Flexible Bayesian MIDAS: time-variation, group-shrinkage, and sparsity

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Motivation & What we do

- Nowcast and forecast models face challenges in presence of large and heterogeneous shocks and non-linearities or shifts over time
  - Large observations can distort parameter estimates and increase uncertainty
  - Changing correlation structure and heterogeneous dynamics across indicators

- Require flexible model features
  - Accounting for time-varying trends and stochastic volatilities (SV) beneficial in UC, DFM, VAR models (Stock and Watson, 2009; Clark, 2011; Antolin-Diaz et al., 2017)
  - Covid-19: Account for extreme observations via t-distr. errors or outliers (Carriero et al., 2021; Lenza and Primiceri, 2022; Antolin-Diaz et al., 2021)
  - Bayesian shrinkage (Carriero et al., 2015; Mogliani and Simoni, 2021)

- We combine time-varying components with multivariate MIDAS (Ghysels et al., 2020) and Bayesian group-shrinkage: Trend-SVt-BMIDAS with GIGG prior

Literature

Trend-SV-t-BMIDAS model

four key features

1. Time-varying unobserved components in the lower-frequency target variable (time-varying Trend, stochastic volatlity (SV), t-distr. errors)
2. Information from high frequency indicators in multivariate MIDAS regression
3. Bayesian group-shrinkage via a global-local prior with three tiers of continuous shrinkage (overall, between indicators, and within lags of an indicator)
4. A new group-wise sparsification algorithm on the posterior
    - Ex-post sparsification motivated by decision theory allows for variable selection and helps interpret signals over time via inclusion probabilities.
    - Approach separates shrinkage and sparsity akin to "illusion of sparsity" (Giannone et al., 2021), while accounting for within-group correlation.

Main Results: empirical application for nowcasting UK GDP growth

- Combination of time-varying components and group-shrinkage GIGG prior improves nowcast performance for UK GDP growth, before and including the pandemic.

  1. GIGG group-shrinkage prior shrinks information set towards a sparse selection indicators, while to a lesser extent also drawing on other indicators.
  2. Time-varying components help to shift between the most meaningful indicators over data release cycle, rather then relying on constant signals.

- Inclusion probabilities inform about signals exploited by the model

  - Early in data release cycle reliance on surveys, then shift to 'hard' indicators.
  - Covid-19 pandemic: shift towards indicators for services, away from production.
  - Other shrinkage priors such as Horseshoe rely on diffuse or broad set of indicators and profit less from time-varying components.

Methodology

The Trend-BMIDAS-SV-t Model

$$
\begin{aligned}
y_t &= \tau_t + \sum_{k=1}^{K} \mathcal{B}(L^{1/m}; \theta_k) x_{k,t} + \sqrt{\lambda_t} e^{\frac{1}{2}(h_0 + w_h \tilde{h}_t)} \tilde{\epsilon}_t^y, \\
\tilde{\epsilon}_t^y &\sim N(0,1), \ \lambda_t \sim IG(\nu/2, \nu/2) \\
\tau_t &= \tau_{t-1} + e^{\frac{1}{2}(g_0 + w_g \tilde{g}_t)} \tilde{\epsilon}_t^\tau, \ \tilde{\epsilon}_t^\tau \sim N(0,1) \\
\tilde{h}_t &= \tilde{h}_{t-1} + \tilde{\epsilon}_t^h, \ \tilde{\epsilon}_t^h \sim N(0,1) \\
\tilde{g}_t &= \tilde{g}_{t-1} + \tilde{\epsilon}_t^g, \ \tilde{\epsilon}_t^g \sim N(0,1).
\end{aligned}
\tag{1}
$$

- τ_t: time-varying trend; τ_t and y_t are lower frequency (quarterly)
- $B(L^{1/m}, \theta_k) x_{t,k} = \sum_{j=1}^{m} \omega(\frac{j-1}{m}; \theta_k) x_{t-(j-1)/m,k}$
  - θ: $(p_k + 1) * K$ parameters that link higher and lower frequency observations.
  - $\omega : \mathbb{R} \times \mathbb{R}^L \to \mathbb{R}$, nests Almon and U-MIDAS Almon
- h_t, g_t: SVs, non-centered (Frühwirth-Schnatter and Wagner, 2010)
- λ_t: enforces a ν-degrees of freedom t-distribution, fat-tailed SV

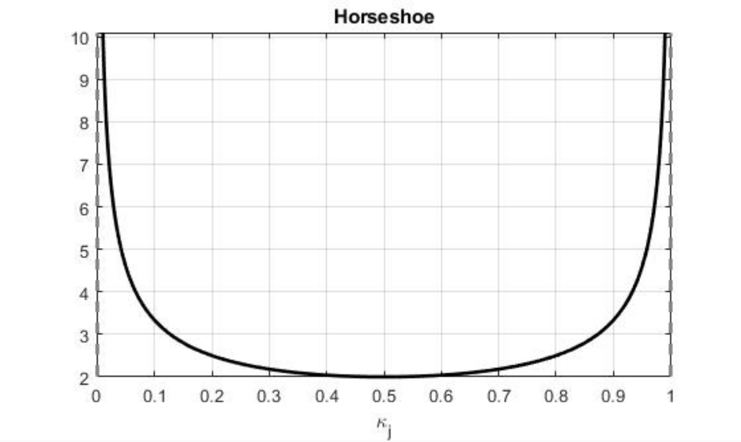
Group-shrinkage prior on multivariate MIDAS component

- GIGG (Group-Inverse-Gamma-Gamma) global-local prior (Boss et al., 2021)
  - Models group-shrinkage + correlation among higher frequency lags
- Each group g has $p_g + 1$ parameters to estimate

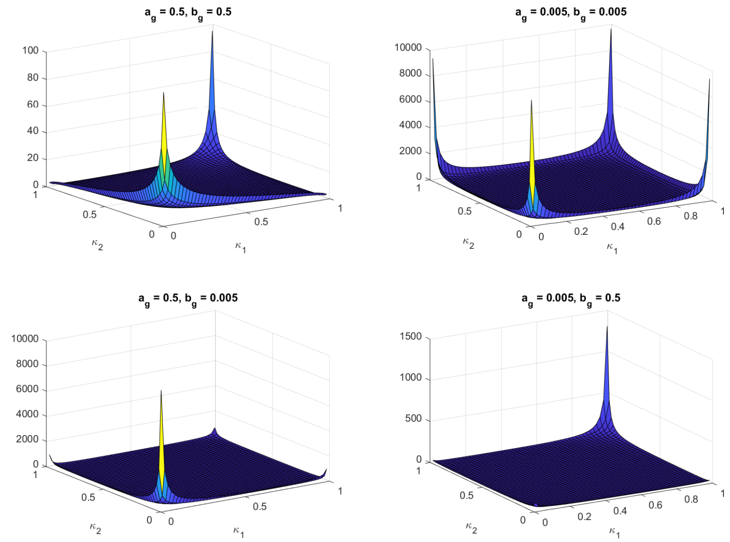
$$\theta_{g,i} \sim N(0, \vartheta^2 \gamma_g^2 \varphi_{g,i}^2), \ \ \forall i \in \{0, \cdots, p_g + 1\}$$
$$\vartheta \sim C_+(0,1), \ \ \gamma_g^2 | a_g \sim G(a_g, 1), \ \ \varphi_{g,i}^2 | b_g \sim IG(b_g, 1), \tag{2}$$

  - ϑ controls the overall level of sparsity
  - $\gamma_g^2(a_g)$ controls sparsity across groups g
  - $\varphi_{g,i}^2(b_g)$ controls correlation of group members i within g
- We set $g = k$, i.e. groups defined as lags of each indicator (can be extended to groupings across indicators if k large)
- At group-size 1, $a_g = b_g = 0.5$, give horseshoe prior (Carvalho et al., 2010)

Univariate shrinkage with global-local prior



GIGG Prior Visualisation: Bi-variate shrinkage hyperparamers



Group-sparsification step on the posterior

- Lack of interpretability with continuous priors: posterior coefficients remain non-zero (Hahn and Carvalho, 2015), impact from indicators on nowcast opaque.
- Solution: Ex-post sparsification algorithm to the posterior θ_g.
  - Decision tool that is separate from regularisation imposed by prior.
  - Minimise a utility function over Euclidean distance between a linear model which penalises group-size (akin to Zou, 2006) and the model's prediction:

$$\mathcal{L}(\tilde{Y}, \alpha) = \frac{1}{2}||XW\alpha - \tilde{Y}||_2^2 + \sum_{k=1}^{K} \phi_k ||\alpha_k||_2, \tag{3}$$

  - penalisation term creates a soft-thresholding effect between [-ϕ_k, ϕ_k]
  - finds smallest subset of groups to achieve predictive performance closest to unsparsified model, coefficients in other groups forced to zero.
- Gives inclusion probabilities that inform about the relative impact of an indicator
  - relative frequency of lag-group k in the sparsified estimate $\alpha^{*(s)}$ over Gibbs draws

Other priors and estimation

Priors for latent states standard: $(\boldsymbol{\tau}, \tilde{\boldsymbol{h}}, \tilde{\boldsymbol{g}})$ joint normal prior as in Chan and Jeliazkov (2009), McCausland et al. (2011) and Kim et al. (1998)

Estimation via Metropolis-within-Gibbs sampler M-H Gibbs

- Recursive sampling from conditional distributions: MIDAS parameters θ, GIGG hyperparameters, latent states $(\boldsymbol{\tau}, \tilde{\boldsymbol{h}}, \tilde{\boldsymbol{g}})$ (non-recursively, as in Chan and Jeliazkov (2009)), λ_t, degrees of freedom ν
- sampling of ν requires Metropolis step
- 5000 burn-in iterations, retain further 5000 for inference

Empirical Application

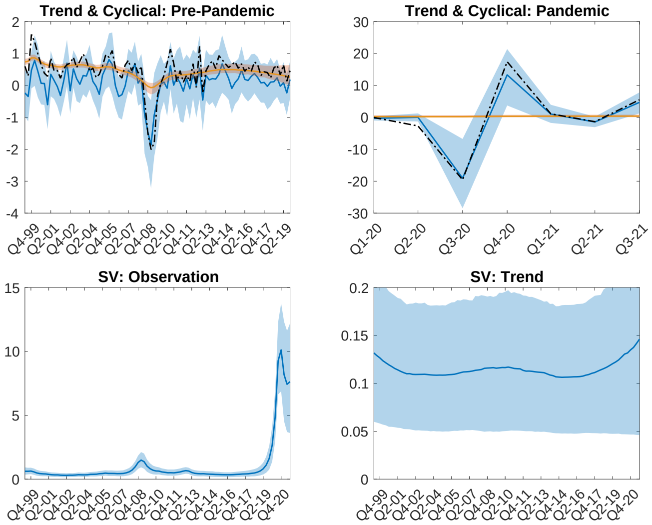
Nowcast quarterly UK GDP growth, 1999-2021

- Setup
  - In-Sample Start: Q1 1999, Nowcast Start: Q1 2011
  - Nowcast End: "pre-pandemic" Q4 2019, "including pandemic" Q3 2021
- Monthly indicators
  - indices of services and production, trade
  - surveys (CBI, PMI, GfK)
  - labour market (unemployment rate, employment, vacancies, hours)
  - mortgage approvals, VISA consumer spending
- Nowcast evaluation
  - pseudo-real-time calendar: 20 nowcasts per quarter around data releases calendar
  - each nowcast has new information set, latest available 6 monthly obs. of each indicator
- Metrics
  - Point: Root-mean-squared forecast error (RMSFE)
  - Density: Average cumulative rank probability score (CRPS)

Three Sets of Results

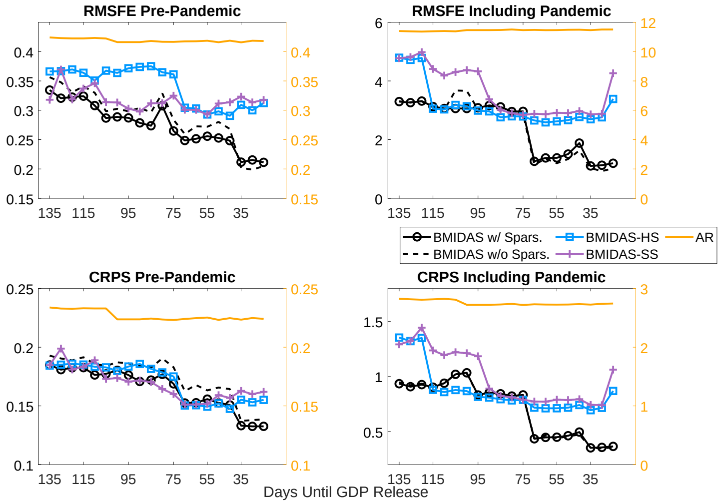
1. Posterior estimates time-varying trend, cyclical component, and stochastic volatilities
2. Nowcast evaluation
   - prior alternatives, all with time varying components
   - prior alternatives, without time varying components
3. Inclusion probabilities Signals exploited over the data release cycle and over time

Time-varying components - Posterior estimates

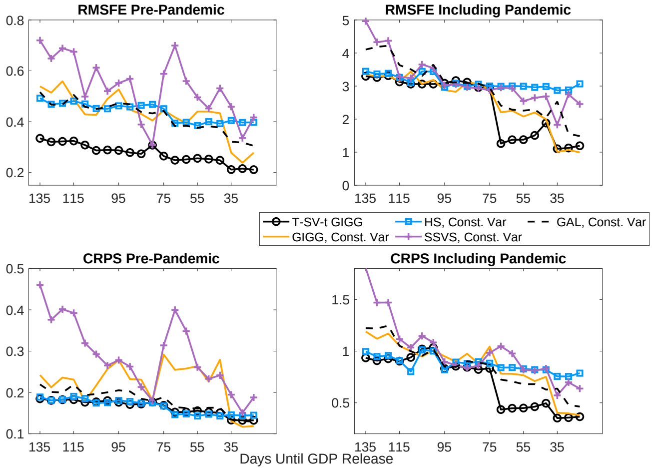


sensitivity

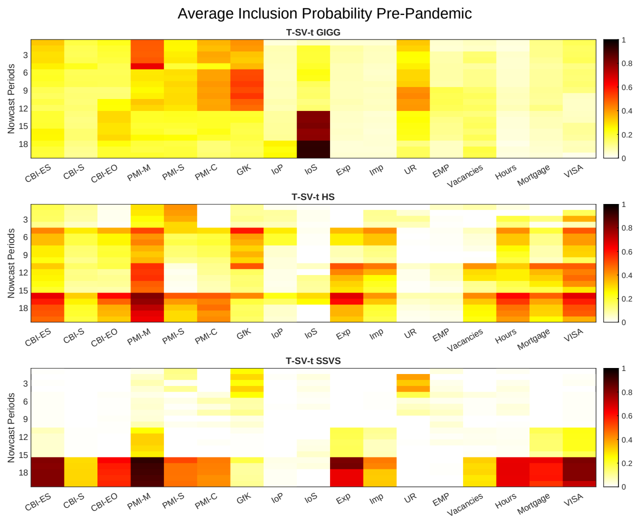
Role of group-shrinkage - Nowcast evaluation across prior choices



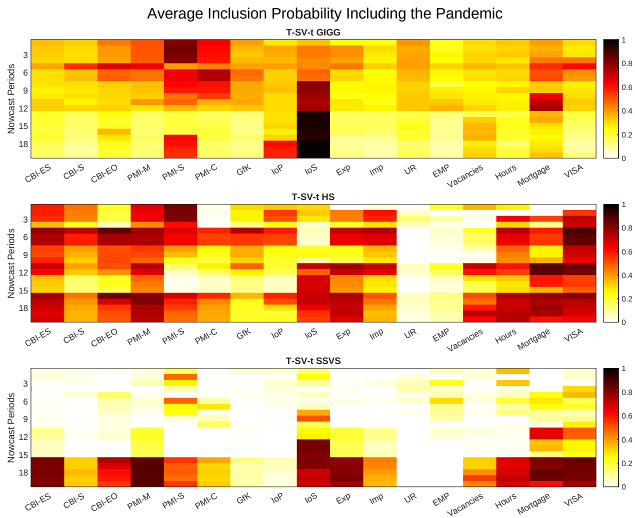
Role of group-shrinkage prior + shutting down time-variation



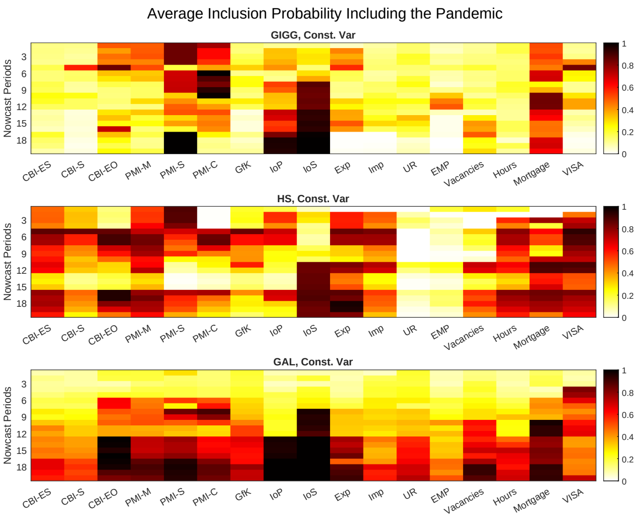
Intuition - signal extraction across priors (pre-pandemic period)



Intuition - signal extraction across priors (incl. pandemic period)



Intuition - signal extraction, no time-varying components (incl. pandemic period)



Conclusions

- Time-varying components + Bayesian MIDAS + flexible group-shrinkage prior.
    - Group-shrinkage and time-varying components (trends, volatilities, large errors) jointly lead to strong nowcasting performance for UK GDP growth.
- Approach brings new insights to literature on density vs sparsity in macroeconomic forecasting (Giannone et al., 2021)
    - grouping + time variation in shrinkage important to separate the relevant sub-group of indicators in each nowcast period and over time (e.g. Covid-19)
    - Shrinkage and sparsification are split apart: no a priori sparsification via prior, but on the posterior to enhance interpretability
- Can be relevant to a range of exercises where group shrinkage matters (e.g. also disaggregated data).

Thank you

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Appendix

Links to the literature

- The model nests / compares to existing models when shutting down model features
  - multivariate BMIDAS with horseshoe (Kohns and Bhattacharjee, 2022) or spike-and-slab group shrinkage prior (Mogliani and Simoni, 2021) asymptotically
  - BMIDAS model with SV (Carriero et al., 2015)
  - Trend-SV-outl. DFM (Antolin-Diaz et al., 2021)

- MIDAS literature (Ghysels et al., 2007, 2020; Foroni et al., 2015)
- Machine learning for nowcasting (Babii et al., 2022)
- Global-local shrinkage priors (Polson and Scott, 2010; Polson et al., 2014; Carvalho et al., 2010) and group-lasso priors (Casella et al., 2010; Xu and Ghosh, 2015) and spike-and-slab (Ishwaran et al., 2005; Piironen et al., 2017)

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Almon lag polynomial restricted MIDAS

- U-MIDAS (Foroni et al., 2015) involves many parameters and can lead to erratic weight profiles
- Restrict coefficients via Almon lag-polynomials on θ_i: assuming a $p_k << L_k$ polynomial process of the coefficients across high-frequency observations

Almon Lag MIDAS

Assume lags $i = 0, \cdots, L$ can be represented by a 3rd degree polynomial, then each HF parameter process, θ_i can be written as:

$$\theta_i = \beta_0 + \beta_1 i + \beta_2 i^2 + \beta_3 i^3 \tag{4}$$

We add economically relevant restrictions (Smith and Giles, 1976)

$$\begin{aligned} \theta'_L &= 0 \\ \theta_L &= 0 \end{aligned} \tag{5}$$

But: Smoothness of Almon-polynomial increases parameter correlation back

Metropolis-within-Gibbs sampling algorithm

1. Sample $\theta|\bullet \sim p(\theta|\mathbf{y}, \bullet)$
2. Sample hyper-parameters $\vartheta, \gamma_k^2, \varphi_{kj}^2, \nu_p$ in one block
   2.1 $\vartheta^2 \sim p(\vartheta^2|\mathbf{y}, \bullet)$
   2.2 $\gamma_k^2 \sim 1/p(\gamma_k^{-2}|\mathbf{y}, \bullet)$
   2.3 $\varphi_{kj}^2 \sim p(\varphi_{kj}^2|\mathbf{y}, \bullet)$
3. sample $\tilde{\boldsymbol{\tau}} \sim p(\tilde{\boldsymbol{\tau}}|\mathbf{y}, \bullet)$ and $\tau_0 \sim p(\tau_0|\mathbf{y}, \bullet)$
4. sample $\tilde{\mathbf{h}} \sim p(\tilde{\mathbf{h}}|\mathbf{y}, \bullet)$, $h_0 \sim p(h_0|\mathbf{y}, \bullet)$ and $w_h \sim p(w_h|\mathbf{y}, \bullet)$
5. sample $\tilde{\mathbf{g}} \sim p(\tilde{\mathbf{g}}|\mathbf{y}, \bullet)$, $g_0 \sim p(h_0|\mathbf{y}, \bullet)$ and $\sim p(w_g|\mathbf{y}, \bullet)$
6. Sample $\{\lambda_t\}_{t=1}^T \sim p(\lambda_t|\mathbf{y}, \bullet)$
7. Sample $\nu \sim p(\nu|\mathbf{y}, \bullet)$ with a Metropolis step back

- sampling technique of Chan and Jeliazkov (2009) allows drawing steps 3.-5. in a non-recursive fashion which increases efficiency and can be sped up using sparse-matrices

Pseudo Real Time Calendar for UK Nowcast Application

Nowcast	Quarter	Days to GDP	Month	Timing within month	Release	Publication Lag
1		135	1	1st of month	PMIs	m-1
2		125	1	End of 2nd week	IoP, IoS, Ex, Im	m-2
3		120	1	3rd week	Labour market data	m-2
4		115	1	3rd Friday of month	Mortgage & Visa	m-1
5		110	1	End of 3rd week	CBIs & GfK	m
6	Reference	105	2	1st of month	PMIs	m-1
7	quarter	97	2	Mid of 2nd week	Quarterly GDP	q-1
8	(nowcast)	95	2	End of 2nd week	IoP, IoS, Ex, Im	m-2
9		90	2	3rd week	Labour market data	m-2
10		85	2	3rd Friday of month	Mortgage & Visa	m-1
11		80	2	End of 3rd week	CBIs & GfK	m
12		75	3	1st of month	PMIs	m-1
13		65	3	End of 2nd week	IoP, IoS, Ex, Im	m-2
14		60	3	3rd week	Labour market data	m-2
15		55	3	3rd Friday of month	Mortgage & Visa	m-1
16		50	3	End of 3rd week	CBIs & GfK	m
17		45	1	1st of month	PMIs	m-1
18	Subsequent	35	1	End of 2nd week	IoP, IoS, Ex, Im	m-2
19	quarter	30	1	3rd week	Labour market data	m-2
20	(backcast)	25	1	3rd Friday of month	Mortgage & Visa	m-1

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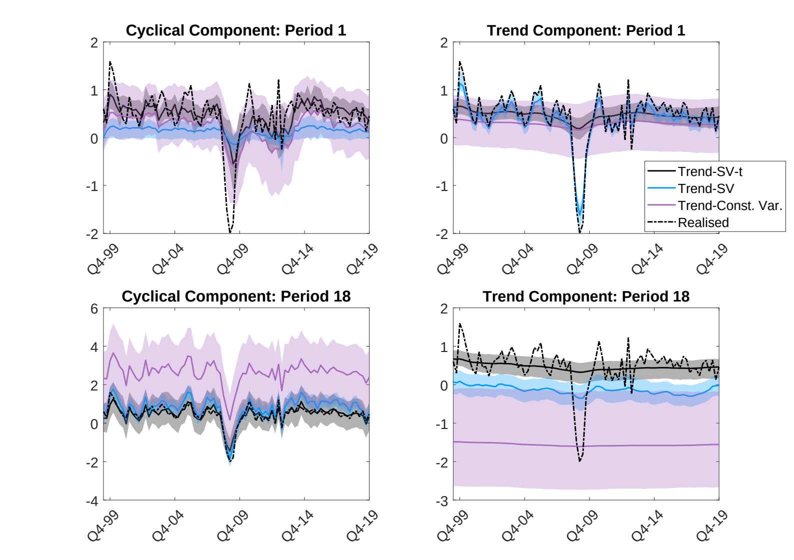
Trend and SV posterior estimates, alternative models.



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Point nowcast evaluation results, RMSFE relative to AR(2).

	Evaluation pre-pandemic				Evaluation incl. pandemic period			
Nowcast Periods	Average	6	13	18	Average	6	13	18
	RMSFE				RMSFE			
AR(2) benchmark (abs. RMSFE)	0.42	0.42	0.42	0.42	11.45	11.4	11.47	11.48
T-SV-t BMIDAS, GIGG w/ Spars. (rel. RMSFE)	0.66***	0.68**	0.60*	0.51**	0.21***	0.27	0.11	0.10
Alternatives to T-SV-t (all BMIDAS, GIGG w/ Spars)								
T-SV	0.75***	0.81*	0.72	0.44**	0.21***	0.27	0.17	0.08
T, Constant variance	0.76***	0.84	0.68	0.47**	0.21***	0.28	0.16	0.07
No T, SV-t	0.78***	0.90	0.70	0.46**	0.21***	0.31	0.10	0.08
No T, SV	0.81***	0.91	0.77	0.46**	0.22***	0.31	0.15	0.08
No T, Const. var.	1.03***	1.01	1.00*	0.67	0.22***	0.27	0.19	0.09
Alternatives to BMIDAS (all with T-SV-t, GIGG w/ Spars)								
U-BMIDAS	0.71***	0.74**	0.65**	0.60**	0.24***	0.29	0.18	0.18
MF-DFM	0.67***	0.75	0.67**	0.63**	0.32***	0.33	0.32	0.34
Combination univar. MIDAS	0.68***	0.68**	0.67**	0.67**	0.36***	0.37	0.34	0.33
Alternatives priors on BMIDAS (all with T-SV-t)								
GIGG w/out Spars.	0.69***	0.71*	0.62	0.49*	0.21***	0.32	0.10	0.09
Horseshoe (HS)	0.81***	0.87	0.73	0.74	0.28***	0.28	0.23	0.24
Spike and Slab (SS)	0.76***	0.74**	0.72*	0.78	0.32***	0.38	0.25	0.25

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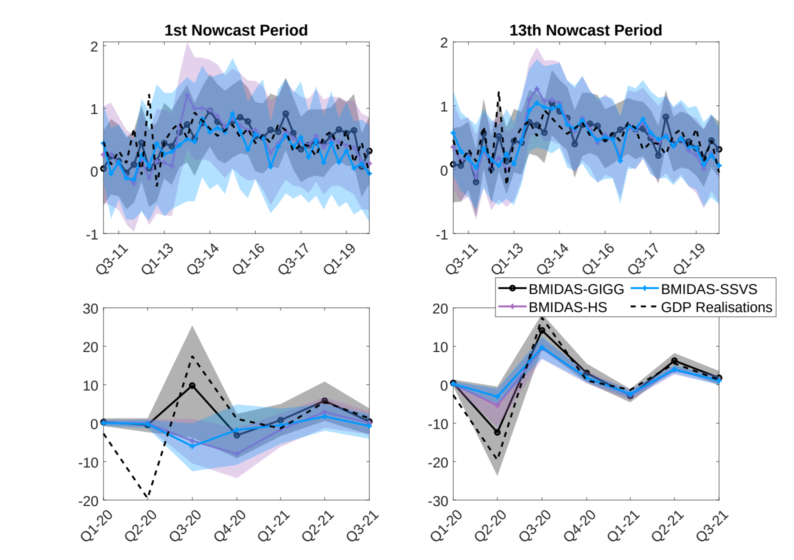
Posterior mean and density nowcasts, alternative priors.



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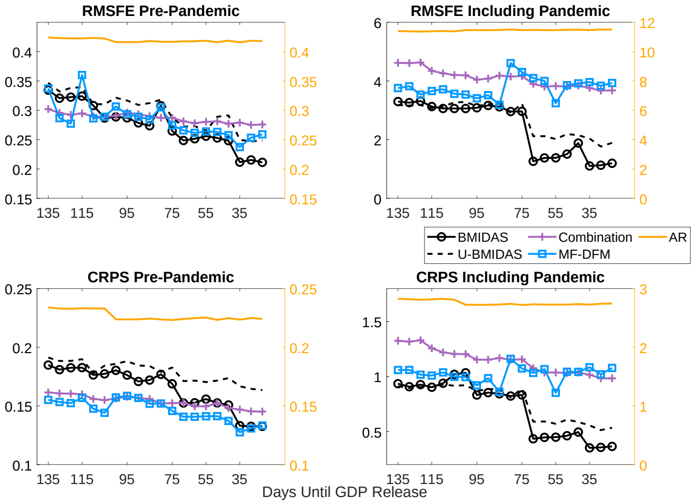
2b) Performance against alternatives to multivariate MIDAS



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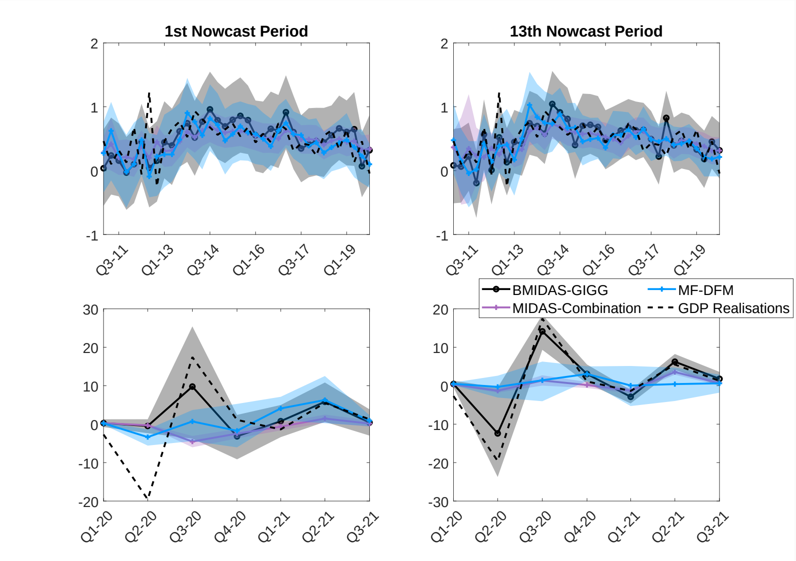
Posterior mean and density nowcasts, alternatives to BMIDAS.



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