

Automation with Heterogeneous Agents: the Effect on Consumption Inequality

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Introduction

Many occupations at high risk of automation in near future

(Frey and Osborne 2017, Arntz et al 2016, Brynjolfsson and McAfee 2014)

Importance of new tasks creation (Acemoglu and Restrepo (2018a))

- **Automation** = displacement effect + productivity effect
- **New tasks** = reinstatement effect + productivity effect

Heterogeneity of the Effects

Two dimensions: **skills** and level of **capital** owned.

Relative skill demand:

- Tasks performed by low-skills workers more likely to be automated (e.g. Acemoglu and Autor 2011)
- Creation of new tasks increases demand for high-skill workers more than for low-skill workers (evidence ➡)

Return of Capital:

- Automation increases capital demand and the return of capital
⇒ benefits *more* who owns more capital
- New tasks also change the return of capital by making production more labor intensive

This Paper

Relationship between automation and consumption inequality in a general equilibrium model

Two theoretical frameworks,

Aiyagari incomplete market model
with educational choice

Task-based model

Main features

- Endogenous heterogeneous capital accumulation
 - Increase in return to capital amplifies effect on inequality
(More 1 ➡, More 2 ➡)
- Educational choice
 - Buffers increase in inequality
- Compute transitional dynamics \implies short/long-run effects of automation

Literature (skip)

1. Literature on the determinants of inequality

- Heckman et al. (1998); Hubmer, Krusell, and Smith Jr (2016); Kaymak and Poschke (2016)
- Moll et al. (2019)

2. Literature on the effects of automation

- Acemoglu and Restrepo (2018a); Hemous and Olsen (2018)
- Sachs and Kotlikoff (2012); Sachs et al. (2015)
- Acemoglu and Restrepo (2019); Dauth, Findeisen, Suedekum, and Woessner (2021)

Model

Households' side

- Ayiagari incomplete market model + educational choice: low skill/high skill
- An agent is born with a level of capital and chooses education.
Permanent decision
- During life, (1) consumption/saving decisions, (2) cannot borrow, (3) exogenous labor supply
- Dying probability, d , in every period \rightarrow offspring of the agent inherits capital the agent had in last period
- Agents face skill-specific not insurable idiosyncratic shock
- Different shocks' histories \implies heterogeneity within skill type

Households' problem

New born agent

$$v_t^0(k) = \max \left\{ \mathbb{E}_\varepsilon \left\{ v_t^h(k, \varepsilon^h) \right\} - \theta(k), \quad \mathbb{E}_\varepsilon \left\{ v_t^\ell(k, \varepsilon^\ell) \right\} \right\}$$

- $\theta(k)$: disutility cost of going to college

Households' problem

New born agent

$$v_t^0(k) = \max \left\{ \mathbb{E}_\varepsilon \left\{ v_t^h(k, \varepsilon^h) \right\} - \theta(k), \quad \mathbb{E}_\varepsilon \left\{ v_t^\ell(k, \varepsilon^\ell) \right\} \right\}$$

- $\theta(k)$: disutility cost of going to college

Agent i skill-type j

$$v_t^j(k_{i,t}, \varepsilon_{i,t}^j) = \max_{c_t, k_{t+1}} \left\{ u(c_{i,t}) + \beta(1-d) \sum_{\varepsilon_{t+1}^j} \pi(\varepsilon_{t+1}^j | \varepsilon_{i,t}^j) v_{t+1}^j(k_{i,t+1}, \varepsilon_{i,t+1}^j) \right\}$$

$$\text{s.t. } c_{i,t} + k_{i,t+1} = (1 + r_t - \delta)k_{i,t} + w_t^j \cdot \varepsilon_{i,t}^j, \quad \text{and } k_t > 0$$

- d : probability of dying.
- $w_t^j = w_t^h$ with education, $w_t^j = w_t^\ell$ without education
- $\varepsilon_{i,t}^j$: labor endowment shock, $\varepsilon_{i,t}^j \sim$ Markov process

Production Side

Unique final good produced with a continuum of tasks:

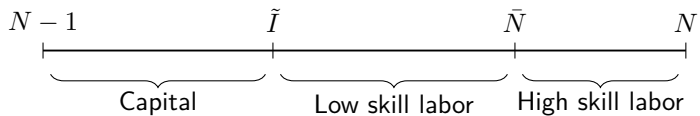
$$\ln Y = \int_{N-1}^N \ln[y(x)] dx$$

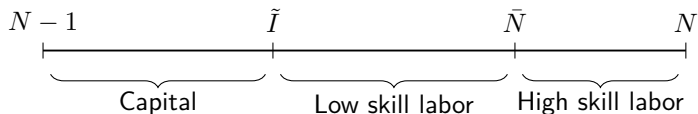
$$y(x) = \gamma_m(x)m(x) + \gamma_\ell(x)l(x) + \gamma_h(x)h(x)$$

Where:

- $m(x)$ amount of capital used to produce task x
- $\gamma_m(x)$ productivity of capital in task x

Comparative advantage structure + factor prices \implies allocation of factors to tasks (More \Rightarrow)

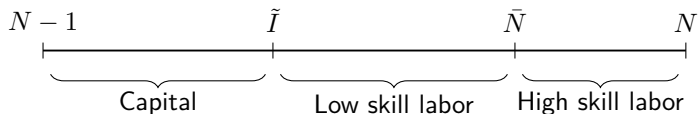




Automation:

- \tilde{I} : highest-indexed task that is *optimal* to produce with machines
- I : highest-indexed task that is *feasible* to produce with machines
- Highest-indexed task automated in equilibrium is,

$$I^* = \min\{I, \tilde{I}\}$$

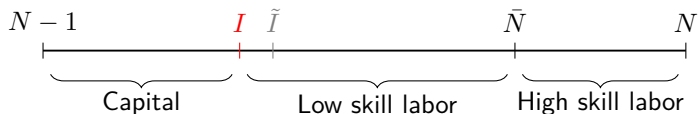


Automation:

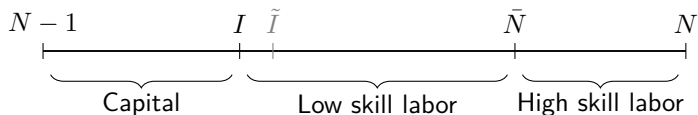
- \tilde{I} : highest-indexed task that is *optimal* to produce with machines
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$$I^* = \min\{I, \tilde{I}\}$$

Automation shock: increase in I when constraint is binding



Prod. Fun. ➡ Diagram ➡



$$Y = G \left(\frac{K}{I - N + 1} \right)^{I - N + 1} \left(\frac{L}{\bar{N} - I} \right)^{\bar{N} - I} \left(\frac{H}{N - \bar{N}} \right)^{N - \bar{N}}.$$

$$r = Y \cdot \frac{I - N + 1}{K}.$$

$$w_\ell = Y \cdot \frac{\bar{N} - I}{L}.$$

$$w_h = Y \cdot \frac{N - \bar{N}}{H}.$$

Equilibrium 1/2

- Given $\{I_t\}_{t=0}^{\infty}$ and $\{N_t\}_{t=0}^{\infty}$
- A RCE are sequences of value functions $\{v_t^h\}_{t=0}^{\infty}$ $\{v_t^l\}_{t=0}^{\infty}$ and $\{v_t^0\}_{t=0}^{\infty}$
- policy functions $\{c_t^h, k_{t+1}^h\}_{t=0}^{\infty}$ and $\{c_t^l, k_{t+1}^l\}_{t=0}^{\infty}$
- firm's choices $\{L_t, H_t, K_t\}_{t=0}^{\infty}$
- prices $\{w_t^l, w_t^h, r_t\}_{t=0}^{\infty}$
- distributions $\{\lambda_t\}_{t=0}^{\infty}$

such that, for all $t \dots$

Equilibrium 2/2

- Given prices, the policy functions solve the agents' problems and the associated value functions are $\{v_t^h\}_{t=0}^\infty$, $\{v_t^\ell\}_{t=0}^\infty$ and $\{v_t^0\}_{t=0}^\infty$
- Given prices and $\{I_t\}_{t=0}^\infty$ and $\{N_t\}_{t=0}^\infty$, the firm chooses optimally labor inputs and capital
- Labor markets clear,

$$H_t = \left[(\Pi_*^h)^T \cdot \varepsilon^h \right] \times S_t^h$$
$$L_t = \left[(\Pi_*^\ell)^T \cdot \varepsilon^\ell \right] \times (1 - S_t^h)$$

- Capital market clears,

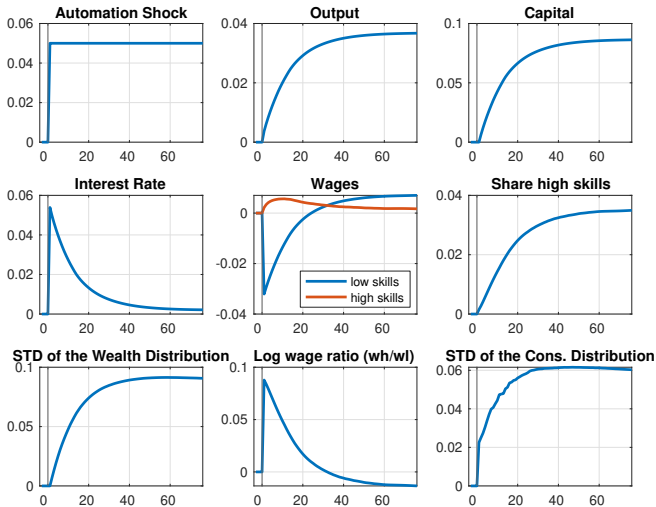
$$K_t = \int_{A \times E} k_{t+1}(k_t, \varepsilon_t) d\lambda_t.$$

Quantitative Analysis - roadmap

1. Calibration → United States, 1978-1981 (Go ➡)
2. **Discussion of Mechanisms**
3. Estimation of the sequences $\{I_t, N_t\}$ (Go ➡)
4. Transition with estimated sequences: (Go ➡)
 - One MIT shock
 - Comparison with data
5. Decomposition exercise:
 - The role of automation and new tasks in the increase in inequality
 - The role of the return to wealth and the role of education choice

Transitional dynamics ΔI

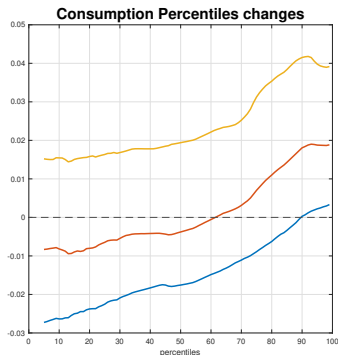
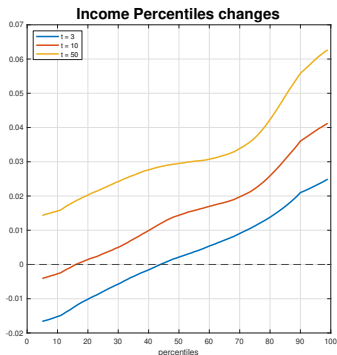
Initial steady state = 0



Interpret with equations ➡ Return to wealth ➡ New tasks ➡

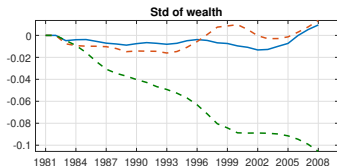
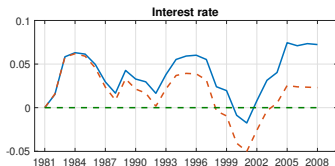
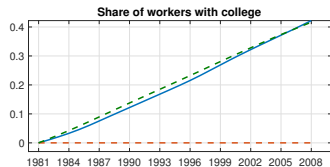
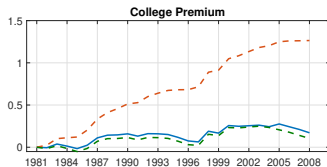
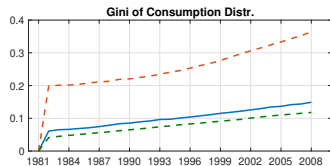
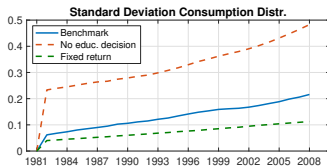
Transitional dynamics ΔI , inequalities

Initial steady state = 0



- Percentiles' change after shock
- Each line indicates a different period

Decomposition, the role of the return of wealth and of education decision



Conclusion

Effect of **automation on consumption inequality**.

Unified framework to account for two main channels,

- Complementarity and substitutability \implies wage inequality
- Return of wealth \implies capital income inequality

Automation shock

- Decreases the labor income of uneducated workers in short-run
- Increase in the return to wealth counteracts drop for the uneducated rich
- Some agents lose in short-run, everybody better off in long-run but increased polarization

Quantitatively important mechanisms

- Return to wealth
- Educational choice

Thank you!

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New tasks

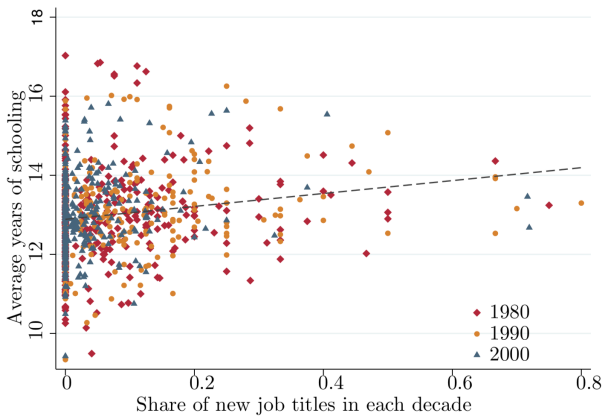


Figure: From A&R 2018a

$$\Delta Y_{it} = \beta N_{it} + \delta_t + \Gamma_t X_{it} + \varepsilon_{it} \quad (1)$$

	Dep. var: Change in average years of schooling.					
	Over 10 years		Over 20 years		Over 30 years	
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Change in occupations with new job titles.						
Share of new job titles at the start of decade	-0.085 (0.123)	-0.103 (0.118)	-0.219 (0.179)	-0.203 (0.173)	-0.452** (0.215)	-0.411** (0.176)
R-squared	0.32	0.53	0.12	0.33	0.02	0.29
Observations	989	989	659	659	329	329
Occupations	330	330	330	330	329	329
Panel B: Change in occupations with more educated workers.						
Average years of education at the start of decade	-0.030*** (0.006)	-0.077*** (0.011)	-0.028** (0.013)	-0.102*** (0.017)	-0.083*** (0.016)	-0.149*** (0.021)
R-squared	0.36	0.59	0.14	0.41	0.17	0.43
Observations	990	990	660	660	330	330
Occupations	330	330	330	330	330	330
<i>Covariates:</i>						
Decade fixed effects	✓	✓	✓	✓	✓	✓
Demographics × decade effects		✓		✓		✓

Figure: From A&R 2018a



Figure: From A&R 2018a

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Median value of net worth for families with holdings By education of head

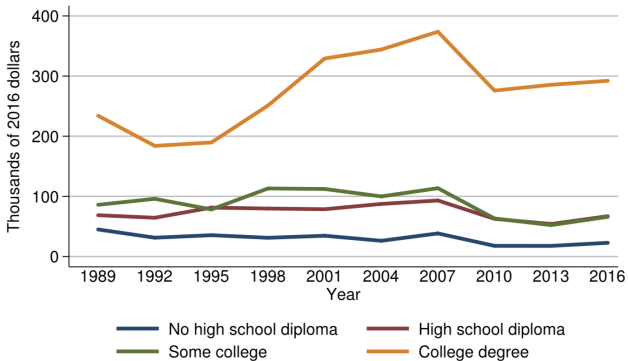


Figure: Survey of Consumer Finances.

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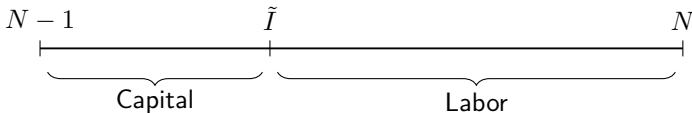
Division between capital and labor

Assumption 1:

$$\frac{d}{dx} \left(\frac{\gamma_\ell(x)}{\gamma_m(x)} \right) > 0 \quad \text{and} \quad \frac{d}{dx} \left(\frac{\gamma_h(x)}{\gamma_m(x)} \right) > 0.$$

Labor has a comparative advantage in higher indexed tasks.

A1 + perfect substitutability of factors in each task \implies range of tasks divided in two areas.



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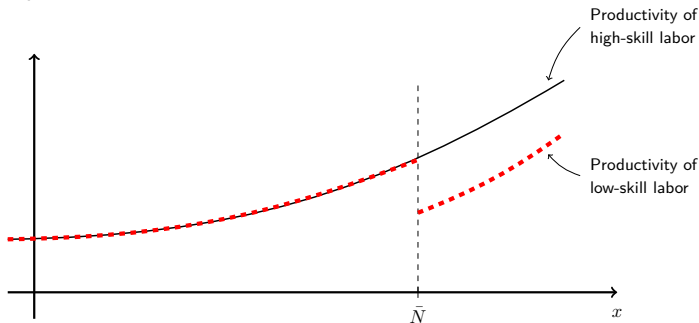
Division between labor inputs

Assumption 2:

$$\gamma_l(x) = \begin{cases} \gamma_h(x) & x \leq \bar{N} \\ \gamma_h(x) \cdot \Gamma & x > \bar{N} \end{cases}$$

Where $\Gamma \in [0, 1]$.

An example:



High skill labor has an advantage in higher indexed tasks with respect to low skills.

Division between labor inputs

Assumption 2:

$$\gamma_l(x) = \begin{cases} \gamma_h(x) & x \leq \bar{N} \\ \gamma_h(x) \cdot \Gamma & x > \bar{N} \end{cases}$$

Where $\Gamma < 1$.

Define \tilde{x} as the threshold that divides tasks produced with low skills and tasks produced with high skills.

Assumption 3:

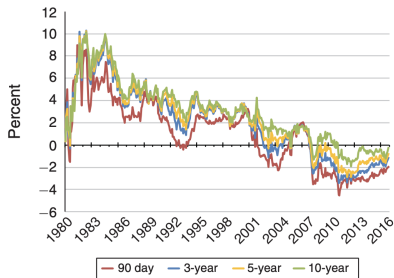
Restrict the attention to the case in which $\tilde{x} = \bar{N}$. This is true when,

$$\frac{w_l}{w_h} > \Gamma.$$

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Proof Intuition ➡

Panel A. Real return US treasuries



Panel B. Real return to US capital

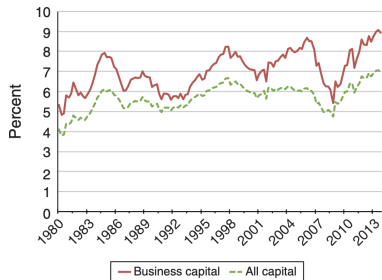


Figure: From Moll et al. 2019

Treasury rates \downarrow but return to entire US capital stock \uparrow .

(Caballero-Farhi-Gourinchas, 2017; Gomme-Ravikumar-Rupert, 2011)

Back to intro \blackrightarrow

Back to transition \blackrightarrow

College Premium

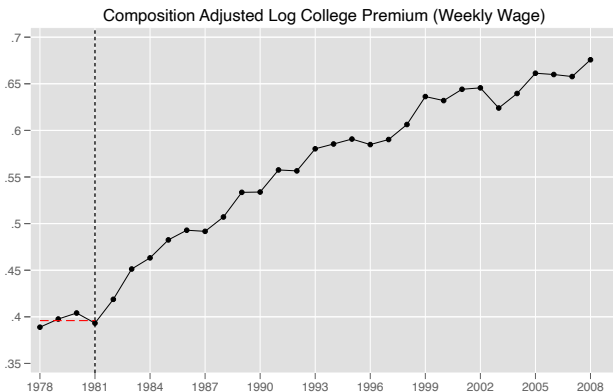


Figure: As in Acemoglu and Autor 2011

CP not affected by changes in experience, gender composition or average level of schooling.

Moment Condition

Issue: the plotted statistics is the *difference between the average log wages* for the two categories, \widehat{CP} .

In the model, w_ℓ is the average wage by construction and the average of log wages is,

$$\begin{aligned} \pi_1^{*,\ell} \log(w_\ell \varepsilon_1^\ell) + \pi_2^{*,\ell} \log(w_\ell \varepsilon_2^\ell) + \pi_3^{*,\ell} \log(w_\ell \varepsilon_3^\ell) &= \\ \log(w_\ell) \underbrace{(\pi_1^* + \pi_2^* + \pi_3^*)}_1 + (\Pi^{*,\ell})^T \log(\boldsymbol{\varepsilon}^\ell) &= \\ \log(w_\ell) + (\Pi^{*,\ell})^T \log(\boldsymbol{\varepsilon}^\ell). \end{aligned}$$

Hence, the moment condition is

$$\overline{\log(w_h)} - \overline{\log(w_\ell)} = \log(w_h) - \log(w_\ell) + (\Pi^{*,h})^T \log(\boldsymbol{\varepsilon}^h) - (\Pi^{*,\ell})^T \log(\boldsymbol{\varepsilon}^\ell) = \widehat{CP}$$

Which, given the expression of the wage ratio is

$$\log\left(\frac{N - \bar{N}}{\bar{N} - I} \cdot \frac{(1 - S_h)}{S_h}\right) + (\Pi^{*,h})^T \log(\boldsymbol{\varepsilon}^h) - (\Pi^{*,\ell})^T \log(\boldsymbol{\varepsilon}^\ell) = \widehat{CP}$$

Calibration Procedure

The expression of the wage ratio is

$$\frac{w_h}{w_\ell} = \frac{N - \bar{N}}{\bar{N} - I} \cdot \frac{L}{H} = \frac{N - \bar{N}}{\bar{N} - I} \frac{1 - S_h}{S_h}$$

Which implies

$$\bar{N} = \frac{N + I \frac{w_h}{w_\ell} \frac{S_h}{1 - S_h}}{1 + \frac{w_h}{w_\ell} \frac{S_h}{1 - S_h}}$$

I compute \bar{N} with this expression. I plug the value of the wage ratio and the share of college/non-college that I see in the data. Given \bar{N} I compute m by solving

$$\hat{z} = \frac{\exp(mN) - \exp(m\bar{N})}{\exp(m\bar{N}) - \exp(mI)} \cdot \frac{\bar{N} - I}{N - \bar{N}}$$

This leaves me with $(q_y, \tilde{\gamma}, \tilde{\theta})$ that I calibrate with **SMM** by matching (Cost Saving of automation, K/Y , S_h).

Aggregate Productivity G

$$G = \exp \left(\int_{N-1}^{\bar{I}} \ln(\gamma_m) dx + \int_{\bar{I}}^{\bar{N}} \ln(\gamma_\ell(x)) dx + \int_{\bar{N}}^N \ln(\gamma_h(x)) dx \right)$$

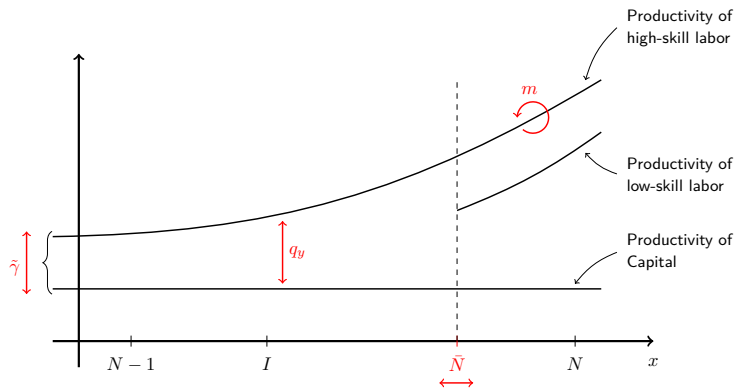
Back ➡

Productivity schedules

$$\gamma_h(x) = \tilde{\gamma} \cdot q_y \cdot e^{m(x - \frac{I+N}{2})},$$

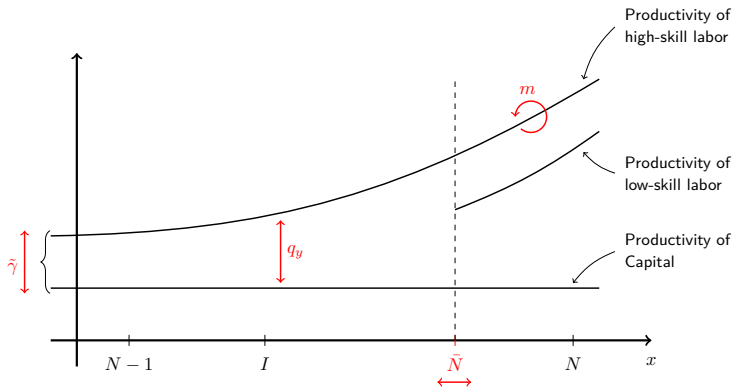
$$\gamma_l(x) = \begin{cases} \gamma_h(x) & x \leq \bar{N} \\ \gamma_h(x) \cdot \Gamma & x > \bar{N} \end{cases}$$

$$\gamma_m(x) = \tilde{\gamma}$$



Productivity schedules

- \bar{N} → composition adjusted $\widehat{CP} \left(\frac{w_h}{w_\ell} = \frac{N-\bar{N}}{\bar{N}-I} \cdot \frac{(1-S_h)}{S_h} \right)$, (More ➡)
- m → productivity ratio college/non-college (Hellertein et al. 1999)
- $\tilde{\gamma}$ → capital output ratio
- q_y → cost saving of automation at the margin $\left(\frac{w_\ell}{\gamma_\ell(I)} / \frac{r}{\gamma_m} \right)$



More ➡

Back ➡

DESCRIPTION	VALUE	TARGET/SOURCE
<i>PREFERENCES</i>		
σ Risk Aversion	2	Standard
β Discount	0.95	Standard
δ Depreciation	6%	Standard
d Death probability	3%	33 years average working life
$\tilde{\theta}$ Education Cost	15.04	Share of workers with col. degree
<i>TECHNOLOGY</i>		
N Highest-indexed task	1	Normalization
I Highest-indexed automated task	0.35	Labor share = 0.66
$\tilde{\gamma}$ Productivity	0.12	$K/Y = 3$
q_y Productivity of labor 1	0.7	Cost saving = 30%
m Productivity of labor 2	1.66	$\hat{z} = 1.67$
\bar{N} Highest-indexed task non-college	0.84	Log college premium = 0.43

Table: Calibrated parameters of the model. 1978-1981, US economy.

The Markov transition matrices are 9×9 . The parameters of the labor income risk are calibrated using estimates from Guvenen (2009).

Targeted moments

Expressions		Data	Model
$\log(w_h/w_\ell)$	log wage ratio	0.43	0.43
\hat{z}	prod. ratio	1.67	1.67
K/Y	capital/output	3	2.46
$\frac{w_\ell}{\gamma_\ell(I)} / \frac{r}{\gamma_m}$	cost saving autom.	30%	30%
S_h	college share	14%	14%

Untargeted moments

Gini coefficients	Data	Model
consumption	0.24	0.15
wealth	0.77	0.44

Sources: Aguiar and Bils 2015, Kuhn et al. 2018

Interpretation of the results, 1

Interest rate, capital and output

Exogenous variation: invention and adoption of automation technology

$\uparrow I$

$$r = Y \cdot \frac{\uparrow I - N + 1}{K}.$$

$\implies \uparrow r$ as the reaction of capital is sluggish.

As agents start accumulating more capital, the interest rate goes back down.

Output increase as capital increases.

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Interpretation of the results, 2

Workers type shares and college premium

$$\frac{w_h}{w_\ell} = \frac{N - \bar{N}}{\bar{N} - \uparrow I} \cdot \frac{(1 - S_h)}{S_h}$$

New born worker problem

$$v_t^0(k) = \max \left\{ \uparrow \mathbb{E}_\varepsilon \{v_t^h(k, \varepsilon^h)\} - \theta(k), \quad \mathbb{E}_\varepsilon \{v_t^\ell(k, \varepsilon^\ell)\} \right\}$$

$\theta(k)$: disutility cost of going to college.

The share of high skill agents increases for two reasons.

1. The college premium increases.
2. The per capita capital increases \implies new born workers are, on average, richer.

Point 2 explains why in the final steady state, the share of high skill workers is higher despite the college premium being lower.

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Interpretation of the results, 3

Wealth and consumption inequality

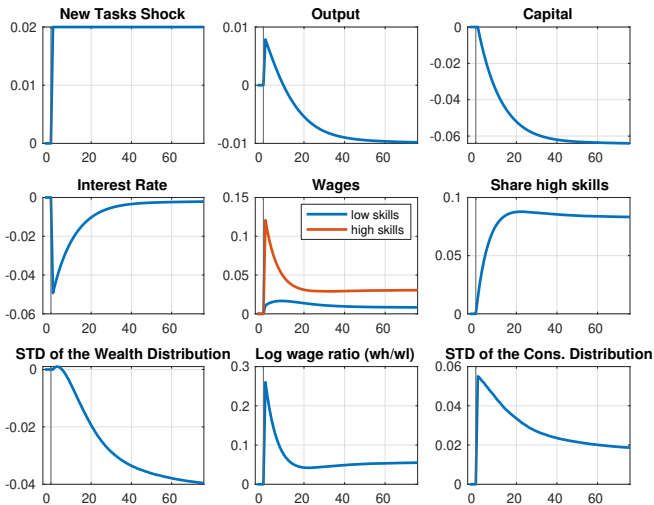
$\uparrow r \implies$ increase in the s.d. of the wealth distribution ($\uparrow SD_{\text{wealth}}$).

$\uparrow SD_{\text{wealth}} \implies$ increase in the s.d. of the consumption distribution ($\uparrow SD_{\text{cons}}$).

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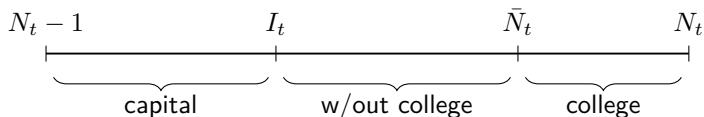
Transitional dynamics ΔN

Initial steady state = 0

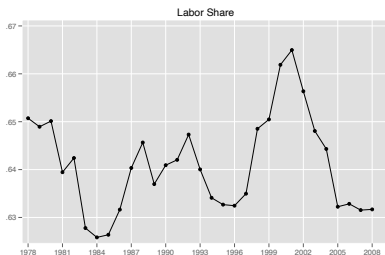


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Estimation of I and N



Use the series of the labor share (BEA).



- Assume that increase in I and N are not contemporaneous.
- Use the model expression $N_t - I_t = \text{LABOR SHARE}_t$ (More \Rightarrow). If LS increases, N increases, if LS decreases I increases.

Estimation of I and N

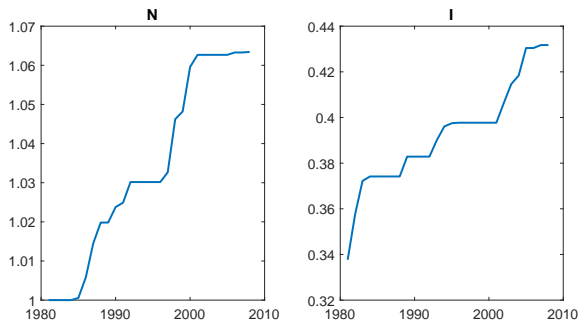
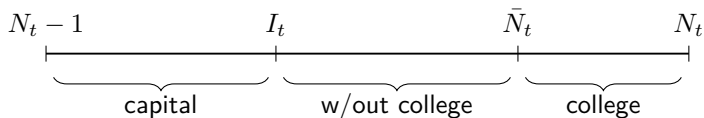


Figure: Estimated I_t and N_t

Labor Share

Expression

$$LS = \frac{w_h H + w_\ell L}{Y}$$

$$LS = \frac{Y(N - \bar{N}) + Y(\bar{N} - I)}{Y} = N - I$$

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Transition with estimated sequences

Implementation:

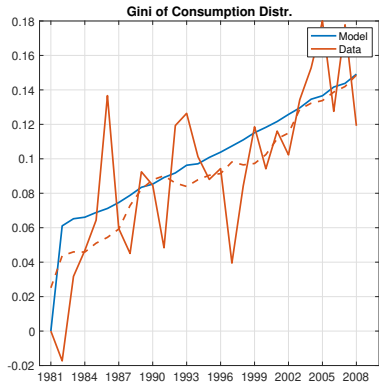
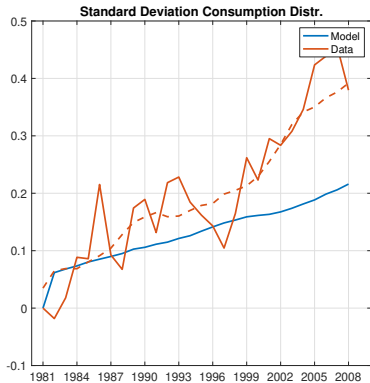
1. Initial calibrated steady-state
2. Estimated sequences $\{I_t, N_t\}$, 28 years
3. Extend the estimated sequences with linear trends for 50 years
4. After the linear increase, I and N remain constant to allow steady-state convergence

Data sources

- Consumption data → Consumer Expenditure Survey, measure constructed by Aguiar and Bils (2015)
- Composition adjusted college premium and share of workers with college degree → March CPS

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Comparison with data



Comparison with data

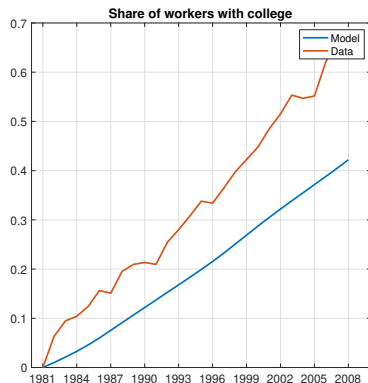
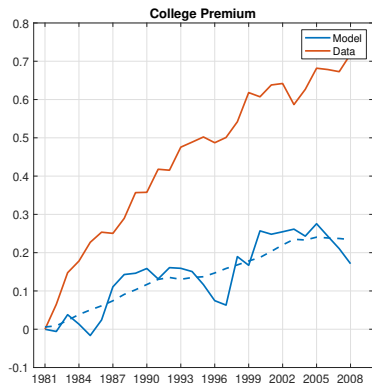
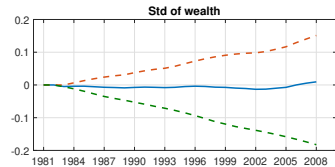
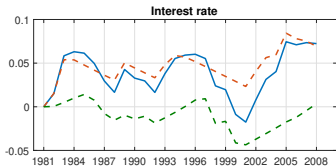
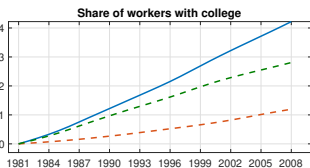
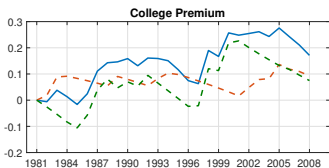
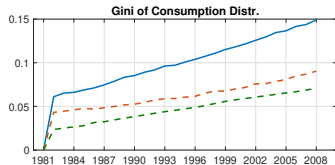
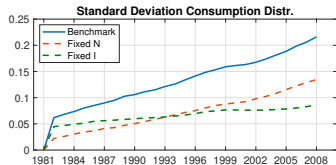


Figure: $\left(\frac{w_h}{w_\ell} \propto \frac{N - \bar{N}}{\bar{N} - I} \cdot \frac{(1 - S_h)}{S_h} \right)$

Decomposition, the role of task automation and new tasks introduction



$$\ln Y = \int_{N-1}^N \ln[y(x)] dx$$

$$y(x) = \begin{cases} \gamma_k(x)k(x) + \gamma_\ell(x)l(x) + \gamma_h(x)h(x) & \text{if } x \in [0, I] \\ \gamma_\ell(x)l(x) + \gamma_h(x)h(x) & \text{if } x \in (I, N] \end{cases}$$

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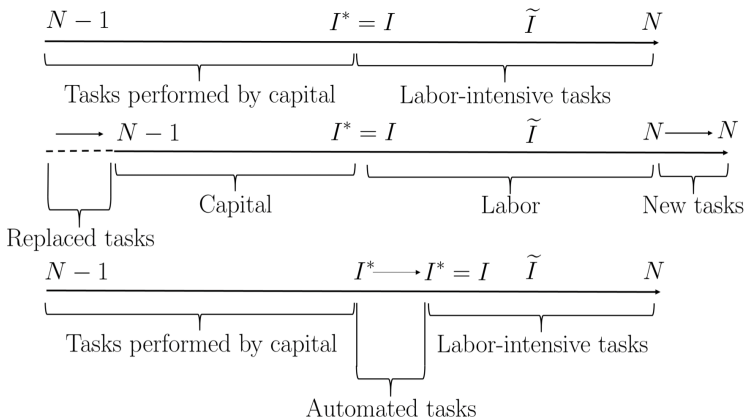


Figure: From A&R 2018a

The following condition on wages,

$$\frac{w_\ell}{w_h} > \Gamma.$$

Implies,

$$\begin{cases} \frac{w_\ell}{\gamma_\ell(x)} < \frac{w_h}{\gamma_h(x)} & x \leq \bar{N} \\ \frac{w_\ell}{\gamma_\ell(x)} > \frac{w_h}{\gamma_h(x)} & x > \bar{N}. \end{cases}$$

Indeed,

$$\gamma_l(x) = \begin{cases} \gamma_h(x) & x \leq \bar{N} \\ \gamma_h(x) \cdot \Gamma & x > \bar{N}. \end{cases}$$

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