the elasticity of substitution between y^N and y^T in the cost function, is set to 1.5. A number larger than 1 guarantees that F is convex and, thus, ensures that the production possibilities frontier is concave.

In Appendix B.5 we support our choices of σ^N , σ^T , and ρ by studying how varying these

¹⁵We can calibrate this parameter externally because we target the value of h .

¹⁶However, the estimates in Ostry and Reinhart (1992) are in the range of 1.22–1.27 and our calibration implies an elasticity equal to 2. In Appendix B.5, we study how our results change when setting $\sigma^N = \sigma^T = 1.5$, which implies that the goods are complements with an elasticity of 0.66.

parameters affects the response of macroeconomic variables to shocks to the terms of trade. Though we find that many results are robust to the choice of parameter values, the reactions of debt spreads, inflation, output and exports favor our benchmark calibration.

Finally, the steady-state value of p^T is 1. Our calibration strategy, described below, is such that other parameters pin down all external variables. Any value of p^T delivers the same observables, except for the exchange rate, which we do not target. When we allow the terms of trade to evolve stochastically, we calibrate the stochastic process for $\ln p_t^T$ to match the data.

5.3 Calibration of the steady-state economy

We next discipline a set of parameters that are calibrated *jointly* to match a set of long-run averages. We use data collected by the World Bank for Argentina, Brazil, Chile, Colombia, Mexico, Peru, and Uruguay from 1991 to 2018 due to data availability. A significant fraction of the parameters can be calibrated in a version of the model without aggregate shocks (other than extreme value shocks), thus reducing the time it takes to calibrate the model and allowing it to match the targets exactly. We will mention the critical parameter that reproduces each moment as we explain the choice of targets for exposition. Table 9 in Appendix B.2 shows the marginal reaction of moments to parameters.¹⁷

Table 2: Calibration parameters and targets

| Parameter | Value | Statistic | Target/Non-stochastic Model |
|------------|--------|------------------------|-----------------------------|
| A | 1.4575 | Real GDP | 1.000 |
| β | 0.8675 | Inflation, % | 3.800 |
| γ | 0.1082 | Transfers/GDP | 0.117 |
| α^N | 2.6888 | Exports/GDP | 0.209 |
| α^H | 0.9265 | Employment/Population | 0.587 |
| α^G | 0.4240 | Gov. Consumption/GDP | 0.133 |
| B^D | 0.1854 | Debt/GDP | 0.185 |
| ω_1 | 0.0228 | Haircut, Share of Debt | 0.305 |
| κ | 0.0235 | Default, % | 0.700 |

We set the steady-state value of productivity, A , so that the steady-state real GDP equals 1, making some statistics easier to read. When we allow the productivity to evolve stochastically, we calibrate the stochastic process for $\ln A_t$ to match the data.

The value of the discount factor β helps the model produce an annual 3.8% inflation rate. The parameter γ matches the ratio of transfers to GDP, which in the data average 11.7%. The value of α^H allows the model to hit the long-run average for the employment-to-population ratio, 0.59. The weight in the utility of the government consumption good, α^G , delivers government

¹⁷The corresponding expressions for each target are presented in Appendix B.1.

consumption over GDP of 13.3%. The parameter α^N takes a value that allows the model to reproduce the ratio of exports to GDP, which is 21% in the data.

The values of ω_1 and B^D determine the costs and benefits of default, respectively. Thus, they are used to reproduce the implied haircut obtained by the country in default, which is 30.5% (Dvorkin et al., 2021), and the external debt-to-GDP ratio, which is 18.5%.

The scale parameter in the distribution of taste shocks, κ , determines the risk of sovereign default in the steady-state and is calibrated to reproduce a default rate of 0.7% annual. We choose a default rate target that is lower than the more typical 2% since in this version of the model, default only occurs due to the extreme value “non-fundamental” shocks.¹⁸

5.4 Calibration of model with aggregate shocks

We now introduce aggregate shocks to the economy. We consider two cases: one with shocks to the price of exports, p^T , and the other with shocks to productivity, A . We start from the benchmark calibration of the previous section and modify it so that the models with shocks predict a higher default rate and fit the targeted levels of debt and haircuts. The remaining moments, which we targeted in the previous step, do not change significantly when adding aggregate shocks—see Appendix B.3. The only exception is inflation, which rises sharply during debt crises, as we explain below; hence, average inflation increases when targeting a higher default probability, though it is still close to the target for the non-stochastic economy.

For the economy with terms of trade shocks we assume p^T follows the process $\ln(p_{t+1}^T) = \rho_p \ln(p_t^T) + \varepsilon_{p,t+1}$, where $\varepsilon_{p,t+1} \sim N(0, \sigma_p^2)$. For the economy with TFP shocks, we assume that A follows the process $\ln(A_{t+1}) = \rho_A \ln(A_t) + \varepsilon_{A,t+1}$, where $\varepsilon_{A,t+1} \sim N(0, \sigma_A^2)$. The values presented in Table 3 are the averages from estimations of these processes for each of the seven countries we use for our calibration.¹⁹

Table 3 presents the recalibrated parameters and how we match the corresponding targets. We first need to adjust the values of B^D and ω_1 . The former targets debt over GDP and the latter the debt haircut in the event of default. These two parameters interact and cannot be calibrated independently. We chose values that minimize the distance between models, and between model and target statistics.

The new parameter to be calibrated is ω_2 , which determines how the cost of default depends on the state of the economy. This parameter played no role in the non-stochastic economy but is critical for determining the economy’s default rate with aggregate shocks. We set it so that

¹⁸See the numbers calculated by Tomz and Wright (2013) for different sets of countries. Alternatively, we could target the average EMBI for these countries. Matching the average for about 300 basis points would require a higher value of κ and a smaller value of β . This alternative calibration yields similar results.

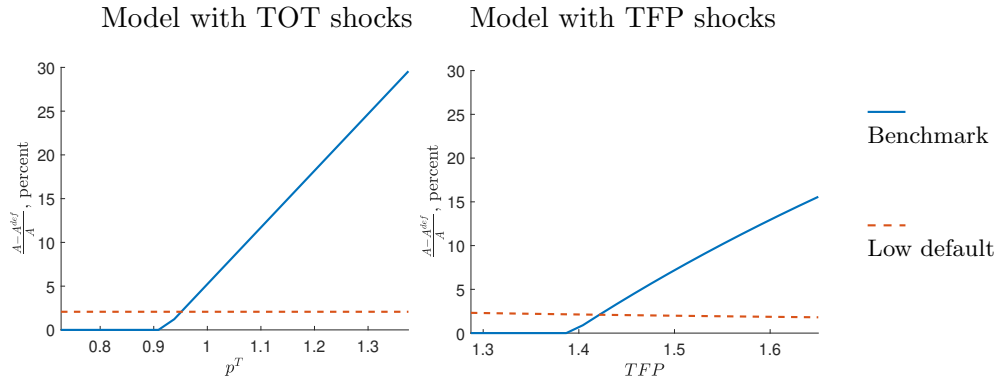
¹⁹See the details in Appendix C.2, where we also show that using commodity prices yield similar results than terms of trade.

Table 3: Recalibrated and new parameters and targets

| Parameter | Shock | | Statistic | Target | Shock | |
|------------|-------|-------|-------------------------------------|--------|-------|-------|
| | p^T | A | | | p^T | A |
| B^D | 0.149 | 0.160 | Debt/GDP | 0.185 | 0.173 | 0.169 |
| ω_1 | 0.087 | 0.068 | Haircut/Debt | 0.305 | 0.257 | 0.230 |
| ω_2 | 0.955 | 1.450 | Default, % | 2.000 | 2.140 | 2.010 |
| ρ_s | 0.880 | 0.863 | Estimation details in Appendix C.2. | | | |
| σ_s | 0.076 | 0.031 | Estimation details in Appendix C.2. | | | |

the model replicates a default rate of 2%, which is standard in the sovereign default literature. The blue lines in Figure 3 show the cost of default as a function of p^T and A . Our choice of ω_2 implies critical values for the term-of-trade and productivity, both below their corresponding steady state value. Below this critical point, the productivity penalty of default is zero; above it, it is increasing in the value of the stochastic variable. The red dashed lines in Figure 3 correspond to a low-default case, which we describe in Section 6.5.

Figure 3: Cost of default in terms of reduction in TFP



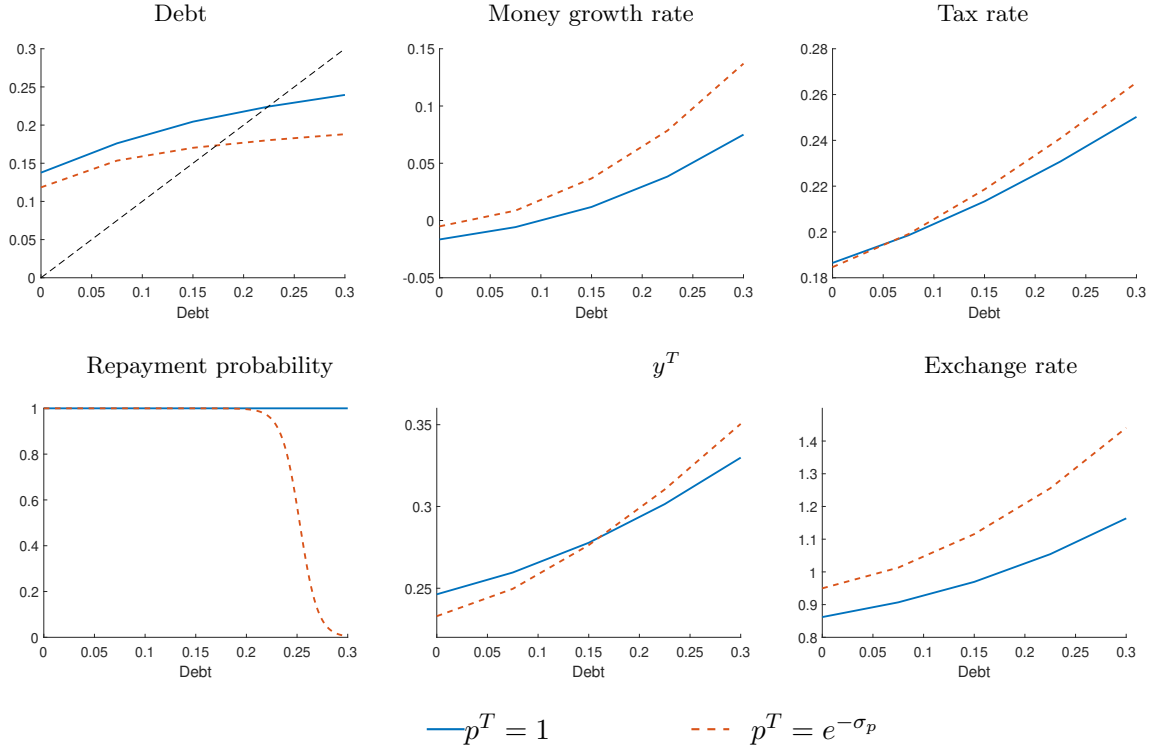
5.5 Equilibrium policies

Figure 4 shows equilibrium policies conditional on repayment for the model with terms-of-trade shocks.²⁰ The x-axis represents the beginning-of-period debt. Each policy function is plotted for two values of export prices: the steady state value, i.e., $p^T = 1$, and one standard deviation below that, i.e., $p^T = e^{-\sigma_p}$. Conditional on repayment, debt choice, the money growth rate, the tax rate and the exchange rate are all increasing in debt. Note, however, that the higher the debt, the lower the amount of net new debt being issued. Similarly, the repayment probability decreases in debt as the incentives to default become larger. Overall, higher debt requires larger distortions to finance it, which implies faster money printing and higher taxation.

We measure inflation as the growth rate in the consumer price index—see Appendix B.1.

²⁰The economy with productivity shocks features very similar qualitative properties, so we omit its presentation for brevity.

Figure 4: Equilibrium policies as functions of debt (p^T shock model)



Notes: Equilibrium policies conditional on repayment for the model with terms-of-trade shocks as functions of beginning-of-period debt and for two values of p^T .

As such, inflation has two components: non-tradable and imported. Although, the money growth rate when repaying is increasing in debt, expected inflation is non-monotonic in debt. It is increasing for low values of debt and decreasing for higher values of debt. The expected inflation in nontradable goods increases with debt because the money growth rate increases with debt, a standard result. However, expected inflation in imported goods is decreasing in debt for large debt values. This pattern occurs because conditional on repayment, an increasing proportion of debt must be repaid, resulting in exchange rate appreciation. Combining these two forces results in the non-monotonicity of overall expected inflation as a function of debt.

In terms of allocations c^N , c^T , g , y^N and h are decreasing in debt (not shown in this Figure), while y^T is increasing in debt. Higher debt implies larger domestic policy distortions (higher μ and τ) and a higher exchange rate, discouraging imports and promoting exports.

Figure 4 also shows the response to a terms-of-trade shock. Consider an economy at the steady-state level of p^T and the associated steady-state level of debt (b around 0.22). Imagine this economy suffers a negative shock to p^T , shifting from the blue solid lines to the red dashed policy functions. The default probability increases due to the adverse shock, which increases the interest on the debt. In response, the government reduces its indebtedness. To finance this deleveraging, the government increases the money growth rate, μ , and the income tax rate, τ ,

while the nominal exchange rate depreciates. Notably, the response of tradable production, y^T , to a shock to p^T depends on the level of debt. When the economy has a high level of debt and suffers a negative shock to its terms of trade, y^T has to increase to reduce the level of debt even more than before the shock. This behavior is in sharp contrast to an economy with a low level of debt. In this case, the economy can still borrow to smooth the negative shock, though less than before the shock, and produce lower tradable output in response to the negative terms-of-trade shock. In the sections below, we show that these response patterns match the data.

5.6 Seigniorage

Though not directly calibrated, an important moment in our economy is the amount of revenue raised by seigniorage, i.e., by the creation of money. We define seigniorage as the change in the supply of money divided by nominal output. Following Aisen and Veiga (2008) and Kehoe, Nicolini, and Sargent (2020), we estimate that the use of seigniorage in countries we used in our calibration averages between 2% and 4% of output—see Appendix B.7 for details.

In our model, seigniorage is defined as μ_t/Y_t , i.e., the growth of the money supply divided by nominal GDP (recall that Y_t is normalized by the money stock, as defined in Appendix B.1). For our calibration, seigniorage is 2.5% in steady state. With either terms-of-trade or TFP shocks, we get 2.7% on average. These figures are all in the estimated empirical range.²¹

5.7 Business cycles statistics

A standard practice is to measure standard deviations and correlations of key macroeconomic variables in the model-simulated data and compare these with the data from emerging markets.²² Table 4 shows that the benchmark model generates significant variation in the trade balance-to-GDP ratio and spreads.²³ Though not targeted in the calibration, the benchmark model also reproduces the correlations between output and other relevant variables we observe in the data. The model replicates three salient properties of emerging markets, namely, that the trade balance-to-GDP, exports-to-GDP and bond spreads are all counter-cyclical.

6 Aggregate Fluctuations in Emerging Markets

In this section, we study how our model and actual emerging economies respond to shocks to the terms of trade and productivity. First, we present evidence that there is an increase in

²¹Note that a small modification of the model, such as adding a velocity parameter in the cash-in-advance constraint, as we do in Appendix A.5, would allow us to perfectly match the steady state with any specific target in the empirically plausible range.

²²We use time series for the seven countries in our sample to make this comparison. We detrend GDP using a band-pass filter as in Christiano and Fitzgerald (2003) to separate a time series into trend and cyclical components. We set the minimum and maximum periods of oscillation of cyclical component at 2 and 16 years, respectively.

²³We define spreads as the yield of the bond maturing this period, given the level of debt, B , and the realization of shocks, s , but before the realization of the extreme value shocks ε , and minus the yield of a risk-free bond. That is, $\text{Spread}(B, s) = \left(\mathcal{P}(B, s) + (1 - \mathcal{P}(B, s)) \frac{B^D}{B} Q^D(s) \right)^{-1} - 1$.

Table 4: Business Cycles Statistics

| | Data | Model with: | |
|------------------------------|--------|--------------|------------|
| | | p^T shocks | TFP shocks |
| Std. Dev. (trade bal./Y) | 0.035 | 0.017 | 0.015 |
| Std. Dev. (spreads) | 3.923 | 3.303 | 2.315 |
| Std. Dev. (exports/Y) | 0.052 | 0.021 | 0.015 |
| Correlation(trade bal./Y, y) | -0.357 | -0.177 | -0.492 |
| Correlation(spreads,y) | -0.362 | -0.073 | -0.187 |
| Correlation(exports/Y,y) | -0.178 | -0.140 | -0.556 |

Note: Data for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay from 1980 to 2018. Y refers to nominal GDP; y is the cyclical components of real GDP per capita, respectively. See appendix C.1 for details.

the rates of inflation and currency depreciation in periods of debt crises, a pattern that is also present in the model. Second, we show that the model can reproduce how interest rate spreads, inflation and currency depreciation respond to shocks to the price of exports and productivity. Third, we analyze the impact on the real economy. Fourth, we argue that default risk is a significant factor in explaining the dynamics of inflation and currency depreciation in emerging markets. Last, we show that the model captures the cyclical properties of domestic policies in emerging countries and how these are mainly driven by sovereign default risk.

6.1 Inflation and currency depreciation during debt crises

We conduct an event study to understand how inflation behaves during a sovereign debt crisis, defined as an episode in which there is a sudden increase in sovereign debt spreads. We follow Calvo et al. (2006) and consider spikes on spreads exceeding two standard deviations above the prevailing sample mean. We then simulate our model for the cases with terms-of-trade and productivity shocks and identify debt crises in the same way. We measure the impact as the percentage point change in the rates of inflation and currency depreciation for the year of the spread spike, relative to the preceding year. Recall that in the model, inflation has two components, non-tradable and imported. Appendix C.3 contains all the details of this exercise.

Table 5 shows that there is significantly higher inflation during a debt crisis. Inflation increases on average by 6.7 percentage points in the year of the crisis. When the sample is restricted to Latin America, which is closer to the countries in our calibration, inflation increases by 4.8 percentage points. We find similar patterns in the model, though with significant differences, depending on the type of shock that hits the economy. When the shock is to the price of exports, inflation increases by 4.4 percentage points, whereas when the shock is to TFP, the impact is larger, about 12.7 percentage points.

The last column in Table 5 presents the results for the rate of currency depreciation. In the

Table 5: Inflation and Currency Depreciation during Debt Crises

| | Data | | Model with | |
|-----------------------|---------------|---------------|--------------|------------|
| | All countries | Latin America | p^T shocks | TFP shocks |
| Inflation | | | | |
| 1-year interval | 6.7 | 4.8 | 4.4 | 12.7 |
| 2-year interval | 3.8 | 2.6 | 2.2 | 7.2 |
| Currency depreciation | | | | |
| 1-year interval | 9.0 | 7.0 | 16.8 | 17.0 |
| 2-year interval | 9.3 | 6.9 | 6.3 | 7.3 |

data, sovereign debt crises are associated with significant depreciation of the currency. Note that this depreciation is more significant than inflation both in the data and in the model, indicating a real depreciation during the crisis year. The difference between depreciation and inflation is particularly large in the model with shocks to the price of exports, which should not be surprising given that this shock implies a real depreciation prior to any policy reaction. Finally, note that debt crises in the data are a combination of potentially multiple shocks, not just shocks to the terms-of-trade and productivity.

6.2 Dynamics with term-of-trade and productivity shocks

We now analyze how spreads, inflation and currency depreciation respond to shocks to the terms-of-trade, p^T , and productivity, A . Following Jordà (2005) we use local projections to estimate these responses in the data and the simulated model. The results are presented in Figure 5. Appendix C.4 explains the regressions in more detail.

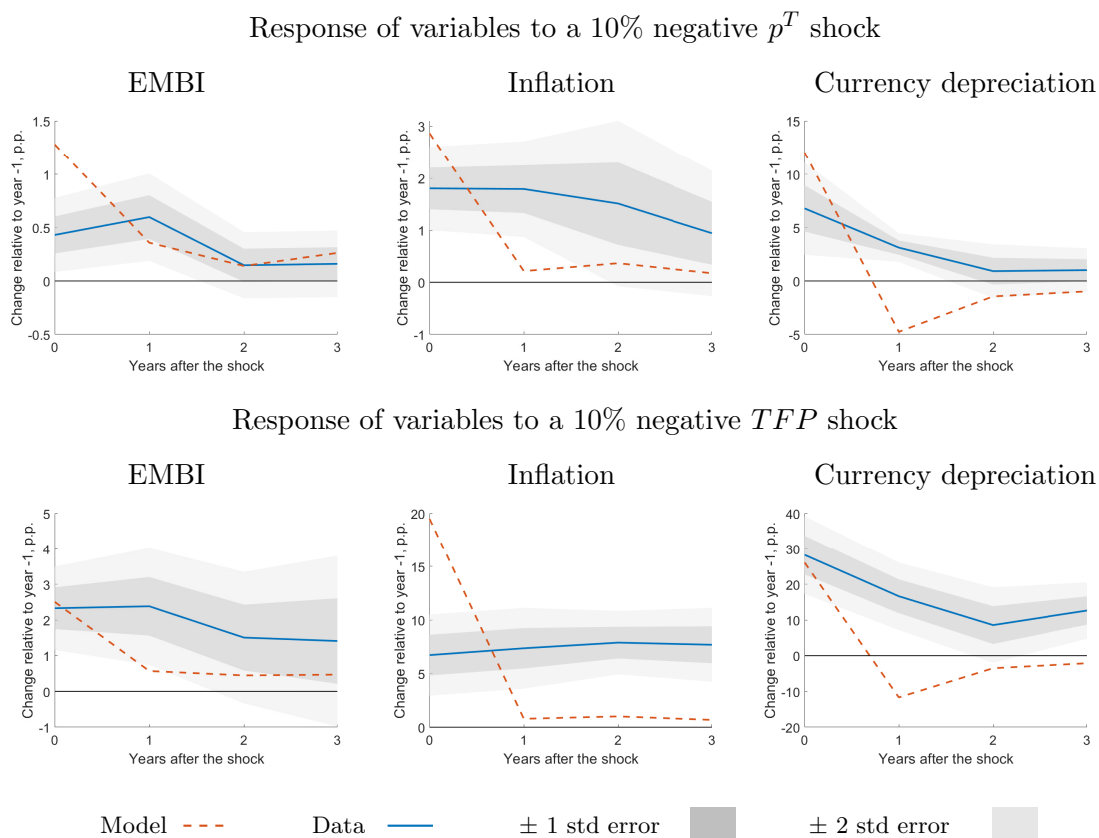
The top panels of Figure 5 show how spreads, inflation and currency depreciation respond when the terms of trade fall by 10%. This analysis resembles the work of Drechsel and Tenreyro (2017), who estimates the contemporaneous response of Argentina’s spread to terms-of-trade shocks. They argue that it is reasonable to assume that international commodity prices are exogenous to developments in Argentina’s economy. They find that “a 10 percent deviation of commodity prices from their long-run mean can move Argentina’s real interest spread by almost 2 percentage points” (i.e., 200 basis points). We use a shorter time series, but we include seven countries and analyze the impact on more variables. To capture the differences in the international prices relevant to each country, we use terms of trade instead of commodity prices.²⁴

The top left plot in Figure 5 shows the response of the EMBI spread to a terms-of-trade

²⁴As we use terms of trade instead of commodity prices our argument for exogenous shocks is weaker than in Drechsel and Tenreyro (2017) and the exercise can be interpreted as validating the correlations in the model and the data.

shock. In the data, we find that a 10% decline in terms of trade increases the EMBI spread by about 50 basis points, an effect that persists over the next year and then declines to zero.²⁵ The dashed red line corresponds to the estimation using model-simulated data. The response of spreads is larger in the model on impact, but smaller in the second period. The overall response has the same sign and a similar magnitude in the data and the model.

Figure 5: Effect of shocks on inflation and devaluation



Next, we compare the reaction of inflation to a shock to the terms of trade (top middle plot in Figure 5), which is helpful to test our assumptions for the inclusion of money in the model. The results suggest that a 10% fall in the term of trade in the data implies an increase in inflation of slightly less than 2 percentage points.²⁶ The response of inflation is larger in the model on impact, close to 3 percentage points, but also less persistent. The similarity between the data and the model is reassuring of our modeling choice for price determination and inflation.

The top-right plot shows the impact of a decline in the terms of trade on the rate of currency depreciation. As with spreads and inflation, currency depreciation reacts more on impact in

²⁵Our estimates are more conservative than Drechsel and Tenreyro (2017) because we include more countries with fewer debt crises than Argentina. If we consider only Argentina, the estimates are more alike.

²⁶We truncated inflation at 50% annual. Otherwise, hyperinflation episodes in Argentina and Brazil dominate the value of any statistic.

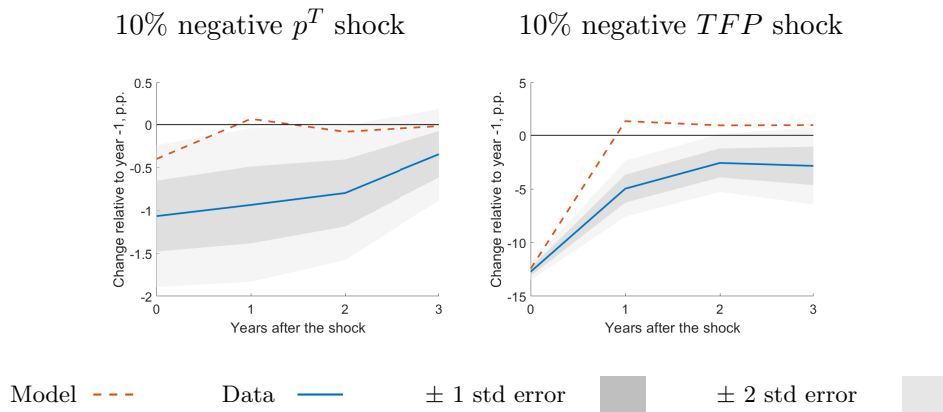
the model. After that, the depreciation rate of the currency is negative in the model, indicating some overshooting when the shock hits.

The bottom panels of Figure 5 show how spreads, inflation and currency depreciation react to 10% decline to productivity. All variables increase on impact, both in the data and the model, but there are some differences in the dynamics. In the data the effects of a productivity shock are more persistent than in the model. Notably, the effect on inflation in the model is concentrated in the initial period; note, however, that cumulative inflation during the first three years is similar in the data and the model.

6.3 Impact on economic activity

Figure 6 shows the reaction of real GDP growth to shocks to the terms of trade and productivity, using the local projections we described above. The effect of terms-of-trade shocks on real GDP growth in the data, shown in the left panel, is significantly different from zero but has large standard errors. On average, real GDP growth falls by about 1 percentage points in the year that terms of trade fall by 10%. Replicating such an effect in the model is challenging. Kehoe and Ruhl (2008) show that in a multi-sector model, the first-order effect of changes in terms of trade on real GDP is zero. Our model allows for a novel mechanism, as policy distortions need to increase to repay the sovereign debt when the terms of trade deteriorate. This mechanism generates a 0.5 percentage points decline in the year of the shock, which is smaller than the point estimate in the data but within 2 standard errors. In the literature, there exist other channels to generate a more prominent effect of terms-of-trade shocks on output.²⁷

Figure 6: Effect of shocks on GDP growth



Note: See Appendix C.4 for more information on the regressions.

The right panel of Figure 6 shows the response of real GDP growth to productivity shocks. The overall message does not change relative to the case with terms-of-trade shocks, though the magnitudes are now significantly larger. A 10% decline in TFP generates a 12% decline in

²⁷See Kohn et al. (forthcoming).

output growth, which reproduces the dynamics of growth in the data remarkably well.

Table 6 shows the volatility of real GDP for the data and the model variants we study. Although productivity shocks have a smaller standard deviation than terms-of-trade shocks, real output is more volatile with the former. We find that fluctuations in export prices account for 18% of the variance in real output, while TFP shocks explain 83%.²⁸ In the empirical literature, the percent of fluctuations in output accounted for by term-of-trade shocks varies between about 10% to 40% mainly depending on the country and period considered (Drechsel and Tenreyro, 2017; Schmitt-Grohé and Uribe, 2018).

Table 6: What Drives GDP Fluctuations?

| | Data | Models with | | | |
|------------------------|-------|--------------|-------------|------------|-------------|
| | | p^T shocks | | TFP shocks | |
| | | benchmark | low default | benchmark | low default |
| Standard deviation (y) | 0.038 | 0.016 | 0.005 | 0.035 | 0.035 |

Note: Data for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay from 1980 to 2018. y refers to the cyclical components of real GDP per capita. See appendix C.1 for details.

6.4 Nominal wage growth

Schmitt-Grohé and Uribe (2016) presents evidence that suggest some degree of wage rigidity in emerging countries while Na, Schmitt-Grohé, Uribe, and Yue (2018) argues that a devaluation (and associated inflation) may be an optimal response to shocks that require a downward adjustment in real wages. Our framework assumes all domestic prices are flexible, so a concern is whether our results would go through if there were downward nominal wage rigidities. To this effect, we take our simulations of the models with term-of-trade and TFP shocks, used in the exercises above, and compute annual nominal wage growth. For each economy, Table 7 shows how frequently nominal wages fall and how frequently they fall by more than 4% yearly, which is the preferred measure of wage rigidity in Schmitt-Grohé and Uribe (2016) and Na, Schmitt-Grohé, Uribe, and Yue (2018).

Table 7: Frequency of nominal wage contractions, %

| | Model with | |
|------------------|--------------|------------|
| | p^T shocks | TFP shocks |
| All contractions | 2.92 | 13.16 |
| By more than 4% | 0.04 | 0.98 |

With terms-of-trade shocks, the nominal wage growth falls below -4% only 0.04% of the time, while it is below zero with a frequency of 2.92%. Hence, our results would only change

²⁸We use variances to be able to add the volatility of shocks. However, note that these two numbers do not need to add to 100%.

slightly if we introduced a constraint on nominal wage growth. There are two reasons for this finding. First, annual inflation is about 4.1% on average and so, real wages can decline even when nominal wages do not contract. Second, in periods of distress, the government in our model increases the money growth rate; thus, inflation increases, which allows for real wages to fall even if nominal wages were to also grow at a faster rate, provided they do not keep up with inflation.

Downward nominal wage rigidity appears to be a potentially relevant concern in the presence of productivity shocks. In our simulations, nominal wages contract 13.16% of the time. However, they fall more than 4% only 0.98% of the time. Thus, downward wage rigidity would only be relevant if we had a strict no-negative growth constraint, but not a particular concern if we used the estimate in Na, Schmitt-Grohé, Uribe, and Yue (2018).

6.5 The role of sovereign default risk

Our model has two key elements: distortionary domestic policies and sovereign default. We studied theoretically the role of domestic policies in Section 4.4. Here, we analyze the role of default risk quantitatively, by comparing how our benchmark calibration and an economy with low sovereign default risk react to shocks to the price of exports and productivity.

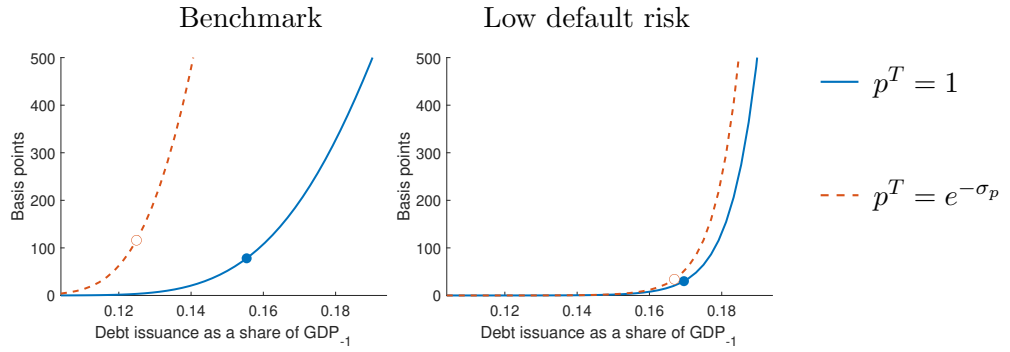
The model is recalibrated to have significantly lower sovereign default risk. Specifically, we change the default-cost function so that the government is not forced to reduce debt in bad times. The dashed red lines in Figure 3 from Section 5.4 represent the new cost of default. The key difference with the benchmark calibration is that the cost of default is independent of the shock value. As a result, the default probability drops to 0.8%, which is similar to that in the economy without aggregate shocks. The remaining target statistics are close to those of the benchmark calibration with aggregate shocks—see Appendix B.3.

To illustrate how the economy with “low default risk” works, Figure 7 shows the relationship between spreads and the amount of debt issued changes after a shock to the terms-of-trade.²⁹ The size of the shock is again one standard deviation. The left panel shows the benchmark calibration. The blue line shows the debt-spread relationship before the p^T shock and the dashed red line shows it after the shock. The dots mark the level of debt chosen by the government in each case. After the p^T shock hits, the spread as a function of debt moves up and consequently, the government reduces its debt—note that even at spreads of 500 basis points, the government is not able to rollover the same amount of debt over GDP. The key mechanism at work here is the same as in models of sovereign default in the tradition of Arellano (2008).

The right panel in Figure 7 shows the case with low default risk. Given the same shock to p^T

²⁹A similar pattern is observed for productivity shocks.

Figure 7: Spreads as a function of debt and p^T , benchmark vs low default



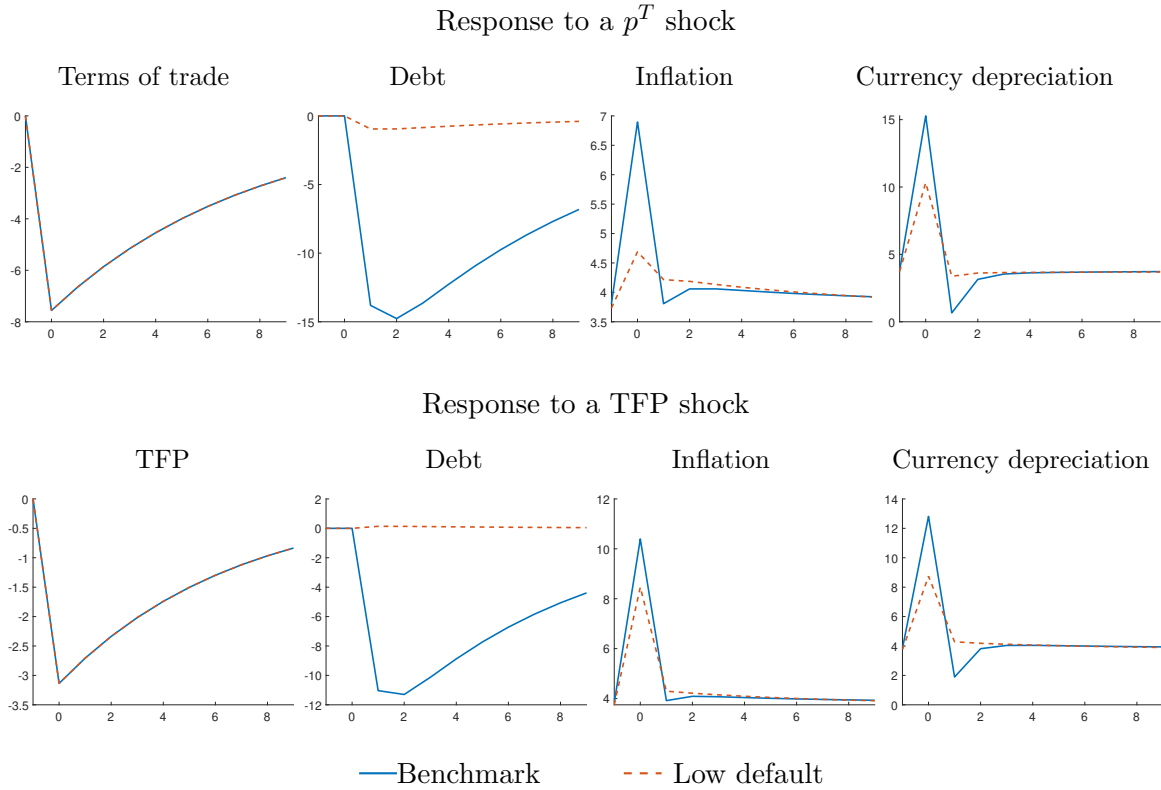
as in the benchmark economy, the relationship between spreads and debt moves considerably less, so that deleveraging is significantly smaller. Recall that in this case, the cost of default is independent of the state of the economy. This specification of the default cost, which is closer Aguiar and Gopinath (2006), will be helpful to understand which of our results derive from sovereign default risk.

Figure 8 shows the model dynamics of debt, inflation, and currency depreciation after adverse shocks to the terms of trade (top panels) and productivity (bottom panels). Debt is shown as deviations from the pre-shock level. The solid blue line represents the benchmark model, and the dashed-red line the model with low default. We assume that the shock hits each of the economies after many periods of the exogenous variables (p^T and A) being at their mean. The size and persistence of each shock correspond to the estimates values in the calibration of the model.

A crucial characteristic of the low-default economy is that debt does not react significantly to a shock. The second row of plots in Figure 8 confirms that feature of the low-default economy for both types of shocks. In contrast, in the benchmark economy, debt decreases significantly after an adverse shock since the government deleverages when facing a steep increase in the cost of issuing debt. By construction, this increase in spreads does not occur in the low-default economy. The different reactions of spreads and debt highlight a key mechanism in the sovereign default literature: emerging markets are forced to repay part of their debt to the rest of the world after a negative shock. This mechanism is essentially absent when the risk of default is low enough. Naturally, default risk has implications for policy. The effect of adverse shocks on inflation and currency depreciation is significantly muted when default risk is low. Thus, this exercise illustrates the importance of including a sizable sovereign default risk to understand the response of nominal variables such as inflation and currency depreciation in emerging markets.

To further quantify the importance of sovereign risk for nominal variables, we compute the variance of inflation and currency depreciation in the simulated time series. In the model with

Figure 8: Debt, Inflation and Depreciation after a shock



Note: Terms of trade, TFP and debt are expressed as percentage deviations from steady state, while inflation and currency depreciation are in percent. The x-axis represents periods after the shock.

p^T shocks, the model with low default-risk generates a variance of inflation that is only 11% of the variance in the benchmark model and a variance of currency depreciation that is 34% of what it is in the benchmark model. Thus, most of the fluctuations in inflation and depreciation in the model with p^T come from sovereign default risk. Similarly, the variances of inflation and depreciation for the models with TFP shocks are significantly smaller in the case of low default risk. In that case, this ratio of variances is 52% for inflation and 27% for currency depreciation, again suggesting that sovereign default risk is essential for accounting for inflation and currency depreciation dynamics.

Table 6, presented in the previous section, also shows that default risk plays an important role in explaining the impact of shock to the terms of trade to the real economy. The standard deviation of output is significantly smaller in the model specification with low default. In contrast, default risk is not relevant to understand the contribution of productivity shocks to output volatility.

6.6 Cyclical properties of domestic policies

We conclude our analysis by studying the cyclical properties of domestic policies. First, consider monetary policy. Following Vegh and Vuletin (2015), we compute the inflation tax

rate as $\text{inflation}/(1+\text{inflation})$.³⁰ We then compare the models with shocks to the terms of trade and productivity, with the average for the seven Latin American countries in our sample, in terms of the standard deviation of the inflation tax and the correlation between the cyclical components of the inflation tax and real GDP. Table 8 shows that the volatility in monetary policy generated by our models is close to that in the data. We also find a negative correlation between the cyclical components of the inflation tax and output, which the model replicates. That is, during recessions, when real output falls, the inflation tax increases.

Table 8: Policy over the business cycle

| | Data | Model, shocks | |
|---|--------|---------------|--------|
| | | p^T | TFP |
| Std. Dev. (inflation tax) | 0.036 | 0.030 | 0.053 |
| Correlation (inflation tax, y) | -0.342 | -0.530 | -0.656 |
| Std. Dev. (personal income tax) | 0.025 | 0.009 | 0.008 |
| Correlation (personal income tax, y) | -0.171 | -0.122 | -0.440 |

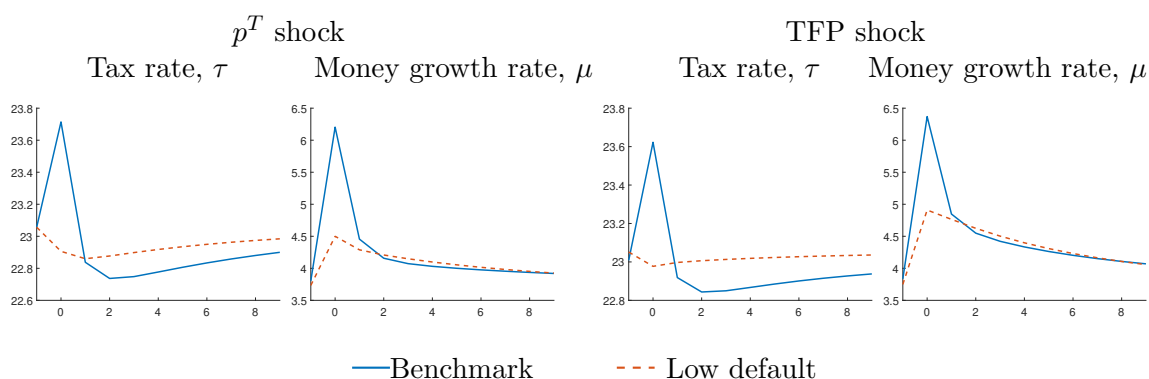
Note: Data are the average of Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cyclical component of real GDP per capita.

Next, we study the behavior of taxation by looking at the cyclical components of the personal income tax rate and real GDP. The data on taxes for the seven Latin American countries in our sample comes from Vegh and Vuletin (2015). Table 8 shows that government increases taxes during bad times; i.e., it engages in austerity during recessions. This policy is optimal in our model since the government must repay part of the debt to the rest of the world in bad times, which, as argued above, is a typical feature of emerging markets. The cyclical properties of fiscal policy stand in stark contrast with the behavior in developed economies, where deficits rise during recessions (see Frankel et al., 2013).

We now analyze how domestic policies interact with default risk, again focusing on the shocks we presented in Figure 8. Here, Figure 9 compares the behavior of the tax and money growth rates in the benchmark economy and the version with low default. The two leftmost plots show the response of taxes and the money growth rate to a shock to terms of trade. Recall from Figure 8 that the debt falls considerably in the benchmark economy, while it barely changes in the low-default economy. The implications of the smaller deleveraging on domestic policies are significant: in the model with low default risk, taxes and money growth react much less; and taxes respond in the opposite direction. A similar result is observed for the policy response to productivity shocks, as shown in the two rightmost plots.

³⁰We detrend the series using Christiano and Fitzgerald (2003) as explained before.

Figure 9: Response to a Shock in a Simulated Path



Note: The tax and money growth rates are expressed in percentages. Shocks are the same as in Figure 8. The x-axis represents periods after a shock.

7 Concluding remarks

Emerging economies experience recurrent debt crises, in part, due to their tendency to overborrow during good times. Their fragility to adverse shocks is likely also a consequence of inadequate economic policy frameworks; for example, a lack of fiscal discipline and excessive reliance on seigniorage and currency depreciation.

Our paper connects domestic policies to sovereign default and economic outcomes. We modeled fiscal and monetary policies as inherently distortionary and assumed the government lacks commitment to both external credit repayment and the conduct of its domestic policies. Our framework led to new insights into the tradeoffs faced by governments when deciding their level of indebtedness, the probability of repayment and the determination of domestic policies. We then showed that the model reproduces standard business cycle statistics, dynamic policies, and macroeconomic aggregate responses to terms-of-trade and productivity shocks in emerging markets. The two features considered in this model, risk of sovereign default and distortionary domestic policies, are essential for the model's success in replicating data from emerging markets.

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Online Appendix

This material is for a separate, on-line appendix and not intended to be printed with the paper.

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A Theory

A.1 Derivations

In order to characterize the solution to the household's problem, let χ and ψ denote the Lagrange multipliers associated with constraints (7) and (2), respectively. The necessary first-order conditions with respect to (c^N, c^T, h, m') for an interior solution are

$$u_N - p^N(\chi + \psi) = 0, \quad (33)$$

$$u_T - e\chi = 0, \quad (34)$$

$$-v_\ell + \chi(1 - \tau)w = 0, \quad (35)$$

$$\beta \mathbb{E} [V'_m | B, \mathcal{I}, s] - \chi(1 + \mu) = 0, \quad (36)$$

where V_m denotes the partial derivative of V with respect to the individual state variable, m . The corresponding envelope condition implies that $V_m = \chi + \psi$. From (33) and (34) we can solve for the Lagrange multipliers,

$$\chi = \frac{u_T}{e}, \quad (37)$$

$$\psi = \frac{u_N}{p^N} - \frac{u_T}{e}. \quad (38)$$

Replacing these expressions in (35) and (36) yields (8). Using (6) to replace e in (38) and imposing $\psi \geq 0$ yields (12).

The government budget constraint

Take the government budget constraint (5), multiply both sides by $F_N c^N$, use the market clearing for labor, $h = F(y^N, y^T)$, and conditions (9), (10) and (11) to obtain

$$\tau F(y^N, y^T) + F_N(\mu c^N - g - \gamma) - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Next, replace the tax rate, τ , using (13) and the money growth rate, μ , using (14) to obtain the government budget constraint in a competitive equilibrium,

$$[1 - (F_T/p^T)(v_\ell/u_T)]F(c^N + g, y^T) - F_N(c^N + g + \gamma) + \beta(F_T/p^T)\mathbb{E}[u'_N c^{N'} | B, \mathcal{I}, s] / u_T - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Since $F(y^N, y^T) = F_N y^N + F_T y^T = F_N(c^N + g) + F_T y^T$ we obtain

$$(F_T/p^T) \{c^T - (v_\ell/u_T)F(c^N + g, y^T)\} + \beta \mathbb{E}[u'_N c^{N'} | B, \mathcal{I}, s] / u_T - \gamma F_N \geq 0,$$

which after multiplying both sides by $u_T(p^T/F_T)$ implies (15).

A.2 Value Functions and Extreme Value Shocks

Given the policy functions, we can define the value functions $V^P(B, s)$, $V^D(s)$ as follows:

$$\begin{aligned} V^P(B, s) &= u(\mathcal{C}^N(B, s), \mathcal{C}^T(B, s)) + v(1 - F(\mathcal{C}^N(B, s) + \mathcal{G}(B, s), \mathcal{Y}^T(B, s))) + \vartheta(\mathcal{G}(B, s)) \\ &\quad + \beta \mathbb{E}[\mathcal{V}(\mathcal{B}(B, s), s')|s], \\ V^D(s) &= u(\bar{\mathcal{C}}^N(s), \bar{\mathcal{C}}^T(s)) + v(1 - F(\bar{\mathcal{C}}^N(s) + \bar{\mathcal{G}}(s), \bar{\mathcal{Y}}^T(s))) + \vartheta(\bar{\mathcal{G}}(s)) \\ &\quad + \beta \mathbb{E}[\delta \mathcal{V}(B^D, s') + (1 - \delta)V^D(s')|s], \end{aligned}$$

for all (B, s) .

As described in the main text, the utility shocks ε^P and ε^D affect the government's decision to repay or default. The assumptions on the distribution of these shocks imply that their difference has mean zero and is distributed logistic; i.e., $\varepsilon = \varepsilon^P - \varepsilon^D$ follows

$$\mathcal{F}(\varepsilon) = \frac{\exp[\varepsilon/\kappa]}{1 + \exp[\varepsilon/\kappa]},$$

where $\kappa > 0$ is the scale parameter of the distribution, which will be useful to control the variance of the ε shocks.

Recall that the repayment probability $\mathcal{P}(B, s)$ can be expressed as

$$\mathcal{P}(B, s) = \Pr(V^P(B, s) - V^D(s) \geq \varepsilon^D - \varepsilon^P).$$

Following McFadden (1973), this expectation has the following closed-form expression:

$$\mathcal{F}(V^P(B, s) - V^D(s)) = \frac{\exp[(V^P(B, s) - V^D(s))/\kappa]}{1 + \exp[(V^P(B, s) - V^D(s))/\kappa]}$$

and so,

$$\mathcal{F}(V^P(B, s) - V^D(s)) = \frac{\exp[V^P(B, s)/\kappa]}{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]}.$$

Therefore,

$$\mathcal{P}(B, s) = \frac{\exp[V^P(B, s)/\kappa]}{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]}, \quad (39)$$

which, in turn, implies

$$\frac{\partial \mathcal{P}(B, s)}{\partial B} = \frac{\partial V^P(B, s)}{\partial B} \frac{\mathcal{P}(B, s)(1 - \mathcal{P}(B, s))}{\kappa}. \quad (40)$$

Next, we can derive a closed-form expression for the expectation of the value function with respect to the utility shocks:

$$\mathcal{V}(B, s) = E_\varepsilon[\hat{\mathcal{V}}(B, s, \varepsilon^P, \varepsilon^D)] = \kappa \ln \{ \exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa] \}. \quad (41)$$

Using this expression, we can easily see that

$$\frac{\partial \mathcal{V}(B, s)}{\partial B} = \mathcal{P}(B, s) \frac{\partial V^P(B, s)}{\partial B}. \quad (42)$$

We use (39) and (41) in the formulation of the government's problem. We use (40) and (42) when characterizing the choice of debt.

A.3 Full characterization of the government's problem

We characterize the problem of the government allowing for transfers to be positive, i.e., $\gamma \geq 0$, and the non-negativity constraint (19) to be potentially bind. These more general assumptions do not alter the characterization of debt choice, (29). To simplify some of the notation below, let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T (F_N / F_T)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma/dc^T = \Gamma \times (u_{TT}/u_T) < 0$, while the convexity of F implies that $\Gamma_N = \Gamma_g = \Gamma \times (F_{NN}/F_N - F_{NT}/F_T) > 0$ and $\Gamma_y = \Gamma \times (F_{NT}/F_N - F_{TT}/F_T) < 0$. Recall that $\Phi \equiv v_\ell - v_{\ell\ell} F(c^N + g, y^T) > 0$.

Since the problems in repayment and default are functionally identical with respect to (c^N, c^T, y^T, g) , we focus on (PP)–(19). Let ξ , λ and ζ be the Lagrange multipliers associated with the constraints (17), (18) and (19), respectively. The necessary first-order conditions with respect to (c^N, c^T, y^T, g) are

$$u_N - v_\ell F_N - \lambda(F_N \Phi + \gamma \Gamma_N) + \zeta(u_{NN} - \Gamma_N) = 0, \quad (43)$$

$$u_T - \xi + \lambda(u_T + u_{TT} c^T - \gamma \Gamma_T) - \zeta \Gamma_T = 0, \quad (44)$$

$$-v_\ell F_T + \xi p^T - \lambda(F_T \Phi + \gamma \Gamma_y) - \zeta \Gamma_y = 0, \quad (45)$$

$$-v_\ell F_N + \vartheta_g - \lambda(F_N \Phi + \gamma \Gamma_g) - \zeta \Gamma_g = 0. \quad (46)$$

Since $\Gamma_N = \Gamma_g$, (43) and (46) imply

$$\zeta = -\frac{(u_N - \vartheta_g)}{u_{NN}} \quad (47)$$

Suppose $\zeta = 0$. Then, (47) implies $u_N = \vartheta_g$ and (43) and (45) solve for the remaining Lagrange multipliers

$$\begin{aligned} \lambda &= \frac{u_N - v_\ell F_N}{F_N \Phi + \gamma \Gamma_N}, \\ \xi &= \frac{u_N F_T \Phi + \gamma [v_\ell F_T \Gamma_N + (u_N - v_\ell F_N) \Gamma_y]}{p^T (F_N \Phi + \gamma \Gamma_N)}, \end{aligned}$$

Hence, (44) implies

$$F_T \Phi (u_N - \Gamma) - \gamma [(u_T p^T - v_\ell F_T) \Gamma_N - (u_N - v_\ell F_N) \Gamma_y] = p^T (u_N - v_\ell F_N) (u_T + u_{TT} c^T - \gamma \Gamma_T), \quad (48)$$

where we used the definition of Γ to simplify the expression.

When $\zeta > 0$, (47) implies $u_N > \vartheta_g$. Conditions (43) and (45) solve for the remaining Lagrange multipliers

$$\lambda = \frac{\vartheta_g - v_\ell F_N + (u_N - \vartheta_g)(\Gamma_N/u_{NN})}{F_N \Phi + \gamma \Gamma_N}$$

$$\xi = \frac{1}{p^T(F_N \Phi + \gamma \Gamma_N)} \left[\vartheta_g(F_T \Phi + \gamma \Gamma_y) - \gamma v_\ell(F_N \Gamma_y - F_T \Gamma_N) + \left(\frac{u_N - \vartheta_g}{u_{NN}} \right) (F_T \Gamma_N - F_N \Gamma_y) \Phi \right]$$

Hence, (44) implies

$$\begin{aligned} & u_T(F_N \Phi + \gamma \Gamma_N) + (\vartheta_g - v_\ell F_N)(u_T + u_{TT}c^T - \gamma \Gamma_T) \\ & + \left(\frac{u_N - \vartheta_g}{u_{NN}} \right) [\Gamma_N(u_T + u_{TT}c^T - \gamma \Gamma_T) - (F_T \Gamma_N - F_N \Gamma_y)(\Phi/p^T) + (F_N \Phi + \gamma \Gamma_N)\Gamma_T] \\ & = \frac{1}{p^T} [\vartheta_g(F_T \Phi + \gamma \Gamma_y) - \gamma v_\ell(F_N \Gamma_y - F_T \Gamma_N)] \end{aligned} \quad (49)$$

A.4 Proofs

Proof of Proposition 1. (i) Consider the case in which $\gamma = 0$. Note that (19) implies $u_N - \Gamma \geq 0$. Since $\Phi > 0$, the left-hand side of (27) is non-negative. Next, $\lambda > 0$ implies $u_N - v_\ell F_N > 0$. Hence, $u_N - \Gamma > 0$ if and only if $u_T + u_{TT}c^T > 0$, while $u_N - \Gamma = 0$ if and only if $u_T + u_{TT}c^T = 0$. If preferences are such that $u_T + u_{TT}c^T < 0$, then (27) cannot be satisfied—a contradiction. In this case, $\zeta > 0$ and therefore, (19) binds.

(ii) Consider the case in which $\gamma > 0$ suppose $\zeta = 0$. From (24) we can write $\xi = u_T + \lambda(u_T + u_{TT}c^T - \gamma \Gamma_T)$ and then rearrange (25) as follows

$$F_T(v_\ell + \lambda \Phi) - p^T u_T = p^T \lambda [u_T + u_{TT}c^T - \gamma(\Gamma_T + \Gamma_y)]. \quad (50)$$

From (23) $F_T(v_\ell + \lambda \Phi) = (u_N - \lambda \gamma \Gamma_N)(F_T/F_N)$. Thus, (50) implies

$$u_N(F_T/F_N) - p^T u_T = p^T \lambda [u_T + u_{TT}c^T + \gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y]]. \quad (51)$$

Recall that $\Gamma_N > 0$, $\Gamma_T < 0$ and $\Gamma_y < 0$. Hence, if $\frac{-u_{TT}c^T}{u_T} \leq 1$, then $u_T + u_{TT}c^T \geq 0$ and so, the right hand-side of (51) is strictly positive. Then, $u_N(F_T/F_N) - p^T u_T > 0$ and (19) is satisfied with strict inequality.

Now suppose (19) is satisfied with equality while $\zeta = 0$. Then (51) implies $u_T + u_{TT}c^T = -\gamma[\Gamma_N(F_T/F_N) - \Gamma_T - \Gamma_y] < 0$ and so, $\frac{-u_{TT}c^T}{u_T} > 1$. Therefore, it follows by continuity that there exists some $\hat{\sigma}^T > 1$ such that (19) is satisfied with equality for all $\frac{-u_{TT}c^T}{u_T} \leq \hat{\sigma}^T$. In this case, policy is away from the Friedman rule if $\frac{-u_{TT}c^T}{u_T} < \hat{\sigma}^T$. \square

Proof of Proposition 2. The envelope condition of problem (PP) implies $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. The derivative of (16) with respect to B' is:

$$\frac{\partial [Q(B',s)B']}{\partial B'} = \mathbb{E} \left\{ \frac{\mathcal{P}(B',s')}{1+r} \left[1 + \frac{\partial V^P(B',s')}{\partial B'} \frac{(1 - \mathcal{P}(B',s'))(B' - Q^D(s')B^D)}{\kappa} \right] \Big|_s \right\}. \quad (52)$$

Using (40)–(42) and (52), we obtain the following expressions for the first and second terms of (28)

$$\begin{aligned}\frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} &= -\mathbb{E}[\mathcal{P}(B', s')\xi'|s] \\ \frac{\partial Q(B', s)B'}{\partial B'} &= \mathbb{E}\left\{\frac{\mathcal{P}(B', s')}{1+r}\left[1 - \frac{(1 - \mathcal{P}(B', s'))(B' - Q^D(s')B^D)\xi'}{\kappa}\right]\middle|s\right\}.\end{aligned}$$

The last term in (28) requires additional work. Given that $\mathcal{P}(B', s')$ is the probability of transitioning from $\mathcal{I} = P$ to $\mathcal{I}' = P$ for all (B', s') , we can write

$$\mathbb{E}[u'_N c^{N'}|P, s] = \mathbb{E}[\mathcal{P}(B', s')u'_N c^{N'} + (1 - \mathcal{P}(B', s'))\bar{u}'_N \bar{c}^{N'}|s],$$

where $u'_N c^{N'}$ corresponds to the repayment state tomorrow, $\mathcal{I}' = P$, and $\bar{u}'_N \bar{c}^{N'}$ corresponds to the default state tomorrow, $\mathcal{I}' = D$. Note that the expectation on the right-hand side (only conditional on s) is taken with respect to s' . We can take the derivative of the expression above with respect to B' to obtain

$$\frac{\partial \mathbb{E}[u'_N c^{N'}|P, s]}{\partial B'} = \mathbb{E}[\mathcal{P}(B', s')(u'_N + u'_{NN}c^{N'})\mathcal{C}_B^{N'} + (u'_N c^{N'} - \bar{u}'_N \bar{c}^{N'})\mathcal{P}'_B|s],$$

where $\mathcal{C}_B^{N'}$ and \mathcal{P}'_B denote the derivatives of $\mathcal{C}^N(B', s')$ and $\mathcal{P}(B', s')$ with respect to B' . Recall that, when in default, allocations are not a function of B , i.e., $\bar{C}^N(s)$ and so $\bar{C}_B^{N'} = 0$. From (40), we have an analytical expression for \mathcal{P}'_B and from the envelope condition, $\frac{\partial V^P(B, s)}{\partial B} = -\xi$.

Thus, we obtain

$$\frac{\partial \mathbb{E}[u'_N c^{N'}|P, s]}{\partial B'} = \mathbb{E}\left\{\mathcal{P}(B', s')\left[(u'_N + u'_{NN}c^{N'})\mathcal{C}_B^{N'} - \frac{(u'_N c^{N'} - \bar{u}'_N \bar{c}^{N'})(1 - \mathcal{P}(B', s'))\xi'}{\kappa}\right]\middle|s\right\}.$$

We now have all the elements to write the equation characterizing debt choice. \square

Proof of Proposition 3. Consider the EG real allocation $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ that solves the problem (PPEP) where lump-sum, unconstrained taxes \mathcal{T} make the government budget constraint becomes (31). In order to prove this result, we first solve the problem (PPEP) and then we construct the monetary policy and taxes $(\hat{\mu}, \hat{\tau})$ as well as the prices $(\hat{p}^N, \hat{e}, \hat{w})$ that support this allocation as an equilibrium in our setting.

The necessary first-order conditions characterizing the EG real allocation are

$$\hat{u}_N = \hat{v}_\ell \hat{F}_N, \quad (53)$$

$$\hat{v}_\ell \hat{F}_T = p^T \hat{u}_T, \quad (54)$$

$$\hat{\vartheta}_g = \hat{v}_\ell \hat{F}_N, \quad (55)$$

which imply $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$; i.e., the non-negative constraint (19), which we ignore to derive

the EG real allocation, is satisfied with equality. The balance of payment implies

$$p^T \hat{y}^T - \hat{c}^T + \hat{Q}(B', s) \hat{B}' - \hat{B} = 0. \quad (56)$$

We now construct the policies and prices that support the EG real allocation, we have that the price of non-tradable goods and wages are determined by

$$\begin{aligned} \hat{p}^N &= \frac{1}{\hat{c}^N}, \\ \hat{w} &= \frac{\hat{p}^N}{\hat{F}_N}, \end{aligned}$$

while the exchange rate is determined by

$$\hat{e} = \frac{\hat{p}^N \hat{F}_T}{p^T \hat{F}_N},$$

The monetary policy has to be tailored so that

$$\hat{\mu} = \frac{\beta \mathbb{E} [\hat{u}'_N \hat{c}^{N'} | B, \mathcal{I}, s]}{\hat{u}_T \hat{c}^N p^T (\hat{F}_N / \hat{F}_T)} - 1,$$

as it has to decentralize money holdings such that $m' = m = 1$. Since $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$, we obtain

$$\hat{\mu} = \frac{\beta \mathbb{E} [\hat{u}'_N \hat{c}^{N'} | B, \mathcal{I}, s]}{\hat{u}_N \hat{c}^N} - 1. \quad (57)$$

On the other hand, taxes are given by

$$\hat{\tau} = 1 - \frac{\hat{v}_\ell \hat{F}_T}{\hat{u}_T p^T} = 0.$$

Finally, lump-sum transfers are designed to make the budget constraint of the government (31) hold so that

$$\hat{\mathcal{T}} = \hat{p}^N (\hat{g} + \gamma) - \hat{\mu} + \hat{e} (p^T \hat{y}^T - \hat{c}^T).$$

□

Proof of Proposition 4. As shown in Proposition 3, the real EG allocation implies zero labor taxes when monetary policy is given by (32). Thus, combining the balance of payments with the government budget constraint when lump-sum taxes are not available implies

$$\hat{e} [\hat{Q}(\hat{B}', s) \hat{B}' - B] = \hat{p}^N (\hat{g} + \gamma) - \hat{\mu}, \quad (58)$$

i.e., a non-linear first-order difference equation in domestic debt, B .

First, consider a steady state in an environment with no aggregate shocks s . Since $\hat{g} + \gamma \geq 0$ and $\hat{\mu} \leq 0$ (see (57) above), imply $\hat{e} > 0$ and $\hat{Q}(\hat{B}', s) < 1$, it follows that any steady state would require $B < 0$; i.e., the government must accumulate a sufficiently large amount of assets to finance its expenditures. This asset position would never be reached, as (58) implies that the

amount of B is strictly increasing and positive when the initial stock of debt is positive.

Consider now the general case in a stochastic environment. Observe that since $\hat{p}^N(\hat{g} + \gamma) - \hat{\mu} \geq 0$, then $B > 0$ implies that $B' > 0$ as

$$\frac{\hat{B}'}{(1+r)} \geq \hat{Q}(\hat{B}', s)\hat{B}' = B + \frac{\hat{p}^N}{\hat{e}}(\hat{g} + \gamma) - \frac{\hat{\mu}}{\hat{e}}, \quad (59)$$

and so $\hat{B}' \geq (1+r)B + (1+r)\left(\frac{\hat{p}^N}{\hat{e}}(\hat{g} + \gamma) - \frac{\hat{\mu}}{\hat{e}}\right)$. Therefore, as $r > 0$, the sequence of debt for this allocation is strictly increasing and unbounded as long as $B_0 > 0$. We argue that this cannot be an equilibrium path. To see this, define \bar{y}^T as the unique solution to $F(0, \bar{y}^T) = 1$, i.e., the highest level of the tradable good that can be produced as $h = 1$ and $\hat{y}^N = \hat{c}^N + \hat{g} = 0$.

Let $\bar{p}^T = \max p^T$ and conjecture that $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{p}^T\bar{y}^T$ and all s . From the balance of payments, non-default tradable consumption can be written as

$$\hat{c}^T = \hat{Q}(\hat{B}', s)\hat{B}' + p^T\hat{y}^T - B \leq \max_{B'}\{\hat{Q}(\hat{B}', s)\hat{B}'\} + \bar{p}^T\bar{y}^T - B \leq \frac{1}{1+r} \frac{(1+r)}{r}\bar{p}^T\bar{y}^T + \bar{p}^T\bar{y}^T - B. \quad (60)$$

Therefore, if $B = \frac{(1+r)}{r}\bar{p}^T\bar{y}^T$, then (60) implies that non-default tradable consumption cannot be positive and leads to a contradiction, as the EG allocation is interior and consequently the outcome must be default; i.e., $Q\left(\frac{(1+r)}{r}\bar{p}^T\bar{y}^T, s\right) = 0$ for all s . Therefore, since Q is decreasing, $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{p}^T\bar{y}^T$ and all s and validates the conjecture.

To conclude the proof, observe that as the sequence of debt would be strictly increasing and unbounded, it would be larger than $\frac{(1+r)}{r}\bar{p}^T\bar{y}^T$ in finite time and thus contradicts (59) since $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r}\bar{p}^T\bar{y}^T$ and all s . \square

A.5 General cash-in-advance constraint

In this section we consider a version of our model with a more general cash-in-advance constraint. Following the rationale provided by Lucas (1980) one could think of an agent in our economy as being split in three: a shopper for non-tradable goods, an importer and a worker. Shopper and worker set off at the same time, which means that the shopper cannot use money acquired in the period to make purchases, i.e., needs to use money acquired in previous periods as rationalized by constraint (2). In our benchmark model, the importer is assumed to trade on credit, so money is not necessary to purchase imported goods. Alternatively, if the importer sets out after the worker comes back or if imports are purchased on credit but required to be settled in domestic money by the end of the period, then a cash-in-advance constraint on the purchase of imported goods would not bind, as some money would still need to be carried over to purchase non-tradable goods at the beginning of the following period. The budget constraint would thus be sufficient to guarantee that the household has enough money to purchase and settle imports. Hence, for a cash-in-advance constraint to also be relevant for imported goods,

we must assume that the importer sets out at the same time as the shopper and is required to settle purchases in domestic currency immediately and not on credit.³¹

In the analysis that follows, we assume that proportions θ^N and θ^T of expenditures on non-tradable and imported goods, respectively, must be supported by previously accumulated money balances. These proportions could be lower, equal or greater than one. One important caveat is that θ^N and θ^T must be different since, otherwise, the problem is not well defined. For the symmetric case when $\theta^N = \theta^T$, we study the limiting case as $\theta^T \rightarrow \theta^N = \theta$. We find that this case implies that monetary policy follows the Friedman rule, which would be inconsistent with the data on monetary policy and inflation in emerging markets.

Consider then a general version of the cash-in-advance constraint:

$$\theta^N p^N c^N + \theta^T e c^T \leq m. \quad (61)$$

where $\theta^N \geq 0$, $\theta^T \geq 0$ and $\theta^N \neq \theta^T$.

The solution to the representative household's problem must necessarily satisfy

$$\frac{(1-\tau)w}{\theta^N - \theta^T} \left(\frac{u_T \theta^N}{e} - \frac{u_N \theta^T}{p^N} \right) = v_\ell \quad (62)$$

$$(1+\mu) \left(\frac{u_T \theta^N}{e} - \frac{u_N \theta^T}{p^N} \right) = \beta \mathbb{E} \left[\frac{u'_N(1-\theta^T)}{p^{N'}} - \frac{u'_T(1-\theta^N)}{e'} \middle| B, \mathcal{I}, s \right] \quad (63)$$

plus constraints (61) and the budget constraint (7).

In equilibrium, we obtain:

$$p^N = \frac{F_N}{\theta^N c^N F_N + \theta^T c^T F_T / p^T}. \quad (64)$$

$$w = \frac{1}{\theta^N c^N F_N + \theta^T c^T F_T / p^T}. \quad (65)$$

$$e = \frac{F_T / p^T}{\theta^N c^N F_N + \theta^T c^T F_T / p^T}. \quad (66)$$

Finally, the Lagrange multiplier associated with the cash-in-advance constraint must be non-negative, which implies the following equilibrium condition:

$$\left(\frac{1}{\theta^N - \theta^T} \right) \left(\frac{u_N}{F_N} - \frac{u_T}{F_T / p^T} \right) \geq 0. \quad (67)$$

Note that this condition would not be well-defined if $\theta^T = \theta^N = \theta$.

The expressions for the tax rate and the money growth rate are, respectively,

$$\tau = 1 - \frac{(\theta^N - \theta^T)v_\ell}{\theta^N p^T (u_T / F_T) - \theta^T (u_N / F_N)}, \quad (68)$$

³¹Note that the argument still applies if imports are purchased with foreign currency but currency exchange markets are subject to similar timing and settlement assumptions we described.

$$\mu = \frac{\beta \mathbb{E} \left\{ \left[\theta^N c^{N'} F'_N + \theta^T c^{T'} (F'_T/p^{T'}) \right] \left[(1 - \theta^T)(u'_N/F'_N) - (1 - \theta^N) p^{T'}(u'_T/F'_T) \right] \mid B, \mathcal{I}, s \right\}}{\left[\theta^N c^N F_N + \theta^T c^T (F_T/p^T) \right] \left[\theta^N p^T (u_T/F_T) - \theta^T (u_N/F_N) \right]} - 1. \quad (69)$$

Finally, following the steps described in Section A.1 for the benchmark case, we obtain the government budget constraint in a monetary equilibrium:

$$\begin{aligned} & \left[\theta^N p^T (u_T/F_T) - \theta^T (u_N/F_N) \right] \left[(1 - \theta^N) F_N c^N + (1 - \theta^T) (F_T/p^T) c^T - F_N \gamma \right] - (\theta^N - \theta^T) v_\ell F(y^N, y^T) \\ & + \beta \mathbb{E} \left\{ \left[\theta^N c^{N'} F'_N + \theta^T c^{T'} (F'_T/p^{T'}) \right] \left[(1 - \theta^T)(u'_N/F'_N) - (1 - \theta^N) p^{T'}(u'_T/F'_T) \right] \mid B, \mathcal{I}, s \right\} \geq 0. \end{aligned} \quad (70)$$

To summarize, policy rates τ and μ are given by (68) and (69), respectively. Implementability is restricted by the non-negativity constraint, (67), the government budget constraint (70) and the balance of payment (4).

We now consider the limiting case, $\theta^T \rightarrow \theta^N = \theta$. The inequality condition (67) implies that

$$\lim_{\theta^T \rightarrow \theta^N} \left(\frac{u_N}{F_N} - \frac{u_T}{F_T/p^T} \right) = 0. \quad (71)$$

If this were not the case, the limiting Lagrange multiplier on the cash-in-advance constraint would not be a finite number. We can now derive expressions for taxes and money growth. Starting with (68), we rewrite it as

$$\tau = 1 - \frac{(\frac{\theta^N}{\theta^T} - 1) v_\ell}{\frac{\theta^N}{\theta^T} p^T (u_T/F_T) - (u_N/F_N)}. \quad (72)$$

When $\frac{\theta^N}{\theta^T} = 1$, the equation above is not well-defined as (71) leads to 0/0 when $\frac{\theta^N}{\theta^T} = 1$. To analyze the case with $\frac{\theta^N}{\theta^T} = 1$, we apply the L'Hôpital's rule, which implies that

$$\tau = 1 - \frac{v_\ell F_T}{p^T u_T} = 1 - \frac{v_\ell F_N}{u_N} \quad (73)$$

as $\theta^T \rightarrow \theta^N = \theta$. Note that this expression is the same as the one in the benchmark case, equation (13). Also, given (71), the tax rate may also be expressed as $\tau = 1 - \frac{v_\ell F_N}{u_N}$.

As with the tax rate, we can rewrite the expression for the money growth rate using (69) as follows,

$$\mu = \frac{\beta \mathbb{E} \left\{ \left[\frac{\theta^N}{\theta^T} c^{N'} F'_N + c^{T'} (F'_T/p^{T'}) \right] \left[(\frac{1}{\theta^T} - 1)(u'_N/F'_N) - (\frac{1}{\theta^T} - \frac{\theta^N}{\theta^T}) p^{T'}(u'_T/F'_T) \right] \mid B, \mathcal{I}, s \right\}}{\left[\frac{\theta^N}{\theta^T} c^N F_N + c^T (F_T/p^T) \right] \left[\frac{\theta^N}{\theta^T} p^T (u_T/F_T) - (u_N/F_N) \right]} - 1.$$

When $\theta^T = \theta^N = \theta$, it follows that

$$\mu = \frac{\beta \mathbb{E} \left\{ \left[c^{N'} F'_N + c^{T'} (F'_T/p^{T'}) \right] (\frac{1}{\theta} - 1) \left[(u'_N/F'_N) - p^{T'}(u'_T/F'_T) \right] \mid B, \mathcal{I}, s \right\}}{\left[c^N F_N + c^T (F_T/p^T) \right] \left[p^T (u_T/F_T) - (u_N/F_N) \right]} - 1.$$

which by (71) would not be well-defined. Hence, apply the L'Hôpital's rule for $\theta^T \rightarrow \theta^N = \theta$ and obtain

$$\mu = \frac{\beta \mathbb{E} [c^{N'} F'_N p^{T'}(u'_T/F'_T) | B, \mathcal{I}, s]}{c^N F_N p^T(u_T/F_T)} - 1$$

which, given (71), simplifies to

$$\mu = \frac{\beta \mathbb{E} [c^{N'} u'_N | B, \mathcal{I}, s]}{c^N u_N} - 1.$$

As mentioned above, we find that monetary policy implements the Friedman rule when $\theta^N = \theta^T$.

As a final note, we made the derivations above using $\theta^T \rightarrow \theta^N = \theta$. The same results apply when instead using $\theta^N \rightarrow \theta^T = \theta$.

B Quantitative Results

B.1 Definition of macroeconomic aggregates

- Nominal GDP (in pesos, normalized by the money stock),

$$Y_t = e_t p_t^T y_t^T + p_t^N y_t^N.$$

- GDP in foreign currency (USD),

$$Y_t^{USD} = p_t^T y_t^T + \frac{1}{e_t} p_t^N y_t^N.$$

- The GDP deflator (in pesos, normalized by the money stock)

$$P_t^y = \left(\frac{e p_t^T y_t^T}{Y} \right) e_t p_t^T + \left(\frac{p_t^N y_t^N}{Y} \right) p_t^N$$

- Real GDP,

$$Y_t^R = \frac{Y_t}{P_t^y}.$$

- Consumption expenditures (in pesos, normalized by the money stock),

$$C_t = e_t c_t^T + p_t^N c_t^N.$$

- Consumption price index (in pesos, normalized by the money stock),

$$P_t^c = \left(\frac{e c_t^T}{C} \right) e_t + \left(\frac{p_t^N c_t^N}{C} \right) p_t^N.$$

- Inflation, measured as the change in the consumption price index,

$$\pi_t = \frac{P_t^c}{P_{t-1}^c} (1 + \mu_{t-1}) - 1.$$

- Currency depreciation

$$\Delta_t = \frac{e_t}{e_{t-1}} (1 + \mu_{t-1}) - 1$$

Note that inflation and currency depreciation are corrected by the money growth rate, since prices are normalized by the money stock.

B.2 Identification

To provide a heuristic proof of identification, we compute how each parameter would change if we change one target at a time by 10 percent. The results, presented in Table 9, justify the link between parameters and targets mentioned in the calibration section. The first column shows how each parameter changes when we target a default rate 10 percent larger, i.e., 1.1 percent instead of 1 percent. Note that the more significant change is for κ . By increasing κ

Table 9: Percent change in each parameter when a target is increased by 10 percent (p^T shock)

| | Target increased by 10 percent | | | | | | | | |
|------------|--------------------------------|--------------|-------------|--------------|---------------|--------------|-------------|--------------|--------------|
| | Default | Debt | Haircut | G | Hours | Exports | Inflation | Transfers | Real GDP |
| κ | 9.68 | 8.99 | 6.07 | -11.79 | 57.65 | 6.39 | 7.34 | -22.34 | 4.88 |
| B^D | 0.00 | 10.00 | -4.39 | 0.00 | 0.00 | -4.13 | 0.00 | 0.00 | 10.00 |
| ω_1 | 3.39 | 9.33 | 5.85 | -4.95 | 22.31 | 5.95 | 2.74 | -9.61 | 0.00 |
| α^G | 0.02 | 0.23 | 0.01 | 11.72 | -4.22 | -7.92 | -0.27 | 3.17 | 4.88 |
| α^H | 0.01 | 0.07 | 0.00 | -1.80 | -27.77 | -6.86 | 0.33 | -1.52 | 4.88 |
| α^N | 0.02 | 0.23 | 0.01 | 0.53 | -4.22 | -9.32 | -0.27 | 3.17 | 0.00 |
| β | -0.01 | -0.09 | 0.00 | -1.54 | 4.40 | 1.25 | 0.64 | -3.07 | 0.00 |
| γ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | 0.00 | 10.00 | 10.00 |
| A | 0.00 | 0.00 | 0.00 | 0.00 | -9.09 | -1.03 | 0.00 | 0.00 | 10.00 |

Note: Each number represents the percentage change in the parameter when the target is increased by 10 percent.

9.68 percent and adjusting all the parameters (except ω_1) very slightly, the model can replicate all the targets perfectly. Thus, we selected κ as the critical parameter to obtain the default rate.

In the second column of Table 9, we present the percent change in each parameter that would allow the model to replicate a debt to GDP ratio 10 percent larger. In addition to the change in κ , which we already show is key to replicating the default rate, the most substantial change is in B^D followed by ω_1 . Clearly, these parameters are important to determine debt because they determine the benefits and costs of default. We pick B^D for debt because its adjustment is larger and highlights ω_1 for matching haircuts because it is the larger adjustment to match the haircut among the remaining parameters.

Continuing with the same logic, we connect each parameter in Table 9 with a moment.

B.3 Calibration

When we calibrated the models with aggregate shocks we only targeted debt over GDP, the default probability, and the debt haircut when defaulting. As shown in Table 10, other moments are all very close to the data, except for inflation that it is equal to 4.375% for the model with TFP shocks, while it is 3.8% in the data.

B.4 Computational procedure

The equilibrium is solved globally, using the equations derived above. The algorithm uses 21 equally spaced gridpoints for debt, between 0 and 1.5, and 21 gridpoints for p^T or TFP estimated with the Tauchen method with a bandwidth of 2 (i.e., a multiple 2 of the unconditional standard deviation). To compute expectations we interpolate policy functions with a modified Akima piecewise cubic Hermite interpolation in a dense grid of 20001 equally spaced points in debt and 501 equally spaced points in p^T or TFP for which we estimate its corresponding tran-

Table 10: Averages of simulated data in economies with aggregate shocks

| | Data | Shocks to p^T | | Shocks to TFP | |
|-------------------------|-------|-----------------|-------------|---------------|-------------|
| | | Benchmark | Low default | Benchmark | Low default |
| Real GDP | 1.000 | 0.996 | 0.998 | 0.992 | 0.995 |
| Inflation, % | 3.800 | 4.136 | 3.909 | 4.375 | 4.143 |
| Transfers / GDP | 0.117 | 0.118 | 0.117 | 0.118 | 0.118 |
| Exports / GDP | 0.209 | 0.208 | 0.207 | 0.209 | 0.209 |
| Employment / Population | 0.587 | 0.586 | 0.587 | 0.586 | 0.586 |
| Gov Consumption / GDP | 0.133 | 0.133 | 0.133 | 0.134 | 0.134 |
| Debt / GDP | 0.185 | 0.173 | 0.173 | 0.169 | 0.173 |
| Default probability | 0.020 | 0.021 | 0.008 | 0.020 | 0.008 |
| Haircut, Share of Debt | 0.305 | 0.257 | 0.257 | 0.230 | 0.257 |

sition matrix with the Tauchen method.³² We experimented with different grid sizes, different interpolations schemes (e.g., linear), and different ways of computing the expectations (e.g., computing its corresponding integral). The final choice of grid points and methods is the most efficient allocation of computing time. For example, either computing the integral, or increasing the size of the grids, deliver the same solution but require more computing time.

B.5 The choice of ρ , σ , θ_N , and θ_T

This section discusses the choice of $\sigma^N = \sigma^T = 0.5$, $\rho = 1.5$, $\theta_N = 1$, and $\theta_T = 0$ by comparing the results for alternative parameters. In particular, we consider three alternatives: (i) $\sigma^N = \sigma^T = 1.5$; (ii) $\rho = 2$; and (iii) $\theta_N = 0.95$ and $\theta_T = 0.40$. Recall that we set $\sigma^N = \sigma^T = 0.5$ because $\sigma^T < 1$ is sufficient for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers γ are zero). The value of ρ determines the elasticity of substitution between y^N and y^T in the cost function and is set to 1.5. A number larger than 1 ensures that the production possibilities frontier is concave. The cash-in-advance constraint in our benchmark model assumes money is only used to finance the purchases of non-tradable goods. In Section A.5 we presented a generalized version of this constraint. Using the notation from the general version, our benchmark case sets $\theta_N = 1$ and $\theta_T = 0$.

We re-calibrate the model with p^T shocks to ensure that each alternative calibration fits the targeted moments well. Next, we evaluate the non-targeted statistics in Tables 4, 6 and 8. We present all the moments in Table 11, where we added at the bottom the average for all moments of the absolute percentage distance between data and model. This last statistic is revealing of how better our preferred calibration fits these moments. This measure of the fit of non-targeted

³²The interpolated value at a query point is based on a piecewise function of polynomials with degree at most three evaluated using the values of neighboring grid points in each respective dimension. The Akima formula is modified to avoid overshoots.

moments in twice as large for the economy with $\rho = 2$ and a bit more in the economy with $\sigma = 1.5$. It is only slightly higher for the economy with $\theta_N = 0.95$ an $\theta_T = 0.40$.

We find that the economy with $\sigma = 1.5$ does the worse job a fitting data moments because it generates a positive correlation of trade balance/Y with output, a negative correlation of real expenditure with output, no correlation of real consumption and real output, and almost acyclical inflation. In the case of the economy with $\rho = 2$, the poorest fit is due to the fact that it generates a positive correlation of exports/Y with output and pro-cyclical tax rates.

Table 11: Business Cycles and Policy Statistics for Alternative Parameters (p^T model)

| | Data | Benchmark | $\rho = 2.0$ | $\sigma = 1.5$ | $\theta_N = 0.95$ $\theta_T = 0.40$ |
|---|--------|-----------|--------------|----------------|--|
| Std. Dev. (y) | 0.038 | 0.016 | 0.006 | 0.008 | 0.016 |
| Std. Dev. (trade balance/Y) | 0.035 | 0.017 | 0.008 | 0.007 | 0.018 |
| Std. Dev. (c) / Std. Dev. (y) | 1.193 | 2.478 | 3.980 | 1.772 | 2.290 |
| Std. Dev. (spreads) | 3.923 | 3.303 | 2.097 | 1.213 | 4.011 |
| Std. Dev. (exports/Y) | 0.052 | 0.021 | 0.014 | 0.012 | 0.023 |
| Std. Dev. (depreciaton) | 0.196 | 0.102 | 0.086 | 0.102 | 0.112 |
| Correlation(trade balance/Y, y) | -0.357 | -0.177 | -0.288 | 0.275 | 0.062 |
| Correlation(c,y) | 0.846 | 0.589 | 0.723 | -0.064 | 0.374 |
| Correlation(spreads,y) | -0.362 | -0.073 | -0.132 | 0.260 | 0.033 |
| Correlation(exports/Y,y) | -0.178 | -0.140 | 0.107 | 0.302 | -0.058 |
| Correlation(depreciation,y) | -0.252 | -0.226 | -0.316 | 0.278 | -0.006 |
| Correlation(depreciation,spreads) | 0.431 | 0.205 | 0.261 | 0.291 | 0.174 |
| Corr. (inflation tax, y) | -0.214 | -0.530 | -0.665 | 0.108 | -0.255 |
| Corr. (real expenditure, y) | 0.260 | 0.467 | 0.709 | -0.275 | 0.290 |
| Corr. (personal income tax, y) | -0.171 | -0.122 | 0.237 | 0.364 | -0.253 |
| Average of absolute percentage distance to data | - | 0.567 | 1.029 | 1.374 | 0.610 |

Note: Data is the average of the numbers Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cycle of GDP. The inflation tax is defined as in Vegh and Vuletin (2015): inflation/ (1 + inflation).

For the economy with $\theta_N = 0.95$ an $\theta_T = 0.40$, we find that the volatilities of the trade balance-to-GDP ratio, interest rate spreads and exports-to-GDP ratio are similar to the benchmark calibration. However, the alternative economy has counterfactual cyclicity of the trade balance, spreads and exports (i.e., the correlations of these variables with GDP have the opposite sign in the data).

B.6 The role of fiscal and monetary policy distortions

Consider a version of our model in which we remove the monetary policy distortion. Specifically, we modify the problem of the government by imposing that the non-negativity constraint (12) be satisfied with equality. This assumption eliminates the monetary policy distortion as it forces the government to always implement a slack cash-in-advance constraint (i.e., households are satiated with money balances). Analytically, this situation corresponds to the case $\zeta > 0$,

derived in Appendix A.3.

We computed the deterministic steady state of the economy without monetary policy distortions and compared it to our benchmark case. The results displayed in Table 12 show the effect of monetary policy distortions for some key macroeconomic variables. Notably, inflation would be 17 percentage points lower if we remove monetary distortions. Consequently, the tax rate would be significantly higher (12.3 percentage points) in the economy without monetary distortions. Other variables also change as a consequence of the change in the mix of monetary and fiscal financing of government expenditures. The clearest example is that real GDP would be more than 3 percent lower in the economy without monetary distortions. This is a consequence of relying too heavily on taxation to finance government expenditures, i.e., of implementing a suboptimal policy mix.

Table 12: Steady state statistics

| | Benchmark | No monetary distortions | Lump-sum taxes |
|------------------------|-----------|----------------------------|-------------------|
| Real GDP | 1.000 | 0.967 | 1.117 |
| Inflation, % | 3.800 | -13.300 | -13.300 |
| Revenue / GDP | 0.231 | 0.353 | 0.332 |
| Exports / GDP | 0.209 | 0.169 | 0.170 |
| Gov. Consumption / GDP | 0.133 | 0.135 | 0.129 |
| Debt / GDP | 0.185 | 0.174 | 0.164 |
| Haircut, share of debt | 0.305 | 0.304 | 0.358 |
| Default probability, % | 0.700 | 0.689 | 1.051 |

The last column in Table 12 presents steady state moments for the economy with lump-sum taxes. As we show in Section 4.4.2, this economy has no fiscal or monetary policy distortions. Inflation and tax revenue are similar to those in the economy with no monetary policy distortions. The absence of any policy distortions implies the economy is significantly larger: real GDP is about 12% higher than in the benchmark economy. Interestingly, the economy with lump-sum taxes displays the lowest debt-over-GDP ratio and the highest debt haircut and default probability.

B.7 Seigniorage in emerging countries

The most consistent reference for seigniorage in emerging markets is the IMF. Following Aisen and Veiga (2008), seigniorage is measured as the change in reserve money (line 14a IFS) over nominal GDP (line 99a IFS). The first column in the following table provides the values for the countries for which we calibrated the model, taken directly from Aisen and Veiga (2008). On average, the measure they consider for seigniorage is 4.1 percent for the countries in our sample. There are important differences across countries, ranging from 1.9% in Colombia to

Table 13: Seigniorage in Emerging Markets

| | Aisen and Veiga (2008) (original) | Aisen and Veiga (2008) (updated) | Kehoe et al. (2020) (original) |
|-----------|--------------------------------------|-------------------------------------|-----------------------------------|
| Argentina | 0.060 | 0.045 | 0.022 |
| Brazil | 0.036 | 0.033 | 0.019 |
| Chile | 0.069 | 0.029 | 0.021 |
| Colombia | 0.019 | 0.016 | 0.010 |
| Mexico | 0.022 | 0.018 | 0.017 |
| Peru | 0.034 | 0.032 | 0.021 |
| Uruguay | 0.049 | 0.029 | 0.026 |
| Average | 0.041 | 0.029 | 0.019 |

over 6.9% in Chile.

The second column in Table 13 presents our replication and extension of the measure provided by Aisen and Veiga (2008). We extended the sample period from their last observation, 1999, to the more recent observation for each country in the IFS data set. The average measure of seigniorage is now significantly lower, 2.9 percent, indicating that the use of seigniorage has declined over time for these countries. This is to be expected as these countries continue to develop and the quality/independence of their central banks improves. The last measure that we consider is from Kehoe, Nicolini, and Sargent (2020), whose average for the countries we consider is a bit lower than our updated number for Aisen and Veiga (2008).

C Data

C.1 Data Sources

This section lists the sources for all the variables used in the main body of the paper.

Variables in Table 2 and 3:

- “Inflation” is Inflation, consumer prices (annual %) from the World Bank. Indicator Code FP.CPI.TOTL.ZG.
- “Transfers/GDP” constructed as the product of two series from the World Bank. Subsidies and other transfers (% of expense) with indicator code GC.XPN.TRFT.ZS and Expense (% of GDP) with indicator code GC.XPN.TOTL.GD.ZS.
- “Exports/GDP” is Exports of goods and services (% of GDP) from the World Bank. Indicator code NE.EXP.GNFS.ZS.
- “Employment/Population” is Employment to population ratio, 15+, total (%) (modeled ILO estimate). Indicator code SL.EMP.TOTL.SP.ZS.
- “Gov. Consumption/GDP” is General government final consumption expenditure (% of GDP) from the World Bank. Indicator code NE.CON.GOVT.ZS.
- “Debt/GDP” is Public External Debt (%GDP) computed using the ratio of the following two variables from the World Bank. External debt stocks, public and publicly guaranteed (PPG) (DOD, current US\$) with indicator code DT.DOD.DPPG.CD and GDP (current US\$) with indicator code NY.GDP.MKTP.CD.
- “Haircut, Share of Debt” is the median “SZ haircut, HSZ” in Table 1 of Dvorkin et al. (2021).
- “Default rate” is obtained from Tomz and Wright (2013). They construct a database of 176 sovereign entities spanning 1820 to 2012. The frequency of default is sensitive to the sample being analyzed. They mention that their findings are “similar to the 2% default probability that is a target for many calibrated versions of the standard model,” which is the number we use as well. The unconditional probability of a country with positive debt (a borrower) defaulting on debts owed to commercial creditors is 1.7% per year. Nevertheless, this probability is higher in developing countries. Note also in Figure 2 of Tomz and Wright (2013) that in a typical year, there are no defaults or there is one country in default. We considered this fact when calibrating a significantly lower default rate in the model with only ε shocks.

The sources for variables used in Table 4 and 8 are:

- “Real GDP growth” is GDP per capita (constant LCU) from the World Bank. Indicator Code NY.GDP.PCAP.KN.
- “Trade balance” is Trade balance (% GDP) computed using two variables from the World Bank. Trade (% of GDP) with indicator code NE.TRD.GNFS.ZS and the variable Exports of goods and services (% of GDP) mentioned above.
- “Spreads” is the J.P. Morgan Emerging Markets Bond Spread (EMBI+) obtained from the World Bank. Indicator Id: EMBIG.

The additional sources for Table 8 is Vegh and Vuletin (2015). For taxes, we use the file they made available, “data_AEJEP.dta”, and using the variable “individual_tr,” we follow the same detrending procedure to make the correlation more comparable. For this variable, Vegh and Vuletin (2015) present in Figure 11 the correlation in growth rates; i.e., the correlation between the change in the personal income tax rate and GDP growth. If we follow that procedure, our results confirm the similarity of the model and the data—we obtain -0.2319 in the model and -0.1009 in the data.

C.2 Estimation of a stochastic process for terms of trade and productivity

We use data on terms of trade from ECLAC - CEPALSTAT, Economic Indicators and statistics, External sector. The index is called “terms of trade and purchasing power of exports”. We also use the time series of commodity prices used by Drechsel and Tenreyro (2017). Before estimating the autoregressive process, we take logs of the series and subtract the mean. Table 14 presents the results. The time period is 1980 to 2019. The coefficients ρ_p and σ_p are both similar for all the seven countries, so we use the average in our benchmark calibration. It is reassuring that the estimation results for the commodity price index presented at the bottom of Table 14 are also quite similar to the average. We did not de-trend the series before estimating the stochastic process so as to include long-duration cycles in the terms of trade (often referred to as “super-cycles”) in our quantitative exercises and keep the model and the data more comparable. We have also estimated these stochastic processes after de-trending the time series for terms of trade. The main difference is that the resulting value of ρ_p is smaller, which implies that shocks are less persistent.

For productivity we use the Penn World Table version 10.0, variable *rtfpna* (TFP at constant national prices). Before estimating the autoregressive process, we take logs of the series and subtract the linear trend. Table 14 presents the results. The time period is the same as for

terms of trade, 1980 to 2019. The coefficients ρ_{tfp} and σ_{tfp} are both similar for all the seven countries, so we use the average in our benchmark calibration.

Table 14: Estimation of process for shocks: p^T and tfp

| | Number of years | p^T | | TFP | |
|-----------------------|-----------------|--------------------|--------------------|--------------------|--------------------|
| | | ρ_p | σ_p | ρ_{tfp} | σ_{tfp} |
| Argentina | 40 | 0.9303 (0.0568) | 0.0608 (0.0064) | 0.7759 (0.1072) | 0.0433 (0.0062) |
| Brazil | 40 | 0.8746 (0.0760) | 0.0657 (0.0072) | 0.8158 (0.0575) | 0.0270 (0.0027) |
| Colombia | 40 | 0.9187 (0.0585) | 0.0847 (0.0095) | 0.8796 (0.0905) | 0.0139 (0.0015) |
| Mexico | 40 | 0.8216 (0.1339) | 0.0702 (0.0036) | 0.8765 (0.0556) | 0.0251 (0.0023) |
| Chile | 40 | 0.9139 (0.0871) | 0.1021 (0.0106) | 0.9066 (0.0906) | 0.0289 (0.0024) |
| Peru | 40 | 0.9329 (0.0729) | 0.0733 (0.0063) | 0.9171 (0.0533) | 0.0444 (0.0040) |
| Uruguay | 40 | 0.7706 (0.0926) | 0.0717 (0.0072) | 0.8712 (0.0432) | 0.0369 (0.0031) |
| Average | | 0.8804 | 0.0755 | 0.8632 | 0.0314 |
| Commodity price index | 36 | 0.8757 (0.0965) | 0.0910 (0.0134) | | |

Note: Standard errors in parenthesis.

C.3 Event Study Selection

We use quarterly data on EMBI+ Sovereign Spread data from Bloomberg to select the episodes of debt crisis. The series are in basis points and cover 13 emerging market countries and extend from 1997Q1 to 2021Q4, depending on the country. The countries are Argentina, Brazil, Colombia, Croatia, Hungary, Indonesia, Mexico, Peru, Philippines, Russia, South Africa, Turkey, and Ukraine.

Following Calvo et al. (2006), episodes are initially flagged if the EMBI+ spread is larger than the sample mean plus 2 standard deviations. The sample mean and standard deviation are calculated without spread observations above the 95th percentile to avoid increasing these values with extreme observations. Episodes are dropped if the EMBI spread never reaches 500 basis points or if the peak is small relative to previous country events (e.g. Ukraine in 2020). Additionally, episodes are dropped if the spread is within 100 basis points in the prior year without continual increases (up-down scenario). This is done to limit volatile events. For example, assume 2007Q3 is flagged as an event start for a country with a spread of 750 basis points. If the spread in 2007Q1 is 700 basis points and then drops to 600 basis points in 2007Q2 and then increases to 750 basis points in 2007Q3 then we remove the flag. Given the flag is

time $t=0$, this procedure is applied to $t=-4$, $t=-3$, and $t=-2$, i.e., the quarters in the year prior to the event flag, except the one directly before. We do not include the quarter directly before the flag in question ($t=-1$) since it cannot be determined if the spread declined between the two values. To these episodes we added others for which four conditions are satisfied: the spreads increase to over 500 basis points, there are no “up-down” patterns, the peak is not small relative to previous country events, and there is a known event that occurred (the 2007-09 GFC shock). This adds events in 2008 for Peru, Phillipines, and Russia. Once we have an episode, quarters leading to that episode are flagged as part of the event if spreads are significantly increasing at that point. This is measured by if the change in spread from previous quarter is greater than 90 basis points or the year-over-year growth rate is greater than 75 basis points. This is done to better capture the beginnings of some crises. If there are only 1 to 3 unflagged quarters between flagged quarters for a country, we count those two flags as the same event. After we have the episodes selected, the event start ($t=0$) is marked as the first quarter flagged. For example, if an episode has flags from 2005Q1 to 2007Q2 then $t=0$ would be 2005Q1.

The inflation series are end-of-quarter consumer price index values retrieved from either IMF International Financial Statistics (IFS) or the country’s statistical agency via Haver Analytics, depending on availability. The indices are used to calculate year-over-year percent inflation. Croatia, Mexico, and Ukraine inflation data are from IFS. Argentina inflation data are from Cavallo and Bertolotto (2016). All others are from the Emerge database in Haver Analytics. The measure of inflation is truncated at +/- 50 percentage points to reduce the weight of extreme events.

For the table, we presented the difference in inflation (and depreciation) between one period before the shock ($t=-1$) and the period of the shock ($t=0$). Since the model is yearly and the data is quarterly, we take end-of-period inflation values over 4 quarters in the data. We take the 4 quarters preceding the event to be period -1, so the end-of period value is inflation in the quarter prior to the event start. Likewise, we take the 4 quarters including and directly after the event as period 0, so the end-of-period inflation is the value in the third quarter after the event. The same process is implemented to obtain end-of-period depreciation.

For the model, we need to adjust the criteria that the spread must be at least 500 basis points because the period is year. We take the average spread in selected events in the data from the quarter of the event ($t=0$) to the third quarter after the event ($t=3$). The minimum of these averages is 379 basis points, which we use for the model’s lower threshold for event selection.

C.4 Local projections

We consider four alternative left-hand-side variables: inflation, currency depreciation, EMBI spreads, and GDP growth (i.e., $\ln(GDP_t) - \ln(GDP_{t-1})$). We refer to these variables as y_t^i , where i refers to the country and t to the year. The right-hand-side variable of interest is the log(terms of trade), or productivity, and we refer to this variable as lp_t^i .

The difference of a variable δ periods ahead with the same variable one period ago is $\Delta y_{t+\delta,t-1}^i = y_{t+\delta}^i - y_{t-1}^i$. The panel regression we run to obtain the response to terms of trade shocks is

$$\Delta y_{t+\delta,t-1}^i = \alpha^\delta + \beta^\delta \Delta lp_{t,t-1}^i + \text{controls}.$$

We run this regression 32 times: for each of the four alternative left-hand-side variables, for each of the two alternative right-hand-side variables, and for $\delta = \{0, 1, 2, 3\}$. The controls consist of two lags of $\Delta y_{t+\delta,t-1}^i$, two lags of $\Delta lp_{t,t-1}^i$, and country fixed effects.

In Figure 5 and 6, we plot the coefficients β^δ multiplied by -10 to represent a 10 percent decline in the terms of trade or productivity. The standard errors showed by the shaded area in the figure are robust standard errors.

The time period for the regressions is 1980 to 2019 or the latest available observation. The most important exception is the regression for the EMBI spread, which starts in 1997 due to data availability of this variable.

We also conduct this comparison using contemporaneous regressions between these four variables and the terms of trade. The estimated semi-elasticities, similar to those in Drechsel and Tenreyro (2017), are quite similar in the model and the data and resemble the effect at time zero in the analysis presented here.