Decomposing HANK

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Abstract

This paper introduces a decomposition approach for analyzing the responses of macroeconomic variables to aggregate shocks in incomplete market models. The approach decomposes the responses into RANK and redistribution effects. The RANK effects capture the response of a (constructed) representative agent, while the redistribution effects capture the response of the heterogeneous-agent model to a transfer scheme among agents. The redistribution effects are further decomposed into interest rate exposure, income exposure, liquidity, asset price, and tax exposure channels. I apply this approach to monetary policy shocks and quantitatively measure the contribution of each channel to the deviation of HANK from RANK.

1 Introduction

Heterogeneous-agent models have become increasingly popular in the literature on monetary policy. These models incorporate non-insurable idiosyncratic income shocks and borrowing constraints as standard features. They can capture the realistic heterogeneity of households regarding their wealth position, consumption behavior, and exposure to aggregate shocks. By introducing nominal rigidity, heterogeneous-agent New Keynesian (HANK) models can examine the role of household heterogeneity in the transmission of monetary policy.

As a well-known result, HANK can have amplified/dampened general equilibrium responses to aggregate shocks relative to the representative-agent New Keynesian (RANK) model. When the covariance of agents' marginal propensity of consumption (MPCs) and agents' exposures to the aggregate shock is positive, the consumption responses are amplified; and when that covariance is negative, the consumption responses are dampened.

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An important neutral result is studied in Werning (2015), in which he points out that when agents are equally exposed to the aggregate shock, HANK is 'as if' RANK.

A natural question is, what are the sources of unequal exposures in HANK models? This question is trivial in analytical HANK models because the model-builder constructs the unequal exposures in a stylized way to capture the amplification/dampening mechanism. A well-known example is the Two-agent New Keynesian (TANK) model in Bilbiie (2020). In TANK, Savers receive a larger fraction of the counter-cyclical monopolistic profits than Hand-to-Mouth households. Given Hand-to-Mouth households have a larger MPC than Savers, the counter-cyclical income inequality leads to amplification. Quantitative HANK models, however, need to be solved with complicated numerical methods, making the model challenging to interpret. The sources of unequal exposures are more endogenous and less transparent to evaluate in quantitative models. It is not clear how to evaluate the contribution of different redistribution channels to the amplified/dampened responses.

This paper approaches this issue by interpreting the HANK model as a deviation from the RANK model due to the lack of counterfactual transfers to agents. In the case of unequal exposures, I make counterfactual transfers to agents to remove the redistribution induced by the aggregate shock. With the counterfactual transfers, an aggregation result similar to Werning (2015) arises, and the economy can be characterized with only aggregate conditions, which is referred to as the 'representative-agent' ('RA') economy. Then by examining the counterfactual transfers, we can have a clear interpretation of the redistribution channels in HANK and quantitatively evaluate the contribution of each channel to the amplified/dampened responses.

Specifically, considering a perfect-foresight HANK economy starting from its stationary equilibrium and its responses to one-time unexpected monetary policy shocks $\boldsymbol{\epsilon} \equiv \{\epsilon_t\}_{t=0}^{\infty}$ which follow a mean-reverting process. For variable x, we can define its impulse responses as

$$\tilde{x}_t \equiv x_t(\boldsymbol{\epsilon}) - x^*,$$

where $x_t(\boldsymbol{\epsilon})$ is x's realization at time t following the monetary policy shocks and x^* denotes its value in the stationary equilibrium. At the same time, consider counterfactual transfers to households $\boldsymbol{\omega} = \{\omega(z^t), \forall z^t \in Z^t\}_{t=0}^{\infty}$ where $\omega(z^t)$ is the transfer received by the household conditional on its history of idiosyncratic shocks $z^t = (z_0, z_1, \cdots, z_t)$. The key observation is that, by appropriately constructing the transfers, the redistribution induced by the monetary policy shocks can be removed, and the dynamics of aggregates can be characterized with only aggregate conditions. The heterogeneous-agent economy has an equivalent representative-agent representation. Given this observation, the impulse responses of HANK $\tilde{x}_t$ can be decomposed into the RANK effects $\tilde{x}_t^A$ and the redistribution effects $\tilde{x}_t^R$. The RANK effects are the model's responses to two sequences of shocks: the monetary policy shocks $\boldsymbol{\epsilon}$ and the transfer shocks $\boldsymbol{\omega}$:

$$\tilde{x}_t^A \equiv x_t(\boldsymbol{\epsilon}, \boldsymbol{\omega}) - x^*.$$

And the redistribution effects $\tilde{x}_t^R$ are the model's responses to the negative of the transfer shocks $-\boldsymbol{\omega}$.

$$\tilde{x}_t^R \equiv x_t(\mathbf{0}, -\boldsymbol{\omega}) - x^*.$$

Then to the first order,

$$\tilde{x}_t = \tilde{x}_t^A + \tilde{x}_t^R.$$

By examining the sources of transfers, I further decompose the redistribution effects into five channels: income exposure, interest rate exposure, asset price, tax exposure, and liquidity channels.

$$\tilde{x}_t^R = \tilde{x}_t^{income} + \tilde{x}_t^{interest} + \tilde{x}_t^{asset} + \tilde{x}_t^{tax} + \tilde{x}_t^{liquidity}$$

Under empirical-plausible model calibrations, the income exposure, interest rate exposure, asset price, and liquidity channels amplify the consumption responses, while the tax exposure channel dampens the consumption responses. For an expansionary monetary policy shock, the income exposure channel operates through the higher exposure of high-MPC workers. The interest rate exposure channel operates through the redistribution between debtors and creditors from an interest rate cut, while debtors have higher MPCs than creditors. The asset price increases benefit asset sellers and hurt asset buyers. Asset sellers are households that experience negative income shocks, and asset buyers are households that experience positive income shocks. On average, the former have a higher MPC than the latter. Regarding the change in tax payment, however, high-MPC workers benefit less from the tax reduction, as they pay a smaller share of the aggregate tax. The liquidity channel is muted with a constant path of government debt. In incomplete market economies, public debt serves as private liquidity. The cyclical asset supply influences the liquidity in the economy. The paper demonstrates that, under uniform taxation, the liquidity channel corresponds to borrowing constraint shocks induced by the time-varying path of government debt. Intuitively, more liquidity is injected into the economy following an expansionary shock, and the borrowing conditions are relaxed, amplifying consumption response.

In the quantitative analysis, I consider the model's response to 25bp monetary policy shock with the persistence of 0.61. The redistribution effects amplify the responses of output and consumption and dampen the responses of investment and real interest rates. On impact, the consumption in HANK rises by 0.5%. Regarding the decomposition, the RANK effects account for 0.4% of the consumption increase, the interest exposure channel for 0.065%, the income exposure channel for 0.045%, the liquidity channel for 0.015%, and the tax exposure channel for -0.025%. The effects of the asset price channel are close to zero. The decomposition for output is qualitatively similar to the decomposition of consumption. Quantitatively, the redistribution effects are of a smaller magnitude because if the redistribution from one channel amplifies the consumption response, it will dampen the investment response. The net effects on output are smaller than on consumption.

The decomposition approach proposed in this paper is mostly related to Werning (2015). Werning (2015) analyzes cases where the incomplete economy can be aggregated as an 'as if' representative agent economy, which corresponds to the RANK effects defined in this paper. For more general cases where the 'as if' result does not hold, I use counterfactual transfers to preserve the aggregation. I then interpret the heterogenous-agent economy as a deviation from the 'as if' representative agent economy due to the absence of counterfactual transfers. Berger, Bocola and Dovis (2019) exploit the idea that a representative agent representation with wedges can represent the heterogeneous-agent economy to quantify the implications of risk sharing. However, this approach does not identify the specific economic mechanisms that generate the endogenous and time-varying wedges, which is the main objective of this paper. By examining the origin of the wedges, I can quantify the contributions of specific economic mechanisms.

This paper builds on the earlier work of Hagedorn (2020), who also employ counterfactual transfers to construct 'as if' agents to examine price determinacy in incomplete market models. Hagedorn et al. (2019) use counterfactual transfers to assess the imbalance between aggregate demand and aggregate supply in the context of the forward guidance puzzle. However, this paper has a different objective: I apply this approach to decompose the effects of aggregate shocks in general equilibrium. For this aim, I assume those transfers depend on the history of individual productivity shocks. I also show that the transfer profile is not unique, in contrast to Hagedorn (2020). The quantification reveals important insights for developing HANK models. For instance, given the interpretation of the liquidity channel, the deficit-financed fiscal rule assumed in Kaplan, Moll and Violante (2018) implies a counter-cyclical liquidity supply, which is not intuitive. I instead assume that the public debt increases following an expansionary shock, capturing the increasing asset supply and relaxed borrowing conditions for households.

In section 2, I illustrate the idea of the decomposition in a simple TANK model from Bilbiie (2020). In section 3, I introduce the decomposition approach to a HANK model and discuss the source of the counterfactual transfers. In section 4, I add investment to the model. In section 5, I quantitatively implement the decomposition.

2 Aggregate Shock Decomposition: A TANK Example

The idea of the decomposition approach can be easily illustrated in the Two-Agent New Keynesian (TANK) model. For comparison, the TANK model used here is kept identical to Bilbiie (2020). I briefly describe the environment and characterize the equilibrium conditions. Details of the model can be found in Bilbiie (2020).

2.1 Model description

There are two types of households with total unit mass. A fraction of λ households is hand-to-mouth H, who are excluded from financial markets and consume their current income. The budget constraint of H is given by

$$C_t^H = W_t N_t^H + D_t^H$$

where W_t is real wage, H_t is H's labor supply, and D_t^H is the firm's profits received by H. The remaining fraction $1-\lambda$ of households are savers S, trading one-period riskless real bonds. The budget constraint of S is given by

$$C_t^S + \frac{B_{t+1}}{1+r_t} = B_t + W_t N_t^S + D_t^S$$

where N_t^S is S's labor supply and D_t^S is the firm's profits received by S. All households maximize their discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$ subject to the sequence of their budget constraints. The utility function takes the form $U(C,N) = C^{1-1/\sigma}/(1-\sigma) - N^{1+\varphi}/(1+\varphi)$.

The supply side is standard. There is a continuum of firms, and each firm produces a differentiated good with linear technology $Y_t(i) = A_t N_t(i)$. In each period, firms have the possibility of θ to reset the price. The demand for each good is $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$ where $P_t = (\int_0^1 P_t(i)^{1-\epsilon} di)^{1/(1-\epsilon)}$ is the aggregate price index and Y_t is the aggregate output. The standard supply-side implies the canonical representation of the log linearized Philips Curve: $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$ where y_t is the log deviation of output from steady state.

The government implements standard NK optimal subsidy inducing marginal cost pricing financed by a lump-sum tax on the firms' profits. The profit function is $D_t(i) = (1+\tau) P_t(i) Y_t(i)/P_t - W_t N_t(i) - T_t^F$. With the optimal subsidy, $\tau = 1/(\epsilon - 1)$, firms' steady-state profits are zero. in the stationary equilibrium, households have the same income and consumption.

The central bank conducts monetary policy in the form of the Taylor rule: $i_t = r^* + \phi_\pi \pi_t + \epsilon_t$ where r^* is the steady state real interest rate, and ϵ_t is an exogenous monetary policy shock.

The key assumption in TANK is the distribution rule of the firm's profits. The government redistributes τ^D share of profits to H: $D_t^H = \tau^D D_t / \lambda$, and $1 - \tau^D$ share of profits to S: $D_t^S = (1 - \tau^D) D_t / (1 - \lambda)$. When $\tau^D = \lambda$, H and S receive the same profits, and their income and consumption have the same responses in equilibrium. When $\tau^D \neq \lambda$, TANK deviates from this representative-agent benchmark.

Denote log deviations of variables from their steady-state values except for interest rates by small letters. After imposing the market clearing condition, the aggregate Euler equation of TANK is derived as

$$c_t = E_t c_{t+1} - \delta^{-1} \sigma (r_t - r^*) \tag{1}$$

where $\delta^{-1} = (1 - \lambda)/(1 - \lambda \chi)$ and $\chi = 1 + \varphi(1 - \tau^D/\lambda)$. Though H have no access to financial markets and their consumption does not price the bond, one can infer the quantitative relation between their consumption and interest rates from the relation between H and S's equilibrium consumption.

From the aggregate Euler equation (1), we can see the amplifying/dampening mechanism in TANK. As already mentioned, if $\tau^D = \lambda$, it follows $\chi = 1$ and $\delta^{-1} = 1$. The elasticity of contemporaneous aggregate consumption to interest rates is the same as RANK. In equilibrium, the income and consumption responses of H and S are the same. If $\tau^D < \lambda$, H receive a smaller amount of profits than S. With counter-cyclical profits, it implies that H's consumption responds more than S's consumption. As a weighted sum, aggregate consumption also responds more than S's consumption, and its elasticity to interest rates is larger than the consumption elasticity of S: $\delta^{-1}\sigma > \sigma$. For a given change in real interest rates, the aggregate consumption response in TANK is amplified relative to RANK.

With the full characterization of the equilibrium, I now consider the output response to an exogenous monetary policy shock and decompose the output response in two different ways. I refer to the first decomposition as a *partial equilibrium decomposition*, which is used in the existing literature; and the second one as a *general equilibrium decomposition*, which is the decomposition in this paper. For illustration, here I consider a monetary policy shock that lasts only one period: $E_t \epsilon_{t+1} = 0$. Given a monetary policy shock ϵ_t, the output response of TANK is

$$y_t = -\frac{\delta^{-1}\sigma}{1 + \delta^{-1}\sigma\phi_\pi\kappa} \epsilon_t. \tag{2}$$

In the case of amplifying, $\delta^{-1} > 1$, and the output response is larger (in abstract value) than that in RANK. In the case of dampening, $\delta^{-1} < 1$, the output is less responsive to monetary policy shocks relative to RANK.

2.2 Partial Equilibrium Decomposition

One way to decompose the response of output y_t is to decompose it into substitution effects and income effects as in Auclert (2019) (also phrased as 'direct effects' and 'indirect effects' in Kaplan, Moll and Violante (2018)). The substitution effects are the response of aggregate consumption keeping the income of households unchanged. When interest rates fall, households save less for the future and consume more today due to intertemporal substitution. The income effects are the response of aggregate consumption keeping the interest rates unchanged.

The above decomposition is partial equilibrium in nature: it solves households' partial equilibrium optimization problems. Given an exogenous shock, it does not permit us to solve the equilibrium response of interest rates and output response. Most importantly, it is not closely related to the amplifying/dampening mechanism of TANK. After some algebra, it can be shown that

$$c_t^{sub} = \beta(1 - \lambda\chi)y_t$$
$$c_t^{inc} = [1 - \beta(1 - \lambda\chi)]y_t$$

The sizes of substitution effect c_t^{sub} and income effect c_t^{inc} depend on H's measure λ and the amplifying/dampening parameter χ. For example, in the case of proportional distribution of firm profits ($\tau^D = \lambda$, $\chi = 1$ and $\delta = 1$), the economy's response is equivalent to RANK. But the size of substitution effects simply varies with H's measure λ. Only conditional on H's measure λ, we can infer the parameter χ from the decomposition result.

2.3 General Equilibrium Decomposition

I apply a 'general equilibrium decomposition' to the output response y_t, which consists of **RANK effects** y_t^A and **redistribution effects** y_t^R, such that $y_t = y_t^A + y_t^R$. This decomposition is based on the observation that monetary policy shocks in TANK induce a redistribution between H and S due to their unequal exposures to the countercyclical profits. This affects the income and consumption elasticities of H and S to aggregate output and hence the aggregate response to monetary policy shocks. In a counterfactual scenario where this redistribution is eliminated, TANK behaves the same as RANK. To achieve this scenario, I construct counterfactual lump-sum transfers to households. The difference between TANK and RANK is then attributed to the absence of these transfers.

Let ω_t^H, ω_t^S be the counterfactual transfers to H and S, respectively, that eliminate the redistribution effects of a monetary policy shock ϵ_t. The RANK effects of the shock on output y_t^A are the response of output to the shock and the transfers $(\epsilon_t, \omega_t^H, \omega_t^S)$; and the redistribution effects of the shock on output y_t^R are the response of output to the opposite of the transfers $(-\omega_t^H, -\omega_t^S)$. The counterfactual transfers are purely redistributive: $\lambda\omega_t^H + (1-\lambda)\omega_t^S = 0$, where λ is the fraction of H in the population.

RANK effects. By construction, the RANK effects y_t^A are equal to the output response in RANK:

$$y_t^A = -\frac{\sigma}{1+\sigma\phi_\pi\kappa}\epsilon_t. \tag{3}$$

In RANK effects, S and H have the same consumption response. However, this is inconsistent with the households' budget constraints in TANK, where there is an endogenous redistribution through firms' profit distribution. To eliminate this redistribution, I construct an exogenous transfer scheme between S and H. With lump-sum transfers, the budget constraints of households are

$$c_t^{H,A} = w_t^A + n_t^{H,A} + \frac{\tau^D}{\lambda}d_t^A + \omega_t^H \tag{4}$$

$$c_t^{S,A} = w_t^A + n_t^{S,A} + \frac{1-\tau^D}{1-\lambda}d_t^A + \omega_t^S \tag{5}$$

where ω_t^S and ω_t^H are the transfers (as a percentage of steady state output Y^*) to S and H, respectively. Assuming that both households satisfy their optimal labor supply condition in equilibrium, so $c_t^{S,A} = c_t^{H,A}$ implies $n_t^{S,A} = n_t^{H,A}$[1], the budget constraints require:

$$\omega_t^H = (1-\frac{\tau^D}{\lambda})d_t^A \tag{6}$$

$$\omega_t^S = (1-\frac{1-\tau^D}{1-\lambda})d_t^A \tag{7}$$

With this transfer scheme, S and H have the same consumption response, and the aggregate Euler equation holds.

Redistribution effects. Consider an exogenous transfer shock such that $\lambda T_t^H + (1-\lambda)T_t^S = 0$ where T_t^H and T_t^S are the transfers (as the percentage of steady-state output Y^*) to H and S, respectively. The Appendix shows that the output response of TANK to a transfer shock is

$$y_t = -\frac{1}{\sigma\phi_\pi\kappa+\delta}\frac{1}{1+(\sigma\varphi)^{-1}}T_t^S \tag{8}$$

[1]Otherwise, the transfers also need to compensate for the difference between households' labor income **given** $c_t^{S,A} = c_t^{H,A}$.

To obtain the redistribution effects, I input the negative of the transfers $\{-\omega_t^H, -\omega_t^S\}$ into the model. Letting $T_t^S = -\omega_t^S$,

$$y_t^R = \frac{1-\delta}{\sigma\phi_\pi\kappa + \delta} y_t^A.$$

Discussion. Expressing the output response (2) of TANK y_t in terms of RANK effects (3) y_t^A:

$$y_t = \frac{1+\sigma\phi_\pi\kappa}{\delta + \sigma\phi_\pi\kappa} y_t^A.$$

It can be verified that $y_t = y_t^A + y_t^R$. The output response can be decomposed into RANK and redistribution effects in this linear system. In the case of amplification ($\tau^D < \lambda$, $\chi > 1$ and $\delta < 1$), the redistribution effects are in the same direction as RANK effects, and the total effects are greater than RANK effects (in absolute value). The endogenous redistribution through firms' profit distribution τ^D/λ in TANK amplifies the output response. To see this, consider an expansionary monetary policy shock $\epsilon_t < 0$, from (7) it follows $\omega_t^H < 0$ and $\omega_t^S > 0$. The negative of the transfers $\{-\omega_t^H, -\omega_t^S\}$ subsidize H by taxing S. In TANK, fiscal stimulus in the form of transfers from S to H is itself a policy instrument that stimulates the economy (see Bilbiie, Monacelli and Perotti (2013)). In the case of dampening ($\tau^D > \lambda$, $\chi < 1$ and $\delta > 1$), the negative of the transfers tax H and subsidize S, which will dampen the economy's response. In TANK, the RANK effects are a natural benchmark to evaluate the amplifying/dampening mechanism. When extending this decomposition approach to HANK, I also use the RANK effects as the benchmark.

From this application, one can also see the difference between the general equilibrium decomposition and the partial equilibrium decomposition. Consider the case of proportional distribution of firm profits ($\tau^D = \lambda$, $\chi = 1$ and $\delta = 1$). The general equilibrium decomposition implies zero redistribution effects $y_t^R = 0$. All output response is due to RANK effects regardless of the mass of hand-to-mouth households because, in equilibrium, S and H have the same responses. This result contrasts with the partial equilibrium decomposition, in which the relative strength of substitute effects and income effects is sensitive to the mass of constrained households.

One drawback of the partial equilibrium decomposition is that it does not capture the critical amplifying/dampening mechanism in TANK and, more generally, the transmission mechanism of monetary policy in heterogeneous-agent New Keynesian (HANK) models. The partial equilibrium decomposition mainly captures the heterogeneous MPCs across households rather than the correlation between households' MPCs and income. The main task of this paper is to address this concern. In the following sections, I extend the above decomposition to HANK and discuss under what conditions the general equilibrium dynamics of HANK deviate from RANK.

3 Decomposing HANK

This section introduces the general equilibrium decomposition approach for incomplete-market models. I consider a one-asset HANK model and decompose the economy's response to monetary policy shocks. I describe the model and define the decomposition in section 3.1 and 3.2. I discuss the RANK and redistribution effects in section 3.3 and 3.4. I assume that (i) agents have perfect foresight about the evolution of aggregate shocks; (ii) in the infinite horizon, the economy is back to its initial stationary equilibrium.

3.1 Model Description

The model is a heterogeneous-agent version of the textbook New Keynesian model similar to McKay, Nakamura and Steinsson (2016). Time is discrete and runs to the infinite horizon. The economy is populated by households, firms, a government, and a monetary policy authority. In this economy, households face idiosyncratic uncertainty on incomes and have access to one-period risk-less government bonds, subject to exogenous borrowing constraints. There is price stickiness in the firm's price setting. The government collects taxes from households to pay interest on the debt. A monetary authority operates a Taylor rule, and I analyze the economy's response to innovation to this Taylor rule.

Households. There is a unit continuum of households that face idiosyncratic productivity shocks $z_t \in Z_t$. Let $z^t = (z_0, z_1, \cdots, z_t)$ be a history of idiosyncratic states up to period t. For ease of notation, the initial state z_0 also indexes the initial bond holdings. At $t = 0$, the economy inherits an initial distribution over idiosyncratic states and bonds $\Phi_0(z_0)$. The stochastic process then induces a distribution $\Phi_t(z^t)$ over histories $z^t \in Z^t$. Households are infinitely lived and have preferences over consumption $c(z^t)$ and labor supply $n(z^t)$ given by the utility function

$$E[\sum_{t=0}^{\infty} \beta^t u(c(z^t), n(z^t))] \tag{9}$$

where β is the subjective discount factor. I also assume that the period utility function is given by

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \varphi \frac{n^{1+\nu}}{1+\nu}$$

Households derive utility from consumption and dis-utility from working. Households face the budget constraints

$$c(z^t) + b(z^t) = (1 + r_t) b(z^{t-1}) + W_t z_t n(z^t) + \pi(z) - \tau(z^t) \tag{10}$$

for all $t = 0, 1, \cdots$ and histories $z^t \in Z^t$. Households face labor income risks so that if they work $n(z^t)$, they supply $z_t n(z^t)$ to firms and receive labor income $W_t z_t n(z^t)$, where W_t is the real wage. The idiosyncratic productivity z_t evolves according to the first-order auto-regressive process $\log z_t = \rho_e \log z_{it-1} + \sigma_e e_{it}$ with normal innovations $e_{it} \sim \boldsymbol{N}(0, 1)$. Households also receive (type-speific) profits $\pi(z)$ from intermediate firms and pay taxes $\tau(z^t)$ to government.

The financial markets are incomplete. Households have access to a risk-free government bond with a real interest rate r_{t+1} between periods t and $t+1$. However, households' bond holdings are subject to the constraints

$$b(z^t) \geq \phi, \tag{11}$$

where ϕ is the exogenous borrowing limit and is strictly higher than natural borrowing limits.

Firms. A competitive final-good firm produces a final good from intermediate goods, indexed by j, according to the production function $Y_t = (\int y_{j,t}^{1/\mu} dj)^{\mu}$. The intermediate goods are produced by monopolistic competitive firms using labor as the only input with linear technology $y_{j,t} = A n_{j,t}$, where $n_{j,t}$ denotes the labor hired by firm j in period t.

Each intermediate firm sets its price to maximize profits subject to quadratic price adjustment costs as in Rotemberg (1982)

$$\Theta_t(P_{j,t}, P_{j,t-1}) = \frac{\mu}{\mu - 1} \frac{1}{2\kappa} [\log(P_{j,t}/P_{j,t-1})]^2 Y_t$$

where $\kappa > 0$. The corresponding Philips curve can be derived as

$$\log(1 + \pi_t^P) = \kappa \left(\frac{W_t}{A} - \frac{1}{\mu} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}^P),$$

where π_t^P is the inflation. The price adjustment creates real costs Θ_t, and profits equal output net of labor expenditure and price adjustment costs $\Pi_t = Y_t - W_t N_t - \Theta_t$.

Fiscal Policy. The government collects taxes from households to pay interest on the debt, giving the budget constraint

$$T_t + B = (1 + r_t)B,$$

where T_t is the aggregate tax. Currently, I assume that government maintains a constant level of debt and adjusts the taxes to balance its budget. Later I will allow the government to adjust the level of outstanding debt and document a 'liquidity' channel of monetary policy shocks.

Monetary policy. The monetary authority sets the nominal interest rates on government bonds i_t according to a Taylor rule $i_t = r^* + \phi_\pi \pi_t^P + \epsilon_t$. The ex-post real interest rates satisfy Fisher equation $r_t = i_{t-1}/\pi_t^P$.

Equilibrium. Given a sequence of exogenous monetary policy shocks $\{\epsilon_t\}$, an equilibrium consists of the path for aggregates $\{r_t, W_t, C_t, Y_t, \pi_t^P, \Pi_t, T_t\}$, profits distribution and tax payment rules $\{\pi(z), \tau(z^t)\}$, and households choices $\{c(z^t), n(z^t), b(z^t)\}$ such that:

(i). households optimization: given initial bond holdings, the path of aggregates, and profits distribution and tax payment rules, households choose $\{c(z^t), n(z^t), b(z^t)\}$ to maximize their utility function (9) subject to the budget constraints (10) and borrowing constraints (11); The Philips Curve holds; government budget constraint holds; nominal interest rates evolve according to the Taylor rule;

(ii). market clearing: for $t = 0, 1, \cdots$ the good, labor and bond markets clear:

$$C_t + \Theta_t = Y_t,$$
$$N_t = L_t,$$
$$B_t^d = B;$$

(iii). aggregation: the aggregate quantities are consistent with household quantities,

$$\int zn(z^t) d\Phi_t(z^t) = N_t,$$
$$\int b(z^t) d\Phi_t(z^t) = B_t^d,$$
$$\int c(z^t) d\Phi_t(z^t) = C_t,$$
$$\int \tau(z^t) d\Phi_t(z^t) = T_t,$$
$$\int \pi(z) d\Phi_t(z^t) = \Pi_t.$$

In the economy's stationary equilibrium, aggregate quantities and prices are constant, and inflation is zero. Variable x's stationary equilibrium value is denoted as x^*, and x's deviation from its stationary equilibrium value is denoted as $\tilde{x}$. The percentage deviation is denoted as $\hat{x}$.

3.2 Transition Dynamics and Decomposition

Assuming the economy starts from the stationary equilibrium and considering the economy's response to one-time unexpected monetary policy shocks $\boldsymbol{\epsilon} = \{\epsilon_t\}_{t=0}^{\infty}$. In general equilibrium, x_t, the realization of variable x, is a function of the path of the shock $x_t = x_t(\boldsymbol{\epsilon})$. The response of x_t to the monetary policy shocks $\boldsymbol{\epsilon}$ is given as

$$\tilde{x}_t \equiv x_t(\boldsymbol{\epsilon}) - x^*.$$

As in the last section, I decompose the response $\tilde{x}_t$ into RANK and redistribution effects. To do this, I construct exogenous transfer shocks $\boldsymbol{\omega} = \{\omega(z^t), \forall z^t \in Z^t\}_{t=0}^{\infty}$ where $\omega(z^t)$ is the transfer received by the household conditional on the productivity path z^t. The households' budget constraints with the counterfactual transfers then read

$$c(z^t) + b(z^t) = (1 + r_t)b(z^{t-1}) + W_t z_t n(z^t) + \pi(z) - \tau(z^t) + \omega(z^t).$$

The transfers $\boldsymbol{\omega}$ are exogenous to households but endogenous to the monetary policy shocks $\boldsymbol{\epsilon}$: they remove the redistribution induced by the shocks $\boldsymbol{\epsilon}$. So the transfers are a function of the shock path: $\boldsymbol{\omega} = \boldsymbol{\omega}(\boldsymbol{\epsilon})$. I omit the argument $(\boldsymbol{\epsilon})$ when it does not lead to confusion.

Definition 1. The RANK effects of the monetary policy shocks $\boldsymbol{\epsilon}$ on variable x_t is the response of x_t to two sequences of shocks: monetary policy shocks $\boldsymbol{\epsilon}$ and the transfer shocks $\boldsymbol{\omega}$:

$$\tilde{x}_t^A \equiv x_t(\boldsymbol{\epsilon}, \boldsymbol{\omega}) - x^*;$$

The redistribution effects of the monetary policy shocks $\boldsymbol{\epsilon}$ on variable x_t is the response of x_t to $-\boldsymbol{\omega}$, the negative of transfer shocks:

$$\tilde{x}_t^R \equiv x_t(-\boldsymbol{\omega}) - x^*.$$

The implicit assumption behind this decomposition is that the impulse response function is linear in vanishingly small shocks:

$$\tilde{x}_t(\boldsymbol{\epsilon}, \boldsymbol{\omega}, -\boldsymbol{\omega}) = \tilde{x}_t(\boldsymbol{\epsilon}, \boldsymbol{\omega}) + \tilde{x}_t(-\boldsymbol{\omega})$$

To the first order, the response of x_t to multiple shocks is the sum of its responses to each shock. In the following, x_t^A denotes the value of x_t in RANK effects $x_t^A = x_t(\boldsymbol{\epsilon}, \boldsymbol{\omega})$, and x_t^R denotes the value of x_t in redistribution effects $x_t^R = x_t(-\boldsymbol{\omega})$.

3.3 RANK effects

The key for the decomposition is the construction of the transfers $\boldsymbol{\omega}$. Proposition 1 shows that, for given monetary policy shocks $\boldsymbol{\epsilon}$, there exist counterfactual transfers $\boldsymbol{\omega}$ such that the heterogeneous-agent model is 'as if' a representative-agent model, as in Werning (2015).

Proposition 1. For a given sequence of monetary policy shocks $\boldsymbol{\epsilon}$, there exist counterfactual transfers $\boldsymbol{\omega}$ such that:

(i) The dynamics of aggregates can be characterized with only aggregate conditions:

- Aggregate Euler equation $(C_t^A)^{-\sigma} = \beta^A(1 + r_{t+1}^A)(C_{t+1}^A)^{-\sigma}$, where $\beta^A \equiv 1/(1 + r^*)$;
- Aggregate labor supply condition $W_t^A(C_t^A)^{-\sigma} = \varphi^A(N_t^A)^{\nu}$, where $\varphi^A \equiv W^*(C^*)^{-\sigma}(N^*)^{-\nu}$;
- The Philips curve, government budget constraint, Taylor rule, and market clearing conditions.

(ii) The individual consumption and labor supply satisfy:

$$c^A(z^t)/c^*(z^t) = C_t^A/C^*;$$
$$n^A(z^t)/n^*(z^t) = N_t^A/N^*.$$

(iii) The transfers sum to zero crosssectionally $\int \omega(z^t)d\Phi_t(z^t) = 0$.

Proof. See Appendix.

I will use the terminology 'as if' representative-agent equilibrium to refer to the equilibrium in Proposition 1. The 'as if' representative agent's subjective discount factor is the steady-state real discount rate $1/(1 + r^*)$. Aggregate labor supply is the sum of individual labor supply given individual consumption $c^A(z^t)$. With frictionless labor markets, the aggregate labor supply condition coincides with the representative-agent case.

From the aggregate conditions in Proposition 1, we can obtain the path of aggregates $\{r_t^A, W_t^A, C_t^A, Y_t^A, \pi_t^{P,A}, \Pi_t^A, T_t^A\}$ given the monetary policy shock $\boldsymbol{\epsilon}$. The path of aggregates determines the household's consumption $c^A(z^t)$, labor income $W_t^A z_t n^A(z^t)$, profits income $\pi^A(z)$, and tax payment $\tau^A(z^t)$. The bond demand function $b^A(z^t)$ is also needed to back up the transfer term $\omega(z^t)$ from the household budget constraint. In the proof of Proposition 1, I impose the bond demand function $b^A(z^t) = b^*(z^t)$. The next proposition shows that the bond demand function $\{b^A(z^t), \forall z^t \in Z^t\}$ and the corresponding transfers $\boldsymbol{\omega}$ are actually not unique.

Proposition 2. For bond demand function $b^A(z^t)$ satisfying $\forall z^t \in Z^t$,

(i) Borrowing constraint and complementary slackness condtion: $b^A(z^t) \geq \phi, =$ if $u'(c^*(z^t)) > \beta(1 + r^*)E[u'(c^*(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 b^A(z^t) u'(c^A(z^t)) = 0$;

(iii) Bond market clearing: $\int b^A(z^t) d\Phi_t(z^t) = B$,

the transfer term $\omega(z^t)$ is given by

$$\omega(z^t) = c^A(z^t) + b^A(z^t) - (1 + r_t^A) b^A(z^{t-1}) - W_t^A z_t n^A(z^t) - \pi^A(z) + \tau^A(z^t). \tag{12}$$

Proof. See Appendix.

The bond demand function $b^A(z^t)$ is indeterminate. This follows from unconstrained households using bonds to smooth consumption. The income loss at time t can be compensated by future or past income, and unconstrained households can use the bond market to implement the consumption and labor supply plan given by Proposition 1. However, constrained households have a fixed bond demand at the borrowing limit ϕ.

Proposition 1 also holds with permanent heterogeneity in discount factors and can be extended to include frictional labor supply. The restriction imposed on labor supply is that households have the same consumption responses. Consider a simple case for illustration. Assuming a fixed cost of working in the style of Broer et al. (2020): $u(c, n) = c^{1-\sigma}/(1-\sigma) - n^{1+\varphi}/(1+\varphi) - \theta \mathbb{1}_{n>0}$. Some households optimally choose not to work because of the fixed cost of working and their high consumption or low productivity levels. Let n' be the labor supply implied by the first-order condition. In this case, for given aggregate consumption C_t^A and wage W_t^A, the aggregate labor supply N_t^A is

$$N_t^A \equiv \int z_t n^A(z^t) d\Phi_t(z^t)$$
$$\text{where } n^A(z^t) = \begin{cases} n', \text{ if } u(c^A(z^t), n') \geq u(c^A(z^t) - W_t^A z_t n', 0) \\ 0, \text{ otherwise} \end{cases}$$

Households supply n' if and only if $u(c^A(z^t), n') \geq u(c^A(z^t) - z_t W_t n', 0)$. Otherwise, the household's labor supply is zero. The aggregate labor supply is non-linear because of the non-linear individual's labor supply. This example shows that household heterogeneity in labor supply implies a deviation from representative-agent models on the supply side.

In the case of linear labor income taxes, the aggregate labor supply condition is $(1 - \Gamma_t^A) W_t^A (C_t^A)^{-\sigma} = \varphi^A (N_t^A)^\nu$, where $\varphi^A \equiv (1 - \Gamma^*) W^* (C^*)^{-\sigma} (N^*)^{-\nu}$ and Γ is the tax rate. Aggregate tax is $T_t^A = \Gamma_t^A W_t^A N_t^A$ and individual tax payments are $\tau^A(z^t) = \Gamma_t^A z_t W_t^A n^A(z^t)$.

3.4 Redistribution effects

Consider the response of the economy to $-\omega$, the negative of the transfers. The general equilibrium generally requires numerical solution. However, we can gain insights from the partial equilibrium analysis[2]. Heterogeneous marginal propensity of consumption (MPCs) across households is a crucial feature of incomplete-market models. Auclert (2019) and Patterson et al. (2019) are primarily interested in the correlation between MPCs and household income. The consumption response is amplified if households with higher MPCs receive higher transfers. For a transient shock lasting for only one period, the aggregate consumption response in partial equilibrium to the transfer shock $-\boldsymbol{\omega}$ is

$$\partial C_0 := \int MPC_{i0} \times (-\omega_{i0}) di = cov_I(MPC_{i0}, -\omega_{i0}).$$

The equation follows from the re-distributive nature of the transfers: $\int -\omega(z^t) d\Phi_t(z^t) = 0$. The cross-sectional covariance between households' MPCs and $-\omega(z^t)$, the negative of their transfers, is the partial equilibrium response of consumption. In the case of amplifying, $cov_I(MPC_{i0}, -\omega_{i0}) > 0$; in the case of dampening, $cov_I(MPC_{i0}, -\omega_{i0}) < 0$.

The appendix shows that the negative of the transfers can be further decomposed as following

$$\begin{aligned} -\omega(z^t) = & \underbrace{(\hat{y}^A(z^t) - \hat{Y}_t^A) y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B)(r_t^A - r^*)}_{\text{interest rate exposure}} \\ & + \underbrace{(T_t^A - T^*) - (\tau^A(z^t) - \tau^*(z^t))}_{\text{tax exposure}} \\ & + \hat{C}_t^A (y^*(z^t) - c^*(z^t)) \qquad (13) \\ & + (b^*(z^t) - b^A(z^t)) - (1 + r_t^A)(b^*(z^{t-1}) - b^A(z^{t-1})) \qquad (14) \end{aligned}$$

where I define $y \equiv Wzn + \pi$ as the household's income. From the above expression, I define three sources of redistribution: the income exposure channel, the interest rate exposure channel, and the tax exposure channel. There are also two residual terms (13) and (14). The term (13) is not zero because even if the transfers compensate for the redistribution from the previous channels, the budget constraints of households can not be scaled. In the stationary equilibrium, the net saving $y^*(z^t) - c^*(z^t)$ is generally not zero, and the term (13) is used to compensate for the scaling of the net saving. However, the effects of this term are negligible quantitatively, so I ignore this channel. The term (14) is due to the undetermined bond demand function. Note that after imposing the bond demand function $b^A(z^t) = b^*(z^t)$, the term (14) is zero. Since $b^*(z^t)$ is the bond demand function in the stationary equilibrium, it satisfies Proposition 2.

[2] Auclert and Rognlie (2018) point out that the partial equilibrium and general equilibrium responses are closely related under standard assumptions about the behavior of monetary policy, following an inequality shock (similar to the redistribution shock in this paper). I take this approach and focus the analysis on the partial equilibrium response of consumption ∂C_0.

The income exposure channel is defined as

$$(\hat{y}^A(z^t) - \hat{Y}_t^A)y^*(z^t). \tag{15}$$

For households whose income increases more (less) than the aggregate income increasing (in percentage terms), the income exposure channel is positive (negative). The income exposure channel is the main source of heterogeneity in Bilbiie (2020). On the empirical side, Patterson et al. (2019) documents a positive covariance between workers' MPCs and their elasticities of earnings to GDP $cov(MPC, \hat{y}/\hat{Y}) > 0$. Broer, Kramer and Mitman (2020) finds workers at the bottom of the income distribution are more exposed to aggregate earnings risk in general and to monetary policy shocks specifically. In my framework, this implies (15) is positive for low-income households and negative for high-income households. The income exposure channel taxes high-income workers and subsidizes low-income workers. Low-income workers have higher MPCs, so the income exposure channel amplifies the consumption response.

The interest exposure channel is defined as

$$(b^*(z^{t-1}) - B)(r_t^A - r^*). \tag{16}$$

Following an expansionary shock, the interest rate decreases $r_t^A < r^*$. This channel is negative for a household whose net bond holdings are positive (the creditor), and positive for a household whose net bond holdings are negative (the debtor). As interest rates decrease, creditors lose, and debtors benefit. Note that for bondholders, the net bond position $b^*(z^t) - B$, rather than the gross position $b^*(z^t)$, determines their exposure to the interest rate shock. This is because the interest income from the aggregate component of their bond holdings B is used to pay the aggregate tax T_t. The aggregate tax also decreases when the interest rates decrease, which can be seen from the government's budget. On the aggregate level, the economy does not benefit or lose from the interest change $\int b^*(z^{t-1})d\Phi_{t-1}(z^{t-1}) = B$, so the cross-sectional sum of (16) is zero. The interest exposure channel taxes creditors and subsidizes debtors. Considering that creditors have lower MPCs and debtors have higher MPCs, the redistribution from the interest rate exposure channel contributes to a positive correlation between $MPCs(z^t)$ and $-\omega(z^t)$, and the amplification of the consumption response.

The tax exposure channel is defined as

$$(T_t^A - T^*) - (\tau^A(z^t) - \tau^*(z^t)). \tag{17}$$

In the case of uniform taxation $\tau^A(z^t) - \tau^*(z^t) = T_t^A - T^*$, the tax exposure channel is muted because all households benefit equally from the tax reduction. In quantitative HANK models, assuming linear taxation on labor income is common. In this case, equation (17) can be simplified to $(1 - z_t n^*(z^t)/N^*)(T_t^A - T^*)$[3]. The tax exposure channel is positive for high-income households with $z_t n^*(z^t) > N^*$ because they pay a higher fraction of taxes in the stationary equilibrium and benefit more from the tax deduction. Households who pay fewer taxes in the stationary equilibrium $z_t n^*(z^t) < N^*$ also benefit less from the tax reduction, so the term of the tax exposure channel for them is negative. The tax exposure channel taxes low-income and subsidizes high-income workers, contributing to a negative correlation between $MPCs(z^t)$ and $-\omega(z^t)$ and dampening the consumption response.

3.5 Liquidity Channel

In the baseline model, I assume that government maintains a constant level of debt. Previous studies in the quantitative HANK literature have found that the fiscal policy response is crucial for determining the effects of aggregate shocks[4]. The government can also adjust the amount of debt B_t. In this section, I attribute the effects induced by the varying paths of government debt to the liquidity channel.

In incomplete market models, the amount of liquidity will affect the equilibrium interest rates. An economy with a large amount of liquidity is closer to a representative agent economy, and an economy with zero liquidity is essentially an autarky, and all households are hand-to-mouth. Following an interest rate cut, if the government changes the asset supply through fiscal policy response, it changes the liquidity in the economy, which will have real effects. This is due to the failure of Ricardian equivalence: when the government alters the timing of taxes, it directly affects the consumption of non-Ricardian households. The liquidity channel can be of interest independent of monetary policy shocks. Wolf (2021*a*) and Wolf (2021*b*) study the role of deficit-financed lump-sum fiscal transfer as a stimulating policy tool, which is essentially the liquidity channel defined here. Auclert, Rognlie and Straub (2018) discuss the fiscal multiplier under different financing policies. The deficit-financed fiscal multiplier is greater than 1, which is also due to the increasing asset supply following the government spending shock.

Let $\bar{b}^A(z^t)$, $\bar{\tau}^A(z^t)$ and $\bar{T}_t^A$ denote the bond demand function, individual tax payment, and aggregate tax, respectively, when the government debt is constant. The appendix shows

[3] From $\Gamma_t \int W_t z_t n(z^t) d\Phi_t(z^t) = T_t$ we have $\tau(z^t)/T_t = z_t n(z^t)/N_t$. Since $n^A(z^t)/N_t^A = n^*(z^t)/N^*$, it follows $\tau^A(z^t) - \tau^*(z^t) = (z_t n^*(z^t)/N^*)(T_t^A - T^*)$.

[4] See Kaplan, Moll and Violante (2018), Alves et al. (2020), Auclert, Rognlie and Straub (2018), Hagedorn, Manovskii and Mitman (2019), Wolf (2021*a*), Wolf (2021*b*).

that the negative of the transfer term $-\omega(z^t)$ can be decomposed as follows in this case:

$$\begin{aligned}
-\omega(z^t) = & \underbrace{(\hat{y}^A(z^t) - \hat{Y}_t^A)y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B)(r_t^A - r^*)}_{\text{interest rate exposure}} \\
& + \underbrace{(\bar{T}_t^A - T^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t))}_{\text{tax exposure}} \\
& + \underbrace{(\bar{b}^A(z^t) - b^A(z^t)) - (1 + r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t))}_{\text{liquidity}} \\
& + \hat{C}_t^A(y^*(z^t) - c^*(z^t)) \\
& + (b^*(z^t) - \bar{b}^A(z^t)) - (1 + r_t^A)(b^*(z^{t-1}) - \bar{b}^A(z^{t-1})) \qquad (18)
\end{aligned}$$

The income exposure, interest exposure, and tax exposure channels are defined as before and are independent of the path of government debt.

The liquidity channel is defined as

$$(\bar{b}^A(z^t) - b^A(z^t)) - (1 + r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t)). \qquad (19)$$

As in the previous section, after imposing the bond demand function $\bar{b}^A(z^t) = b^*(z^t)$, the term (18) is zero.

The liquidity channel may seem obscure at first. To understand it better, consider the subgroup of households that remain constrained $\bar{b}^A(z^t) = b^A(z^t) = \phi$ and uniform taxation $\bar{\tau}^A(z^t) - \tau^A(z^t) = \bar{T}_t^A - T_t^A$. For these households, the term (19) is $\bar{T}_t^A - T_t^A$. The liquidity channel captures the effects of altering the timing of taxes. When the government shifts the timing of taxes by deficit financing, it transfers income across time for households. In partial equilibrium, the consumption of unconstrained households hardly changes because the net present value of tax change is zero; and the consumption of constrained households responds one-to-one to the change in their tax payment. Then, in general equilibrium, the interest rate will adjust to clear the bond markets, and the unconstrained households will absorb the change in government debt.

To link the above mechanism more closely with the concept of 'liquidity', I show that in the case of uniform taxation, the liquidity channel can be proxied by counterfactual shocks to the borrowing constraint ϕ.

Proposition 3. Off the constant-debt path, assuming (i)uniform taxation $\tau^A(z^t) - \bar{\tau}^A(z^t) = T_t^A - \bar{T}_t^A$; (ii) counterfactual borrowing constraint $\phi_t^A = \phi + B_t^A - B^*$. For bond demand function $\bar{b}^A(z^t)$ satisfying the conditions in Proposition 2, the shifted bond demand function $b^A(z^t) \equiv \bar{b}^A(z^t) + B_t^A - B^*$ satisfies $\forall z^t \in Z^t$

(i) Borrowing constraint and complementary slackness condition: $b^A(z^t) \geq \phi_t^A, =$ if $u'(c^*(z^t)) > \beta(1 + r^*)E[u'(c^*(z^{t+1}))|z^t]$;

(ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 b^A(z^t) u'(c^A(z^t)) = 0$;

(iii) Bond market clearing: $\int b^A(z^t) d\Phi_t(z^t) = B_t^A$,

The transfers $\omega(z^t)$ are invariant to the path of government debt.

Proof. See Appendix.

The idea is similar to the argument in Aiyagari (1994) and Bhandari et al. (2017), which can be viewed as a heterogeneous-agent version of Ricardian equivalence. Suppose government debt increases by δ. For the same consumption choice $c^A(z^t)$, the household now holds more bonds by δ units to the next period, implying that the wealth distribution is shifted for each household in all states. To satisfy the complementary slackness condition of constrained households, the borrowing limit is also shifted by the same amount δ.

Proposition 3 demonstrates that in the case of uniform taxation, we can also use counterfactual shocks to the borrowing constraint to proxy the liquidity channel. In this case, the term (19) is zero, and the effects of the liquidity channel are the economy's response to the (negative) of borrowing constraint shocks $-\Delta\boldsymbol{\phi} \equiv -\{B_t^A - B^*\}_{t=0}^{\infty}$. A common specification of fiscal policy in the quantitative HANK literature is to use government debt to offset the fiscal imbalance in the short run and use taxes to restore the debt in the long run. This fiscal rule implies that, after a decrease in interest rates, the government debt drops on impact and gradually returns to its steady-state level. When assessing the effects of the liquidity channel, $-\Delta\boldsymbol{\phi}$ is exactly the deleveraging shock in Guerrieri and Lorenzoni (2017). The binding borrowing constraint compels poor households to deleverage, even though they may benefit from other channels. The deleveraging shock lowers equilibrium real interest rates and dampens the consumption response.

Note that the uniform-taxation condition is only required away from the constant-debt path. The tax exposure channel is not necessarily muted. This is the case when

$$\bar{\tau}^A(z^t) - \tau^*(z^t) \neq \bar{T}_t^A - T^* \tag{20}$$

$$\tau^A(z^t) - \bar{\tau}^A(z^t) = T_t^A - \bar{T}_t^A \tag{21}$$

The tax exposure channel functions because households benefit differently from the tax reduction. The liquidity channel alters the timing of households' tax payments.

In the case of non-uniform taxation, however, counterfactual shocks to borrowing constraints are not enough to proxy the liquidity channel defined in (19). Consider the bond demand function $b^A(z^t)$ given by

$$\bar{b}^A(z^t) - b^A(z^t) = (1 + r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) - (\bar{\tau}^A(z^t) - \tau^A(z^t)). \tag{22}$$

When government debt deviates from the constant-debt path $\bar{\tau}^A(z^t) \neq \tau^A(z^t)$, households absorb the change of tax payment through bond holdings $b^A(z^t)$. If the bond demand function $b^A(z^t)$ given by (22) satisfies the transversality condition, we can use path-dependent counterfactual borrowing constraint shocks to proxy the liquidity channel. The path-dependent borrowing constraints $\phi^A(z^t)$ satisfy

$$b^A(z^t) \geq \phi^A(z^t), = \text{ if } u'(c^*(z^t)) > \beta(1+r^*)E[u'(c^*(z^{t+1}))|z^t]. \tag{23}$$

However, and more generally, the transversality condition does not hold when the net present value of the change in tax payments is not zero for some households. Consider the case that tax payments are proportional to households' productivity. Then households who receive low productivity during tax increases and high productivity during tax decreases gain from the change in tax timing. Correspondingly, households who receive high productivity during tax increases and low productivity during tax decreases lose from the change in tax timing. In this case, the bond demand function given by (22) will diverge for some paths $z^t \in Z^t$, and the transversality condition does not hold.

The effects of changing the timing of taxes, of course, depend on the taxation scheme. Households may gain or lose from the change in tax timing, depending on their histories of tax payments. In the current decomposition framework, I attribute all effects, including those 'real' redistributive effects, caused by the varying path of government debt to the liquidity channel for two reasons: (i) the idiosyncratic productivity shocks are persistent in standard calibrations; (ii) For most households, their MPCs seem to be unrelated to whether they (expect themselves to) gain or lose from the change in tax timing[5].

3.6 Households' Problem in Recursive Form

First, I impose the following bond demand function $b^A(z^t)$ in the 'as if' representative agent equilibrium,

$$b^A(z^t) = g_t(b^*(z^t)) \equiv \phi + \frac{b^*(z^t) - \phi}{B^* - \phi}(B_t^A - \phi).$$

When government debt is constant, $B_t^A = B^*$ and $b^A(z^t) = b^*(z^t)$. When government debt changes, the function $g_t(\cdot)$ shrinks or stretches the stationary-equilibrium bond demand function, keeping the lower bound of bond demand at the borrowing limit. The bond demand function $b^A(z^t)$ satisfies the transversality condition and the bond market clearing condition if the stationary bond demand function satisfies those conditions.

[5]For households with lowest (highest) productivity, they can only move to higher (lower) productivity levels. Households in the middle can move in either direction.

To compute the model's response to the (negative of) transfers, I write the household's problem with transfers in recursive form. Let $c^*(z, b^{ss})$ and $b'^*(z, b^{ss})$ be the household's consumption and bond demand policy function in the stationary equilibrium, where b^{ss} is the household's wealth in the stationary equilibrium. Note that from the path of aggregates and the household's states in the stationary equilibrium, transfers are fully pinned down:

$$\omega_t(z, b^{ss}) = \frac{C_t^A}{C^*} c^*(z, b^{ss}) + g_t(b'^*(z, b^{ss})) - (1 + r_t^A) g_{t-1}(b^{ss}) - W_t^A z \frac{N_t^A}{N^*} n^*(z, b^{ss}) - \pi_t^A(z) + \tau_t^A(z)$$

I use the household's wealth in the stationary equilibrium b^{ss} as an exogenous state variable to summarize an individual's history relevant to determining the transfers he receives. The household's problem with state-dependent transfers $\omega_t(z, b^{ss})$ in recursive form is:

$$\begin{aligned} V_t^A(z, b, b^{ss}) &= \max_{\{c,n,b'\}} u(c, n) + E[V_{t+1}^A(z', b', b'^{ss}) | z, b^{ss}] \\ s.t. \quad c + b' &= (1 + r_t) b + W_t z n + \pi_t(z) - \tau_t(z) + \omega_t(z, b^{ss}) \\ b' &\geq \phi \end{aligned}$$

The law of motion for the exogenous state b^{ss} is the bond demand policy function in the stationary equilibrium $b'^{ss} = b'^*(z, b^{ss})$. Then by construction, along the equilibrium path, the household's policy function satisfies

$$b_t'^A(z, b, b^{ss}) = g_t(b'^*(z, b^{ss})), \text{and } c_t^A(z, b, b^{ss}) / c^*(z, b^{ss}) = C_t^A / C^*, \text{for } b = g_{t-1}(b^{ss}).$$

4 Adding Investment

In this section, I add investment to the model and discuss the decomposition. The quantitative analysis in the next section is implemented on the model presented in this section. In Appendix C, I consider a model without inverstment and its repsonses to real rate shocks.

4.1 Model Description

Households. Households can also trade in firm shares $v(z^t)$ with price p_t, which provides a dividend stream D_t each period. The household's budget constraint is

$$c(z^t) + b(z^t) + p_t v(z^t) = (1 + r_t) b(z^{t-1}) + (p_t + D_t) v(z^{t-1}) + z_t W_t n(z^t) + \pi(z^t) - \tau(z^t).$$

Households are subject to the non-borrowing constraints

$$b(z^t) + p_t v(z^t) \geq 0.$$

Non-arbitrage condition requires that $r_t = (p_t + D_t)/p_{t-1}$ from $t = 1$. Define total wealth $a(z^t) \equiv b(z^t) + p_t v(z^t)$, then from $t = 1$ the constraints faced by households can be written as

$$\begin{aligned} c(z^t) + a(z^t) &= (1 + r_t)a(z^{t-1}) + z_t W_t n(z^t) + \pi(z^t) - \tau(z^t), \\ a(z^t) &\geq 0. \end{aligned}$$

At $t = 0$, the return on bonds and equity can be different. The return on bonds is subject to unexpected inflation, and the return on equity is subject to unexpected capital gains:

$$\begin{aligned} c(z_0) + a(z_0) &= (1 + r_0)b_{-1} + (p_0 + D_0)v_{-1} + z_0 W_0 n(z_0) + \pi(z_0) - \tau(z_0), \\ a(z_0) &\geq 0. \end{aligned}$$

Labor Market. The modeling of the labor market is non-standard, borrowed from Alves et al. (2020) to simplify the labor-supply analysis. Households supply the same amount of labor $n(z^t) = N_t$ to firms, and the aggregate labor supply follows the wage schedule,

$$W_t = W^* (\frac{N_t}{N^*})^{\epsilon_w}.$$

If $\epsilon_w = 0$, wages are perfectly rigid, and employment is determined by only labor demand. If $\epsilon_w > 0$, there is downward pressure on wages whenever employment is below its steady-state level.

Firms. The intermediate goods firms have a Cobb Douglas production function $y_{j,t} = A k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}$. The Philips Curve is similar to the last section,

$$\log(1 + \pi_t^P) = \kappa(mc_t - \frac{1}{\mu}) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1 + \pi_{t+1}^P).$$

with marginal cost $mc_t = (r_t^K/\alpha)^{\alpha}(W_t/(1-\alpha))^{1-\alpha}/A$.

Firms own capital K_{t-1} and choose investment I_t to obtain the capital of the next period $K_t = (1 - \delta)K_{t-1} + I_t$, subject to quadratic capital adjustment cost. Dividends equal capital products plus post-tax monopolistic profits net of investment, capital adjustment cost, and price adjustment cost,

$$D_t = r_t^K K_{t-1} + \alpha \Pi_t - I_t - \frac{\Psi}{2}(\frac{I_t}{K_{t-1}} - \delta^K)^2 - \Theta_t.$$

Firms choose investment to maximize $p_t + D_t$. Tobin's Q and capital evolve according to the standard Q-theory of investment:

$$\begin{aligned} \frac{I_t}{K_{t-1}} - \delta^K &= \frac{1}{\Psi}(Q_t - 1), \\ (1 + r_{t+1})Q_t &= r_{t+1}^K - \frac{I_{t+1}}{K_t} - \frac{\Psi}{2}(\frac{I_{t+1}}{K_t} - \delta^K)^2 + \frac{K_{t+1}}{K_t} Q_{t+1}. \end{aligned}$$

I assume the monopolistic profits Π_t are taxed, so firms only receive an α fraction of the monopolistic profits. The remaining $1-\alpha$ fraction is paid to households as a lump-sum transfer in proportion to household productivity. This profit distribution scheme fully neutralizes the impact of countercyclical markups and generates reasonable asset price responses.

Fiscal Policy. I assume a non-standard fiscal policy rule to capture the increasing liquidity and relaxed borrowing conditions following an expansionary shock:

$$B_t + T_t = (1+r_t)B_{t-1},$$
$$T_t = T^* + \rho_B(B_{t-1} - B^*) + \epsilon_t^B.$$

Following the monetary policy shock ϵ_t, there is also a negative shock to the level of aggregate tax $\epsilon_t^B = \rho^B \epsilon_t$, which implies an increase in the government deficit and asset supply. In the long run, the government increases taxes to bring the debt back.

Equilibrium. In the equilibrium, households and firms optimize, government budget constraint holds, nominal interest rates evolve according to the Taylor rule, and markets clear:

$$\int a(z^t)d\Phi_t(z^t) = B_t + p_t,$$
$$C_t + I_t + \frac{\Psi}{2}(\frac{I_t}{K_{t-1}} - \delta)^2 + \Theta_t = Y_t.$$

4.2 Decomposition

The appendix shows that the negative of the transfer term $-\omega$ can be decomposed as follows

$$\begin{aligned}
-\omega(z^t) = &\underbrace{(\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A})y^{C,*}(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(r_t^A - r^*)}_{\text{interest rate exposure}} \\
&+ \underbrace{(p_t^A - p^*)(v^*(z^{t-1}) - v^*(z^t))}_{\text{asset price}} + \underbrace{(\bar{T}_t^A - T^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t))}_{\text{tax exposure}} \\
&+ \underbrace{(\bar{b}^A(z^t) - b^A(z^t)) - (1+r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t))}_{\text{liquidity}} \\
&+\hat{C}_t^A(y^{C,*}(z^t) - c^*(z^t)) \\
&+(b^*(z^t) - \bar{b}^A(z^t)) - (1+r_t^A)(b^*(z^{t-1}) - \bar{b}^A(z^{t-1})) \\
&+p_t^A(v^*(z^t) - v^A(z^t)) - p_t^A(v^*(z^{t-1}) - v^A(z^{t-1}))
\end{aligned} \tag{24}$$

where I define $y^C \equiv zWn + \pi + Dv_-$ as the household's income, including non-dividend income $zWn + \pi$ and dividend income Dv_-. On the aggregate level, aggregate income Y^C

equals aggregate consumption C. The residual term (24) is due to the undetermined equity demand. After imposing $v^*(z^t) = v^A(z^t)$, this term is zero.

There is a new asset price channel, which is defined as

$$(p_t^A - p^*)(v^*(z^{t-1}) - v^*(z^t)) \tag{25}$$

Following an expansionary shock, assuming asset prices increase $p_t^A > p^*$. Then For those who sell assets $v^*(z^{t-1}) > v^*(z^t)$, the term (25) is positive; for those who buy assets $v^*(z^{t-1}) < v^*(z^t)$, the term (25) is negative. Rising asset prices benefit sellers rather than holders, which is consistent with the argument made in Fagereng et al. (2022). Theoretically, sellers are households that experience a negative income shock, and buyers are those who experience a positive income shock. On average, sellers should have a higher MPC than buyers, so this channel results in amplification.

Due to the investment response, there is a redistribution between dividend-income and non-dividend-income earners, which is captured by the income exposure channel. The appendix shows that the term of income exposure channel has two parts:

$$(\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A})y^{C,*}(z^t) = (\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - \frac{y_t^{ND,*}(z^t)}{Y^{ND,*}}) + (\hat{y}^{ND,A}(z^t) - \hat{Y}^{ND,A})y^{ND,*}(z^t)$$

where I define $y^{ND} \equiv zWn + \pi$ as the non-dividend income. The second part,

$$(\hat{y}^{ND,A}(z^t) - \hat{Y}_t^{ND,A})y^{ND,*}(z^t),$$

is the same as the last section and captures the redistribution between households who experience a higher non-dividend-income increase and a lower non-dividend-income increase. The first part,

$$(\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - \frac{y^{ND,*}(z^t)}{Y^{ND,*}}),$$

captures the redistribution between dividend-income earners (households with $v^*(z^{t-1}) > y^{ND,*}(z^t)/Y^{ND,*}$) and non-dividend-income earners ($v^*(z^{t-1}) < y^{ND,*}(z^t)/Y^{ND,*}$). If dividends are less responsive than consumption $\hat{D}_t^A < \hat{C}_t^A$, dividend-income earners loss, and non-dividend-income earners gain.

With the presence of investment, dividend and investment responses are negatively correlated. From the expression of the dividend,

$$D_t = r_t^K K_{t-1} + \alpha\Pi_t - I_t - \frac{\Psi}{2}(\frac{I_t}{K_{t-1}} - \delta)^2 - \Theta_t.$$

Notice $r_t^K K_{t-1} + \alpha\Pi_t = \alpha Y_t$. Omitting the capital adjustment cost and price adjustment cost, we have $D_t = \alpha Y_t - I_t = \alpha C_t - (1-\alpha)I_t$. Dividends are less responsive than consumption

$\hat{D}_t^A < \hat{C}_t^A$ if and only if the investment is more responsive than consumption $\hat{I}_t^A > \hat{C}_t^A$. For typical calibrations, investment is more responsive than consumption in the short run and less responsive than consumption in the long run, which implies a redistribution from dividend-income earners to non-dividend-income earners in the short run and the reverse in the long run. Considering that the constrained households are non-dividend income earners, the redistribution induced by the investment response amplifies the consumption response[6].

Note that, given the specification that households supply the same amount of labor and the distribution of the profits is proportional to productivity,

$$y^{ND}(z^t) = z_t WN + z_t(1-\alpha)\Pi = z_t Y^{ND}. \tag{26}$$

The second part is zero as $\hat{y}^{ND,A}(z^t) = \hat{Y}^{ND,A}$. The first part is $(\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - z_t)$. All the effects of the income exposure channel are due to the investment responses.

5 Quantitative Analysis

In this section, I implement the decomposition quantitatively. I calibrate the model and then consider the model's response to a one-time unexpected monetary policy shock. At the time $t = 0$, there is a quarterly innovation to the Taylor rule of $\epsilon_0 = -0.25$ percent (-1 percent annually) with the persistence of 0.61. I use the Sequence-Space approach developed in Auclert et al. (2021) and Boppart, Krusell and Mitman (2018) to solve the model. To implement the decomposition, I first solve the model's stationary equilibrium in the absence of transfers and build the law of motion of the exogenous state (z, b^{ss}) from the households' bond demand policy function.

5.1 Calibration

I consider a model with an annual real interest rate of 5% in the stationary equilibrium. The coefficient of risk aversion σ is set to 1. For the idiosyncratic income process, I use $\rho_e = 0.966$ and $\sigma_e^2 = 0.017$, as in McKay, Nakamura and Steinsson (2016) and Guerrieri and Lorenzoni (2017). On the supply side, the slope of the Phillips Curve is $\kappa = 0.1$. The value of

[6] When unconstrained households accumulate capital for future consumption, constrained households consume additional income in the current period. In the future, constrained households will have to cut their consumption when the economy de-invests and consumes the accumulated capital. Essentially, the redistribution allows constrained households to move their future consumption to today, which has a similar flavor to the liquidity channel discussed in the last section. From this perspective, the investment-induced income exposure channel can also be interpreted as the liquidity channel of productive assets.

government debt to output B/Y is set to 1. The depreciation rate of capital is $\delta^K = 0.07$, and the capital-to-output ratio is set to 2.8. The value of aggregate household wealth to output is 4.2, which includes holdings of government bond 1, and equity of firms 3.2. The capital share satisfies $rp = \alpha Y - \delta^K K$, which gives $\alpha = 0.236$. Since the capital stock is 2.8, the capitalized markup is 0.4. The steady-state markup μ satisfies $\alpha(1 - 1/\mu)/r = 0.4$, giving $\mu = 1.022$. The Taylor rule coefficient ϕ is set to 1.25. Table 1 summarizes the parameter values.

In the baseline calibration, I assume household tax payments are proportional to productivity $\tau(z) \sim z$. The firm profits are distributed to households proportional to their productivity $d(z) \sim z$, as in Kaplan, Moll and Violante (2018). The household portfolio is undetermined. I assume that the individual portfolio is the same as the aggregate portfolio: $b(z^t)/(b(z^t) + p_t v(z^t)) = B_t/(B_t + p_t)$.

To match the US wealth distribution and aggregate MPC, I introduce permanent heterogeneity in the discount factor, as in Carroll et al. (2017) and Auclert, Rognlie and Straub (2020). The discount factor and measure of each group are listed below:

Household group	1	2	3	4	5	6
Population share	Bottom 50%	Next 20%	Next 10%	Next 10%	Next 5%	Top 5%
Discount factors	0.975	0.977	0.979	0.982	0.984	0.987

5.2 RANK effects

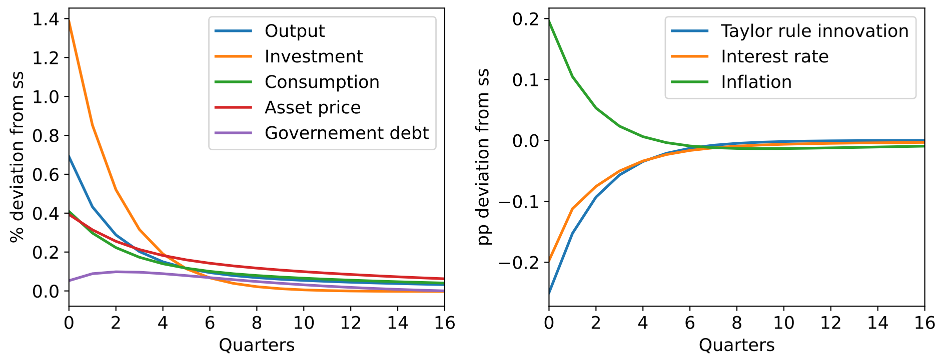

Figure 1: RANK effects

Figure 1 shows the responses of the 'as if' representative agent model. In response to an expansionary monetary policy shock, the real interest rates decrease, stimulating consumption and investment. Given the sticky price, the rising aggregate demand leads to an increase in output and inflation. The investment is more responsive than consumption in the short run and less responsive than consumption in the long run, which implies a redistribution from dividend-income earners to non-dividend-income earners in the short run and the reverse in the long run. The asset price responses imply a redistribution from asset buyers to asset sellers. The government debt increases, and more liquidity is injected into the economy after the expansionary shock.

5.3 Decomposition

Figure 2 shows the decomposition of output, consumption, investment, and real interest rates. The solid blue line is the response of HANK; the yellow dashed line is the response of the 'as if' representative agent model, which is the RANK effects; and the green dotted line is the response of HANK to the negative of the transfers $-\omega$, which is the redistribution effects. If we use the response in RANK as the benchmark, then the redistribution effects amplify the responses of output and consumption and dampen the response of investment and real interest rates. On impact, redistribution effects account for around one-eighth of output response and one-fifth of consumption response.

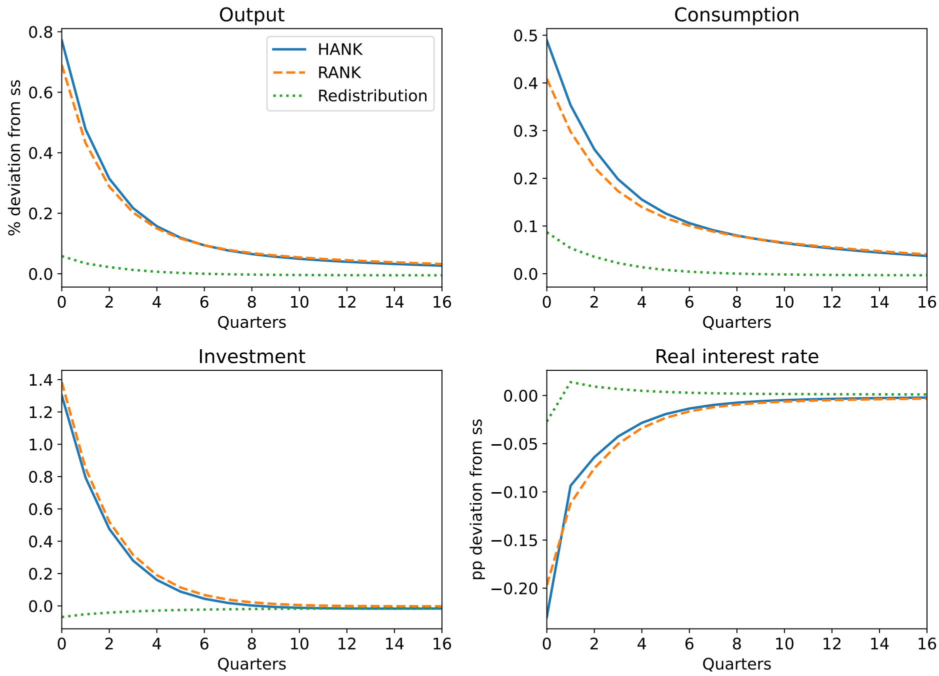

Figure 2: Monetary Policy Shock decomposition

Figure 3 further illustrates the redistribution effects at the channel level. The green line shows the effects of the interest exposure channel, which capture the redistribution between creditors and debtors. A lower interest rate benefits debtors at the expense of creditors, and debtors tend to have a higher MPC than creditors, which amplifies the consumption response. The yellow line shows the effects of the income exposure channel, which captures the redistribution between households that experience a higher income increase and those that experience a lower income increase. The income here includes dividend and non-dividend income (labor and profits income). On average, poor households have a larger share of non-dividend income, and rich households have a larger share of dividend income. Due to the investment response, poor households experience a higher income increase than rich households, which amplifies the consumption response. The purple line shows the effects of the liquidity channel. The liquidity channel captures the effects of cyclical asset supply. Following an interest rate cut, if the government increases the asset supply through fiscal policy response, households can self-insure better, and aggregate spending will increase. The red line shows the effects of the asset price channel. When asset prices increase, asset sellers gain, and asset buyers lose. Rising asset prices benefit sellers rather than holders. In theory, sellers are households that experience a negative income shock, and buyers are those who experience a positive income shock. On average, sellers should have a higher MPC than buyers, so this channel should result in amplification. But quantitively, the effects are negligible. The blue line shows the effects of the tax exposure channel, which is to capture the effects of redistribution between households who benefit more from the tax reduction and who benefit less from the tax reduction. Following an interest cut, aggregate taxes decrease; low-income workers benefit less from the tax reduction as they pay a smaller share of the aggregate tax. The tax exposure channel dampens the consumption response and amplifies the investment response.

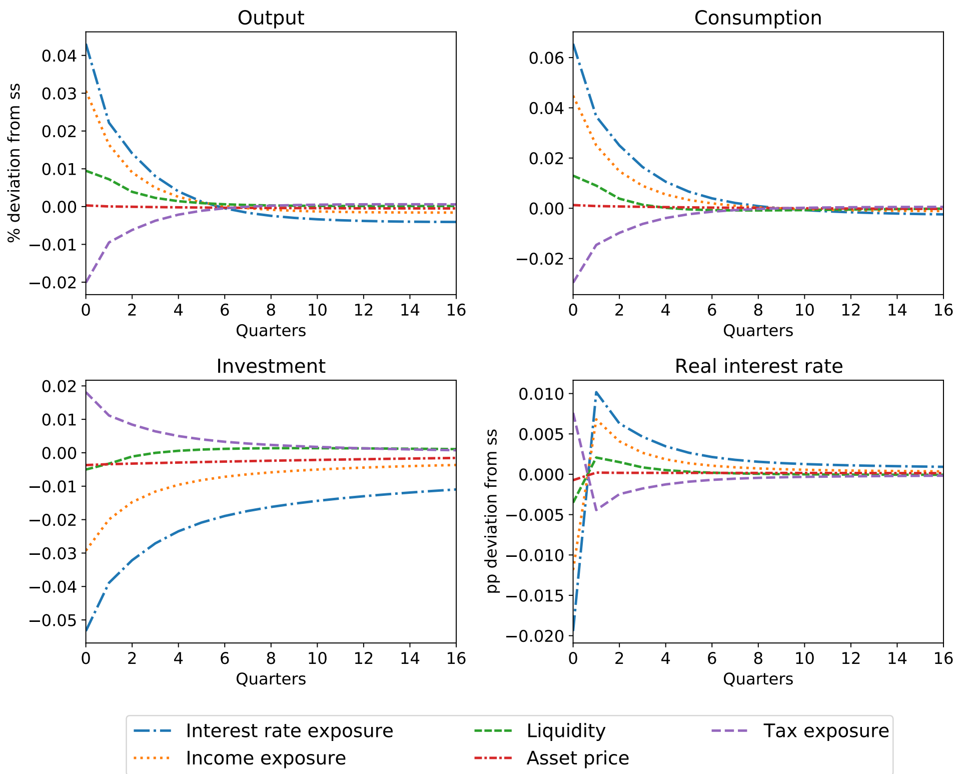

Figure 3: Channel-level decomposition

Another point worth noting is that if one channel amplifies the consumption response, it will dampen the investment response: households who lose from the redistribution consume the capital stock. So, in the long run, the amplification of consumption and output response can be reverted because the capital stock decreases as households consume more and accumulate less. This is clear from the interest exposure channel's effects on output. From quarter 6, the output response is negative because capital stock decreases, and the economy produces less.

Figure 4 shows the household-level consumption responses decomposition (on impact). I omit the asset price channel since the effects are close to zero across the wealth percentile. From Figure 4, we can see that poor households' average consumption responses are higher than rich households. As pointed out previously, the redistribution effects account for one-fifth of the consumption response on the aggregate level. On the individual level, however, the redistribution effects can account for a much larger share of the total consumption responses. For households at the lowest wealth percentile, around 60% of the consumption responses are due to redistribution effects. And for the richest households, the redistribution effects are negative and dampen their consumption responses by more than 50%.

The interest exposure channel amplifies the consumption response of poor households and dampens the consumption response of rich households: the interest rate cut benefits debtors and hurts creditors. The income exposure channel works in a similar manner: on average rich households account for a larger share of dividend income, and poor households account for a larger share of non-dividend income. The investment response benefits poor households and hurts rich households, but the magnitude is smaller than the interest rate exposure channel. The liquidity channel relaxes the borrowing conditions of constrained households and amplifies their consumption. Unconstrained households lend to constrained households relatively homogeneously: the median and rich households exhibit similar consumption cuts. The tax exposure channel dampens low-income households' consumption and amplifies high-income households' consumption (in terms of non-dividend income). Since income and wealth are highly correlated, we can also see this pattern from the responses across the wealth percentile.

6 Conclusion

This paper tries to understand quantitative HANK models by decomposing the incomplete-market model's response to aggregate shocks into two parts: the response of a constructed representative agent and the response of the heterogeneous-agent model to a transfer scheme among agents. By further decomposing the latter, I quantitatively evaluate how different channels contribute to the deviations of HANK from RANK.

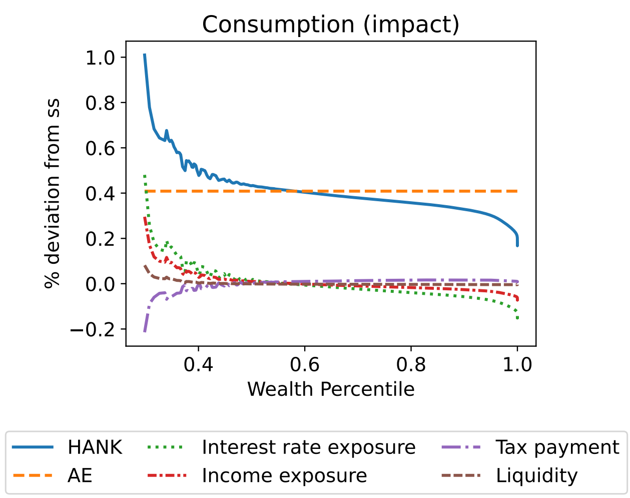

Figure 4: Household-level decomposition

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Appendix A. Proofs

Decomposition in TANK. The equilibrium of TANK can be characterized by the following equations

$$
\begin{aligned}
c_t &= E_t c_{t+1} - \delta^{-1}\sigma(r_t - r^*) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa c_t \\
i_t &= r^* + \phi_\pi \pi_t + \epsilon_t
\end{aligned}
$$

For a transient shock $E_t\epsilon_{t+1} = 0$ and $E_t c_{t+1} = E_t \pi_{t+1} = 0$. The solution is simply

$$
y_t = -\frac{\delta^{-1}\sigma}{1 + \delta^{-1}\sigma\phi_\pi\kappa}\epsilon_t
$$

The RANK effects are obtained by letting $\delta = 1$,

$$
y_t^A = -\frac{\sigma}{1 + \sigma\phi_\pi\kappa}\epsilon_t
$$

Expressing y_t in terms of y_t^A,

$$
y_t / y_t^A = \frac{\delta^{-1}(1 + \sigma\phi_\pi\kappa)}{1 + \delta^{-1}\sigma\phi_\pi\kappa} = \frac{1 + \sigma\phi_\pi\kappa}{\delta + \sigma\phi_\pi\kappa}
$$

Consider a transfer shock such that $\lambda T_t^H + (1-\lambda)T_t^S = 0$ where T_t^H and T_t^S are the amount of transfers (measured as the percentage of steady-state output Y^*) H and S receive, respectively. From the budget constraint of S, one could derive the relation between the S's consumption c_t^S, output y_t, and T_t^S

$$
\begin{aligned}
c_t^S &= w_t + n_t^S + \frac{1-\tau^D}{1-\lambda}d_t + T_t^S \\
&= (1 - \frac{1-\tau^D}{1-\lambda})w_t + \varphi^{-1}(w_t - \sigma^{-1}c_t^S) + T_t^S \\
[1 + (\sigma\varphi)^{-1}]c_t^S &= (1 - \frac{1-\tau^D}{1-\lambda} + \varphi^{-1})w_t + T_t^S \\
c_t^S &= \delta y_t + \frac{1}{1 + (\sigma\varphi)^{-1}}T_t^S
\end{aligned}
\tag{27}
$$

From S's Euler equation, Philips Curve and Taylor rule it follows $c_t^S = -\sigma(r_t - r^*) = -\sigma\phi_\pi\kappa y_t$. Substituting into (27), the output response to the transfer shock is

$$
y_t = -\frac{1}{\sigma\phi_\pi\kappa + \delta}\frac{1}{1 + (\sigma\varphi)^{-1}}T_t^S \tag{28}
$$

To obtain redistribution effects, note that the transfer S receive is

$$
T_t^S = -\omega_t^S = -(1 - \frac{1-\tau^D}{1-\lambda})d_t^A = (1 - \frac{1-\tau^D}{1-\lambda})(\sigma^{-1} + \varphi)y_t^A \tag{29}
$$

substituting (29) into (28) it follows

$$
\begin{aligned}
y_t^R &= -\frac{1}{\sigma\phi_\pi\kappa+\delta}\frac{1}{1+(\sigma\varphi)^{-1}}(1-\frac{1-\tau^D}{1-\lambda})(\sigma^{-1}+\varphi)y_t^A \\
&= \frac{1-\delta}{\delta+\sigma\phi_\pi\kappa}y_t^A
\end{aligned}
$$

We can verify that $y_t = y_t^A + y_t^R$.

Proof of Proposition 1. First, I impose the bond demand function $b^A(z^t) = b^*(z^t)$ and verify the F.O.C with respect to the bond demand

$$
(c^A(z^t))^{-\sigma} \geq \beta(1+r_{t+1}^A)E[(c^A(z^{t+1}))^{-\sigma}|z^t], = \text{if } b^A(z^t) > \phi. \tag{30}
$$

To see this

$$
\begin{aligned}
\frac{(c^A(z^t))^{-\sigma}}{E[(c^A(z^{t+1}))^{-\sigma}|z^t]} &= \frac{(C_t^A/C^*)^{-\sigma}(c^*(z^t))^{-\sigma}}{(C_{t+1}^A/C^*)^{-\sigma}E[(c^*(z^{t+1}))^{-\sigma}|z^t]} && (31)\\
&\geq \beta^A(1+r_{t+1}^A)\beta(1+r^*) && (32)\\
&= \beta(1+r_{t+1}^A).
\end{aligned}
$$

Equation (32) holds because in the stationary equilibrium

$$
(c^*(z^t))^{-\sigma} \geq \beta(1+r^*)E[(c^*(z^{t+1})^{-\sigma}|z^t]. \tag{33}
$$

In the case of $b^A(z^t) > \phi$, it can only be the case that equation (33) holds with equality, so equation (32) also holds with equality.

Second, I prove the aggregate labor supply condition. The individual labor supply condition is

$$
W_t^A z_t(c^A(z^t))^{-\sigma} = \varphi(n^A(z^t))^\nu \tag{34}
$$

Divide equation (34) by labor supply condition in the stationary equilibrium.

$$
\begin{aligned}
\frac{W_t^A z_t(c^A(z^t))^{-\sigma}}{W^* z_t(c^*(z^t))^{-\sigma}} &= \frac{\varphi(n^A(z^t))^\nu}{\varphi(n^*(z^t))^\nu} \\
\frac{W_t^A}{W^*}(\frac{c^A(z^t)}{c^*(z^t)})^{-\sigma} &= (\frac{n^A(z^t)}{n^*(z^t)})^\nu \\
\frac{W_t^A}{W^*}(\frac{C_t^A}{C^*})^{-\sigma} &= (\frac{N_t^A}{N^*})^\nu
\end{aligned}
$$

Third, the transfer is recovered from the budget constraint:

$$
\omega(z^t) = c^A(z^t) + b^A(z^t) - (1+r_t^A)b(z^{t-1}) - W_t^A z_t n^A(z^t) - \pi^A(z) + \tau^A(z^t).
$$

Finally, aggregating over transfers $\omega(z^t)$,

$$\int \omega(z^t)d\Phi_t(z^t) = \int [c^A(z^t) + b^A(z^t) - (1+r_t^A)b(z^{t-1}) - W_t^A z_t n^A(z^t) - \pi^A(z) + \tau^A(z^t)]d\Phi_t(z^t)$$
$$= C_t^A + B - (1+r_t^A)B - W_t^A N_t^A - D_t^A + T_t^A.$$

From the market clearing condition and the government's budget constraint in the 'as if' representative agent equilibrium, it turns out $\int \omega(z^t)d\Phi_t(z^t) = 0$.

Proof of Proposition 2. The borrowing constraint condition holds by construction. To satisfy the F.O.C (30), notice the following corollary from equation (31):

Corollary. Households are (un)constrained in the 'as if' representative agent equilibrium if and only if they are (un)constrained in the stationary equilibrium.

In the case of $b^A(z^t) > \phi$, it can only be the case that equation (33) holds with equality, so equation (32) also holds with equality. The transversality condition follows from the necessary condition of households optimization, and the bond market clearing condition follows from the market clearing in general equilibrium.

Proof of Proposition 3. It's easy to verify that $b^A(z^t)$ satisfies the conditions in Propositition 3 if $\bar{b}^A(z^t)$ satisfies the conditions in Propositition 2. To see that the transfers are invariant to the path of government debt,

$$b^A(z^t) - (1+r_t^A)b^A(z^{t-1}) + \tau^A(z^t)$$
$$=(\bar{b}^A(z^t) + B_t^A - B^*) - (1+r_t^A)(\bar{b}^A(z^{t-1}) + B_{t-1}^A - B^*) + \bar{\tau}^A(z^t) + T_t^A - \bar{T}_t^A$$
$$=\bar{b}^A(z^t) - (1+r_t^A)\bar{b}^A(z^{t-1}) + \bar{\tau}^A(z^t).$$

So

$$\omega(z^t) = c^A(z^t) + b^A(z^t) - (1+r_t^A)b^A(z^{t-1}) - W_t^A z_t n^A(z^t) - \pi^A(z) + \tau^A(z^t)$$
$$= c^A(z^t) + \bar{b}^A(z^t) - (1+r_t^A)\bar{b}^A(z^{t-1}) - W_t^A z_t n^A(z^t) - \pi^A(z) + \bar{\tau}^A(z^t).$$

decomposition at the channel level. Subtracting $\omega(z^t)$ from the household's budget constraint in the stationary equilibrium

$$
\begin{aligned}
-\omega(z^t) =& c^*(z^t) - c^A(z^t) + b^*(z^t) - b^A(z^t) - [(1+r^*)b^*(z^{t-1}) - (1+r_t^A)b^A(z^{t-1})] \\
&- (y^*(z^t) - y^A(z^t)) + (\tau^*(z^t) - \tau^A(z^t)) \\
=& \hat{y}^A(z^t)y^*(z^t) - \hat{C}_t^A c^*(z^t) + (b^*(z^t) - \bar{b}^A(z^t) + \bar{b}^A(z^t) - b^A(z^t)) \\
&- [(1+r^*)b^*(z^{t-1}) - (1+r_t^A)(b^A(z^{t-1}) - b^*(z^{t-1}) + b^*(z^{t-1}) - \bar{b}^A(z^{t-1}) + \bar{b}^A(z^{t-1})] \\
&+ (\tau^*(z^t) - \bar{\tau}^A(z^t) + \bar{\tau}^A(z^t) - \tau^A(z^t)) \\
=& (\hat{y}^A(z^t) - \hat{Y}_t^A)y^*(z^t) + \hat{Y}_t^A y^*(z^t) - \hat{C}_t^A c^*(z^t) + b^*(z^{t-1})(r_t^A - r^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t)) \\
&+ (\bar{b}^A(z^t) - b^A(z^t)) - (1+r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t)) \\
&+ (b^*(z^t) - \bar{b}^A(z^t)) - (1+r_t^A)(b^*(z^{t-1}) - \bar{b}^A(z^{t-1})).
\end{aligned} \tag{35}
$$

From the government budget constraint

$$B(r_t^A - r^*) = \bar{T}_t^A - T^*. \tag{36}$$

Combing equation (35) and (36)

$$
\begin{aligned}
-\omega(z^t) =& (\hat{y}^A(z^t) - \hat{Y}_t^A)y^*(z^t) + \hat{C}_t^A(y^*(z^t) - c^*(z^t)) \\
&+ (b^*(z^{t-1}) - B)(r_t^A - r^*) \\
&+ (\bar{T}_t^A - T^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t)) \\
&+ (\bar{b}^A(z^t) - b^A(z^t)) - (1+r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t)) \qquad (37) \\
&+ (b^*(z^t) - \bar{b}^A(z^t)) - (1+r_t^A)(b^*(z^{t-1}) - \bar{b}^A(z^{t-1})). \qquad (38)
\end{aligned}
$$

In the case that government debt is constant, we have $\bar{b}^A(z^t) = b^A(z^t)$ and $\bar{\tau}^A(z^t) = \tau^A(z^t)$, the term (37) is zero. In the case of $b^*(z^t) = \bar{b}^A(z^t)$, the last term (38) is zero.

decomposition with outside assets. Assume the budget constraints of households are

$$c(z^t) + p_t v(z^t) = (p_t + D_t)v(z^{t-1}) + zWn(z^t) + \pi_t + \omega(z^t).$$

Define $y^C \equiv zWn + \pi + Dv_-$ as the individual income, including non-dividend income $zWn + \pi$ and dividend income Dv_-. Define $Y^C = WN + (1-\alpha)\Pi + D$ as the aggregate income, and we have $C_t = Y_t^C$. The negative of the transfer is

$$-\omega(z^t) = p_t^A v^A(z^{t-1}) + y^{C,A}(z^t) - p_t^A v^A(z^t) - c^A(z^t). \tag{39}$$

Subtracting the budget constraint in the stationary equilibrium from equation (39)

$$
\begin{aligned}
-\omega(z^t) =& p_t^A(v^*(z^{t-1}) + v^A(z^{t-1}) - v^*(z^{t-1})) - p^* v^*(z^{t-1}) + \hat{y}^{C,A}(z^t) y^{C,*}(z^t) \\
& - (p_t^A(v^*(z^t) + v^A(z^t) - v^*(z^t)) - p^* v^*(z^t)) - \hat{C}_t^A c^*(z^t) \\
=& (p_t^A - p^*) v^*(z^{t-1}) + (\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A}) y^{C,*}(z^t) - (p_t^A - p^*) v^*(z^t) \\
& + p_t^A(v^A(z^{t-1}) - v^*(z^{t-1})) - p_t^A(v^A(z^t) - v^*(z^t)) + \hat{C}_t^A(y^{C,*}(z^t) - c^*(z^t)) \\
=& (\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A}) y^{C,*}(z^t) + (p_t^A - p^*)(v^*(z^{t-1}) - v^*(z^t)) \\
& + \hat{C}_t^A(y^{C,*}(z^t) - c^*(z^t)) \\
& + p_t^A(v^A(z^{t-1}) - v^*(z^{t-1})) - p_t^A(v^A(z^t) - v^*(z^t))
\end{aligned} \tag{40}
$$

In the case of $v^A(z^t) = v^*(z^t)$, the last term (40) is zero.

Define $y^{ND} \equiv zWn + \pi$ as the individual non-dividend income and $Y^{ND} \equiv WN + (1 - \alpha)\Pi$ as the aggregate non-dividend income then

$$
\begin{aligned}
& (\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A}) y^{C,*}(z^t) \\
=& \hat{D}_t^A D^* v^*(z^{t-1}) + (\hat{y}^{ND,A}(z^t) - \hat{Y}_t^{ND,A}) y^{ND,*}(z^t) + \hat{Y}_t^{ND,A} y^{ND,*}(z^t) - \hat{Y}_t^{C,A} y^{C,*}(z^t) \\
=& (\hat{D}_t^A - \hat{C}_t^A) D^* v^*(z^{t-1}) + (\hat{y}^{ND,A}(z^t) - \hat{Y}_t^{ND,A}) y^{ND,*}(z^t) + (\hat{Y}_t^{ND,A} - \hat{C}_t^A) y^{ND,*}(z^t) \\
=& (\hat{y}^{ND,A}(z^t) - \hat{Y}_t^{ND,A}) y^{ND,*}(z^t) + (\hat{D}_t^A - \hat{C}_t^A) D^* v^*(z^{t-1}) + (\hat{Y}_t^{ND,A} - \hat{C}_t^A) y^{ND,*}(z^t).
\end{aligned} \tag{41}
$$

From $C_t = Y_t^{ND} + D_t$ we have $\hat{C}_t C^* = \hat{Y}_t^{ND} Y^{ND,*} + \hat{D}_t D^*$. Then

$$
(\hat{D}_t^A - \hat{C}_t^A) D^* + (\hat{Y}_t^{ND,A} - \hat{C}_t^A) Y^{ND,*} = 0,
$$

and

$$
(\hat{Y}_t^{ND,A} - \hat{C}_t^A) y^{ND,*}(z^t) = -(\hat{D}_t^A - \hat{C}_t^A) D^* \frac{y^{ND,*}(z^t)}{Y^{ND,*}}. \tag{42}
$$

Substituting equation (42) into (41) we get the equation in the main text.

Appendix B. Quantitative Results

The calibration of the quantitative HANK model is listed below.

Parameter	Description	Value
r^*	steady state real interest rate	0.0125
σ	Risk aversion	1
Z	TFP	0.784
α	Capital share	0.24
ϵ_I	Capital adjustment cost parameter	3.3
δ^K	Depreciation of capital	0.07
K/Y	Capital to GDP	2.8
B/Y	Government debt to GDP	1
p/Y	Equity to GDP	3.2
ρ_e	Autocorrelation of earnings	0.966
σ_e	Innovation variance	0.017
κ	Slope of Phillips curve	0.1
ϵ^w	Wage elasticity	0.1
ϕ_π	Coefficient on inflation	1.25
ρ_B	Debt reverting rate	0.1
ρ^B	Coefficient of taxes shock	1

Table 1: Calibration

Figure 5 shows the decomposition of asset price and inflation responses. Figure 6 shows the corresponding channel-level decomposition. Redistribution effects dampen the asset price responses. This is because, on the one hand, redistribution effects dampen the response of real interest rates; on the other hand, redistribution effects dampen the response of dividends. The channel-level decomposition of asset price responses is close to the channel-level decomposition of investment responses. The redistribution effects amplify the response of inflation since it amplifies the response of consumption and output.

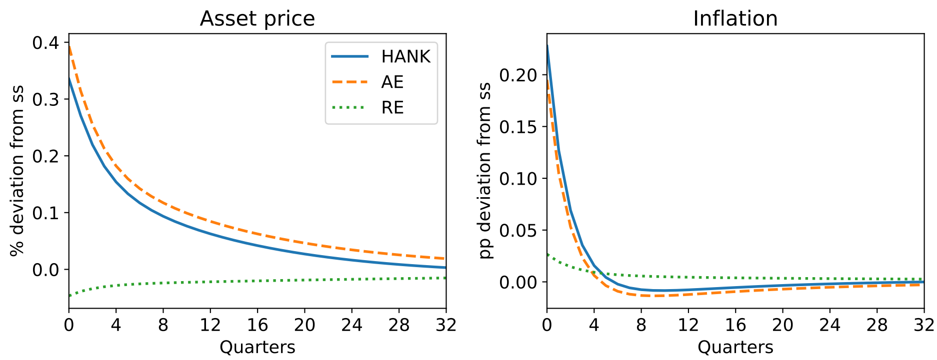

Figure 5: Asset price and inflation decomposition

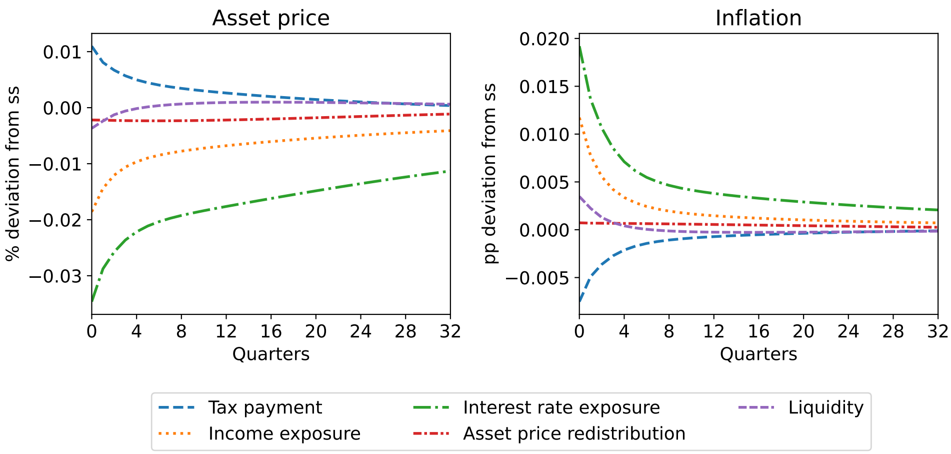

Figure 6: Asset price and inflation decomposition at channel level

Appendix C. Decomposition Without Investment

I implement the decomposition on the model presented in Section 3, where there is no productive capital and investment. I make a small modification to the budget constraints of households:

$$c(z^t) + \frac{b(z^t)}{1+r_t} = b(z^{t-1}) + z_t W_t n(z^t) + \pi_t(z) - \tau_t(z),$$

and we don't need to make a distinction between ex-ante interest rates and ex-post interest rates. The channel level decomposition is, instead,

$$\begin{aligned} -\omega(z^t) = & \underbrace{(\hat{y}^A(z^t) - \hat{Y}_t^A)y^*(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^t) - B)(\frac{1}{1+r^*} - \frac{1}{1+r_t^A})}_{\text{interest rate exposure}} \\ & + \underbrace{(\bar{T}_t^A - T^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t))}_{\text{tax exposure}} \\ & + \underbrace{\frac{\bar{b}^A(z^t) - b^A(z^t)}{1+r_t^A} - (\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t))}_{\text{liquidity}} \\ & + \hat{C}_t^A(y^*(z^t) - c^*(z^t)) \\ & + \frac{b^*(z^t) - \bar{b}^A(z^t)}{1+r_t^A} - (b^*(z^{t-1}) - \bar{b}^A(z^{t-1})), \end{aligned}$$

Where $y \equiv zWn + \pi$ is household income, including labor income zWn and profit income π.

To make the exercise more transparent, I assume that the central bank directly control the real interest rate. At time $t = 0$ there is a quarterly real rate shock $\tilde{r}_0 = -0.25$ percent with the persistence of 0.61. Then by contruction, the output response in the 'as if' representative agent equilibrium is given by the aggregate Euler equation:

$$(C_t^A)^{-\sigma} = \beta^A(1+r_t)(C_{t+1}^A)^{-\sigma}.$$

The redistribution effects, are the economy's response to the transfer shocks keeping the real interes rate at the steady state level.

In the first two exercises, I assume a balanced budget fiscal policy. In the third exercise, I let the government adjust the outstanding debt to illustrate the liquidity channel.

C.1 Calibration

I consider a model with an annual real interest rate of 2% in the stationary equilibrium. The coefficient of risk aversion σ is set to 2. The Frisch elasticity of labor supply is $1/\nu = 0.5$, following Chetty (2012). For the idiosyncratic income process, I use $\rho_e = 0.966$ and $\sigma_e^2 = 0.017$, as in McKay, Nakamura and Steinsson (2016) and Guerrieri and Lorenzoni (2017). The supply of government bonds B is set to match the ratio of aggregate liquid assets to output $B/Y = 5.6$, as in McKay, Nakamura and Steinsson (2016). The borrowing constraint is zero $\phi = 0$. The discount factor $\beta = 0.98$ and disutility from labor $\varphi = 0.933$ are calibrated to deliver the values of annual real interest and unit quarterly output. On the supply side, the slope of the Phillips Curve is $\kappa = 0.1$ and the parameter of the markup of intermediate firms is $\mu = 1.2$. The Taylor rule coefficient ϕ is set to 1.25. In the baseline calibration, I assume that household tax payments are uniform. The firm dividends are distributed to households proportional to their productivity $d(z) \sim z$, as in Kaplan, Moll and Violante (2018). Table 2 summarizes the parameter values.

Parameter	Description	Value	Target
β	Discount factor	0.98	2 percent annual interest rate
σ	Risk aversion	2	
$1/\nu$	Frisch elasticity	1/2	Chetty (2012)
φ	Disutility of labor	0.933	Output
ρ_e	Autocorrelation of earnings	0.966	McKay, Nakamura and Steinsson (2016)
σ_e	Innovation variance	0.017	McKay, Nakamura and Steinsson (2016)
B	Supply of assets	5.6	Aggregate liquid assets
μ	Markup of intermediate firms	1.2	Christiano, Eichenbaum and Rebelo (2011)
κ	Slope of Phillips curve	0.1	Christiano, Eichenbaum and Rebelo (2011)
ϕ	Coefficient on inflation	1.25	
$\pi(z)$	Profits distribution		Proportional to productivity
$\tau(z)$	Tax payment		Uniform across households
ρ_B	Debt reverting rate	0.1	

Table 2: Calibrated Parameter Values

C.2 Purely Transient Shocks

To begin, consider a real rate shock that lasts for only one period (the persistence $\rho = 0$), in the same spirit of the thought experiment in Auclert (2019). The result is shown in Figure 7. The real interest rates decrease and stimulate consumption. Given the sticky price, the rising aggregate demand leads to an increase in both output and inflation.

Regarding decomposition, redistribution effects amplify the output response. Under transient monetary policy shocks, RANK effects last for only one period, the same as in a representative-agent model. In contrast, the redistribution effects affect the economy for a long time, and all the economy's responses after time 0 are due to redistribution effects.

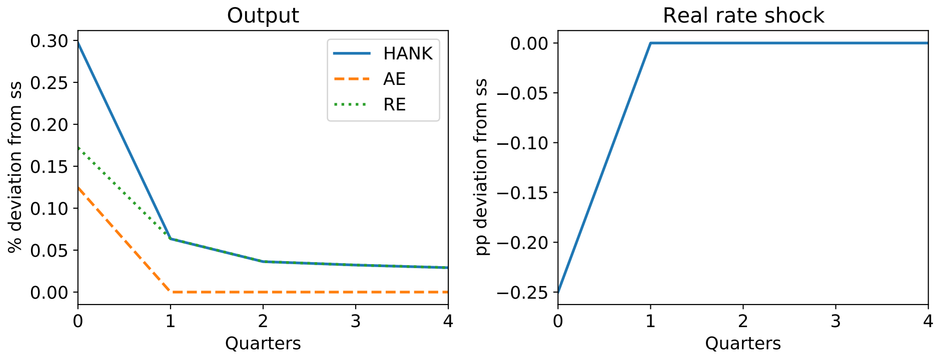

Figure 7: Decomposition of transient real rate shocks

Figure 8 shows the transfers ω_{i0} as a function of the household's wealth and productivity. The left panel of Figure 8 shows the transfers ω_{i0} as a function of wealth at four different productivity levels. The right panel of Figure 8 shows ω_{i0} as a function of household productivity at the wealth distribution's 20th, 40th, 60th, and 80th percentiles. The transfers ω_{i0} increase with the household's wealth and (weakly) with productivity. Transfers increase with wealth because to eliminate the exposure to the interest rate cut, creditors need positive transfers, and debtors need negative transfers. The transfers increase with productivity because profits are countercyclical. The income of household is $y = zWn + zD = z(WNn/N + D)$. Due to labor supply heterogeneity, high-income households have a higher share of profit income, which is countercyclical. High-income households' income increase less and need positive transfers.

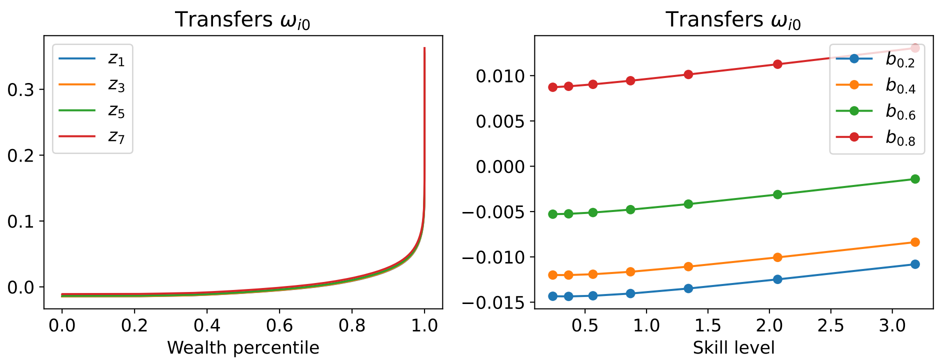

Figure 8: Transfers as a function of households characteristics

Overall, the negative of the transfers $-\omega$ is making positive transfers to poor households by taxing rich households, similar to TANK. Since poor households have higher MPCs, it follows $cov_I(MPC_{i0}, -\omega_{i0}) > 0$. The redistribution effects stimulate aggregate consumption.

C.3 Persistent shocks

Consider the economy's response to the persistent real rate shocks. I apply the decomposition, and the result is shown in Figure 9.

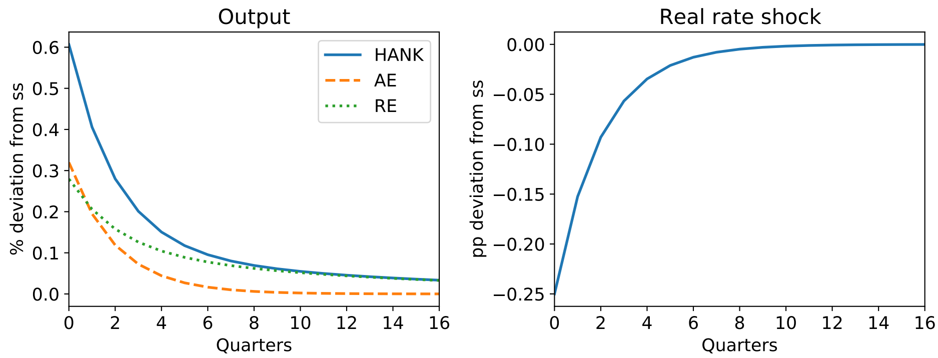

Figure 9: Decomposition of persistent real rate shocks

Output increases by 0.6% on impact. The decomposition result is qualitatively similar to the decomposition of the transient shock in Figure 7. Redistribution effects amplify the output's response to real rate shocks. On impact, RANK effects increase output by 0.31%, and redistribution effects increase output by 0.29%. The redistribution effects amplify the elasticity of output to real interest rates.

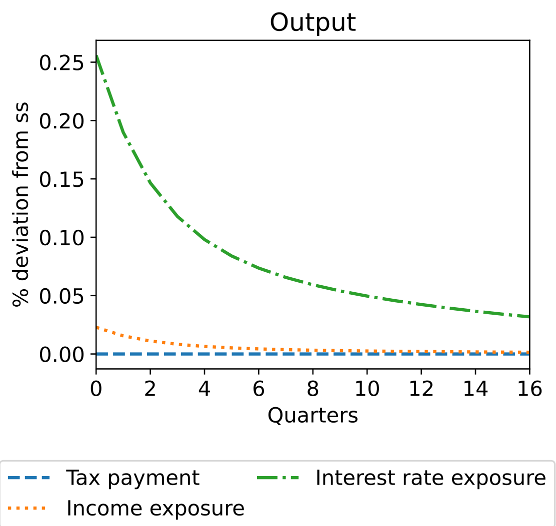

Figure 10: Channel-level decomposition

Figure 10 further decomposes the redistribution effects into different channels for output. Quantitatively, the interest exposure channel accounts for most of the redistribution effects. On impact, the interest exposure channel increases consumption by 0.25%. The interest rate cuts tax creditors and subsidize debtors. Given that debtors have higher MPCs, the interest rate exposure channel stimulates the economy. The income exposure channel slightly contributes to the output amplification. Since I assume uniform taxation, all households benefit equally from the tax reduction, and the tax exposure channel is muted.

C.4 Including Liquidity Channel

Assuming the fiscal policy takes the following rule:

$$T_t = T^* + \rho_B * (B_{t-1} - B^*). \tag{43}$$

The government uses debt to absorb most of the fiscal imbalance in the short run. In the long run, the government uses taxes to bring the debt back to its initial level. Similar fiscal policy specifications are assumed in Kaplan, Moll and Violante (2018), Alves et al. (2020), and Auclert, Rognlie and Straub (2018).

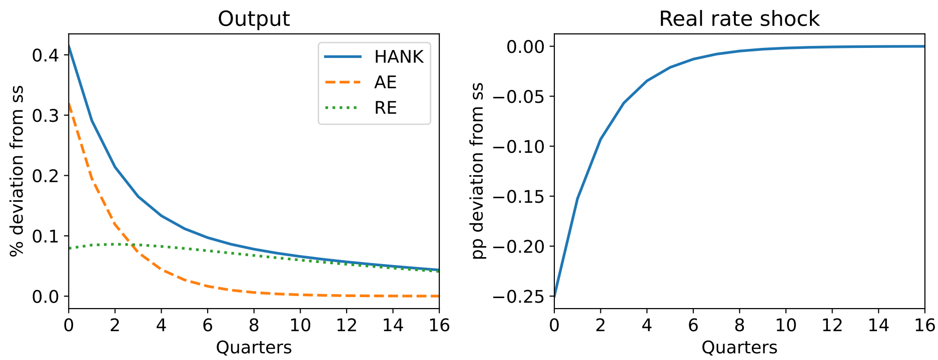

Figure 11: Decomposition with liquidity channel

The decomposition result is shown in Figure 11. The redistribution effects are smaller than Figure 9. On impact, redistribution effects increase output by less than 0.1%, rather than close to 0.3% under a balanced fiscal policy.

Figure 12 further decomposes the redistribution effects into different channels. The interest exposure, income exposure, and tax exposure channels are invariant to the path of government debt, so their effects in Figure 12 are identical to those in Figure 10. However, the liquidity channel decreases output by 0.2%. The liquidity channel explains why the response of HANK with the fiscal policy of (43) is dampened compared to the balanced fiscal policy case.

The fiscal rule (43) implies a countercyclical asset supply. As proved in section 3.5, the liquidity channel is equivalent to a deleveraging shock in the case of uniform taxation. The output needs to decrease to clear the bond market. I show the household-level decomposition in Figure 13 to support this argument. Figure 13 shows the decomposition of the households' on-impact consumption responses. The interest rate exposure channel increases the

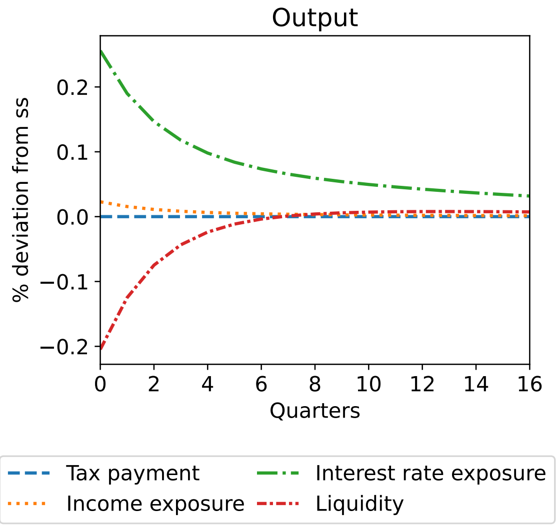

Figure 12: Channel-level Decomposition with liquidity channel

consumption of poor households and decreases the consumption of rich households. However, the liquidity channel forces the constrained households to hold the additional income from other channels. As a result, the redistribution effects on poor households' consumption are smaller.

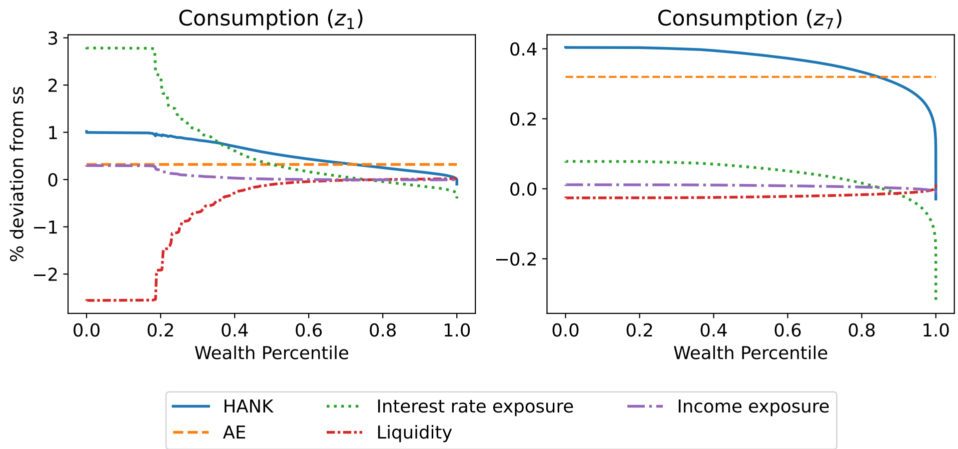

Figure 13: Household-level decomposition (on impact)