Conservative Holdings, Aggressive Trades: Learning, Equilibrium Flows, and Risk Premia

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Motivation

Periods of **high uncertainty** are frequently associated with:

- A **flow of risky assets** from institutional to individual investors' 
  - Institutions **sell**/individuals **buy** when uncertainty is high

- An **increase in risk premia**
  - E.g. FOMC, macro, or earning announcements
Existing explanations

- Flows:
  - Portfolio constraints, information asymmetry...
  - Liquidity provision by individual investors
  - “Attention-induced” trading

- Risk premium:
  - Rational expectations: counter-cyclical **objective** risk premium (habits, long-run risks, disasters, ...)
    (survey: Cochrane, 2017)
  - Parameter uncertainty and **learning**: dynamics of **subjective risk premium** ("out-of-sample")
    (Lewellen and Shanken, 2002, Collin-Dufresne, Johannes, and Lochstoer, 2016b, Nagel and Xu, 2022, ...
This paper

- Alternative explanation for equilibrium flows and risk premia that relies on two channels:
  1. Learning about the underlying parameters of the economy
     * Economy with iid-normal dividend, unknown mean and variance
  2. Heterogeneity in agents’ confidence in parameter estimates
     * Ambiguity-neutral vs. Ambiguity-averse investors (Knightian uncertainty/robustness)

- Key channels to explain the effects of cash flow “surprises” on:
  - portfolio flows
  - risk premia
Main results

- **Equilibrium flows**
  - Ambiguity-averse: conservative holdings but aggressive trades after dividend surprises
  - Ambiguity-neutral: aggressive holdings but conservative after dividend surprises

- **Equilibrium risk premia**
  - Endogenously time-varying subjective risk-premium: increasing following large dividend surprises
    - Variance estimate increases after cash flow surprises
  - Skewness in objective risk premium:
    - Increases more after negative dividend surprises
    - Left-skewed price innovations

- **Methodology**: show how to handle learning about variance in an infinite-horizon OLG economy
  - Bayesian updating with “truncated” priors

- **Empirically**: provide support of predictions using institutional investors data
Outline

- Intuition in a simple **two-period** model
- An **infinite-horizon** overlapping generations (OLG) model
  - (Unknown mean, **known variance** $\implies$ no portfolio flows)
  - Unknown mean, **unknown variance** $\implies$ portfolio flows
- **Empirical** evidence
- Conclusion
Two-period model – Economy

- **Risky asset** in finite supply: produces perishable dividends $\tilde{d} \sim N(\mu, \sigma^2)$
  - $\mu$ unknown
  - $\sigma$ known

- **Riskless asset** in infinite supply—exogenous risk-free rate $r$

- No initial consumption

- At initial date agents have observed a history of $t$ dividends
  - time series average: $m$ and **standard error** $s = \frac{\sigma}{\sqrt{t}}$

- **Two types** of CARA agents with same risk aversion $\gamma > 0$
  - Type-S: **ambiguity neutral** (Subjective Expected Utility)
  - Type-A: **ambiguity averse**
Two-period model – Beliefs

- Type $S$ subjective distribution of the dividend ("single-prior")
  \[ \tilde{d} \sim^S \mathcal{N} \left( \mu^S, \sigma^2 \left( \frac{t + 1}{t} \right) \right), \quad \text{where} \quad \mu^S = m \]

- Type $A$ subjective distribution of the dividend ("multi-prior")
  \[ \tilde{d} \sim^A \mathcal{N} \left( \mu^A, \sigma^2 \left( \frac{t + 1}{t} \right) \right), \quad \text{where} \quad \mu^A \in \mathcal{P} \equiv [m - \kappa s, m + \kappa s] \]

with $\kappa > 0$ coefficient of ambiguity aversion

- Classical statistics, $\kappa = \text{quantile of a distribution}$ (see, e.g., Bewley, 2011)
- $\kappa$ captures heterogeneity between agents: $\kappa = 0 \implies A = S$
- Note: size of set of priors $\mathcal{P}$ depends on standard error $s$
Two-period model – Optimal portfolios

- **Type-S portfolio problem**

\[
\max_{\theta_S} \mathbb{E} \left[ -\frac{1}{\gamma} e^{-\gamma \tilde{W}^S} \right], \quad \text{s.t.} \quad \tilde{W}^S = W^S(1 + r) + \theta_S(\tilde{d} - p(1 + r))
\]

- **Type-A portfolio problem** (Gilboa and Schmeidler (1989))

\[
\max_{\theta^A} \min_{\mu^A \in \mathcal{P}} \mathbb{E}^\mathcal{A} \left[ -\frac{1}{\gamma} e^{-\gamma \tilde{W}^A} \right], \quad \text{s.t.} \quad \tilde{W}^A = W^A(1 + r) + \theta^A(\tilde{d} - p(1 + r))
\]

\[
\mathcal{P} = [m - \kappa s, m + \kappa s]
\]
Two-period model - Demand and Equilibrium

- Portfolio demand
- $p^*$
- $m - \kappa s$
- $1 + r$
- $m + \kappa s$
- $1 + r$

- $S$'s demand, $\theta^S$
- $A$'s demand, $\theta^A$
- Aggregate demand
- Aggregate supply

- $p^* < \frac{m - \kappa s}{1 + r}$: both participate
- $p^* > \frac{m - \kappa s}{1 + r}$: $A$ does not participate
Two-period model – Equilibrium

- **Equilibrium price**: \( p^* \) s.t. \( \theta^A + \theta^S = 1 \)

\[
p^* = \frac{1}{1 + r} m - \lambda,
\]

with

\[
\lambda = \begin{cases} 
\frac{\gamma}{2} \left( \frac{t+1}{t} \right) \sigma^2 + \frac{\kappa}{2} \frac{\sigma}{\sqrt{t}} & \text{if } \kappa \leq \kappa^* \text{ (A&S participate)} \\
\gamma \left( \frac{t+1}{t} \right) \sigma^2 & \text{if } \kappa > \kappa^* \text{ (Only S participates)}
\end{cases}
\]

- Agent A participates only if ambiguity aversion \( \kappa \) is **sufficiently low** \( (\kappa \leq \kappa^*) \)

- If agents A participate, risk premium **linear-quadratic** in \( \sigma \)
  - Ambiguity aversion has a **first-order effect** on asset prices
  - A is locally not risk neutral ("First-order risk aversion")

- \( \lambda \) is S’s **subjective risk premium**: \( \lambda = \mu^S - (1 + r)p^*, \mu^S = m \)
Two-period model – Equilibrium portfolio holdings

- **Equilibrium risky holdings:** Replace equilibrium \( p^* \) in agents’ risky asset demands

\[
\theta^A = \frac{1}{2} - \frac{\kappa}{2\gamma} \left( \frac{\sqrt{t}}{t+1} \right) \frac{1}{\sigma} \\
\theta^S = \frac{1}{2} + \frac{\kappa}{2\gamma} \left( \frac{\sqrt{t}}{t+1} \right) \frac{1}{\sigma}
\]

- An **increasing** dividend **volatility** \( \sigma \) leads to:
  - an increase in \( \theta^A \) (“aggressive A trades”)
  - a decline in \( \theta^S \) (“conservative S trades”)

**Note:** equilibrium portfolio weights \( \theta \)s **independent** of beliefs about dividend **mean** \( \mu \)
Equilibrium Portfolios

Ambiguity-averse equilibrium holdings $\theta^A$ increase with dividend volatility

Ambiguity-neutral equilibrium holdings $\theta^S$ decrease with dividend volatility
Intuition

- Because of ambiguity aversion \( A \) holds less risky asset than \( S \)
  - \( A \) has more marginal “risk capacity” than \( S \)
  - \( A \)’s risky-asset demand shows less risk sensitivity

- Following an increase in variance:
  - \( A \) requires less return compensation than \( S \) to keep the same portfolio
  - As a “response to an increase” in volatility (handwaving argument, more rigorous later)
  - \( \Rightarrow A \) buys and \( S \) sells risky asset

- Ambiguity aversion (\( A \) agents) implies
  - “conservative” holdings: \( \theta^A < \theta^S \) BUT
  - “aggressive” trades: \( \Delta \theta^A > 0, \Delta \theta^S < 0 \)

- Caveat: comparative statics w.r.t. \( \sigma \neq \) portfolio flows!
An Overlapping-Generations (OLG) model

- **Infinite horizon.** Two types of agents:
  - Ambiguity averse: $A$
  - Ambiguity neutral (subjective expected utility): $S$

- Agents live for **two periods** with overlapping generations, CARA utility.

- Risky and risk-free assets. No first-period consumption.

- iid dividend process $d_t \sim \mathcal{N}(\mu, \sigma^2)$:
  - $\mu, \sigma$ constant but **unknown**

- Portfolios of generation-$t$: $\theta^i_t$, $i = A, S$

- Inter-generational **flows**

$$\Delta \theta^i_t = \theta^i_t - \theta^i_{t-1}, \quad i = A, S$$
Why learning about volatility?

- For **tractability**, literature focused mainly on uncertainty about mean, not variance
  - High-frequency observations $\Rightarrow$ variance estimated precisely (Merton 1980)

- Learning about variance relevant if information arrives in “**chunks**”, e.g., FOMC announcements

- Weitzman (2007, AER): “for asset pricing implications […] the most critical issue involved in Bayesian learning […] is the unknown variance”
  - Fat tails of predictive distribution can reverse macro-finance puzzles (risk premium, riskfree, volatility)
Why not stochastic volatility?

- Starkly different implication for equilibrium flows.

- **Stochastic and observable volatility**
  - any new dividend observation reduces the standard error of the mean and hence its confidence interval

- **Unobservable volatility and learning**
  - a change in the estimated variance implies a change in the perceived information quality of all historically observed dividends
  - any new dividend observation can both increase or decrease the standard error of the mean
Two issues

1) OLG with **unknown variance** $\implies$ subjective $d_{t+1}$ is Student-t (fat tails!)
   - $\mathbb{E}_t[u(d_{t+1})] = \mathbb{E}_t \left[ -\frac{1}{\gamma} e^{-\gamma d_{t+1}} \right]$ does not exist!! (see Geweke, 2001)
   - Solution: Assume $\sigma \in [\sigma, \bar{\sigma}]$, precision $\phi = 1/\sigma^2$ is **truncated Gamma**
     - Bakshi and Skoulakis (2010) provide Bayesian updating theory that preserves **conjugacy**
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\[\begin{align*}
\text{normal} & \quad \text{t} & \quad \text{dampened t} \\
\text{density} & \quad \text{density} \\
-8\sigma & \quad -3\sigma & \quad -3\sigma
\end{align*}\]
Two issues (cont.)

2) OLG with constant parameters \((\mu, \sigma)\) \implies\ learning eventually irrelevant

- Perpetual learning is relevant when there is “leakage” in information transfer from generation \(t\) to generation \(t + 1\)

- Model information leakage as shocks that “blur” priors on \(\mu\) and \(\sigma\)

- Similar to “fading memory” (Nagel and Xu, 2021) or “age-related experiential learning” (Malmendier and Nagel, 2016, Collin-Dufresne, Johannes, and Lochstoer, 2016a, Ehling, Graniero, and Heyerdahl-Larsen, 2018, Malmendier, Pouzo, and Vanasco, 2020)
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⇒ **OLG with unknown variance and perpetual learning**

\[
\begin{align*}
  m_t &= \frac{n_{t-1}}{n_t} m_{t-1} + \frac{1}{n_t} d_t \\
  b_t &= b_{t-1} + \frac{n_{t-1}}{n_t} (d_t - m_{t-1})^2 \\
  m_{t+1} &= \frac{n_t}{n_{t+1}} m_t + \frac{1}{n_{t+1}} d_{t+1} \\
  b_{t+1} &= b_t + \frac{n_t}{n_{t+1}} (d_{t+1} - m_t)^2 \\
  d_{t+2} &\sim^i t_{\nu_{t+1}}^{D}[\mu_{t+1}^{i}, b_{t+1}, \frac{n_{t+1}+1}{n_{t+1}}]
\end{align*}
\]
OLG with unknown variance and perpetual learning: Equilibrium portfolios and risk premia

As in the two-pd model, an increase in $\hat{\sigma} \equiv \sqrt{b_t/n}$ leads to:

- an increase in $\theta_t^A$ (“aggressive A trades”) and a decrease in $\theta_t^S$
- an increase in risk premium

Portfolios, $\theta_t^S, \theta_t^A$

Risk premium $\Lambda(b_t)$
Equilibrium portfolios and dividend surprises

- **Dividend surprise:** \( e^{i}_{t+1} = d_{t+1} - \mu^i_t \). **Deviation** from subjective mean belief

- Equilibrium flows large positive and negative surprises \( \implies A \) buys and \( S \) sells the risky asset
Equilibrium portfolio flows and risk premia vs. dividend surprises

- Large surprises $\Rightarrow$ A buys ($\Delta\theta_t^A > 0$)
- Large surprises $\Rightarrow$ high risk premium $\Lambda_t$ (right axis)
Return predictability

- **Objective** risk premium. Using the equilibrium price $p_t = \frac{1}{r}m_t - \Lambda(b_t)$, we have

$$\Lambda_t^{\text{obj}} \equiv \frac{\mu}{r} - p_t = \begin{cases} \frac{1}{r}(\mu - m_t) + \Lambda(t), & \text{if known variance} \\ \frac{1}{r}(\mu - m_t) + \Lambda(b_t), & \text{if unknown variance} \end{cases}$$

$b_t$: measure of variance

- $\mu - m_t$ detectable **ex-post** but not exploitable **ex-ante** (Lewellen and Shanken, 2002)

- **Subjective** risk premium $\Lambda(b_t)$ **not countercyclical**, increases with estimated **volatility** (Nagel and Xu, 2022)

- **Skewness**: asymmetry of $\Lambda_t^{\text{obj}}$ response to news shocks:
  - Bad news $\Rightarrow$ High $b_t$ and low $m_t$ $\Rightarrow$ amplification effect on $\Lambda_t^{\text{obj}}$
  - Good news $\Rightarrow$ High $b_t$ and high $m_t$ $\Rightarrow$ dampening effect on $\Lambda_t^{\text{obj}}$
Empirical analysis (in progress . . .)

Two key challenges:

1) How to map agents in the model to observable market participants?
   ▶ Individual more averse to uncertainty than institutions (Li, Tiwari, and Tong, 2017)
   ▶ Ambiguity aversion influenced by perceived competence (Heath and Tversky, 1991, Fox and Tversky, 1995)
   ▶ ⇒ Type-A ≈ retail investors, Type-S ≈ institutional investors

2) How to measure uncertainty/surprises?
   ▶ Use large market return realizations ($\Delta p_t$) as proxy for increase in uncertainty ($e_t$)
Data and methodology

**Holdings:**
- **Aggregate** level and flow data on corporate equity holdings of **households** and **financial sector**: 1952.Q1–2020.Q4 (Federal Reserve of St. Louis, FRED)
  - 13F reporting institutions, mutual, pension and insurance funds: 274,697 firm-quarter observations

**Returns:**
- CRSP return data of all NYSE, AMEX, and NASDAQ firms, 1965.01–2020.12

**Surprises** in firms’ future profitability:
- Deviation from standardized market returns over a rolling 20-quarter window  
  (Mkt return from Kenneth French’s data library)

**Subjective** risk premium, conditional on surprises
- **Aggregate.** Households and financials holdings return in excess of the 3-month risk-free rate (FRED)
- **Cross sectional.** Fama-MacBeth regression using stock return data (CRSP)
Evidence – institutional investors portfolio flows

- Institutional investors **reduce** risky holdings after extreme surprises

**“Surprise”:** standardized market return $\Delta \theta_t^S$: change in risky asset holding of institutions
Evidence – subjective risk premium

- Large surprises (positive and negative) imply a high subjective risk premium

![Diagram showing lagged standardized market return, lag(z_r) for FRED and CRSP](image-url)
EUR STOXX 50 futures transactions on Eurex

January 2002 to December 2020

824 million trades at a frequency of milliseconds

Three trader types: Agency traders, proprietary traders, market makers

Argue that agency traders (trading for clients) are less ambiguity averse than proprietary traders and market makers

- Desire for robustness as a reaction of market makers’ inventory risk, Routledge and Zin (2009), Easley and O’Hara (2010) and Zhou (2021)

- Proprietary traders and market makers do less informed trades, Menkveld and Saru (2023)

Confirm our model predictions regarding flows and risk premia
Conclusions

- Develop a general equilibrium asset pricing model with
  - learning about the moments of the endowment process
  - heterogeneous confidence in parameter estimates

- Learning about variance (not just the mean)
  - key to understanding risk-premia and portfolio flows around dividend “surprises”

- Can explain features of both asset prices and flows
  - Individuals hold conservative portfolios but buy in response to positive and negative surprises
  - Endogenously time-varying risk premium

- Verify these predictions empirically

- Understanding the dynamics of flows in inelastic markets (“demand-based asset pricing”)
Additional Slides
Intuition–Type-\(S\) agents’ iso-portfolios

- **Iso-portfolios**: pairs \((\sigma, \lambda)\) s.t. \(S\)'s portfolio \(\theta^S\) is constant

- Agents \(S\) are “locally risk neutral” (second-order risk aversion)
Intuition–Type-A agents’ iso-portfolios

- If $\lambda < \kappa \sigma / \sqrt{t} \Rightarrow \text{no participation}$
- Agents A are “locally risk averse” (first-order risk aversion)
Intuition–Market clearing, $\theta^A + \theta^S = 1$

- A iso-lines always **flatter** than $S$ at equilibrium ($\theta^A + \theta^S = 1$)
  - Agents $A$ require a **smaller compensation** for bearing **extra risk** than $S$
Ambiguity averse agents increase $\theta^A$ as $\sigma$ increases.

- $A$ ($S$) iso-lines always intersect $\lambda$ from above (below)


Bibliography II


