# Dual Mandate with Nominal Fiscal Policy: the Case for Price-level Targeting 

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August 28, 2023

## Motivation

■ How to achieve price stability under a dual mandate?

- Price level (PLT) or inflation targeting (IT)?
- Main difference stems from 'history dependence' of the central bank's response.
- PLT: unanticipated shocks to inflation lead to corrective action. (Wicksellian rule)
- IT: deviations from the target are treated as bygones as it aims for average, on-target inflation. (Taylor rule)
- This debate between PLT and IT has re-surfaced: previous decade due to ZLB and now due to high inflation.


## This Paper

- Examines the stability of a Tractable-HANK economy under both PLT and IT, allowing for a dual mandate, and with the government choosing the quantity of nominal debt.
- We show that the equilibrium is not saddle path stable when using a Taylor rule (IT) with a positive response to the output gap.
- We highlight the role of real government spending in driving this result.

■ Main takeaway: to meet its dual objective, the central bank needs to resort to PLT and not IT $\Rightarrow$ follow a Wicksellian rule and not Taylor rule.

## Related Literature

Equilibrium determinacy in New Keynesian models with rational expectations:
■ Wicksellian vs Taylor rules in RANK: Woodford (2003).

- Taylor principle with high debt: Natvik (2009).
- Household heterogeneity: Bilbiie (2008), Galí et al. (2004), Giannoni (2014) and Bilbiie (2021).
- Hagedorn (2021): demand for nominal bonds in incomplete-market economies combined with a nominal debt supply rule leads to price-level determinacy even without an interest rate rule.
- Determinacy is achieved jointly by monetary and fiscal policy, even for arbitrary rules.
- Hence, no Leeper (1991) constraints.
- $=$ FTPL
- Hagedorn et al. (2019a), Hagedorn et al. (2019b).
- Bilbiie (2021): analytical application of Hagedorn (2021)'s model. We extend his analysis by considering the presence of government spending, which turns out to be crucial in driving our results, and by allowing for a dual mandate.
- Idiosyncratic uncertainty: Unconstrained savers $(S)$ and constrained hand-to-mouth $(H)$ with exogenous switching
- $P(S \mid S)=s, P(H \mid S)=1-s ; P(H \mid H)=h, P(S \mid H)=1-h$.
- Focus on stationary equilibria where the mass of $H$ is the unconditional probability: $\lambda=\frac{1-s}{2-s-h}$; mass of $S$ is $(1-\lambda)$.
- The $H$ agents do not save but they make an optimal labour supply decision determining their wage income. The $S$ agents supply labour, save and also receive profits.

■ Insurance: 'Full' within type but 'limited' across types.

- Two assets: Stocks (illiquid) and bonds (liquid, used for self-insurance)
- Positive liquidity: government-provided

■ Fiscal Policy: Quantity nominal debt rule (Hagedorn (2021)). fiscal policy is committed to satisfying the present value budget constraint at all times

## Tractable-HANK: log-linearized around $G^{N}=0$

$$
\begin{aligned}
& c_{t}^{R}=\delta E_{t} c_{t+1}^{R}+A_{1}\left[\zeta_{1}\left(g_{t}^{N}-p_{t}\right)+\zeta_{2}\left(E_{t} g_{t+1}^{N}-E_{t} p_{t+1}\right)\right]+ \\
& +A_{2}\left[\zeta_{1}\left(t_{t}^{N}-p_{t}\right)+\zeta_{2}\left(E_{t} t_{t+1}^{N}-E_{t} p_{t+1}\right)\right]+ \\
& +A_{3}\left[\zeta_{1}\left(b_{t}^{N}+r_{t-1}^{N}-p_{t}\right)+\zeta_{2}\left(E_{t} b_{t+1}^{N}+r_{t}^{N}-E_{t} p_{t+1}\right)\right]-A_{4}\left(r_{t}^{N}-E_{t} \pi_{t+1}\right)
\end{aligned}
$$

IS equation

Philips curve

$$
\pi_{t}=\beta E_{t} \pi_{t+1}+\kappa(\phi+\sigma) c_{t}^{R}+\kappa \phi\left(g_{t}^{N}-p_{t}\right)
$$

Gov. bgc

$$
b_{t+1}^{N}=\frac{1}{\beta}\left(b_{t}^{N}+r_{t-1}^{N}\right)+\frac{g_{t}^{N}}{B^{N}}-\frac{T^{N}}{\beta^{N}} t_{t}^{N}
$$

Debt rule

$$
b_{t+1}^{N}=\Phi_{B P} p_{t}+\Phi_{B B} b_{t}^{N}+\Phi_{B G} g_{t}^{N}+\varepsilon_{b^{N}}^{B}
$$

Price level

$$
p_{t}=\pi_{t}+p_{t-1}
$$

Gov. spending $\quad g_{t}^{N}=\rho g_{t-1}^{N}+\varepsilon_{t}^{G}$

TR (IT)

$$
r_{t}^{N}=\Phi_{\pi} \pi_{t}+\Phi_{y}\left(c_{t}^{R}+\left(g_{t}^{N}-p_{t}\right)\right)+\varepsilon_{t}^{M}
$$

or $\operatorname{WR}(\mathrm{PLT}) \quad r_{t}^{N}=\Phi_{p} p_{t}+\Phi_{y}\left(c_{t}^{R}+\left(g_{t}^{N}-p_{t}\right)\right)+\varepsilon_{t}^{M}$

## Tractable-HANK: Parameters calibration

| Parameter |  | Value |
| :--- | :--- | :--- |
| Discount factor | $\beta$ | 0.99 |
| Inverse IES | $\sigma$ | 1 |
| Inverse Frisch elasticity of labour supply | $\phi$ | 1 |
| Elasticity of substitution between varieties | $\eta$ | 6 |
| Share of Hand to Mouth Agents | $\lambda$ | 0.27 |
| Probability of staying saver | $s$ | 0.98 |
| Coefficient of $\Pi$ in Taylor rule | $\Phi_{\pi}$ | $\in(0,3)$ |
| Coefficient of $P$ in Wicksellian rule | $\Phi_{p}$ | $\in(0,3)$ |
| Coefficient of $Y$ in Taylor/Wicksellian rule | $\Phi_{y}$ | $\in(0,1)$ |
| Coefficient of $B_{t}^{N}$ in fiscal rule | $\Phi_{B B}$ | 0 |
| Coefficient of $G_{t}^{N}$ in fiscal rule | $\Phi_{B G}$ | 0 |
| Coefficient of $P_{t}$ in fiscal rule | $\Phi_{B P}$ | 0 |
| Steady-state price level | $P$ | 1 |
| Steady-state hours | $\bar{N}$ | 0.33 |
| Steady-state nominal debt-to-GDP ratio | $B_{y}$ | 0.57 |
| Steady-state nominal government spending-to-GDP ratio | $G^{N} / Y$ | 0.2 |
| Rotemberg price adjustment parameter | $\xi$ | 42.68 |

## Stability


(a) Taylor rule case

(b) Wicksellian rule case

Figure 1: Stability region for different values of $\Phi_{\pi}, \Phi_{p}$ and $\Phi_{y}$

## Stability without $G^{R}$


(a) Taylor rule case

(b) Wicksellian rule case

Figure 2: Model without $G_{t}^{N}$ : Stability region for different values of $\Phi_{\pi}, \Phi_{p}$ and $\Phi_{y}$

## Intuition

■ + Demand shock: increase in economic activity $\Rightarrow H_{t} \uparrow, \mathbb{M} \downarrow, \uparrow P$

- With $B_{t+1}^{N}=B^{N}, G_{t}^{N}=G^{N}$ then $T_{t}^{N}=T^{N}$.
- $\downarrow G_{t}^{R}=\frac{G_{t}^{N}}{P_{t}} \& \downarrow \frac{T_{t}^{N}}{P_{t}} \& \downarrow \frac{B_{t+1}^{N}}{P_{t}}$.
- What about $C_{t}^{R}$ ?
- $\uparrow$ income effect from lower taxes.
- $\downarrow$ precautionary savings motive.
- ?: $\downarrow(\uparrow)$ substitution effect from $\uparrow(\downarrow)$ interest rate.

■ $\underbrace{Y_{t}^{R}}_{?}=\underbrace{C_{t}^{R}}_{\uparrow}+\underbrace{\frac{G_{t}^{N}}{P_{t}}}_{\downarrow}$

- We may end up with lower actual economic activity as private demand and public demand are not perfect substitutes.
- This might already wipe out some of the initial increase without any intervention. Real government spending acts as an automatic (de)stabilizer.


## Intuition

■ However, if this mechanism is strong enough $\downarrow Y_{t}^{R}$ requiring the central bank to lower interest rates to stimulate demand and discourage savings.

- $\downarrow \frac{B_{t+1}^{N}}{P_{t}}$ : lower value of households savings $\Rightarrow \downarrow C_{t+1}^{R}$. This would call for lower interest rates.
- While the orgiginal increase in prices (and inflation) require higher interest rates, lower goods demand and high saving demand need lower interest rates to clear the goods and the asset market.
- With such a trade-off, with dual mandate and IT, the increase in the nominal rates will be less than the case where the Central Bank only targets inflation.
- This will keep $P_{t}$ high, pushing $\downarrow G_{t}^{R}=\frac{G_{t}^{N}}{P_{t}}, \downarrow Y_{t}^{R}$, hence output stabilization would not be possible, leading to instability.


## Intuition

- Without $G_{t}^{R}$ then $\underbrace{Y_{t}^{R}}_{\uparrow}=\underbrace{C_{t}^{R}}_{\uparrow}$
- The difference between Taylor and Wicksellian rules come from the difference in history dependence.

■ PLT: "bygones are not bygones". This ensures the stability of real government spending.

- IT: is a growth target and not a level target, and implies a different level of prices.


## Robustness

- Closed form solutions in a simplified version of the model. ©details
- Results are robust to:

1. Interest rate smoothing
2. Endogenous fiscal rules
3. Fiscal policy setting taxes instead of debt
4. Sticky wages
5. Sensitivity of parameters like high levels of debt to GDP ratios...

## Conclusion

- Incomplete markets coupled with fiscal rule choosing the quantity of nominal debt ensures price level determinancy even with an interest rate peg. (Hagedorn (2021))
- This significantly expands the set of (Monetary and Fiscal) policies that can be evaluated.

■ We show that this result is retained (breaks down) when the Central Bank follows a Wicksellian (Taylor) rule with a 'dual mandate'.

- We highlight the crucal role of real government spending in driving these results.
- Policy implication: Central banks with a dual mandate might need to consider price level targeting.


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## Appendix

## Households

- At the beginning of the period, the family head pools resources within the island. The aggregate shocks are revealed and the family head determines the consumption/saving choice for each household in each island.

■ Then households learn their next-period participation status and have to move to the corresponding island accordingly, taking only (equally-split fraction of) bonds with them from the current island. No transfers after agents know their next period state.

- $B_{t}^{R, j}$ is the per-capita, real, beginning-of-period- $(t+1)$ bonds on island $j=S, H$, after the consumption-saving choice, and also after changing state and pooling.
■ $Z_{t}^{R, j}$ is the per-capita, real, end-of-period- $(t)$ values: after the consumption-saving choice but before agents move across islands.
- The flow of bonds across islands:

$$
\begin{gather*}
\mathbf{B}_{t+1}^{R, S}=(1-\lambda) B_{t+1}^{R, S}=(1-\lambda) s Z_{t+1}^{R, S}+\lambda(1-h) Z_{t+1}^{R, H}  \tag{1}\\
\mathbf{B}_{t+1}^{R, H}=\lambda B_{t+1}^{R, H}=(1-\lambda)(1-s) Z_{t+1}^{R, S}+\lambda h Z_{t+1}^{R, H} \tag{2}
\end{gather*}
$$

## Households' problem - I

$$
\begin{array}{r}
W\left(\frac{B_{t}^{N, S}}{P_{t-1}}, \frac{B_{t}^{N, H}}{P_{t-1}}, \Omega_{t}\right)=\max _{\left\{C_{t}^{R, S}, C_{t}^{R, H}, Z_{t+1}^{R, S}, Z_{t+1}^{R, H}, \Omega_{t+1}\right\}}\left[(1-\lambda) U\left(C_{t}^{R, S}, N_{t}^{S}\right)+\lambda U\left(C_{t}^{R, H}, N_{t}^{H}\right)\right] \\
\\
+\beta E_{t} W\left(\frac{B_{t+1}^{N, S}}{P_{t}}, \frac{B_{t+1}^{N, H}}{P_{t}}, \Omega_{t+1}\right)
\end{array}
$$

subject to:

$$
\begin{gather*}
C_{t}^{R, S}+Z_{t+1}^{R, S}+V_{t} \Omega_{t+1}^{R}=W_{t}^{R} N_{t}^{S}+R_{t-1}^{N} \frac{P_{t-1}}{P_{t}} \frac{B_{t}^{N, S}}{P_{t-1}}+\Omega_{t}^{R}\left(V_{t}+D_{t}^{R}\right)-\frac{T_{t}^{N, S}}{P_{t}}  \tag{3}\\
C_{t}^{R, H}+Z_{t+1}^{R, H}=W_{t}^{R} N_{t}^{H}+R_{t-1}^{N} \frac{P_{t-1}}{P_{t}} \frac{B_{t}^{N, H}}{P_{t-1}}-\frac{T_{t}^{N, H}}{P_{t}}  \tag{4}\\
Z_{t+1}^{R, S}, Z_{t+1}^{R, H} \geq 0 \tag{5}
\end{gather*}
$$

as well as laws of motion for bond flows (equations 1 and 2).

## Households' problem - II

- To get a simple equilibrium representation, the focus is on equilibria where the constraint of $S$ agents never binds and the constraint of $H$ always binds.
- The household's utility function takes the form:

$$
\begin{equation*}
U\left(C_{t}^{R, j}, N_{t}^{j}\right)=\frac{\left(C_{t}^{R, j}\right)^{1-\sigma}}{1-\sigma}-\nu \frac{\left(N_{t}^{j}\right)^{1+\phi}}{1+\phi} \tag{6}
\end{equation*}
$$

- Define $\Gamma=\frac{C^{R, S}}{C^{R, H}}$ as the steady-state consumption inequality. The household's intertemporal decisions are captured by the (log-linearized) self-insurance equation of $S$ :

$$
\begin{equation*}
c_{t}^{R, S}=\frac{(1-s) \Gamma^{1 / \sigma}}{(1-s) \Gamma^{1 / \sigma}+s} E_{t} c_{t+1}^{R, H}+\frac{s}{(1-s) \Gamma^{1 / \sigma}+s} E_{t} c_{t+1}^{R, S}-\sigma^{-1}\left(r_{t}^{n}-E_{t} \pi_{t+1}\right) \tag{7}
\end{equation*}
$$

- Here, $(1-s)$ is the probability to switch to a bad state next period, which generates a precautionary demand for bonds.


## Firms

- A continuum of monopolistically competitive firms produce differentiated goods $Y_{t}^{R}(k)$ using labor $N_{t}(k)$ according to a constant-returns production function $Y_{t}^{R}(k)=N_{t}(k)$.
- Firms set prices optimally under Rotemberg adjustment costs, parameterized by $\xi$.

■ Subsidy in place to ensure marginal cost pricing, which ensures zero profits in steady-state.

- This price-setting behaviour implies the following (log-linearised) Philips Curve:

$$
\begin{equation*}
\pi_{t}=\beta E_{t} \pi_{t+1}+\kappa \phi y_{t}+\kappa \sigma c_{t}^{R}+u_{t} \tag{8}
\end{equation*}
$$

where $\kappa=\frac{(\eta-1)}{\xi}, \pi_{t}$ is the level of inflation, $y_{t}^{R}$ is the level of output in the economy.

## Government

- Spends a nominal amount, $G_{t}^{N}$, levies lump-sum nominal taxes, $T_{t}^{N}$ and supplies nominal liquid bonds, $B_{t}^{N}$.

■ Distinction between nominal and real fiscal variables: $G_{t}^{R}=\frac{G_{t}^{N}}{P_{t}}, T_{t}^{R}=\frac{T_{t}^{N}}{P_{t}}, B_{t}^{R}=\frac{B_{t}^{N}}{P_{t}}$.

- The log-linearized fiscal block of the model is:

$$
\begin{gather*}
b_{t+1}^{N}=\frac{1}{\beta}\left(b_{t}^{N}+r_{t-1}^{N}\right)+\frac{g_{t}^{N}}{B^{N}}-\frac{T^{N}}{B^{N}} t_{t}^{N}  \tag{9}\\
b_{t+1}^{N}=\Phi_{B B} b_{t}^{N}+\Phi_{B P} p_{t}+\Phi_{B G} g_{t}^{N}+\varepsilon_{b^{N}}^{B}  \tag{10}\\
g_{t}^{N}=\rho_{g} g_{t-1}^{N}+\varepsilon_{t}^{G} \tag{11}
\end{gather*}
$$

- Committed to satisfying the PV budget constraint at all times, for all price levels.


## Market Clearing

- Aggregation of consumption and labour supply between the two types of households gives:

$$
\begin{gather*}
C_{t}^{R}=\lambda C_{t}^{R, H}+(1-\lambda) C_{t}^{R, S}  \tag{12}\\
N_{t}=\lambda N_{t}^{H}+(1-\lambda) N_{t}^{S} \tag{13}
\end{gather*}
$$

- Resource Constraint of the economy:

$$
\begin{equation*}
Y_{t}^{R}=C_{t}^{R}+\frac{G_{t}^{N}}{P_{t}} \tag{14}
\end{equation*}
$$

## Closed form solution

Simplifying assumptions:

1. full consumption insurance in steady-state $\Gamma=\frac{C^{S}}{C^{H}}=1$.
2. Firms are myopic $\Rightarrow$ static New Keynesian Philips curve:

$$
\pi_{t}=p_{t}-p_{t-1}=\kappa(\phi+\sigma) c_{t}^{R}+\kappa \phi\left(g_{t}^{N}-p_{t}\right)
$$

3. Nominal debt fixed at steady-state, $b_{t}^{N}=0$.
4. Forward looking MP rules:

- TR: $r_{t}^{N}=E_{t} \pi_{t+1}+\Phi_{y} E_{t} y_{t+1}^{R}$.
- WR: $r_{t}^{N}=E_{t} p_{t+1}+\Phi_{y} E_{t} y_{t+1}^{R}$.


## Stability

- Assuming further log utility $(\sigma=1)$ and unitary Frisch elasticity of labor supply ( $\phi=1$ ), the economy is now described by a single second-order difference equation in terms of the price-level:

$$
F\left(E_{t} p_{t+1}, p_{t}, p_{t-1}\right)=F^{*}
$$

Table 1: Acceptable domain of parameters

| Parameter | Restriction |
| :--- | :--- |
| Discount factor | $0<\beta<1$ |
| Slope of the NKPC | $\kappa>0$ |
| Probability to stay type S | $0<s<1$ |
| Steady-state debt to GDP ratio | $0<B_{y}<1$ |
| Monetary policy rule coefficient on output | $\Phi_{y} \geq 0$ |
| Mass of type H | $(1-s) \leq \lambda<\lambda^{*}<1$ |

## Taylor Rule case

- The IS equation is

$$
\begin{equation*}
Q_{2} p_{t+1}+Q_{1} p_{t}+Q_{0} p_{t-1}=Q^{*} \tag{15}
\end{equation*}
$$

- The characteristic polynomial of the difference equation (15) is:

$$
\begin{equation*}
J(x)=Q_{2} x^{2}+Q_{1} x+Q_{0} \tag{16}
\end{equation*}
$$

- The necessary and sufficient condition for determinacy, in this case, is $J(1) J(-1)<0$ (Woodford (2003)).


## Stability



Figure 3: Taylor rule case: Characteristic polynomial for different values of $\lambda$ and $\Phi_{y}$

## Stability

$$
\begin{align*}
J(-1) & =\frac{\lambda\left(2\left(-3 \kappa \lambda+\kappa+6 \lambda+\frac{\kappa(2 \lambda+s-1)}{\beta}-2 \kappa s+2 s-4\right)+(\kappa-2)(\lambda-1) \Phi_{y}\right)}{2 \kappa(\lambda-1) \lambda}  \tag{17}\\
& +\frac{B_{y}(2 \lambda+s-1)\left((\kappa-2)(\lambda+s-1) \Phi_{y}-2 \kappa(2 \lambda+s-1)\right)}{\beta 2 \kappa(\lambda-1) \lambda} .
\end{align*}
$$

- $\lambda<1$ is enough to ensure that both denominators are negative.
- Given $\lambda \geq(1-s), B_{y}>0$ and $\kappa>0$ the sum of the two numerators is $<0$ if

$$
\lambda<\lambda_{T}^{\star}\left(\Phi_{y}, \kappa, \beta, s, B_{y}\right)
$$

- $\lambda_{T}^{\star}$ is the threshold for Inverted Aggregate Demand Logic in this model.
- if $\lambda<\lambda_{T}^{\star}\left(\Phi_{y}, \kappa, \beta, s, B_{y}\right)$ then $J(-1)>0$.


## Stability

$$
\begin{equation*}
J(1)=\frac{2 \beta \lambda(1-s)+2(s-1) B_{y}(-\beta \lambda+\lambda+s-1)}{2 \beta(\lambda-1) \lambda}+\frac{\Phi_{y}\left(\beta(\lambda-1) \lambda+(s-1) B_{y}(\lambda+s-1)\right)}{2 \beta(\lambda-1) \lambda} . \tag{18}
\end{equation*}
$$

- The first term is unambiguously negative.
- To get $J(1)<0$ we need the second term to be either negative or smaller than the first one. But the latter is not true unless $\Phi_{y}=0$.
- If $\Phi_{y}>0$ the second term is larger than the first one.
- Hence $\Phi_{y}>0$, we require the term multiplied by $\Phi_{y}$ in (18) to be negative.
- This is possible when its numerator is positive, that is:

$$
\begin{equation*}
\beta(\lambda-1) \lambda+(s-1) B_{y}(\lambda+s-1) \geq 0 \tag{19}
\end{equation*}
$$

For (19) to hold in when $(1-s) \leq \lambda$, we require $B_{y}(\lambda+s-1) \leq(\lambda-1) \beta$ which is a contraddiction.

- Therefore to get stability we need $\Phi_{y}=0$ for the Taylor rule case.


## Kuhn-Tucker Conditions

The set of equations governing the bond-holding decision of $S$ island are:

$$
\begin{gather*}
U^{\prime}\left(C_{t}^{R, S}\right) \geq \beta E_{t}\left\{R_{t+1}\left[s U^{\prime}\left(C_{t+1}^{R, S}\right)+(1-s) U^{\prime}\left(C_{t+1}^{R, H}\right)\right]\right\}  \tag{20}\\
\text { and } 0=Z_{t+1}^{R, S}\left[U^{\prime}\left(C_{t}^{R, S}\right)-\beta E_{t}\left\{R_{t+1}\left[s U^{\prime}\left(C_{t+1}^{R, S}\right)+(1-s) U^{\prime}\left(C_{t+1}^{R, H}\right)\right]\right\}\right] \tag{21}
\end{gather*}
$$

where we use the Fisher relation, $R_{t}=R_{t}^{N} \frac{P_{t-1}}{P_{t}}$.
The set of equations governing the bond-holding decision of $H$ island are:

$$
\begin{gather*}
U^{\prime}\left(C_{t}^{R, H}\right) \geq \beta E_{t}\left\{R_{t+1}\left[(1-h) U^{\prime}\left(C_{t+1}^{R, S}\right)+h U^{\prime}\left(C_{t+1}^{R, H}\right)\right]\right\}  \tag{22}\\
\text { and } 0=Z_{t+1}^{R, H}\left[U^{\prime}\left(C_{t}^{R, H}\right)-\beta E_{t}\left\{R_{t+1}\left[(1-h) U^{\prime}\left(C_{t+1}^{R, S}\right)+h U^{\prime}\left(C_{t+1}^{R, H}\right)\right]\right\}\right] . \tag{23}
\end{gather*}
$$

The equation corresponding to illiquid shares is:

$$
\begin{equation*}
U^{\prime}\left(C_{t}^{R, S}\right) \geq \beta E_{t}\left\{\frac{V_{t+1}+D_{t+1}^{R}}{V_{t}} U^{\prime}\left(C_{t+1}^{R, S}\right)\right\} \tag{24}
\end{equation*}
$$

with $\Omega_{t+1}^{R}=\Omega_{t}^{R}=(1-\lambda)^{-1}$.

## Coefficients of the IS equation

$$
\begin{gathered}
\zeta_{1}=\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}, \\
\zeta_{2}=\frac{(\Gamma(\lambda-1)-\lambda)\left((\lambda-1)(s-1) \Gamma^{1 / \sigma}+\lambda s(\Gamma(\lambda-1)-\lambda)\right)}{(s-1) \Gamma^{1 / \sigma}-s}, \\
A_{1}=\frac{1+\phi}{1-\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}(\sigma+\phi+1)}, \\
A_{2}=\frac{-T_{y}}{1-\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}(\sigma+\phi+1)}, \\
A_{3}=\frac{(1-s) B_{y}}{\lambda \beta\left(1-\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}(\sigma+\phi+1)\right)}, \\
A_{4}=\frac{(1-\lambda)}{\sigma\left(1-\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}(\sigma+\phi+1)\right)}, \\
\delta=\frac{s\left((\Gamma(\lambda-1)-\lambda)(\sigma+\phi+1)\left((\lambda-1) \Gamma^{1 / \sigma}+\Gamma(\lambda-1) \lambda-\lambda^{2}\right)-1\right)-(\lambda-1)(\Gamma(\lambda-1)-\lambda) \Gamma^{1 / \sigma}(\sigma+\phi+1)}{\left(s-(s-1) \Gamma^{1 / \sigma}\right)\left(\lambda(\Gamma(-\lambda)+\Gamma+\lambda)^{2}(\sigma+\phi+1)-1\right)} .
\end{gathered}
$$

