# Cournot Equilibrium and Welfare with Heterogeneous Firms

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# I. Introduction

**General Framework**

- Consider an industry with $N$ firms producing an homogeneous good and competing à la Cournot
- Firms are heterogeneous in both fixed and variable costs
  - Heterogeneity is unobserved by the econometrician, but known by the firm
  - Heterogeneity in the fixed costs cannot generate heterogeneous firm size
  - Heterogeneity in the variable cost function is unable to explain why so many small firms make positive profits

**Contributions of the paper**

- (Re-)State theoretical results applying to Cournot equilibrium with heterogeneous firms:
  - existence and unicity
  - highlight the role played by firm size
- Develop a general but tractable empirical model that can
  - reproduce the observed distribution of firm sizes
  - identify the distribution of firms' fixed and variable costs
  - characterize technologies which allow firms to survive, to grow or force them to exit
  - identify firms which contribute to increase efficiency in the economy

**Related literature**

- Theoretical literature:
  - **Short-run:** Novshek (1985), Gaudet and Salant (1991), Amir (1996), Salant and Shaffer (1999)
  - **Long-run:** Mankiw and Whinston (1986), Acemoglu and Jensen (2013), Amir et al. (2014), Okumura (2015)
- Empirical literature:
  Hsieh and Klenow (2009), Koebel and Laisney (2016), Chen and Koebel (2017), Wooldridge (2019), Baqaee and Farhi (2020), De Loecker (2020), Peters (2020), etc.

**Some stylized facts for France**

Table: Number of active firms and employment by firm size, manufacturing, France and Germany, 2017

| | | | Firm size | | | |
|---|---|---|---|---|---|---|
| | | Total | 0-9 | 10-49 | 50-249 | >250 |
| France | No. of Firms | 193,609 | 162,955 | 23,468 | 5,658 | 1,522 |
| | No. of Employees | 2,832,458 | 259,459 | 488,990 | 601,247 | 1,482,624 |
| Germany | No. of Firms | 234,310 | 170,585 | 43,540 | 15,845 | 4,340 |
| | No. of Employees | 7,040,463 | 336,753 | 939,166 | 1,701,813 | 4,062,731 |

How to explain these gaps?

- Garicano (2016, AER): hampered firm growth as labor laws start to bind on firms with 50 or more employees
- We: imperfect competition and distribution of firms' fixed and variable cost efficiency

- On a given market, goods are homogeneous
- The inverse demand function to the market:

$$p = P(y_n + \sum_{j \neq n}^{N} y_j), \tag{1}$$

  - $p$ denotes the output price level
  - $y_n$ the production of firm $n$
  - $Y_{-n} \equiv \sum_{j \neq n}^{N} y_j$ the total output of firms' n competitors

- Firms are characterized by heterogeneous cost functions

$$c_n(w_n, y_n) = u_n(w_n) + v_{1n}(w_n)y_n + \frac{1}{2}v_{2n}(w_n)y_n^2 \qquad (2)$$
$$= \underbrace{\gamma_n^u u(w_n)}_{\text{Fixed cost}} + \underbrace{\gamma_{1n}^v v_1(w_n)y_n + \frac{1}{2}\gamma_{2n}^v v_2(w_n)y_n^2}_{\text{Variable costs}}$$

  - Input prices are denoted by $w_n$ (labour, capital, intermediate inputs)
  - The unobserved heterogeneity terms are stochastic, satisfying
$$E[\gamma_n^u] = E[\gamma_{1n}^v] = E[\gamma_{2n}^v] = 1.$$
  - The variable cost:
$$v_n(w_n, y_n) = \gamma_{1n}^v v_1(w_n)y_n + \frac{1}{2}\gamma_{2n}^v v_2(w_n)y_n^2$$
  - The variable cost function $v_n$ satisfies
$$v_n(w_n, 0) = 0.$$

- We define variable cost heterogeneity $\gamma_n^v$ as a weighted average of $\gamma_{1n}^v$ and $\gamma_{2n}^v$ as

$$\gamma_n^v = \frac{\gamma_{1n}^v v_1(w_n) y_n + \frac{1}{2}\gamma_{2n}^v v_2(w_n) y_n^2}{v(w_n, y_n)} \tag{3}$$

- this allows to write equivalently:

$$c_n(w_n, y_n) = \gamma_n^u u(w_n) + \gamma_{1n}^v v_1(w_n) y_n + \frac{1}{2}\gamma_{2n}^v v_2(w_n) y_n^2 \tag{4}$$

$$= \gamma_n^u u(w_n) + \gamma_n^v v(w_n, y_n) \tag{5}$$

▸ Assumptions A1-A4

A 5:
- (i) The parameters $\gamma_n \equiv (\gamma_n^u, \gamma_{1n}^v, \gamma_{2n}^v)$ are stochastic and exogenous to the firm
- (ii) Firms know $\gamma_n$ before producing and competing à la Cournot

- In the short run, with fixed number of firms, the Nash equilibrium is characterized by:

$$y_n^b(w_n, Y) = \frac{P(Y) - \gamma_{1n}^v v_1(w_n)}{\gamma_{2n}^v v_2(w_n) - P'(Y)}, \tag{6}$$

$$Y^N = \sum_{n=1}^{N} y_n^b(w_n, Y^N). \tag{7}$$

- Note: $y_n^N$, appearing as an "explanatory variable" in the cost function $c$, is negatively correlated with unobserved heterogeneity
- The quadratic specification allows to obtain an explicit solution for Cournot's equilibrium in terms of (nonnegative) individual and aggregate production levels

# II. Short-run Cournot equilibrium 4

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▸ Proposition 1 (exerpt) At Cournot equilibrium

- the value of marginal cost of production decreases with firm size
- the price markup increases with firm size

▸ Proposition 2 (exerpt) At Cournot equilibrium

- firm $i$'s individual production level decreases with $\gamma_i^v$
- firm $i$'s production level increases with $\gamma_j^v$

# II. Short-run Cournot equilibrium 5

A 6: There is a decreasing relationship between $\gamma^v$ and $\gamma^u$:

$$\gamma_n^v = e(\gamma_n^u) + \eta_n, \quad (8)$$

where $\eta_n$ is an iid random term such that $E[\eta_n|\gamma_n^u] = 0$.

Implications:

- on average, technological progress is not transmitted through simultaneous reductions in both cost parameters $\gamma_n^u$ and $\gamma_n^v$
- there is a trade-off characterized by $e$.
- $cov(\gamma_n^u, \gamma_n^v) < 0$

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Proposition 3 At Cournot equilibrium (and with identical input prices)

- firms' sizes $\{y_m^N\}_{m=1}^M$ are inversely ordered w.r.t $\{\gamma_m^v\}_{m=1}^M$:
  ie. $y_i^N < y_j^N$ iff $\gamma_i^v > \gamma_j^v$
- the biggest firm a lower variable cost and, on average, a higher fixed costs

- Remember: $c_n(w_n, y_n) = \gamma_n^u u(w_n) + \gamma_n^v v(w_n, y_n)$

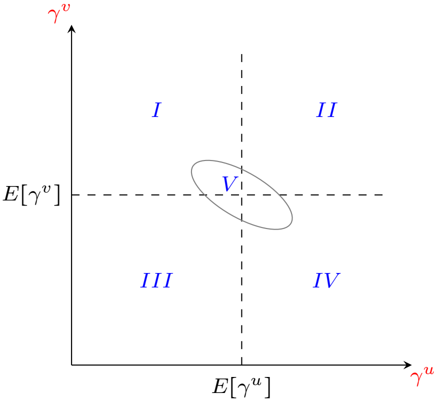


Figure: Five technological zones

- In the long run, the number of firms adjusts. How ?
- The literature investigated several tracks: Ericson and Pakes (1996), Amir and Lambson (2003)
- In the long run, the number of firms adjusts in order to satisfy:

$$E\left[P(Y^N)y_n^N - c_n(w_n, y_n^N)\right] \geq 0, \tag{9}$$

$$E\left[P(Y^N + y_m)y_m - c_m(w_m, y_m)\right] \leq 0, \tag{10}$$

The future technology is random, due to stochastic (Markovian) technological change.

- These equations define the LRCE as the quantities and number of firms:

$$Y^C, N^C, \{y_n^C\}_{n=1}^{N^C}.$$

- We now investigate welfare at LRCE.
- In a setup with identical firms, see Mankiw and Whinston (1986) and Amir et al. (2014)
- Central planer (CP) has to consider technological differences when deciding which firm is allowed to produce and how much
- Assumption: CP knows $\gamma_n$ of each firm
- The welfare function is similar to the one of Mankiw and Whinston (1986):
- The welfare optimizing individual and aggregate productions are denoted by $y_n^W$ and $Y^W$.
- The welfare function is:

$$\begin{aligned} W\left(y_1, \ldots, y_M\right) &= \int_0^{\sum_{m=1}^{M} y_m} P\left(s\right) ds - \sum_{m=1}^{M} c_m\left(w_m, y_m\right), \\ y_m &\geq 0. \end{aligned} \tag{11}$$

- The CP decides about firms' level of production $y_n$
- The values of technological parameters is given $\{\gamma_n\}_{n=1}^{M}$, i.e. no entry/exit
- A firm with $y_n = 0$ bears the fixed cost $u_n$,
- The CP is able to remove inefficiencies introduced by markups and imperfect competition
- Output levels are given such that:

$$W^S \equiv \max_{\{y_n\}_{n=1}^{M}} \left\{ W\left(y_1, \ldots, y_M\right) : \{y_n \geq 0\}_{n=1}^{M} \right\}.$$

- The Short-Run Optimal Welfare (SROW) is characterized by the first order Kuhn and Tucker necessary conditions for an inner maximum for $W$:

$$P\left(\sum_{m=1}^{M} y_m\right) = \frac{\partial c_n}{\partial y_n}\left(w_n, y_n\right) - \lambda_n, \qquad y_n \geq 0, \qquad \lambda_n \geq 0, \qquad \lambda_n y_n = 0, \tag{12}$$

  for $n = 1, \ldots, M$.
- The welfare optimizing individual and aggregate productions are denoted by $y_n^S$ and $Y^S$.

# IV. Short-run optimal Welfare and LRCE 3

$C$ stands for LRCE - Long-run Cournot Equilibrium

$S$ stands for SROW - Short-Run Optimal Welfare

Proposition 4 (exerpt) At LRCE

- Welfare is too low: $W^{C} \leq W^{S}$,
- Profits are too high: $\pi_n^{C} > \pi_n^{S}$
- Big firms produce too little, $y_n^{C} < y_n^{S}$

Proposition 5 At LRCE

- $N^{S} \leq N^{C}$
- $HH^{S} > HH^{C}$

→ Implication: industrial policy should not try to minimize industry concentration at all costs, but the opposite policy would improve welfare in the case of Cournot competition.

# V. Long-run optimal Welfare

- CP selects production technologies active at Long-Run Optimal Welfare (LROW)
- CP is able to replicate technologies
- Here, cost of inactivity bears no fixed cost, CP prevents entry of such a firm
- Formally,

$$W^L \equiv \max_{\{y_n,\gamma_n\}_{n=1}^M} \left\{ W\left(\{y_n\}_{n=1}^M, \{\gamma_n\}_{n=1}^M\right) : \{y_n \geq 0\}_{n=1}^M \wedge \{\gamma_n\}_{n=1}^M \in \Gamma \right\}, \tag{13}$$

  where the technological set $\Gamma \subset \mathbb{R}^2$ denotes the set of all technologies active at LRCE.
- the long-run technological parameters $\gamma^L$ optimal and

$$c^L(w,y) = c(w,y,\gamma^L), \tag{14}$$

▸ Proposition 6 (exerpt)

- the LROW exists and is unique
- at LROW all firms have zero profit and local constant returns to scale
- $W^L \geq W^S$

# VI. Output demand estimation 1

- Consider the output demand addressed to a manufacturing industry $i = 1, \ldots, I$
- Estimate the elasticity of output demand wrt its price
- Aggregate prices and production data
  - $22$ 2-digit industries ($I = 22$)
  - for $1994 - 2016$ ($T = 22$) (loss of one period by differencing),
  - Total of $IT = 484$ observations

# VI. Output demand estimation 1

- Consider the output demand addressed to a manufacturing industry $i = 1, \dots, I$
- Estimate the elasticity of output demand wrt its price
- Aggregate prices and production data
  - 22 2-digit industries ($I = 22$)
  - for $1994 - 2016$ ($T = 22$) (loss of one period by differencing),
  - Total of $IT = 484$ observations
- We consider the following parametric specification for the output demand (Laisney and Koebel, 2016)

$$\ln Y_{it} = \alpha_i + \alpha_Y \ln Y_{i,t-1} + \alpha_p \ln P_{it} + \alpha_{IM} \ln P_{it}^{IM} + \epsilon_{it}, \tag{15}$$

  assuming $E(\alpha_i | Y_{i,t-1}, P_{it}, P_{it}^{IM}) \neq 0$.
- Taking the first-difference eliminates the industry fixed-effects, yielding

$$\Delta \ln Y_{it} = \alpha_Y \Delta \ln Y_{i,t-1} + \alpha_p \Delta \ln P_{it} + \alpha_{IM} \Delta \ln P_{it}^{IM} + \eta_{it}, \tag{16}$$

  with $\eta_{it} = \Delta \epsilon_{it}$.
- Problem: by simultaneity, still $E(\eta_{it} | \Delta \ln Y_{i,t-1}, \Delta \ln P_{it}) \neq 0$

# VI. Output demand estimation 2

- Use supply shifter as instruments to trace out the output demand
- The $(L \times 1)$ vector of instruments, $z_{it}$, includes labor cost, price of intermediate products and export/imports, lagged values (up to lag 3) of endogenous variables

$$z_{it} = (w_{it}, p_{it}^{M}, p_{it}^{X}, p_{it}^{IM}, \{Y_{i\tau}\}_{\tau=1}^{t-3}, \{P_{i\tau}\}_{\tau=1}^{t-3})$$

- Total of 130 moment conditions

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- Total of 130 moment conditions
- Use linear 2-stage GMM, defined by

$$\left(\sum_{i=1}^{I}\sum_{t=1}^{T}\eta_{it}z_{it}^{\top}\right)\mathbf{W}\left(\sum_{i=1}^{I}\sum_{t=1}^{T}z_{it}\eta_{it}\right) = \eta^{\top}\mathbf{ZWZ}^{\top}\eta, \tag{17}$$

- assuming $E[\eta_{it}z_{it}] = 0$
- Apply two-ways clustering to account for
  - heteroskedasticity,
  - contemporaneous dependence between residuals of different industries,
  - temporal dependence within a given industry and consecutive time periods

# VI. Output demand estimation 3

- Price and quantity data for $I = 22$ 2-digit manufacturing industries and $T = 22$ years
- $\ln Y_{it} = \alpha_i + \alpha_Y \ln Y_{i,t-1} + \alpha_p \ln P_{it} + \alpha_{IM} \ln P_{it}^{IM} + \epsilon_{it}$

Table: Output demand estimates

| | FE | FD | FD-GMM |
|---|---|---|---|
| $\alpha_Y$ | 0.92 (0.02) | 0.05 (0.05) | 0.76 (0.06), [0.03] |
| $\alpha_p$ | −0.12 (0.07) | −0.67 (0.17) | −0.64 (0.18), [0.08] |
| $\alpha_{IM}$ | 0.04 (0.07) | 0.55 (0.16) | 0.49 (0.18), [0.07] |
| $OIT$ | - | - | 0.99 |

Notes: HAC robust standard errors are given in parenthesis, clustered standard errors are in brackets. $OIT$: p-value of the over-identification test, for the validity of the 130 orthogonality conditions.

# VI. Output demand estimation 4

- The inverse demand elasticity is obtained by

$$\varepsilon\left(P^d, Y\right) = \frac{1}{\varepsilon\left(Y^d, p\right)} \tag{18}$$

- Setting $Y_{i,t-1} = Y_{i,t}$ we obtain the long-run demand elasticities wrt price

Table: Industry short- and long-run elasticities of output demand

| | Short-run | | Long-run | |
|---|---|---|---|---|
| | $\varepsilon\left(Y^d, p\right)$ | $\varepsilon\left(P^d, Y\right)$ | $\varepsilon\left(Y^d, p\right)$ | $\varepsilon\left(P^d, Y\right)$ |
| Estimate | $-0.64$ | $-1.56$ | $-2.67$ | $-0.37$ |
| s.e. | 0.18 | 0.44 | 0.87 | 0.12 |

- Investigate the relationship between the markup $\mu_{nt}$, and the market share $y_{nt}/Y_t$, parameterized by the inverse demand elasticity:

$$\frac{p_{nt}}{\partial c/\partial y_{nt}(w_{nt}, y_{nt})} = \frac{1}{1 + \varepsilon\left(P^d, Y_t\right) y_{nt}/Y_t} \tag{19}$$

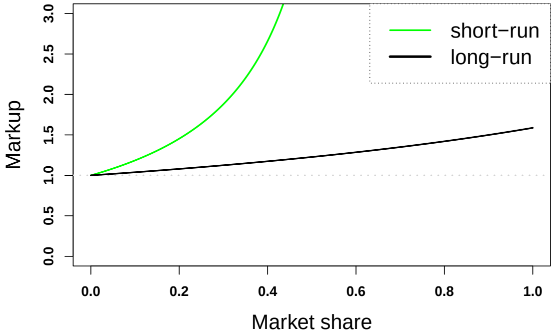


Figure: The estimated relationship between firms' market share and markup

- The markup is monotonically increasing in market share
- Short-run: substantial markup of $1.45 - 1.88$ to firms with biggest market share
- Long-run: markup falls to the interval $1.08 - 1.12$

- French firm-level data 1994-2016 (FICUS/FARE): 176,640 firms, 1,455,383 observations, 184 4-digit manufacturing industries

Table: Statistics by firm size in a typical 4-digit manufacturing industry[a]

| Firm size[b] | # of firms | Share of firms | Share of employees | Share of production |
|---|---|---|---|---|
| 1 | 50 | 14.71 | 0.40 | 0.28 |
| 2-4 | 82 | 24.12 | 1.86 | 1.05 |
| 5-9 | 73 | 21.47 | 3.93 | 2.19 |
| 10-19 | 52 | 15.29 | 5.67 | 3.56 |
| 20-49 | 49 | 14.41 | 12.29 | 9.14 |
| 50-99 | 16 | 4.71 | 8.83 | 6.91 |
| 100-199 | 9 | 2.65 | 10.76 | 9.28 |
| 200-499 | 6 | 1.76 | 14.83 | 14.47 |
| 500+ | 3 | 0.88 | 41.43 | 53.11 |
| Total | 340 | 100.00 | 100.00 | 100.00 |

[a] All figures represent averages over all 4-digit industries and years (1994-2016). Shares are given in %.

[b] Firm sizes are measured by the number of employees.

▸ Included industries

# VII. Data for cost function estimation 2

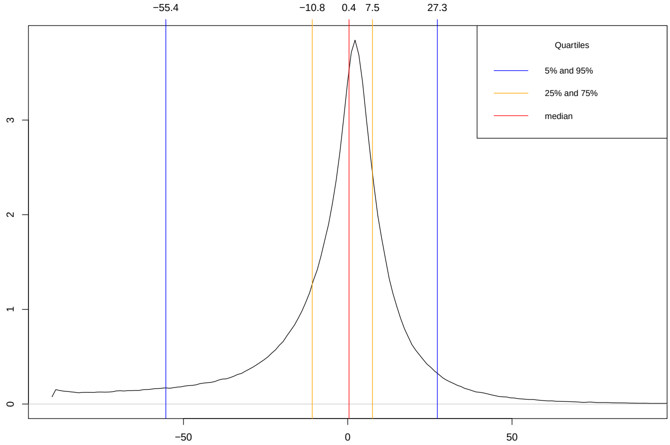


Figure: The profit rates: $(py_{nt} - c_{nt})/c_{nt}$

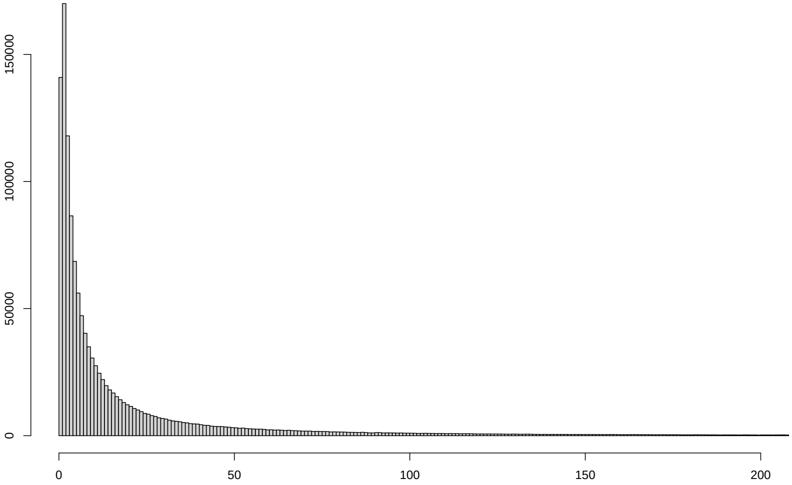


Figure: Production density

# VII. Data for cost function estimation 4

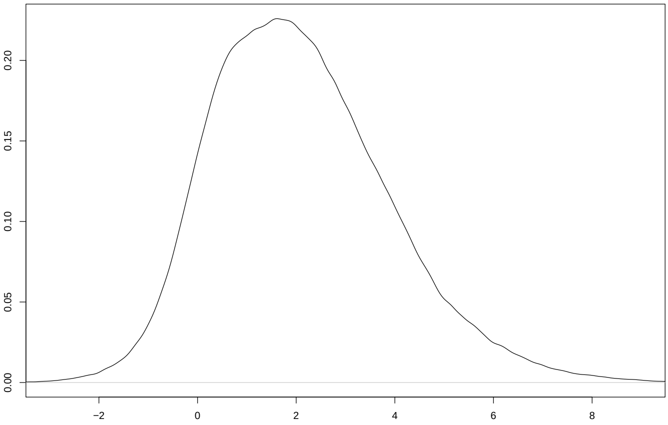


Figure: Log(Production) density

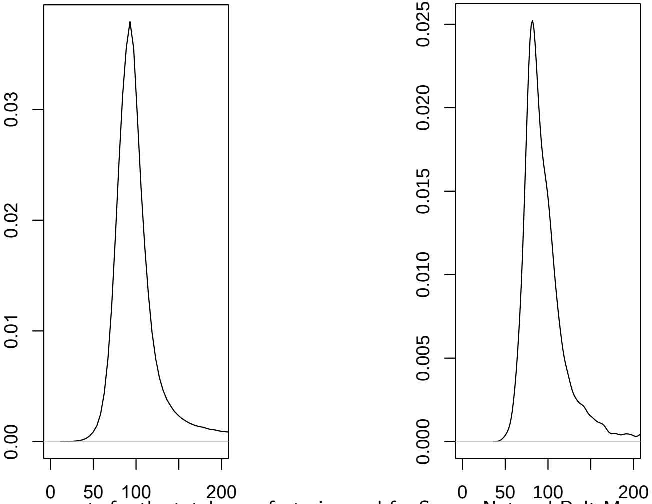


Figure: Average cost: for the total manufacturing and for Screw, Nut and Bolt Manufacturing

# VIII. Cost function estimation 1

- Consider the the cost and marginal cost functions:

$$c_{nt} = u_{nt}(w_{nt}, t; \theta^{u}) + v_{1,nt}(w_{nt}, t; \theta^{v_1}) y_{nt} + \frac{1}{2} v_{2,nt}(w_{nt}, t; \theta^{v_2}) y_{nt}^2 + \varepsilon_{nt}^{c}$$

$$p_{nt}\left(1 + \varepsilon\left(P^d, Y_t\right) y_{nt}/Y_t\right) = v_{1,nt}\left(w_{nt}, t; \theta^{v_1}\right) + v_{2,nt}\left(w_{nt}, t; \theta^{v_2}\right) y_{nt} + \varepsilon_{nt}^{p}$$

- For $p_{nt}$ we use an output price index, available at the 2-digit industry level
- The cost function components $u$, $v_1$, and $v_2$ are FFF in prices $w_{nt}$ and time index $t$
- The fixed cost cannot take negative values, so that we specify:

$$u_{nt}(w_{nt}, t) = \max\left\{\gamma_{nt}^{u} u(w_{nt}, t) + \eta_{nt}^{u}, 0\right\} \tag{20}$$

- For $i = c, p$ and $j = u, v_1, v_2$, we specify:

$$v_{j,nt}(w_{nt}, t) = \gamma_{nt}^{v_j} v_j(w_{nt}, t) + \eta_{nt}^{v_j}, \qquad j = 1, 2. \tag{21}$$

- We rely on a correlated random coefficient approach to account for unobserved heterogeneity (Wooldridge, 2019)

- **Assumption 9**. The unobserved technological random terms satisfy:

$$
\begin{aligned}
E[\gamma^j_{nt}|w_{nt}, t, y_{nt}] &= E[\gamma^j_{nt}|w_{nt}, t, z_{nt}],\\
E[\eta^j_{nt}|w_{nt}, t, y_{nt}] &= E[\eta^j_{nt}|w_{nt}, t, z_{nt}],\\
E[\gamma^j_{nt}|w_{nt}, t, z_{nt}] &= E[\gamma^j_{nt}|z_{nt}] = \gamma^j(z_{nt}) = 1 + (z_{nt} - \overline{z})^\top \beta^j,\\
E[\eta^j_{nt}|w_{nt}, t, z_{nt}] &= E[\eta^j_{nt}|z_{nt}] = \eta^j(z_{nt}) = (z_{nt} - \overline{z})^\top \delta^j, \qquad j = u, v_1, v_2.
\end{aligned}
$$

- Conditionally to $w_{nt}, t, z_{nt}$ our two equations system becomes:

$$
\begin{aligned}
y_{nt} &= y^s(p_t, w_{nt}, t, z_{nt}) + \varepsilon^y_{nt}\\
&= \frac{p_t - \gamma^{v_1}(z_{nt})v_1(w_{nt}, t) - \eta^{v_1}(z_{nt})}{\gamma^{v_2}(z_{nt})v_2(w_{nt}, t) + \eta^{v_2}(z_{nt}) + \varepsilon \frac{p_t}{Y_t}} + \varepsilon^y_{nt},
\end{aligned}
$$

$$
\begin{aligned}
c_{nt} = {} & \gamma^u(z_{nt})u(w_{nt}, t) + \eta^u(z_{nt}) + \gamma^{v_1}(z_{nt})v_1(w_{nt}, t)y^s_{nt} + \eta^{v_1}(z_{nt})y^s_{nt}\\
& + \frac{1}{2}\gamma^{v_2}(z_{nt})v_2(w_{nt}, t)((y^s)^2 + \sigma^2_y) + \frac{1}{2}\eta^{v_2}(z_{nt})((y^s)^2 + \sigma^2_y) + \varepsilon^c_{nt}.
\end{aligned}
$$

# IX. Empirical results 1

We evaluate in turn:

- the distribution of unobserved heterogeneity $\gamma^u$ and $\gamma^v$
- the size of fixed costs
$$\frac{u_{nt}}{c_{nt}}(w_{nt}, t, y_{nt}),$$
- the rate of Returns To Scale (RTS)
$$\frac{\partial \ln c}{\partial \ln y}(w, t, y),$$
- the Rate of Technological Change (RTC)
$$\frac{\partial \ln c}{\partial t}(w, t, y).$$

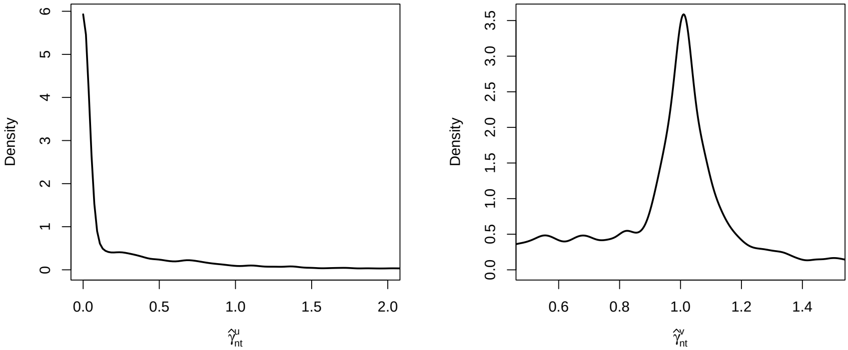


Figure: Unobserved heterogeneity fixed and variable costs

Table: Fixed costs by firm size[a,b]

| Firm size | Share $u_{nt} = 0$ (1) | $Q_{25}(u/c)$ (2) | $Q_{50}(u/c)$ (3) | $Q_{75}(u/c)$ (4) | $Q_{50}(\gamma^u)$ (5) | $Q_{50}(\gamma^v)$ (6) |
|---|---|---|---|---|---|---|
| 1 | 81.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.99 |
| 2-4 | 78.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 |
| 5-9 | 70.76 | 0.00 | 0.00 | 0.12 | 0.00 | 0.97 |
| 10-19 | 57.08 | 0.00 | 0.00 | 0.47 | 0.00 | 0.98 |
| 20-49 | 41.55 | 0.00 | 0.18 | 0.69 | 0.22 | 1.00 |
| 50-99 | 31.21 | 0.00 | 0.37 | 0.79 | 0.79 | 0.99 |
| 100-199 | 22.19 | 0.10 | 0.48 | 0.83 | 2.09 | 0.97 |
| 200-499 | 12.47 | 0.34 | 0.61 | 0.90 | 6.44 | 0.90 |
| 500+ | 2.63 | 0.55 | 0.85 | 1.15 | 32.46 | 0.88 |

[a] Firm sizes are measured by the number of employees.

[b] Column (1) reports the share of firms with zero fixed; $Q_p$ reports the $p^{th}\%$ quantile of the distribution of the variable in parentheses.

Table: RTS and RTC by firm size[a,b]

| Firm size | Returns To Scale $Q_{25}(e(c;y))$ | Returns To Scale $Q_{50}(e(c;y))$ | Returns To Scale $Q_{75}(e(c;y))$ | Rate of Technological Change $Q_{25}(e(c;t))$ | Rate of Technological Change $Q_{50}(e(c;t))$ | Rate of Technological Change $Q_{75}(e(c;t))$ |
|---|---|---|---|---|---|---|
| 1 | 0.79 | 0.99 | 1.09 | -0.26 | -0.01 | 0.15 |
| 2-4 | 0.86 | 1.00 | 1.07 | -0.10 | -0.01 | 0.06 |
| 5-9 | 0.90 | 1.00 | 1.07 | -0.07 | -0.01 | 0.03 |
| 10-19 | 0.91 | 1.00 | 1.06 | -0.06 | -0.01 | 0.02 |
| 20-49 | 0.90 | 1.00 | 1.05 | -0.05 | -0.01 | 0.03 |
| 50-99 | 0.88 | 0.98 | 1.04 | -0.05 | -0.01 | 0.03 |
| 100-199 | 0.86 | 0.96 | 1.03 | -0.04 | -0.01 | 0.02 |
| 200-499 | 0.82 | 0.94 | 1.02 | -0.06 | -0.02 | 0.01 |
| 500+ | 0.75 | 0.90 | 1.00 | -0.40 | -0.02 | 0.01 |

[a] Firm sizes are measured by the number of employees.

[b] $Q_p$ reports the $p^{th}\%$ quantile of the distribution of the variable in parentheses.

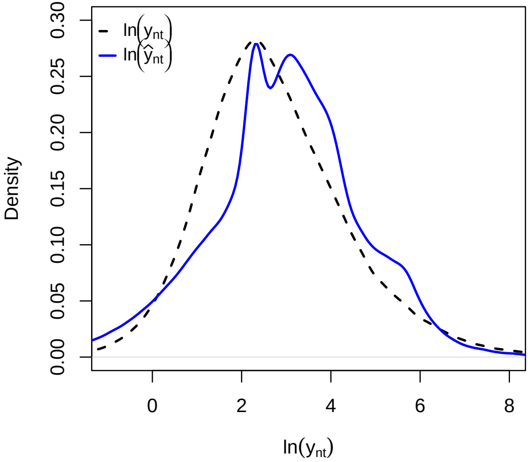


Figure: Firms' size distributions, observed and predicted

## Summary

- We proposed a framework for Cournot competition with heterogeneous firms
- and adapted some results obtained for homogeneous firms and/or symmetric equilibrium
- We found some interesting theoretical results regarding market power and firm size
- Still ongoing:
  - Empirical investigation for the general cost function
  - Which firm shall be shut down or be started-up to increase welfare?

## Empirical challenge

Table: Number of active firms and employment by firm size, manufacturing, France and Germany, 2017

| | | | Firm size | | | |
|---|---|---|---|---|---|---|
| | | Total | 0-9 | 10-49 | 50-249 | >250 |
| France | No. of Firms | 193,609 | 162,955 | 23,468 | 5,658 | 1,522 |
| | No. of Employees | 2,832,458 | 259,459 | 488,990 | 601,247 | 1,482,624 |
| SROW | No. of Firms | | | | | |
| | No. of Employees | | | | | |
| Germany | No. of Firms | 234,310 | 170,585 | 43,540 | 15,845 | 4,340 |
| | No. of Employees | 7,040,463 | 336,753 | 939,166 | 1,701,813 | 4,062,731 |

Welfare gain decomposition:

$$W^S - W^C = \int_{Y^C}^{Y^S} P(s)\, ds + (N^C - N^S)c^C + \left( N^S c^C - \sum_{m \in \mathcal{N}^S} c(w_m, y_m, \gamma_m) \right)$$

Thank you for your attention, suggestions, and comments!

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- Firms behave as profit maximizers,

$$P(Y) + P'(Y)y_n = \frac{\partial c_n}{\partial y_n}(w_n, y_n) = \frac{\partial v_n}{\partial y_n}(w_n, y_n) \quad (22)$$

A 1: The inverse demand function $P$ is nonnegative, continuous, differentiable and decreasing in $Y$.

A 2: The cost function is continuous in $w_n$ and $y_n$, nonnegative, differentiable and increasing in $w_n$ and $y_n$

A 3: There exist firm-level and aggregate production levels $\overline{y}$ and $\overline{Y}$ such that:

(i) the marginal revenue is lower than the marginal cost:

$$P(Y) + P'(Y)\,y < \partial c_n/\partial y_n\,(w_n, y)\,, \quad (23)$$

for any $y > \overline{y}$ and $Y > \overline{Y}$, and any firm $n = 1, \ldots, N$;

(ii) the cost function is not too concave:

$$P'(Y) < \partial^2 c_n/\partial y_n^2\,(w_n, y)\,, \quad (24)$$

for any $y < \overline{y}$ and $Y < \overline{Y}$, and any firm $n = 1, \ldots, N$.

A 4: The marginal revenue function satisfies:

$$P^{'}(Y) + y_n P^{''}(Y) \leq 0, \quad (25)$$

for any value of $y_n \leq Y < N\overline{y}$.
This is Novshek (1985) sufficient condition for existence of Cournot equilibrium.

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# Proposition 1

- Under A1-A4, for given $N$, the Cournot equilibrium exists and is unique
- The backward reaction functions:

$$y_n^b(w_n, Y), \quad \text{and} \quad Y = \sum_{n=1}^{N} y_n^b(w_n, Y). \tag{26}$$

- the Cournot equilibrium is characterized by $Y^N$ and $y_n^N = y_n^b(w_n, Y^N)$.

## Proposition 1

Under A1-A4, at the Cournot equilibrium with fixed number of $N$ firms:

1. The elasticity of inverse demand $\epsilon(P, Y)$ satisfies $-N < \epsilon(P, Y) < 0$
2. Firm's $n$ market share satisfies $y_n^N / Y^N < -1/\epsilon(P, Y)$
3. The value of the marginal cost of production decreases with firm size
4. The price markup increases with firm size
5. For a subset of $N' < N$ active firms, $Y^{N'} < Y^N$ and $y_n^{N'} > y_n^N$.

$$P'(Y^N)(y_n^N - y_m^N) = \frac{\partial c_n}{\partial y}(w_n, y_n^N) - \frac{\partial c_m}{\partial y}(w_m, y_m^N) \tag{27}$$

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## Proposition 2

Under A1-A5, at the short-run Cournot equilibrium with fixed number of firms,

1. firm $n$ individual production level decreases with $\gamma_n^v$
2. firm $n$ production level increases with $\gamma_m^v$
3. the aggregate equilibrium level of production decreases with $\gamma_n^v$
4. individual and aggregate production levels are unaffected by a change in $\gamma_n^u$
5. firm $n$ profit decreases with $\gamma_n^v$ and $\gamma_n^u$
6. firm $n$ profit increases with $\gamma_m^v$

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## Proposition 3

Under A1-A7, we consider two firms at Cournot equilibrium, both with similar input prices $w$. Assume that the cost functions are convex. The Nash equilibrium production levels of firms $m$ and $n$ satisfy $y_m^N < y_n^N$ iff
(i) the biggest firm is more productive: $\gamma_m^v > \gamma_n^v$
(ii) the biggest firm has a lower variable cost for each unit produced:
$v_m\left(w, y_m^N\right)/y_m^N > v_n\left(w, y_n^N\right)/y_n^N$
(iii) on average, bigger firms have higher fixed costs: $\mathrm{E}\left[\gamma_m^u\right] < \mathrm{E}\left[\gamma_n^u\right]$ and $\mathrm{E}\left[u_m\left(w\right)\right] < \mathrm{E}\left[u_n\left(w\right)\right]$;
(iv) on average, bigger firms have a larger efficient scale of production.

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## Proposition 4

We assume A1-A4. In comparison to the LRWM, the LRCE is characterized by
(i) a lower aggregate production and a higher price: $Y^C < Y^W$ and $P\left(Y^C\right) > P\left(Y^W\right)$
(ii) profits which are too high: $\pi_n^C > \pi_n^W$
(iii) big firms which produce too little, $y_n^C < y_n^W$
(iv) small firms with global decreasing returns which produce too much: $y_n^C > y_n^W$ and some of them which should be shut down
(v) small firms with increasing returns which either produce too little, or should be shut down
(vi) only a subset of the firms active at LRCE is still active at the LRWM.

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## Proposition 5

Under A1-A7, we consider firms with similar input prices $w$ at Cournot equilibrium. Assume that the cost functions are convex. Then:
(i) $N^W \leq N^C$
(ii) the Hirschman-Herfindahl index of concentration is higher at the LRWP than at LRCE.

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## Proposition 6

Under A1-A8, we consider firms with similar input prices $w$, and ignore the integer constraint on $N$. Then
(i) the LROW exists and is unique;
(ii) at LROW all firms have zero profit and local constant returns to scale;
(iii) $W^L \geq W^S$;
(iv) the fixed cost is zero at LROW if $e'(\gamma^{uL}) < u(w)/v(w, y^L)$;
(v) It is equivalent to maximize the central planer problem $W^L$ or decentralized profits wrt $(y_n, \gamma_n)$, for given price level $p$, which clears the product market with free entry;

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# Industry output demand and simultaneity

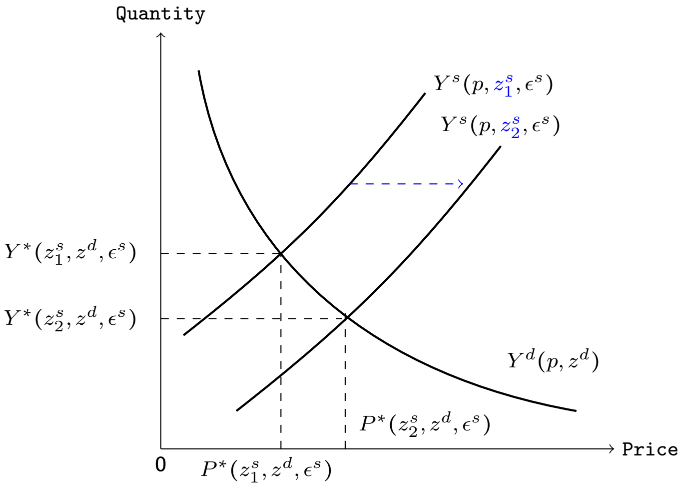


Figure: Identifying industry demand

# Included industries

Table: Description of two-digit industries

| Industry[a] | Description | # Firms[b] | # Obs.[c] |
|---|---|---|---|
| 11 | Beverages | 3,031 | 26,049 |
| 13 | Manufacture of tobacco products | 7,012 | 59,299 |
| 14 | Manufacture of wearing apparel | 15,658 | 82,221 |
| 15 | Manufacture of leather and related products | 3,054 | 22,220 |
| 16 | Manufacture of wood and of products of wood | 13,220 | 109,643 |
| 17 | Manufacture of paper and paper products | 2,825 | 28,447 |
| 18 | Printing and reproduction of recorded media | 21,799 | 174,024 |
| 20 | Manufacture of chemicals and chemical products | 5,204 | 47,581 |
| 21 | Manufacture of basic pharm. products and pharm. preparations | 979 | 8,522 |
| 22 | Manufacture of rubber and plastic products | 8,801 | 86,595 |
| 23 | Manufacture of other non-metallic mineral products | 11,668 | 95,613 |
| 24 | Manufacture of basic metals | 2,042 | 18,767 |
| 25 | Manufacture of fabricated metal products | 34,397 | 326,264 |
| 26 | Manufacture of computer, electronic and optical products | 7,388 | 57,119 |
| 27 | Manufacture of electrical equipment | 5,033 | 42,623 |
| 28 | Manufacture of machinery and equipment | 13,362 | 111,735 |
| 29 | Manufacture of motor vehicles, trailers and semi-trailers | 4,013 | 35,857 |
| 30 | Manufacture of other transport equipment | 1,799 | 12,852 |
| 31 | Manufacture of furniture | 15,355 | 109,952 |
| | Total | 176,640 | 1,455,383 |

a) Statistical classification of economic activities in the European Community, Rev. 2 (2008)

b) # Firms describes the number of firms which were active over the period (it is computed as the total number of different firms identifiers).

c) # Obs. describes the total number of observations.

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