Allowing for Dynamic Student Fixed Effects in Teacher Value-Added Estimation

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- What we do:
  1. We use a TWFE model to estimate TVA while allowing for time-varying student fixed effects (TV-TWFE).
  2. We use our TVA to further analyse *(1)* sorting and segregation patterns *(2)* characteristics associated with TVA estimates.

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- The tests are adaptive and standardized within-subject and grade.
- After basic restrictions, we end up with a data set of 1.1 million observations with 30.000 individual teachers and 380.000 students.

TVA Estimation I - Lagged-Score Value-Added

- TVA is commonly estimated using

$$Y_{gt} = \psi_{j(g,t)} + X'_{gt}\gamma + \varepsilon_{gt}$$

where X's are controls such that TVA estimates are *causal*. That is, to ensure that

$$\mathbb{E}\left[\varepsilon_{gt} | X'_{gt}, \psi_{j(g,t)}\right] = \mathbb{E}\left[\varepsilon_{gt} | X'_{gt}\right]$$

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- This model does not allow for sorting between latent teacher quality and student performance (after controls).

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- Notice that ψ_j is measured using "movers".
- One needs mean independence for valid identification of the TVA.
- Hence, "mobility" needs to be exogenous for ψ_j to be identified.
  - Violated when student moves are determined by time-specific performance.

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$$Y_{gts} = \alpha_{gt} + \psi_{j(g,t,s)} + \varepsilon_{gts}$$

- Student fixed-effects do not need to be time-invariant.
- Hence, symmetric mobility is allowed as long as it is not subject-specific.

Results - Overview

Given valid identification, we present different parameter estimates that are of general interest.

1. Variance decomposition
   - How "important" are teachers?
   - How much do students "sort"?

2. What other characteristics are TVA associated with?
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   - Well-being of the student.

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Table 2: Variance and Sorting Estimates of TVA

	TWFE Model		TV-TWFE Model	
	(1) Not Bias-Corrected	*(2)* Bias-Corrected	*(3)* Not Bias-Corrected	*(4)* Bias-Corrected
Variance of TVA: $\mathbb{V}ar\left(\hat{\psi}\right)$	0.102	0.054	0.394	0.095
Sorting: $\mathbb{C}ov\left(\hat{\alpha}, \hat{\psi}\right)$	-0.0534	-0.0148	-0.3462	-0.0549

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- We argue that including time-specific student effects to account for endogenous mobility is essential for valid estimation of TVA in our setting.

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Decomposition 1: previous decomposition

$$\mathbb{V}ar\left(Y_{gts}\right) = \underbrace{\mathbb{V}ar\left(\alpha_{gt}\right)}_{\text{Variance of student-year effect}} + \underbrace{\mathbb{V}ar\left(\psi_j\right)}_{\text{Variance of TVA}} + 2\underbrace{\mathbb{C}ov\left(\alpha_{gt}, \psi_j\right)}_{\text{Sorting}} + \mathbb{V}ar\left(\epsilon_{gts}\right) \tag{8.1}$$

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- Next, we investigate more detailed variance decompositions (Song et al. (2019), Haltiwanger et al. (2022))

Decomposition 2 - within vs between teachers:

$$\mathbb{V}ar\,(Y_{gts}) = \underbrace{\overbrace{\mathbb{V}ar(\bar{\alpha}_j)}^{\text{Segregation}} + \overbrace{\mathbb{V}ar(\psi_j)}^{\text{Variance of TVA}} + 2\,\overbrace{\mathbb{C}ov(\bar{\alpha}_j, \psi_j)}^{\text{Sorting}}}_{\text{between-teacher variation}} + \underbrace{\mathbb{V}ar(\alpha_{gt} - \bar{\alpha}_j) + \mathbb{V}ar(\epsilon_{gts})}_{\text{within-teacher variation}} \tag{8.2}$$

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$$\underbrace{\mathbb{V}ar(\alpha_{gt} - \bar{\alpha}_j) + \mathbb{V}ar(\epsilon_{gts})}_{\text{within-teacher variation}} \qquad (8.2)$$

- Segregation explains almost half of the student variation

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Decomposition 3 - between vs within school/teacher:

$$
\begin{aligned}
\mathbb{V}ar\,(Y_{gts}) = \underbrace{\begin{array}{l} \mathbb{V}ar(\bar{\psi}_k) \\ +\,2\mathbb{C}ov(\bar{\alpha}_k, \bar{\psi}_k) \\ +\,\mathbb{V}ar(\bar{\alpha}_k) \end{array}}_{\text{between-school}} \; \underbrace{\begin{array}{l} +\;\mathbb{V}ar(\psi_j - \bar{\psi}_k) \\ +\;2\mathbb{C}ov([\bar{\alpha}_j - \bar{\alpha}_k], [\psi_j - \bar{\psi}_k]) \\ +\;\mathbb{V}ar(\bar{\alpha}_j - \bar{\alpha}_k) \end{array}}_{\text{within-school, between-teacher}} \; + \; \mathbb{V}ar(\alpha_{gt} - \bar{\alpha}_j) \quad & \\
& \text{(Sorting)} \\
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+ \underbrace{\mathbb{V}ar(\epsilon_{gts})}_{\text{within-teacher}} \quad & (8.3)
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+\, \mathbb{V}ar(\bar{\alpha}_k) \quad &+ \quad \mathbb{V}ar(\bar{\alpha}_j - \bar{\alpha}_k) && \qquad\qquad \text{(Segregation)} \\
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- The primary sources of segregation and sorting are happening between schools and not within schools.

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Results - What is TVA Associated with?

Table 5: TVA Regressions Using Teacher Characteristics as Regressors

	(1) ψ_j	(2) ψ_j	(3) ψ_j	(4) ψ_j	(5) ψ_j	(6) ψ_j	(7) ψ_j	(8) ψ_j	(9) ψ_j
Subject-specific competences (other)	0.037**								0.0604***
	(0.0153)								(0.0217)
Subject-specific competences (teacher)		-0.0130							0.0498***
		(0.0131)							(0.0183)
Immigrant			-0.108***						-0.1079***
			(0.036)						(0.0405)
SES				0.0002					0.0001
				(0.0003)					(0.0003)
High school GPA					0.010**				0.0022
					(0.0055)				(0.0055)
Teacher college grade						0.0060**			0.0060**
						(0.027)			(0.0026)
Age							0.0024***		-0.0006
							(0.0007)		(0.0010)
Experience								0.0031***	0.0027**
								(0.00086)	(0.0013)
# Observations	726,644	726,644	726,644	726,644	726,644	726,644	726,644	726,644	726,644

Results - Do students perceive high-TVA teachers differently

Table 6: Well-being Regressions Using TVA Scores as Regressors

(a) Teacher-related Questions

Dependent	*Independent* Avr. TVA
Do you like your class	0.0190*** (0.0062)
If there is noise in the classroom, teachers can quickly establish quietness	0.0393*** (0.0061)
Do your teachers help you learn in ways that work?	0.0268*** (0.0063)
The teachers are good at supporting and helping me at school when I need it	0.0147** (0.0062)
The classes are exciting	0.0134** (0.0062)
The teacher makes sure to use the student's ideas in the classes	0.0215*** (0.0063)
Year*X*School*X*Grade FE	X

(b) Non-teacher-related Questions

Dependent	*Independent* Avr. TVA
Do you feel lonely?	0.0092 (0.0063)
Do you like the breaks?	0.0044 (0.0062)
Have you (not) been bullied this school year?	0.0121* (0.0063)
I like the outside areas of my school	0.0058 (0.0060)
The toilets in my school are nice and clean	0.0032 (0.0056)
Year*X*School*X*Grade FE	X

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- (preliminary) results indicate that segregation of students is substantial and that the primary segregation and sorting is happening between schools and not within schools.

- High TVA teachers are associated with higher education performance. Lastly, high TVA teachers are associated with many other well-being outcomes of the student, particularly those related to classroom management.

Future Work

We have several suggestions for future research

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  - Future outcomes?
  - Teacher quality composition associated with educational choice?
  - Interaction effects for teacher characteristics and future outcomes?

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Thanks!

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Appendix - Data & Institutional Setting

Table 1

	(1) Full Sample	(2) Restricted Sample	(3) Largest Connected Sample	(4) Leave-one-out connected sample
Mother:				
Under Education	0.116	0.123	0.125	0.125
Highest Education (yr)	12.70	12.89	12.90	12.90
Total Income	205, 616	217.493	218.167	217.579
Unemployment Length	0.652	0.556	0.561	0.562
Immigrant	0.143	0.148	0.145	0.144
Father:				
Under Education	0.063	0.067	0.068	0.068
Highest Education (yr)	12.51	12.65	12.65	12.64
Total Income	290, 179	305, 642	305, 222	303, 743
Unemployment Length	0.469	0.411	0.414	0.412
Immigrant	0.138	0.141	0.137	0.136
Teacher:				
Female	0.728	0.682	0.698	0.699
Immigrant	0.029	0.029	0.028	0.028
Grade - High-school	7.863	7.868	7.872	7.870
Grade - Teacher Education	7.464	7.441	7.395	7.408
Mother Education (yr)	13.12	13.13	13.09	13.09
Father Education (yr)	13.38	13.39	13.35	13.34
Mother Total Income	220, 603	219, 607	216, 141	216, 436
Father Total Income	319, 628	319, 301	316, 059	316, 492
Age	44.55	44.85	44.88	44.83
Experience	21.51	21.93	21.98	21.96
#Observations	3, 041, 691	1, 108, 436	836, 306	726, 644
#Teachers	37, 697	29, 932	19, 996	16, 534
#Students	853, 593	377, 502	360, 952	270, 214

Estimation I: Lagged-Score Value-Added

- TVA is commonly estimated using "lagged-score value-added" models (Chetty et al. (2014))

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- Identification relies on conditional independence between ψ and ε.
- Hence, sorting is not allowed.
- We propose an alternative model that allows for sorting between teacher quality and student performance.

TVA Estimation - II

- Alternatively, following the seminal Abowd et al. (1999) (AKM), the TWFE value-added model is

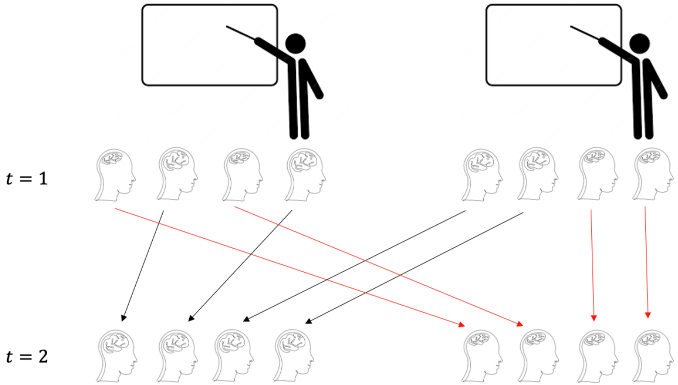
$$Y_{gt} = \alpha_g + \psi_{j(g,t)} + \varepsilon_{gt}$$

- It controls for student-specific effects, α_g, when estimating teacher fixed-effects, ψ_j
- Hence, sorting is allowed between α_g and ψ_j.
- One needs mean independence for valid identification of the TVA.
- Hence, "mobility" needs to be exogenous for ψ_j to be identified.
    - Violated when mobility is determined by time-specific performance.

Estimation

TWFE - Example

- Assume all teachers are equally competent



- One teacher will wrongly be evaluated as better than the other

Estimation

TVA Estimation - III

- We propose controlling for time-specific student effects

$$Y_{gts} = \alpha_{gt} + \psi_{j(g,t,s)} + \varepsilon_{gts}$$

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- Hence, symmetric mobility is allowed as long as it is not subject-specific.

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TV-TWFE - Example

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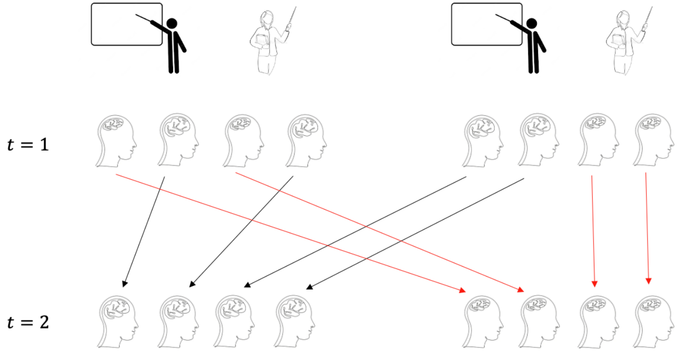

- Teachers are no longer wrongly evaluated.

Estimation

Appendix - Decomposition 2

Table 3: Variance Decomposition - Including Segregation Component

	(1) $\mathbb{V}ar(\bar{\alpha}_j)$ (Segregation)	*(2)* $\mathbb{V}ar(\psi_j)$ (TVA)	*(3)* $\mathbb{C}ov(\bar{\alpha}_j, \psi_j)$ (Sorting)	*(4)* $\mathbb{V}ar(\alpha_{gt} - \bar{\alpha}_j)$ (within teacher)
Variance of test scores: $\mathbb{V}ar\,(Y_{gts})$	0.4155	0.3937	−0.3485	0.6363

Appendix - Decomposition 3

Table 4: Variance Decomposition - Disentangling Within and Between School Components

	(1) Between-school	*(2)* Within-school, between-teacher	*(3)* Within-teacher
(A)	$\mathbb{V}ar(\bar{\psi}_k)$ **[0.2169]**	$\mathbb{V}ar(\psi_j - \bar{\psi}_k)$ **[0.1799]**	$\mathbb{V}ar(\alpha_{gt} - \bar{\alpha}_j)$ **[0.6363]**
(B) Sorting	$\mathbb{C}ov(\bar{\alpha}_k, \bar{\psi}_k)$ **[-0.2049]**	$\mathbb{C}ov([\bar{\alpha}_j - \bar{\alpha}_k], [\psi_j - \bar{\psi}_k])$ **[-0.1439]**	
(C) Segregation	$\mathbb{V}ar(\bar{\alpha}_k)$ **[0.2497]**	$\mathbb{V}ar(\bar{\alpha}_j - \bar{\alpha}_k)$ **[0.1705]**	

Appendix -Robustness I

Table 7: Variance and Sorting Estimates of TVA - Using Alternative Specification

	TWFE Model		TV-TWFE Model	
	(1) Not Bias-Corrected	*(2)* Bias-Corrected	*(3)* Not Bias-Corrected	*(4)* Bias-Corrected
Variance of TVA: $\mathbb{V}ar\left(\hat{\psi}\right)$	0.130	0.066	0.667	0.181
Sorting: $\mathbb{C}ov\left(\hat{\alpha}, \hat{\psi}\right)$	-0.072	-0.018	-0.610	-0.133

Appendix -Robustness II

Table 8: Summary Statistic - Leave-one-out Connected Sample, Individually by Subject

	(1) Sample of Reading Teachers	*(2)* Sample of Math Teachers
Teacher:		
Female	0.836	0.563
Immigrant	0.020	0.036
Grade - High-school	7.889	7.852
Grade - Teacher Education	7.724	7.070
Mother Education (yr)	13.06	13.11
Father Education (yr)	13.28	13.40
Mother Total Income	215,494	217,406
Father Total Income	315,948	317,046
Age	44.65	46.47
Experience	17.03	15.76
Grade - Teacher Education - Reading	6.337	5.623
Grade - Teacher Education - Math	4.526	5.840
#Observations	363,309	363,335

Appendix -Robustness III

Table 9: Variance and Sorting Estimates of TVA - Separate by Subjects and Using TWFE

	TWFE Model - Reading Tests		TWFE Model - Math Tests	
	(1) Not Bias-Corrected	*(2)* Bias-Corrected	*(3)* Not Bias-Corrected	*(4)* Bias-Corrected
Variance of TVA: $\mathbb{V}ar\left(\hat{\psi}\right)$	0.203	0.0429	0.137	0.050
Sorting: $\mathbb{C}ov\left(\hat{\alpha},\hat{\psi}\right)$	-0.158	-0.007	-0.079	0.0004