Costly state verification with ex post participation constraint

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EEA-ESEM, Barcelona, August 30

Introduction

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Research Question

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Related literature:

- **Deterministic inspection**: Townsend (1979), Gale and Hellwig (1985), Halac and Yared (2020).
- **Stochastic inspection**: Border and Sobel (1987), Mookherjee and Png (1989), Baron and Besanko (1984), Ball and Kattwinkel (2019), Palonen and Pekkarinen (2022), Ball and Knoepfle (2023).
- **Monopoly regulation**: Baron and Myerson (1982), Amador and Bagwell (2013), Amador and Bagwell (2022), Laffont and Tirole (1990). Laffont and Tirole (1993).
- **Allocation of an indivisible good**: Ben-Porath et al. (2014), Mylovanov and Zapechelnyuk (2017), Li (2020), Erlanson and Kleiner (2020).
- **Without commitment**: Khalil (1997), Sadakane and Tam (2022).

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   - All equilibria payoffs have semi-separating structures.

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2. Without commitment:
   - All equilibria payoffs have semi-separating structures.
   - The maximum payoff equilibrium has a structure similar to the commitment policy.

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Two players: the principal, and the agent.

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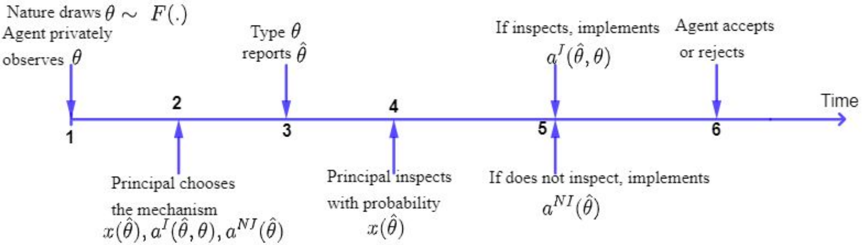
The agent sends a message $m \in \mathbb{M}$.
The principal implements the mechanism.
The agent decides to accept or reject the mandated action.

Timing-commitment

Assume the principal can commit to the mechanism.



Results

The principal's problem is:

$$\max_{x(.),a^{NI}(.),a^{I}(.,.)} \mathbb{E}\Big[(1 - x(\theta))\Big(V(\theta, a^{NI}(\theta))\Big)\mathbb{1}_{a^{NI}(\theta)\geq\theta} + x(\theta)\Big(-\phi + V(\theta, a^{I}(\theta,\theta))\mathbb{1}_{a^{I}(\theta,\theta)\geq\theta}\Big)\Big],$$

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subject to the IC conditions for the agent which is:

$$\theta \in \operatorname*{argmax}_{\hat{\theta}}\Big[(1-x(\hat{\theta}))\Big(u(\theta,a^{NI}(\hat{\theta}))\Big)\mathbb{1}_{a^{NI}(\hat{\theta})\geq\theta} + x(\hat{\theta})\Big(u(\theta,a^{I}(\hat{\theta},\theta))\Big)\mathbb{1}_{a^{I}(\hat{\theta},\theta)\geq\theta}\Big].$$

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$u(\theta,\theta)=0, u_a(\theta,a)>0.$

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For $a\geq\theta$, and $\theta\in[\underline{\theta},\overline{\theta}]$
$V_\theta(\theta,a)\leq 0, V_a(\theta,a)>0$

Results-deterministic inspection

Proposition: the optimal policy

There exist two theresholds θ^*, and θ^{**} such that $\underline{\theta} \leq \theta^* \leq \theta^{**} \leq \overline{\theta}$.
The optimal policy is

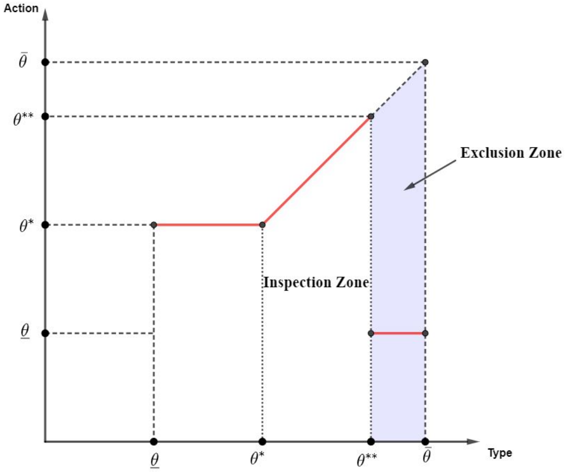
$$a^I(\hat{\theta}, \theta) = \theta,$$

$$x(\hat{\theta}) = \begin{cases} 0 & \hat{\theta} \leq \theta^* \\ 1 & \theta^{**} \geq \hat{\theta} > \theta^* \\ 0 & \hat{\theta} > \theta^{**}, \end{cases}$$

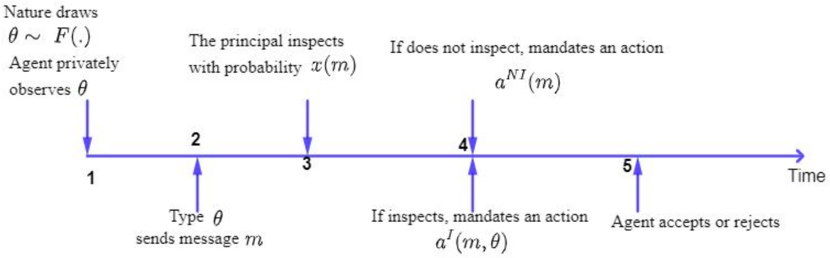
$$a^{NI}(\hat{\theta}) = \begin{cases} \theta^* & \hat{\theta} \leq \theta^* \\ \theta & \theta^{**} \geq \hat{\theta} > \theta^* \\ \underline{\theta} & \hat{\theta} > \theta^{**}, \end{cases}$$

Results-deterministic inspection

The (on the equilibrium path) optimal policy



Timing-without commitment



Without commitment

Equilibrium: PBE.

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principal (receiver):
The payoff if inspects is $V(\theta, a^I(m,\theta))\mathbb{1}_{a^I(m,\theta)\geq\theta}$ which maximizes at

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If does not inspect, should choose

$$a^{NI}(m) \in \underset{\tilde{a}}{\operatorname{argmax}} \int_{\underline{\theta}}^{\overline{\theta}} \left[V(\theta,\tilde{a})\,\mathbb{1}_{\tilde{a}\geq\theta}\right]\beta(\theta|m)\,d\theta.$$

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agent (sender): The agent with type θ chooses message $m(\theta)$ such that

$$m(\theta) \in \underset{\tilde{m}}{\operatorname{argmax}} \Big[\big(1 - x(\tilde{m})\big) u(\theta, a^{NI}(\tilde{m}))\mathbb{1}_{a^{NI}(\tilde{m})\geq\theta} + x(\tilde{m}) u(\theta, a^I(\tilde{m},\theta))\mathbb{1}_{a^I(\tilde{m},\theta)\geq\theta} \Big].$$

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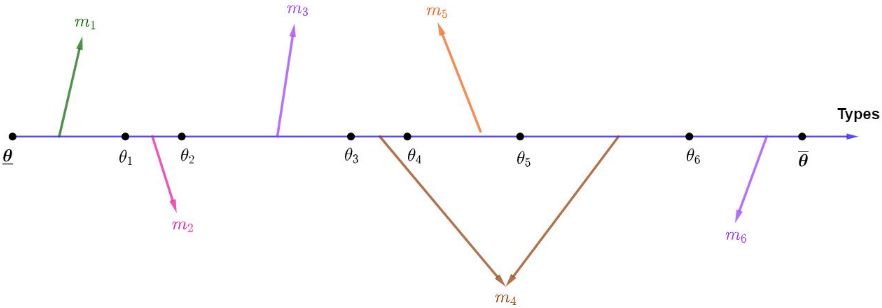

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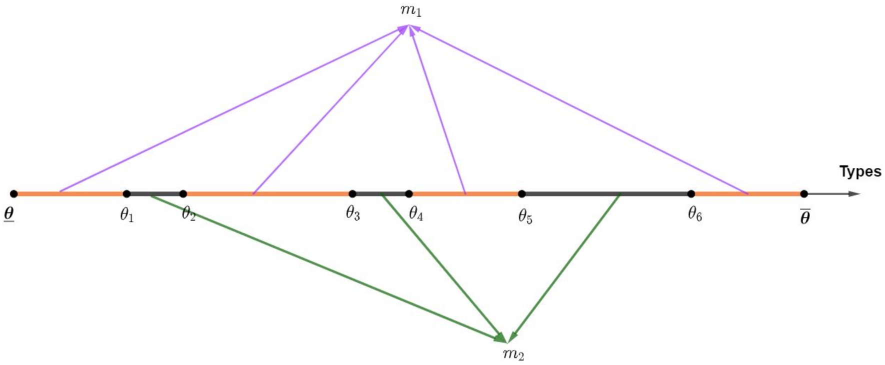

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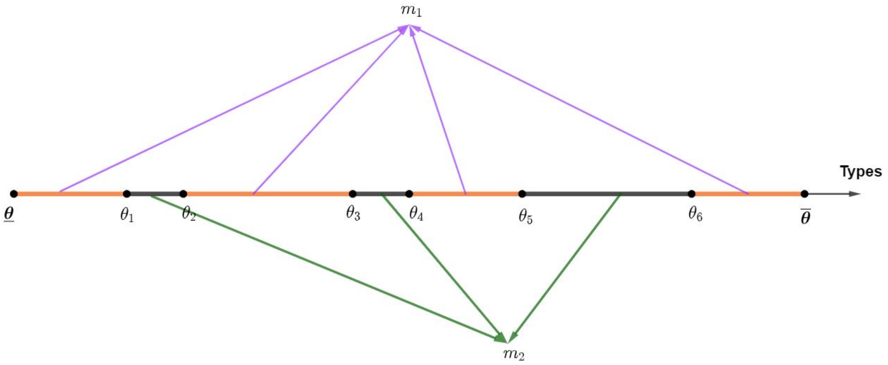

Proposition: Equilibria (ex-ante) payoff for the principal

We can restrict all equilibria payoff to **semi-separating equilibria**.
There are maximum **two groups** of types. Types in each group pool together in one message and they separate by sending different messages.

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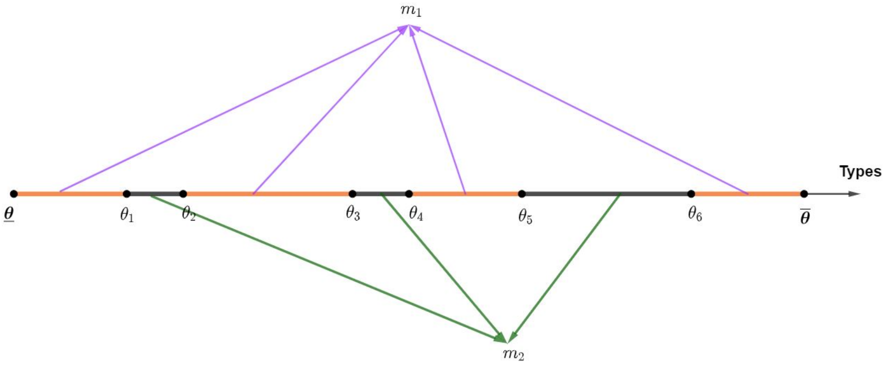

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Two messages: **"inspect me", and "do not inspect me"**.

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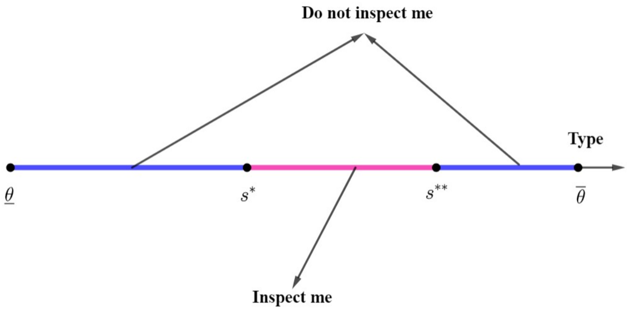

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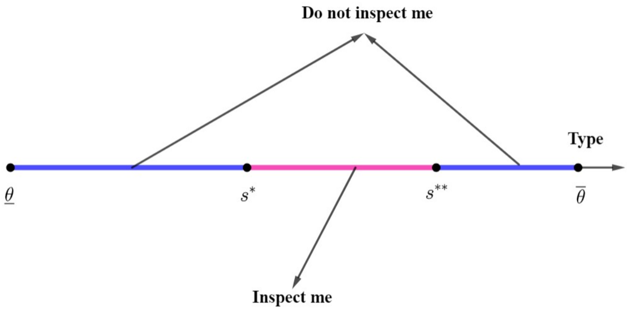

Proposition: Maximum payoff for the principal

If $\frac{\partial^2 V(\theta,a)}{\partial\theta\partial a} \geq 0$ (single-crossing condition), then the maximum payoff equilibrium has the following structure:

$$x(\text{"Inspect me"}) = 1, \;\; x(\text{"Do not inspect me"}) = 0$$

$$a^{NI}(\text{"Do not inspect me"}) = s^*$$

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- In all equilibria payoffs, the agent's strategy is semi-separating.

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Thank You!

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