Decomposing HANK

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Motivation: Unequal Exposures to Aggregate Shocks

- Two ingredients to have amplified or dampened responses in HANK: (i) heterogeneous MPCs; (ii) unequal exposures.
  - Equal exposures: HANK ‘as-if’ RANK, Bilbiie (2018) and Werning (2015).
- What are the sources of unequal exposures in quantitative HANK models?
- How to evaluate the contribution of each redistribution channel to the amplified/dampened IRFs?

Contribution

- Impulse responses $\tilde{x}_t \equiv x_t - x^*$ are decomposed into RANK effects $\tilde{x}_t^A$ and redistribution effects $\tilde{x}_t^R$.

$$\tilde{x}_t = \tilde{x}_t^A + \tilde{x}_t^R$$

  - RANK effects: the GE response of a (constructed) RA;
  - Redistribution effects: the residual.

- 'Channel-level' decomposition of redistribution effects $\tilde{x}_t^R$:

$$\tilde{x}_t^R = \tilde{x}_t^{income} + \tilde{x}_t^{interest} + \tilde{x}_t^{asset} + \tilde{x}_t^{tax} + \tilde{x}_t^{liquidity}$$

▸ Literature

Model

- A HANK model in the style of Werning (2015) combined with supply side.
  - McKay, Nakamura and Steinsson (2016), Auclert and Rognlie (2018), Kaplan and Violante (2022), etc..
- Aggregate MIT shocks.
- Households face idiosyncratic income shocks and borrowing constraints.
- Firms face price adjustment costs and capital adjustment costs.
- Government collects taxes from households to pay the interest on government bonds.
- Nominal interest rates follow the Taylor rule.

HANK: Households

- $z^t = (z_0, z_1, \cdots, z_t)$: the history of idiosyncratic states z_t.
- Household problem:

$$\max_{c,n,b,v} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c(z^t)^{1-\sigma}}{1-\sigma} - \varphi \frac{n(z^t)^{1+\nu}}{1+\nu} \right)$$

$$c(z^t) + b(z^t) + p_t v(z^t) = (1+r_t)\underbrace{b(z^{t-1})}_{\text{bond}} + (p_t + D_t)\underbrace{v(z^{t-1})}_{\text{equity}} + \underbrace{z_t W_t n(z^t)}_{\text{labor income}} + \underbrace{\pi(z_t)}_{\text{profits}} - \underbrace{\tau(z_t)}_{\text{taxes}}$$

$$b(z^t) + p_t v(z^t) \geq \phi.$$

- $1 + r_t = (p_t + D_t)/p_{t-1}$ from $t = 1$. Define wealth $a(z^t) \equiv b(z^t) + p_t v(z^t)$,

$$c(z^t) + a(z^t) = (1+r_t)a(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z_t)$$

$$a(z^t) \geq \phi.$$

HANK: Firms

- The intermediate goods firms: $y_{j,t} = Zk_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}$; quadratic price adjustment cost. The Philips curve

$$\log(1+\pi_t) = \kappa(mc_t - \frac{1}{\mu}) + \frac{1}{1+r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1+\pi_{t+1}).$$

  - $mc_t = (r_t^K/\alpha)^{\alpha}(W_t/(1-\alpha))^{1-\alpha}/Z.$

- Firms own the capital stock and issue equity to households with price p_t and dividends D_t.
  - $D_t = r_t^K K_{t-1} - I_t - \Psi(K_t, K_{t-1}) + \alpha \Pi_t$
  - Firms choose investment I_t to maximize $p_t + D_t$: standard Q-theory of investment.

HANK: Equilibrium

- Labor supply: $W_t / W^* = (N_t / N^*)^{\epsilon_w}$ from Alves et al. (2020).
  - Debortoli and Galí (2017), Auclert and Rognlie (2018).
- Monetary policy $i_t = r^* + \phi_\pi \pi_t^P + \epsilon_t$.
- Government budget constraint $B_t + T_t = (1 + r_t) B_{t-1}$.
  - Fiscal policy $T_t = T^* + \phi_B (B_{t-1} - B^*) + \rho^B \epsilon_t$.
- Standard equilibrium definition.
- Stationary equilibrium: $\{r^*, p^*, D^*, \Pi^*, W^*, C^*, N^*, Y^*, B^*, T^*\}$; $\{c^*(z^t), b^*(z^t), v^*(z^t)\}$.

Aggregate Shock Decomposition

- The transition dynamics following $\boldsymbol{\epsilon} = \{\epsilon_t\}_{t=0}^{\infty}$.

$$\tilde{x}_t \equiv x_t(\boldsymbol{\epsilon}) - x^*.$$

- Path-dependent counterfactual transfer shocks $\boldsymbol{\omega} = \{\omega(z^t), \forall z^t \in Z^t\}_{t=0}^{\infty}$.

$$c(z^t) + a(z^t) = (1 + r_t)a(z^{t-1}) + z_t W_t n(z^t) + \pi(z_t) - \tau(z_t) + \omega(z^t).$$

RANK Effects

- **Proposition 1**. For a given sequence of monetary policy shocks ϵ, there exist counterfactual transfers ω such that
  - The dynamics of aggregates are characterized by aggregate conditions. 'RA' equilibrium.
    - $C_t^A = \beta^A(1 + r_{t+1}^A)C_{t+1}^A$, where $\beta^A \equiv 1/(1 + r^*)$.
  - The individual consumption satisfies

$$c^A(z^t)/c^*(z^t) = C_t^A/C^*.$$

  - The transfers sum to zero cross-sectionally $\int \omega(z^t)d\Phi_t(z^t) = 0$.

Proof Asset demand Extensions

Aggregate Shock Decomposition

- Define the decomposition

$$\tilde{x}_t^A \equiv x_t(\boldsymbol{\epsilon}, \boldsymbol{\omega}) - x^*;$$
$$\tilde{x}_t^R \equiv x_t(\mathbf{0}, -\boldsymbol{\omega}) - x^*.$$

  - To the first order, $\tilde{x}_t = \tilde{x}_t^A + \tilde{x}_t^R$.
- 'RA' equilibrium: $\{r_t^A, p_t^A, D_t^A, \Pi_t^A, W_t^A, C_t^A, N_t^A, Y_t^A, B_t^A, T_t^A\}$; $\{c^A(z^t), b^A(z^t), v^A(z^t)\}$.
- $\hat{x}_t^A$: percentage deviation of x_t^A from x^*.

Redistribution Effects

- Income $y^C \equiv zWn + \pi + Dv_-$.
- $\bar{T}_t$, $\bar{\tau}(z_t)$: aggregate and the individual tax when $B_t = B^*$.

$$
\begin{aligned}
-\omega(z^t) = & \underbrace{(\hat{y}^{C,A}(z^t) - \hat{Y}^{C,A})y^{C,*}(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(r_t^A - r^*)}_{\text{interest rate exposure}} \\
& + \underbrace{(\bar{T}_t^A - T^*) - (\bar{\tau}^A(z_t) - \tau^*(z_t))}_{\text{tax exposure}} + \underbrace{(p_t^A - p^*)(v^*(z^{t-1}) - v^*(z^t))}_{\text{asset price}} \\
& + \underbrace{(b^*(z^t) - b^A(z^t)) - (1 + r_t^A)(b^*(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z_t) - \tau^A(z_t))}_{\text{liquidity}} \\
& + \hat{C}_t^A(y^{C,*}(z^t) - c^*(z^t))
\end{aligned}
$$

- Given $n(z^t) = N_t$ and $\pi(z_t) \sim z_t$, income exposure $(\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - z_t)$.
- Given $\tau(z_t) \sim z_t$, tax exposure $(1 - z_t)(\bar{T}_t^A - T^*)$.

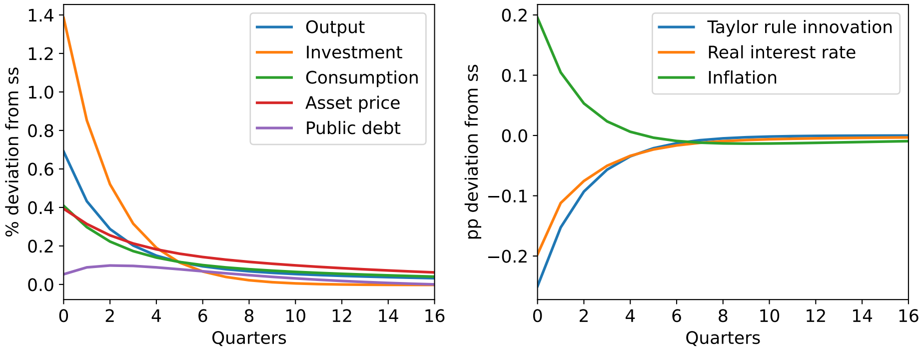
Calibration

- Permanent heterogeneity in discount factor β_i.
- Taxes proportional to productivity; profits proportional to productivity.
- Asset portfolio $b_{it}/(b_{it} + p_t v_{it}) = B_t/(B_t + p_t)$.

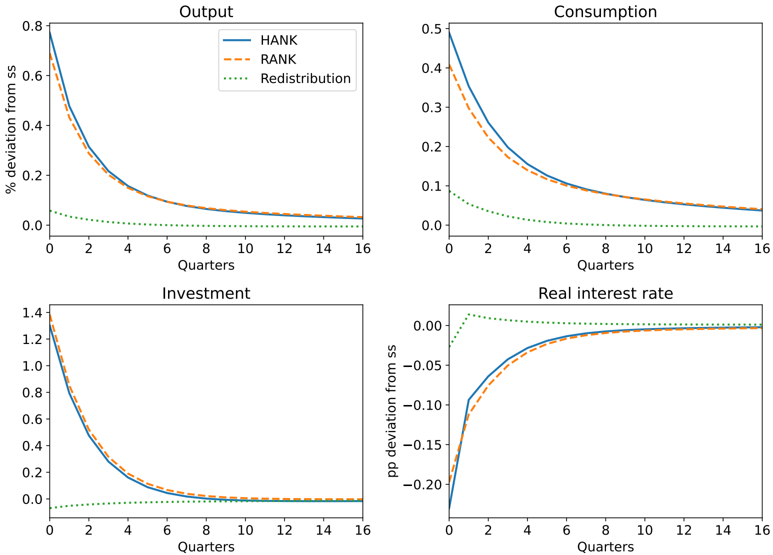
Parameter Value

RANK Effects

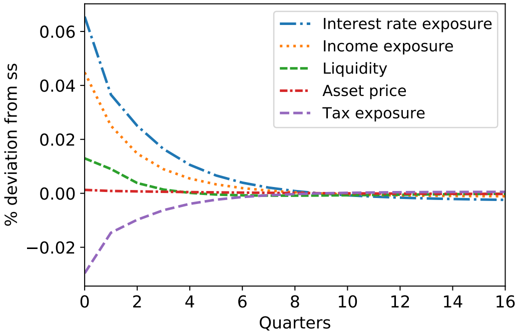
- 25bp monetary policy shocks with the persistence of 0.61;



Monetary Policy Shock Decomposition

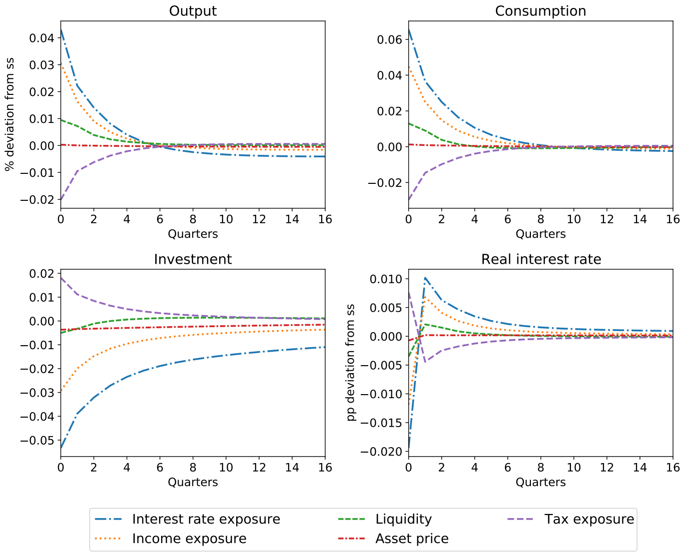


Decomposition of Redistribution Effects (Consumption)



Redistribution channels	Interest rate	Income	Liquidity	Asset price	Tax
Lower MPC	Creditors	Dividend Earners	Unconstrained	Asset buyers	High income
	↓	↓↑	↓↑	↓	↑
Higher MPC	Debtors	Non-dividend Earners	Constrained	Asset sellers	Low income

Decomposition of Redistribution Effects



Conclusion

- RANK effects account for most of the GE responses of HANK.
- Consumption and output responses are amplified; investment responses are dampened.
- Amplification: interest rate exposure, income exposure, liquidity, and asset price channels;
- Dampening: tax exposure channel.

Literature

- Quantitative HANK literature: McKay, Nakamura and Steinsson (2016), Kaplan, Moll and Violante (2018), Auclert (2019), Auclert and Rognlie (2018), Auclert, Rognlie and Straub (2018), Hagedorn et al. (2019), Luetticke (2021), etc...
- Analytical HANK literature: Werning (2015), Bilbiie (2020), Bilbiie (2018), Acharya and Dogra (2020), Ravn and Sterk (2021), Broer et al. (2020), Challe and Ragot (2016), Challe (2020), Berger, Bocola and Dovis (2019), etc..
- The approach taken in this paper is most closely related to Werning (2015) and Hagedorn (2020).
  - 'As-if' result in Werning (2015).
  - Counterfactual transfers in Hagedorn (2020).

◂ Contribution

HANK: equilibrium

- At $t = 0$ the economy inherits a distribution over initial states $\Phi_0(z_0)$. The stochastic process z_t then induces a distribution $\Phi_t(z^t)$ over histories $z^t \in Z^t$.
- Equilibrium: Given the initial distribution $\Phi_0(z_0)$ and the sequence of monetary policy shocks $\{\epsilon_t\}$, an equilibrium consists of the path for aggregates $\{r_t, p_t, \pi_t^p, W_t, C_t, N_t, D_t, Y_t, \Pi_t, T_t\}$, and households choices $\{c(z^t), b(z^t), v(z^t)\}$ such that
  (i). HH optimization; Philips Curve; Q-theory of investemt; government budget constraint; Taylor rule;
  (ii). Market clearing: $C_t + I_t + \Psi(K_t, K_{t-1}) + \Theta_t = Y_t$; $B_t^d = B_t$; $V_t^d = 1$;

HANK: equilibrium

- (iii). Aggregation,

$$\int b(z^t)d\Phi_t(z^t) = B_t^d,$$

$$\int v(z^t)d\Phi_t(z^t) = V_t^d,$$

$$\int c(z^t)d\Phi_t(z^t) = C_t,$$

$$\int \tau(z^t)d\Phi_t(z^t) = T_t,$$

$$\int \pi(z_t)d\Phi_t(z^t) = (1-\alpha)\Pi_t.$$

◂ Model

RANK Effects

- Sketch of proof for Proposition 1.
  - F.O.C w.r.t bond demand:

$$
\begin{aligned}
\frac{(c^A(z^t))^{-\sigma}}{E[(c^A(z^{t+1})^{-\sigma}|z^t]} &= \frac{(C_t^A/C^*)^{-\sigma}(c^*(z^t))^{-\sigma}}{(C_{t+1}^A/C^*)^{-\sigma}E[(c^*(z^{t+1}))^{-\sigma}|z^t]} \\
&\geq \beta^A(1+r_{t+1}^A)\beta(1+r^*) \\
&= \beta(1+r_{t+1}^A).
\end{aligned}
$$

- **Corollary**: HH are constrained in the 'RA' equilibrium iff they are constrained in the stationary equilibrium.

◂ Proposition 1

Asset Demand in 'RA' Equilibrium

- **Proposition 2.** For asset demand function $a^A(z^t)$ satisfying

  (i) Borrowing constraint and complementary slackness condition: $a^A(z^t) \geq \phi, =$ if $u'(c^*(z^t)) > \beta(1+r^*)E[u'(c^*(z^{t+1}))|z^t]$;

  (ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 a^A(z^t)u'(c^A(z^t)) = 0$;

  (iii) Market clearing condition: $\int a^A(z^t)d\Phi_t(z^t) = B_t^A + p_t^A$,

  the transfer term $\omega(z^t)$ is given by

$$\omega(z^t) = c^A(z^t) + a^A(z^t) - (1+r_t^A)a^A(z^{t-1}) - z_t W_t^A n^A(z^t) - \pi^A(z_t) + \tau^A(z_t).$$

Proposition 1

HH Problem in Recursive Form

- Asset demand function normalization: $a^A(z^t) = g_t(a^*(z^t))$
  - $g_t(a^*(z^t)) \equiv \phi + \frac{a^*(z^t)-\phi}{A^*-\phi}(A_t^A - \phi)$
- With (i) the path of aggregates $\{r_t^A, p_t^A, W_t^A, N_t^A, C_t^A, D_t^A, Y_t^A, \Pi_t^A, T_t^A\}$, (ii) HH's states in the stationary equilibrium (z, a^{ss}), transfers $\omega_t(z, a^{ss})$ are fully pin down:

$$\omega_t(z, a^{ss}) = \frac{C_t^A}{C^*} c^*(z, a^{ss}) + g_t(a'^*(z, a^{ss})) - (1 + r_t^A) g_{t-1}(a^{ss}) - zW_t^A N_t^A - \pi_t^A(z) + \tau_t^A(z).$$

HH Problem in Recursive Form

- With transfers $\omega_t(z, a^{ss})$, HH problem in recursive form

$$V_t(z, a, a^{ss}) = \max_{\{c,n,a'\}} u(c, n) + E[V_{t+1}(z', a', a'^{ss})|z, a^{ss}]$$

$$s.t. \quad c + a' = (1 + r_t)a + zW_tN_t + \pi_t(z) - \tau_t(z) + \omega_t(z, a^{ss})$$

$$a' \geq \phi$$

- Law of motion for the exogenous state a^{ss}: $a'^{ss} = a'^{*}(z, a^{ss})$

- Along the equilibrium path, HH policy function satisfies $a_t'^{A}(z, a, a^{ss}) = g_t(a'^{*}(z, a^{ss}))$ and $c_t^A(z, a, a^{ss})/c^*(z, a^{ss}) = C_t^A/C^*$, for $a = g_{t-1}(a^{ss})$.

◀ Proposition 1

Extensions/Limitations

- Extensions:
  - Heterogeneity in discount factor.
  - Friction-less labor supply;
  - Linear labor income taxation;
  - Non-linear labor supply;
- Limitations:
  - Power utility function.

◂ Proposition 1

Aggregate Labor Supply

- Friction-less labor supply. $W_t^A (C_t^A)^{-\sigma} = \varphi^A (N_t^A)^\nu$, where $\varphi^A \equiv W^* (C^*)^{-\sigma} (N^*)^{-\nu}$.
  Proof:

$$\frac{W_t^A z_t (c^A(z^t))^{-\sigma}}{W^* z_t (c^*(z^t))^{-\sigma}} = \frac{\varphi(n^A(z^t))^\nu}{\varphi(n^*(z^t))^\nu}$$
$$\frac{W_t^A}{W^*} (\frac{C_t^A}{C^*})^{-\sigma} = (\frac{N_t^A}{N^*})^\nu.$$

- Linear labor income taxation. $(1 - \Gamma_t^A) W_t^A (C_t^A)^{-\sigma} = \varphi^A (N_t^A)^\nu$, where $\varphi^A \equiv (1 - \Gamma^*) W^* (C^*)^{-\sigma} (N^*)^{-\nu}$.
  - $\tau(z^t) \equiv \Gamma_t W_t n(z^t)$.
  - Tax payment channel: $(1 - z_t n^*(z^t)/N^*)(T_t^A - T^*)$.

Proposition 1

Aggregate Labor Supply

- Fixed-cost of labor supply $u(c, n) = c^{1-\sigma}/(1-\sigma) - n^{1+\nu}/(1+\nu) - \theta \mathbb{1}_{n>0}$

$$N_t^A \equiv \int z_t n^A(z^t) d\Phi_t(z^t)$$

$$n^A(z^t) = \begin{cases} n', \text{if } u(c^A(z^t), n') > u(c^A(z^t) - z_t W_t^A n', 0) \\ 0, \text{otherwise} \end{cases}$$

◂ Proposition 1

Undetermined Asset Demand

- Choose $\bar{b}^A(z^t) = b^*(z^t)$ and $v^A(z^t) = v^*(z^t)$ to remove the last two residuals.

$$
\begin{aligned}
-\omega(z^t) = &\underbrace{(\hat{y}^{C,A}(z^t) - \hat{Y}_t^{C,A})y^{C,*}(z^t)}_{\text{income exposure}} + \underbrace{(b^*(z^{t-1}) - B^*)(r_t^A - r^*)}_{\text{interest rate exposure}} \\
&+ \underbrace{(\bar{T}_t^A - T^*) - (\bar{\tau}^A(z^t) - \tau^*(z^t))}_{\text{tax payment}} + \underbrace{(p_t^A - p^*)(v^*(z^{t-1}) - v^*(z^t))}_{\text{asset price redistribution}} + \\
&+ \underbrace{(\bar{b}^A(z^t) - b^A(z^t)) - (1 + r_t^A)(\bar{b}^A(z^{t-1}) - b^A(z^{t-1})) + (\bar{\tau}^A(z^t) - \tau^A(z^t))}_{\text{liquidity}} \\
&+ \hat{C}_t^A(y^{C,*}(z^t) - c^*(z^t)) \\
&+ (b^*(z^t) - \bar{b}^A(z^t)) - (1 + r_t^A)(b^*(z^{t-1}) - \bar{b}^A(z^{t-1})) \\
&+ p_t^A(v^*(z^t) - v^A(z^t)) - p_t^A(v^*(z^{t-1}) - v^A(z^{t-1}))
\end{aligned}
$$

◂ Redistribution effects

Redistribution Effects: Income Exposure Channel

- Define $y^{ND} \equiv zWn + \pi$ as the non-dividend income.

$$(\hat{y}^{C,A}(z^t) - \hat{Y}^{C,A})y^{C,*}(z^t) = (\hat{y}^{ND,A}(z^t) - \hat{Y}_t^{ND,A})y^{ND,*}(z^t) + (\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - \frac{y^{ND,*}(z^t)}{Y^{ND,*}})$$

- Given $n(z^t) = N_t$ and $\pi(z_t) \sim z_t$,

$$(\hat{y}^{C,A}(z^t) - \hat{Y}^{C,A})y^{C,*}(z^t) = (\hat{D}_t^A - \hat{C}_t^A)D^*(v^*(z^{t-1}) - z_t)$$

- $D_t = r_t^K K_{t-1} - I_t - \Psi(K_t, K_{t-1}) + \alpha \Pi_t$
  - Omitting Ψ, $D_t = \alpha Y_t - I_t$, then $\hat{I}_t^A \geq \hat{C}_t^A \leftrightarrow \hat{D}_t^A \leq \hat{C}_t^A$.
  - For typical calibrations, $\hat{D}_t^A < \hat{C}_t^A$ in the short run and $\hat{D}_t^A > \hat{C}_t^A$ in the long run.
  - Redistribution from dividend earners $(v^*(z^{t-1}) > z_t)$ to non-dividend earners $(v^*(z^{t-1}) < z_t)$ in the short run and the reverse in the long run.

Redistribution effects

Redistribution Effects: Liquidity Channel

- **Proposition 3.** Assuming (i)uniform taxation $\tau^A(z_t) - \bar{\tau}^A(z_t) = T_t^A - \bar{T}_t^A$; (ii) counterfactual borrowing constraint $\phi_t^A = \phi + B_t^A - B^*$. Then for asset demand function $\bar{a}^A(z^t)$ satisfying Proposition 2, the shifted asset demand function $a^A(z^t) \equiv \bar{a}^A(z^t) + B_t^A - B^*$ satisfies $\forall z^t \in Z^t$

  (i) Borrowing constraint and complementary slackness condition: $a^A(z^t) \geq \phi_t^A, =$ if $u'(c^*(z^t)) > \beta(1 + r^*)E[u'(c^*(z^{t+1}))|z^t]$;

  (ii) The transversality condition: $\lim_{t\to\infty} \beta^t E_0 a^A(z^t)u'(c^A(z^t)) = 0$;

  (iii) Bond market clearing: $\int a^A(z^t)d\Phi_t(z^t) = B_t^A + p_t^A$,

  The transfers $\omega(z^t)$ are invariant to the path of government debt.

Redistribution effects

Parameter Value

Parameter	Description	Value
r^*	steady state real interest rate	0.0125
σ	Risk aversion	1
Z	TFP	0.784
α	Capital share	0.24
ϵ_I	Capital adjustment cost parameter	3.3
δ^K	Depreciation of capital	0.07
K/Y	Capital to GDP	2.8
B/Y	Government debt to GDP	1
p/Y	Equity to GDP	3.2
ρ_e	Autocorrelation of earnings	0.966
σ_e	Innovation variance	0.017
κ	Slope of Phillips curve	0.1
ϵ^w	Wage rigidity	1
ϕ_π	Coefficient on inflation	1.25
ρ_B	Debt reverting rate	0.1
ρ^B	Coefficient of taxes shock	1

Calibration

Parameter Value

Household group	1	2	3	4	5	6
Population share	Bottom 50%	Next 20%	Next 10%	Next 10%	Next 5%	Top 5%
Discount factors	0.975	0.977	0.979	0.982	0.984	0.987

Calibration

Household-level Decomposition