Optimal Portfolio Choice with Longevity, Critical Illness and Long-Term Care Insurance*

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Abstract

We develop a rich life cycle model to assess the demand for life annuities, critical illness insurance, and long-term care insurance among retirees in a portfolio-allocation setting. We calibrate our model to reflect a developing-country context with limited public insurance, e.g., urban China. We show that retirees with a low pension allocate at least 30% of their financial wealth at retirement to a life annuity. Retirees with an average pension allocate at least 30% to critical illness insurance. The allocation to long-term care insurance ranges from 5% to 33% across all economic profiles we considered. State-dependent preferences and insurance bundling both increase annuity demand for some retirees. Our results suggest that developing countries should first ensure retirement income security and then focus on covering catastrophic medical expenses, while providing basic long-term care services for all, with special arrangements for females.

Keywords: Annuity; Long-term care insurance; Critical illness insurance; Life cycle saving; Household finance; Retirement; China

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1. Introduction

Improvements in life expectancy and lower fertility rates have led to rapid population ageing in many developing countries such as Brazil, China and Mexico. The growing older population in these countries requires access to pension income, medical services, and long-term care (LTC). In the past, older adults relied mainly on family members for financial support and personal care. However, smaller families, and increased migration and female labour force participation pose challenges to this traditional family support model (e.g., Zhen et al., 2015). At the same time, public insurance programs in many developing countries often provide only basic pension benefits and limited cover for critical illness and LTC. As a result, individuals and their families may face high out-of-pocket medical costs, as critical illnesses such as cancer, heart attack, or stroke often require expensive drugs and advanced medical treatments imported from developed countries (Wagstaff et al., 2018; Liu et al., 2017). An important consequence of the lack of adequate insurance to cover uncertain out-of-pocket health-related costs is that retirees may self-insure and hold on to assets that would otherwise be drawn down to support their living standards in retirement (e.g., De Nardi et al., 2010; Alonso Garcia et al., 2022). However, previous studies on the insurance demand to cover longevity- and health-related risks have largely focused on one type of insurance (e.g., Achou, 2021; Ameriks et al., 2020; Horneff et al., 2020; Reichling and Smetters, 2015), and the impact of critical illness is ignored.

This paper develops a new life-cycle model to study the demand for private life annuities, critical illness insurance and LTC insurance for retirees in a developing-country context with limited public insurance. Specifically, the life annuity provides a lifetime income, the critical illness insurance provides a lump sum benefit when the insured is diagnosed with a critical illness for the first time. The LTC insurance provides regular income when the insured is LTC dependent.

We calibrate the model to reflect the institutional setting in urban China. China is a rapidly ageing country with a basic public insurance system and a developing private insurance market. The public pension has been reformed several times with a large variation in benefits provision. The public health insurance is nearly universal, but the cover is basic, and there is a high chance of catastrophic cost due to limited cover for advanced medical treatments for critical illness (Liu et al., 2017), while LTC programs are being piloted in a number of cities. The private insurance market in China focuses on short-term products for investment purpose, and rarely offers long-term health-related insurance products for retirees. The insurance products considered in this paper are typically not offered in the private market. The optimal portfolios of and welfare gains from access to the three insurance products from our model provide insights into the development of private insurance market.

Our paper makes the following three main contributions. First, we contribute to the literature on household finance focusing on the role of health costs, annuities, or LTC insurance (e.g., De Nardi et al., 2010; Pashchenko, 2013; Reichling and Smetters, 2015; Peijnenburg et al., 2017; Ameriks et al., 2020; Achou, 2021). Most of these studies focus on only one type of insurance (except Wu et al. (2022a) who consider annuities and LTC insurance together), and critical illness insurance is little studied (Schendel, 2014; Hambel, 2020). We are the first to consider life annuities, critical illness insurance, and LTC insurance together in a life-cycle framework, and we study the insurance demand as a portfolio choice. We find that the optimal portfolio with annuities, critical illness insurance and LTC insurance for retirees largely depends on their economic background. For retirees with a low pension, annuities are the most important insurance in their retirement portfolio, while for retirees with an average pension, critical illness insurance is most important.[1] The demand for LTC insurance is relatively lower but positive at all levels of wealth and pensions considered in this study and higher for females. Our results also provide a reference for studies that elicit stated preferences for retirement portfolio allocation that includes longevity and health-contingent insurance (e.g., Wu et al., 2022b; Wan et al., 2023).

Second, our study relates to the household finance literature with state-dependent utility of consumption, that is, whether the marginal utility of consumption is higher or lower in a poor than in a healthy state (e.g., Viscusi and Evans, 1990; Finkelstein et al., 2013). However, their impact is mixed and under-researched. Blundell et al. (2020) found that the consumption fluctuation in old age in the U.S. was largely driven by a reduction of the marginal utility of consumption after a health shock. In contrast, Peijnenburg et al. (2017) showed that neither a higher nor a lower marginal utility of consumption at a poor health state would affect annuity demand in the U.S. We contribute to this literature by studying the impact of state-dependent utility on an optimal portfolio with annuities, critical illness insurance and LTC insurance. We model state-dependent preferences such that the marginal utility of non-medical consumption depends on a retiree's health state, including healthy, critically ill and LTC dependent. This is also the first study to consider two states of poor health and simultaneously allow those states both a lower and a higher marginal utility of consumption, and we find that state-dependent utility affects demand for annuities, critical illness and LTC insurance. We find higher annuity demand for wealthier retirees when taking account of state-dependent utility, regardless of whether a higher or lower weight is assumed for consumption in poor health states. We also find that a higher marginal utility of

[1] In this study, we choose to model the insurance demand of retirees in urban China with low and average public pension incomes. Their pension replacement rates are low and they are financially much more vulnerable than retirees with a high pension. See Section 2 for background on China's pension system.

consumption in both poor health states increased the demand for critical illness and LTC insurance but decreased the annuity demand.

Third, we also contribute to the literature on the 'annuity puzzle' (e.g., Benartzi et al., 2011; Pashchenko, 2013; Reiching and Smetters, 2015; Peijnenburg et al., 2017). We extend this literature focusing on the role of precautionary savings due to health-related risks (e.g., Reichling and Smetters, 2015, Peijnenburg et al., 2017) by showing that when public insurance provides only basic health cover, retirees with an average pension would purchase critical illness insurance instead of an annuity. Furthermore, we show that state-dependent preference and bundled insurance covering risks of longevity, critical illness and LTC could increase annuity demand for more affluent retirees.

Also, previous studies are mostly set in a developed-country context. We also extend the literature to a developing-country context where public health insurance provides only basic cover for critical illness and LTC, and we highlight the importance of critical illness and LTC insurance on annuity demand. The welfare gains with optimal insurance, compared with having no private insurance, are substantially higher for less wealthy retirees.

Our results suggest that policymakers and insurers in developing countries planning for insurance expansion should focus first on providing enough income for those with low pensions and then on covering large expenditures due to critical illness. A basic layer of LTC services should be provided for all, with a higher provision for females. For insurers, we highlight the importance of targeting and provide theoretically optimal insurance levels in a portfolio context that better supports financial advisory services. As well, we suggest that bundled longevity and health-contingent insurance products can increase annuity demand.

Our paper is organised as follows. In Section 2, we provide background on China's public and private insurance in retirement. In Section 3, we describe the life-cycle model of consumption and portfolio allocation for retirement. In Section 4, we present the benchmark results for retirees with different levels of public pensions and financial wealth. In Section 5, we show how key model parameters affect optimal portfolios. Section 6 provides discussion and concluding remarks.

2. Background on China's public and private insurance in retirement

China is ageing with unprecedented speed and magnitude. By the end of 2020, 18.7% of the Chinese population was aged 60 years or above (264 million), and this ratio is projected to increase to 38% (500 million) by 2050 (United Nations, 2022). However, the tradition of elderly support by family members has been dissipated by rapid economic development (Feng et al., 2012). Although there have been several rounds of reform, China's public insurance system provides only basic benefits, and the private market for retirement insurance is immature.[2] The replacement rate of the public pension for urban employees, fell from about 80% of a worker's pre-retirement wage in the 1990s to about 45% of the local average wage in 2019, and it varied substantially with income, ranging from less than 20% for the lowest income group to more than 90% for the highest income group, who were mostly government employees (Chen and Turner, 2021). The private pension cover is close to nil (Fang and Feng, 2020). China's public health insurance system is universal but often excludes expensive imported drugs and medical treatments, and individuals must pay out-of-pocket and often catastrophic expenditures if they desire to access advanced medical services (Liu et al., 2017). Public LTC insurance is still in the pilot phase (with only 49 pilot programs introduced by August 2021).

Despite the government's determination to promote the development of insurance for critical illness, LTC, and retirement income (CBIRC, 2020), the insurance market for the old is still

[2] Please find the review of the public insurance systems in China for pension (Fang and Feng, 2020), healthcare (Pan et al., 2016), and long-term care (Yang et al., 2016).

undeveloped. Few retirement insurance products in China provide long-term benefits or inflation protection. The private annuity market is small and typically products are purchased with regular premiums while working with fixed large payouts at predetermined ages (e.g., 60 or 80 years old) and death benefits. They are mostly framed as wealth management financial products and not designed for longevity risk protection. Long-term private critical illness insurance is typically only available to young adults, rather than the old and guaranteed renewable contracts are also rare for the old. Many critical illness insurance products also set a limit for the insured amount. Private LTC insurance products generally pay lump sum benefits and are designed for investment purposes (Huang et al., 2019). The contracts have limited payouts, short protection periods and adjusted payout amounts depending on age. In addition, A small number of bundled annuity and critical illness and LTC insurance products are on the market. They inherit both the features and limits of the standalone insurance products, and their designs for the value of contracts used for risk protection and investment are more complex.

3. The model

This section introduces a life-cycle model for a single retiree who faces longevity, critical illness, and LTC risks in a developing-country context. We model males and females separately, and assume that the retirees are covered by the public pension and public health insurance for urban employees in China. We model individuals who retire with an initial endowment of financial wealth and public pension. At retirement, the individual decides to use a portion of her wealth to purchase one or more of life annuity, critical illness, and LTC insurance. The remaining wealth is placed in a savings account. In each period while alive, the retiree receives a public pension income, faces stochastic health shocks due to critical illness or LTC, receives insurance payouts (if purchased) and chooses her consumption. At death, the individual leaves a bequest. Different from related studies (De Nardi et al., 2010; Reichling and Smetters, 2015; Peijnenburg et al., 2017; Ameriks et al., 2020), our model features both critical illness and LTC states, and the associated random catastrophic medical expenditure due to critical illness and financial cost of (informal) LTC, which are critical in a developing-country context.

3.1. Heterogeneous individuals

We model males and females with different levels of financial wealth and public pension income because health risks often differ by gender while public pensions and wealth are important factors in financial planning (Inkmann et al., 2011). We consider men aged 60 and women aged 55, in line with the statutory retirement ages for males and white-collar females in China. The individuals retire with financial wealth of W_0 and receive an annual public pension P_t at period t. We derive the optimal retirement portfolio for four types of retirees: i) low pension, low wealth, ii) low pension, high wealth, iii), average pension, low wealth, and iv) average pension, high pension. We choose to model the insurance demand of retirees with low and average public pensions because their pension replacement rates are low and they are financially much more vulnerable than retirees with a high pension as described in Section 2. Based on Zhu and Walker (2018) and the average monthly pensions at the provincial level in 2020,[3] we set the low and the average pension incomes to CNY 1,000 and CNY 3,000, respectively.[4] The wealth values are calibrated around the 30th and 80th percentile of the wealth distribution based on data from the 2018 wave of the nationally-representative China Health and Retirement Longitudinal Survey (CHARLS), which are CNY 150,000 and CNY 1 million, respectively.[5]

[3] The monthly pension at the provincial level can be obtained from http://www.stats.gov.cn/sj/ndsj/2021/indexch.htm (in Chinese).

[4] The exchange rate was 1.00 US Dollars = 6.97 CNY on March 8, 2023.

[5] We did not use a symmetric range because CNY 150,000 is already low, and the even-lower 20th percentile of the wealth could make liquidity a main concern because the relative impacts of other non-modelled risks are larger with limited wealth.

We assume the same public health insurance for urban employees (Employee Basic Medical Insurance) for all four types of retirees since the benefits are almost identical for all scheme participants.[6] The costs of critical illness and LTC in our model are out-of-pocket (see Section 3.4 for more detail).

Our model captures several means-tested government subsidies in a stylised way through a minimum consumption floor S. This subsidy reflects a public safety net. We set the consumption floor according to the government subsidy 'Dibao' – a subsidy for those without sufficient income for necessities. The average subsidy was CNY 687 per month, according to official provincial data in 2020 (Ministry of Civil Affairs, 2020).

3.2. Health and health-related costs

In each period $t > 0$, the retiree can be in one of the four states $H_t \in \{1,2,3,4\}$, where $H_t = 1$ corresponds to healthy, $H_t = 2$ corresponds to critically ill (diagnosed with a critical illness), $H_t = 3$ corresponds to LTC dependent, that is, the retiree cannot undertake without difficulty three or more of the following six activities of daily living (ADLs): bathing, dressing, eating, toileting, continence, transferring in and out of bed,[7] and $H_t = 4$ corresponds to death. We model health as an exogenous Markov process and assume a maximum age of 105. Appendix A describes the estimation of the models for the age-specific health transition probabilities. We estimate separate models for males and females using official tables for mortality and critical illness and estimates for LTC transitions based on CHARLS data. We investigate the sensitivity of the assumptions for health transitions in Section 5.2.

[6] The accumulated amounts in the Individual Medical Account depend on salary history, however, the contribution period is limited for the generation currently close to retirement, and their amounts are not comparable to catastrophic medical costs we considered in the model. Therefore, we assume its impact is minimal.

[7] In China, failing to perform at least three ADLs is usually required to qualify for private long-term care insurance payments.

The individual faces random critical illness $CostCI_t$ and random LTC cost $CostLTC_t$. Both costs are paid out-of-pocket, given that China's public insurance provides only partial cover for critical illness and no widely available cover for LTC (see Section 2). The calibrations of such costs are based on empirical studies and industry reports because of limited data. For simplicity and data limitations, we assume the cost distributions are the same for males and females when they are critically ill or LTC dependent.

3.2.1. Cost of critical illness

The cost of critical illness in China is often immediate and catastrophic to families, especially in the first year after diagnosis as it is often extensive because of urgent life-extending treatments and hospitalisation. To model the distribution of the out-of-pocket cost of critical illness, we follow previous studies and use a lognormal distribution to model the cost of critical illness (e.g., Wu et al., 2018). This distribution captures the fact that the cost is typically right-skewed with a large variation.[8] To calibrate the lognormal distribution for the out-of-pocket cost of critical illness, we use individual medical expenditure data recorded directly by healthcare systems rather than through household surveys. This is because household survey data is likely to underestimate the cost because individuals with severe diseases are less likely to participate, and surveys such as the CHARLS do not provide details of disease severity, nor sufficient measures to estimate full expenses due to critical illness. We estimate the expectation of the lognormal-distributed cost according to the average median of the out-of-pocket costs for critical illness in Beijing (the capital of China) and Zhaoqing (an inland city with a larger reimbursement rate and a larger cap amount than that of an average city in China) in 2014 (Fang et al., 2018). We select these cities as their

[8] The exact cost of critical illnesses is often difficult to access. The average individual inpatient cost is much larger than the median (Zhang et al., 2019), and the variation of the coefficient for personal health expenditure is approximately 7 (Zhao, 2019).

medical expenditure more likely to represent the cost of receiving adequate treatments for illness due to their high income or reimbursement rate and the availability to access advanced medical services in these cities. We assume the cost is exogenous and everyone prefers to receive adequate medical services.[9] We adjust the cost to 2020 prices using medical cost inflation net of the general consumer price index (CPI) from 2015 to 2020 in China.[10] The adjusted cost is CNY 216,000 and assumed to represent a sufficient level of medical expenditure to access adequate medical services.[11]

To calibrate the standard deviation of the lognormal cost of critical illness, we rely on individual-level inpatient billings recorded by the healthcare system and more than 20,000 hospitalisation cases as per Wu et al. (2018). However, the data does not include individuals' ages or disease severity, but the total time for inpatient care is recorded. We include in the estimation sample only those individuals with an inpatient time of more than 25 days, which is about the average length of inpatient time for those aged over 60 (Yin et al., 2019). The calibrated lognormal distribution for the out-of-pocket cost of critical illness is:

$$CostCI \sim \text{Lognormal}\,(11.860, 0.920^2). \tag{4}$$

We further assume a maximum out-of-pocket cost of CNY 800,000. This value is based on an approved reimbursement of CNY 1,500,000 from private medical insurance in 2019.[12]

[9] See Yogo (2016) and reference therein for life-cycle models with endogenous mortality and health investment.

[10] Based on the Willis Towers Watson Global Medical Trends Survey Report 2016, 2017, 2020. The value in 2020 was a forecast. See https://www.willistowerswatson.com/en-US/Insights/2019/11/2020-global-medical-trends-survey-report.

[11] For example, the total medical expenditure for a COVID-19-infected patient with severe symptoms was, on average, approximately CNY 400,000 as reported by the government in 2020, and this amount is considered adequate. Assuming a 50% reimbursement rate, the out-of-pocket cost was around CNY 200,000, which is close to our estimation.

[12] Due to a lack of data, we do not incorporate an auto-regressive process for the out-of-pocket cost of critical illness. Modelling this will likely reduce the demand for critical illness insurance because of a lower expected cost.

3.2.2. Cost of long-term care

We model the cost of LTC as a lognormal function of age and calibrate the cost based on data from the CHARLS Wave 3, collected in 2015.[13] We note that this cost represents the financial cost of informal care as paid public or private LTC services are not yet available in most cities in China. CHARLS reports the hours of informal care each participant received and the cost of hiring a nurse for the participant. In 2020, the minimum hourly wage ranged from CNY 12.5 to CNY 25.3 at the provincial level.[14] We use an hourly wage of CNY 20 to estimate the cost of hours spent for informal care. This approximates the cost of paying a home-health aide, and we add that to the cost of hiring a nurse to calculate the implied total monthly financial costs for LTC. During the estimation, we exclude respondents that were LTC dependent but reported zero cost (e.g., receiving no informal care or not hiring a nurse). We find that a linear model captures the association between the logarithm of the total monthly LTC expenditure and the age of the survey participants reasonably well, and the residual plot shows no evidence of heteroscedasticity. The estimated monthly cost of LTC has the following lognormal distribution with respect to age:

$$CostLTC\,(Age) \sim \text{Lognormal}\,(6.130 + 0.019 * Age,\ 1.460^2). \qquad (5)$$

We also assume a maximum monthly LTC cost of CNY 8,000, which is about twice our model-predicted LTC cost at age 60, and is much higher than the cost of (pilot) institutional care, which is about CNY 5,000 according to Lu et al. (2017). Finally, we multiply the monthly cost by twelve to derive the annual cost of LTC.

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[13] The CHARLS data in 2018 was not available during the model calibration process, and we want to compare our results to the empirical study by Wan et al. (2023) which used the same data.

[14] The data can be found at http://www.mohrss.gov.cn/SYrlzyhshbzb/laodongguanxi_/fwyd/202111/t20211119_428287.html.

3.3. Insurance products

At the start of the model period (at retirement), the retiree can purchase one or more of life annuities, critical illness insurance, and LTC insurance. For simplicity, the retiree cannot sell or terminate the insurance contracts or purchase additional cover in the future. The life annuity pays a fixed amount of real income $Annuity_t$ at each period t until death. Critical illness insurance provides a real lump sum payment CII_t when the insured is diagnosed with a critical illness for the first time. LTC insurance provides a fixed amount of real income $LTCI_t$ in each period when the insured is in the LTC state. The payments from the insurance products can be used for consumption and health-related costs. For example, the annuity income can be saved to cover health costs later. Both the annuity and the critical illness insurance are priced in an actuarially fair way according to age and gender using their respective official tables in China.[15] Since there are no official tables for LTC insurance, we have based our estimates for LTC insurance on the CHARLS data.[16] The pricing details for LTC insurance are described in Appendix B. In addition, we assume a 15% insurance loading for all retirement insurance products, similar to Mitchell et al. (1999). This loading represents the level of administrative costs in China's insurance market (Zhang et al., 2021). In our benchmark model, CNY 10,000 could buy a monthly annuity income of about CNY 35 for males, and CNY 28 for females. Similarly, it could purchase a lump sum payout for critical illness of about CNY 19,496 for males and CNY 21,365 for females, or a regular payment of about CNY 471 and CNY 329 when being LTC dependent for males and females, respectively. The price differences are due to a higher life expectancy of females, comparable cumulative incidence of

[15] We use the official mortality table for pension business to price annuity (CBIRC, 2016), and we use the official table for critical illness incidence rates (CBIRC, 2013) and the mortality table for health insurance business (CBIRC, 2016) to price critical illness insurance.

[16] The insurance industry in China prices annuity and critical illness insurance separately based on regulated official tables. For LTC insurance, both the census data and longitudinal surveys like the CHARLS have been used to estimate the health transitions relevant for pricing. However, the definitions of LTC state are not consistent.

critical illness, and a longer duration of needing LTC, respectively. We investigate the impact of pricing assumptions in Section 5.3.

Retirement wealth not used to buy any of the three insurance products and regular income or payments not used for consumption is placed in a savings account that earns a risk-free rate. We do not consider risky assets because it is not common in the Chinese context with less than 5% of urban individuals close to retirement participating in the stock market (CHARLS, 2018).

Throughout the paper, we assume a real interest rate of R = 2% per year according to the World Bank,[17] and set the real discount rate for insurance pricing at 1.5%, in line with insurance regulations in China. The real discount rate is lower than the real interest rate because the China Banking and Insurance Regulatory Commission (CBIRC) requires that the nominal pricing interest rate cannot be set higher than 3.5% for long-term insurance, and we assume an inflation rate of 2% based on a ten-year average of China's national CPI.

3.4. Preferences

We consider state-dependent preferences. Empirical studies find that the marginal utility of consumption depends on an agent's state of health (e.g., Finkelstein et al., 2013; Viscusi and Evans 1990). Wang and Wang (2020) confirmed this for China. For our benchmark results, we assume the agent's preference is time-separable with subjective discount factor β, and at each period she has a state-dependent utility function similar to Koijen et al. (2016) and Peijnenburg et al. (2017):

$$u(c_t|H_t) = \frac{\eta_{H_t} c_t^{1-\gamma}}{1-\gamma}, \tag{1}$$

[17] The average real interest rate in China from 2010 to 2019 was 2.04%. See https://data.worldbank.org/indicator/FR.INR.RINR?locations=CN.

where c_t is the consumption in period t, γ is the inverse of elasticity of intertemporal substitution or the relative risk aversion coefficient (e.g., Yaari, 1965; Davidoff et al., 2005),[18] η_{Ht} is a parameter weighting the impact of health state on the marginal utility of consumption. This function distinguishes the marginal utility of consumption in different health states, as suggested by Finkelstein et al. (2013), and is a natural extension of Peijnenburg et al. (2017) to allow for state-dependent utilities with more than one unhealthy state. Another way to specify a state-dependent utility is to assume that consumption, rather than utility, is discounted in different health states (Laitner et al., 2018). In this paper, we follow the former approach to incorporate state-dependent utility because i) in our model, the individual chooses consumption after paying health-related costs and, therefore, consumption does not need to be discounted to reflect non-medical consumption, and ii) we are interested in the impact of marginal utility of consumption at the poor health states on portfolio choice. Note that the state-dependent utility given in Equation (1) reduces to the standard case, that is, without state-dependent utility, once $\eta_{Ht} = 1$, and we study this case in Section 5.1.2.

We use $v(M)$ to value bequest motives:

$$v(M) = \frac{bM^{1-\gamma}}{1-\gamma}, \tag{2}$$

where M is the bequest wealth and b is the strength of the bequest motive. This bequest function is used in both simulation studies to identify optimal portfolio allocation (e.g., Friedman and Warshawsky, 1990) and in empirical studies with life-cycle models (e.g., Iskhakov and Keane,

[18] Other studies question the validity of time-separable utility and use the Epstein-Zin-Weil (EZW) preferences (Epstein and Zin, 1989; Weil 1990), which separate the relative risk aversion from the elasticity of inter-temporal substitution, see, e.g., Inkmann et al. (2011).

2021). Its form can be generalised into our state-dependent utility framework by considering death as the terminal poor-health state.[19]

To generate our benchmark results, we calibrate the subjective discount factor β = 0.999 and risk aversion ρ = 3 to represent high patience and average risk aversion among Chinese people, following İmrohoroğlu and Zhao (2018), who study LTC risks and savings in China. These values are comparable to those used in the U.S.-focused literature such as Peijnenburg et al. (2017), who used 0.96 (time preference) and 5 (risk aversion) with the same utility function. We set the strength of the bequest motive b = 50 to reflect a relatively strong bequest motive following Friedman and Warshawsky (1990). Regarding the parameter for state-dependent utility, empirical results are mixed for developed countries (e.g., Finkelstein et al., 2013), and limited for China. Wang and Wang (2020) estimated a linear probability model and found that, for older Chinese, the value of non-medical consumption was around 20% higher when an individual has several chronic diseases and around 30% lower if they were limited by three or more ADLs. Therefore, for η_{Ht} we use 1.2 and 0.7 if the agent is critically ill or needs LTC, respectively. We test the sensitivity of the results to preference parameters in Section 5.1.

3.5. The retiree's objective and decision problem

We model the decision of a healthy retiree who, at retirement, decides to use a portion of her wealth to purchase a life annuity, critical illness insurance, and LTC insurance and chooses her consumption at each future year until death. Her objective is to maximise the expected lifetime utility of consumption in retirement given by the following recursive specification:

[19] Other studies use luxury-bequest motives, see, e.g., De Nardi et al. (2010), Lockwood (2012) and Ameriks et al. (2020).

$$\begin{cases} V_t(M_t, H_t) = \max_{c_t, \omega_a, \omega_c, \omega_l} E_t\left(u(c_t, H_t) + \beta\left[\sum_{H_{t+1}=1}^{4} \pi_t(H_t, H_{t+1}) V_{t+1}(M_{t+1}, H_{t+1})\right]\right) \\ V_t(M_t, 4) = v(M_t) \end{cases}, \text{ s.t.}$$

$$A_t = M_t + P_t + Annuity_t(\omega_a) + CII_t(\omega_c) + LTCI_t(\omega_l) - CostCI_t - CostLTC_t - c_t,$$

$$M_{t+1} = R \times A_t,$$

$$A_t > 0,$$

$$c_t \geq S,$$

$$\omega_a, \omega_c, \omega_l \geq 0,$$

$$\omega_a + \omega_c + \omega_l \leq 1, \tag{3}$$

where E_t is the expectation operator at period t; $u(c_t|H_t)$ is the state-dependent utility of consumption defined in Equation (1); and $\pi_t(H_t, H_{t+1})$ is the transition probability from H_t to H_{t+1} at period t; Consumption c_t depends on the proportions ω_a, ω_c, and ω_l of the initial retirement wealth M allocated to purchase a regular annuity income $Annuity_t(\omega_a)$, a lump sum benefit for critical illness $CII_t(\omega_c)$, and a regular income for LTC $LTCI_t(\omega_l)$, respectively. For a simpler notation, in the remainder of this paper, we omit the notation for the allocated proportions for the corresponding insurance payouts. For example, we use $Annuity_t$ for $Annuity_t(\omega_a)$.

In each period, we assume the following constraints for retirement planning: the retiree cannot sell or cancel the insurance contracts, borrowing is not allowed, and the consumption floor is the government subsidy S. Given the insurance allocation made in the initial period, from period t to $t + 1$, she starts with available retirement wealth M_t, and receives pension P_t and annuity income $Annuity_t$. Depending on the realised health status H_t, she incurs the cost of critical illness $CostCI_t$ if she is critically ill and receives a lump sum payment CII_t if the period t is also the first time she is diagnosed with a critical illness. Alternatively, she incurs the LTC cost $CostLTC_t$ and receives LTC income $LTCI_t$ if she is LTC dependent.

Next, the retiree chooses consumption c_t based on her current cash on hand A_t. Her remaining wealth is placed in a savings account growing at a real risk-free rate R and becomes the available wealth M_{t+1} in the next period. After she chooses consumption in the last period, any remaining wealth in the next period becomes her bequest.

We convert the optimisation problem with several choice variables (3) to sub-problems conditioning on a grid of possible insurance allocation. We solve the sub-optimisation problems for consumption numerically by backward induction using the endogenous gridpoint method (EGM) of Carroll (2006).[20] We then use simulations to determine the optimal portfolio allocation for the retiree. For each retirement portfolio allocation, we calculate the average lifetime utility obtained with 10,000 Monte-Carlo path simulations based on the solved optimal consumption functions. The optimal retirement portfolio is that with the largest average utility.

Similar to Horneff et al. (2020), we calculate the welfare gains from optimal insurance for a retiree as the percentage increase of financial wealth without insurance. We first use simulations to obtain the average lifetime utilities without insurance for an equally spaced sequence of six financial wealth ranging from 100% to 200% of the individual's financial wealth. We then use splines to connect them to obtain a utility curve with respect to financial wealth. The value of financial wealth equalling the utility achieved with optimal insurance reflects the wealth needed to generate the same utility without insurance. We calculate the percentage increase of financial wealth and designate this as the welfare gain for a retiree with optimal insurance. We use the notation '>100%' to denote that the financial wealth needed to generate the same utility with

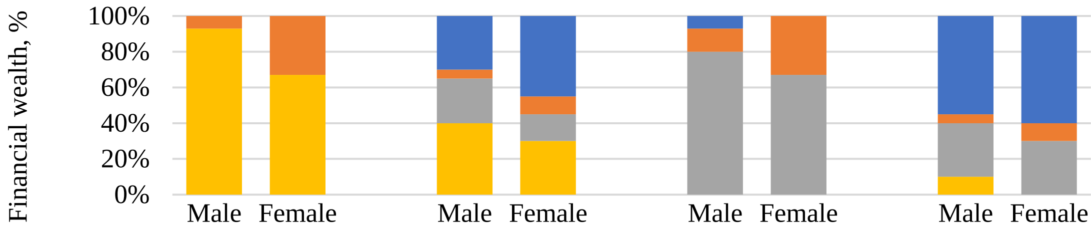
[20] Appendix C provides the solution procedure.

optimal insurance is more than twice the initial financial wealth, and the exact value is beyond this range (extrapolation of the splines outside the fitting range is not used because of potential bias).

4. Main results

In this section, we provide the optimal portfolio choices and welfare gains predicted by the calibrated life-cycle model. For both males and females, we report optimal portfolios for four combinations of financial wealth (low: CNY 150,000, high: CNY 1 million,) and monthly public pension amounts (low: CNY 1,000, average: CNY 3,000). Figure 1 shows the results for males and females. We analyse the impact of preferences, health transitions, pricing, and government subsidy on optimal portfolios in Section 5.

Figure 1: Optimal portfolios for males and females with different pension and financial wealth.



	Male Low pension Low wealth	Female Low pension Low wealth	Male Low pension High wealth	Female Low pension High wealth	Male Avg. pension Low wealth	Female Avg. pension Low wealth	Male Avg. pension High wealth	Female Avg. pension High wealth
Savings account	0%	0%	30%	45%	7%	0%	55%	60%
LTCI	7%	33%	5%	10%	13%	33%	5%	10%
CII	0%	0%	25%	15%	80%	67%	30%	30%
Annuity	93%	67%	40%	30%	0%	0%	10%	0%
Welfare gain	65%	89%	32%	42%	39%	25%	12%	7%

Notes: Results are for males at age 60 and females at age 55 with different pension and financial wealth levels. Welfare gain is on the secondary axis and markers have no colour. LTCI: LTC insurance; CII: critical illness insurance.

We first consider males and females with a *low* monthly pension and *low* financial wealth. Figure 1 shows that for both males and females, the optimal portfolio at time zero comprises only an annuity, and some LTC insurance, but no critical illness insurance and no savings. The welfare gain with this optimal insurance strategy, compared to the baseline with no private insurance, is 65% of initial financial wealth[21] for males and 89% for females. The welfare gains for the least affluent retirees are the largest among all four economic profiles we considered.

Next, we consider retirees with a *low* pension and *high* wealth. Figure 1 shows that the optimal portfolios for males and females comprise all three insurance products and some savings. The welfare gain from optimal insurance is 32% for males and 42% for females. Compared with those with the same low pension level, retirees with higher retirement savings can afford to buy more annuity cover (e.g., for males: 40% * CNY 1 mil = CNY 400,000 vs. 93% * CNY 150,000 = CNY 139,500) and buy other insurances.

Next, we consider retirees with an *average* pension and *low* wealth. Figure 1 shows that in this case, the optimal initial portfolio allocation for males and females includes mostly critical illness insurance, some LTC insurance, some savings (for males), but no annuity. These results suggest that those with an average pension can focus on insuring out-of-pocket health-related costs, especially for critical illness, rather than purchasing annuities. The welfare gains are 39% for males and 25% for females.

Finally, we consider males and females with an *average* pension and *high* wealth, the most affluent group considered in this study. The optimal allocation for this group includes critical illness insurance, some LTC insurance, some annuities (for males), and savings. Of all four economic profiles, these affluent retirees have the highest percentage of wealth allocated to savings, showing the impact of bequest motives and the potential of self-insurance. The welfare gain from optimal insurance in this case is 12% for males and 7% for females.

[21] This means that for males who just retired with CNY 150,000 savings and CNY 1,000 pension, the expected lifetime utility from the optimal portfolio with the three insurance products is equivalent to if they retired with a portfolio with CNY 247,500 (150,000 * 165%) savings without any private insurance.

Overall, our model predicts a substantial demand for life annuities, critical illness and LTC insurance, but this demand varies substantially with respect to a retiree's gender and financial circumstances. Annuity demand is high for those with a low public pension, and critical illness insurance demand is high for those with an average pension. All economic profiles considered in this study predict a positive demand for LTC insurance, while the demand for the life annuity and critical illness insurance can be zero depending on pension level. LTC insurance demand is higher for females[22] and for those with low financial wealth. This signals a unique role of LTC insurance in retirement financial planning. We also find that the welfare gain from optimal insurance is larger for retirees with lower wealth or pension. This means that optimal retirement planning has a relatively larger impact for less affluent individuals.

5. Sensitivity tests

In this section, we show alternative assumptions tested for preferences (risk aversion, time preference, bequest motives, state-dependent utility), health transitions, product pricing, and minimum consumption floor, and we summarise their impact on the optimal portfolio. The tests for risk aversion, time preference, and consumption floor are conducted for both males and females while the remaining tests are only conducted for males.

5.1. Preferences

We first summarise the results for risk aversion, time preference and bequest motives, then we show the results for state-dependent utility. Overall, the results confirm that the optimal portfolio

[22] This is likely caused by the fact that the chance of needing LTC is much higher for females, while the chance of being critically ill is comparable to that of males, as reported in Section 3.3. These health-related risks are the primary risks compared with the longevity risk.

is largely determined by the economic profile such that assumptions of these three preferences modify the benchmark results at small and moderate levels.

5.1.1. Risk aversion, time preference and bequest motives

We vary the risk aversion parameter γ from 3 to 2 and 9, the subjective discount factor β from 0.999 to 0.985 and 0.96, and the strength of bequest motives b from 50 to 0 (no bequest motives), and 100. These values are within the range of parameters in other studies of retirement insurance (e.g., Friedman and Warshawsky, 1990; Peijnenburg et al., 2017; Imrohoro˘glu and Zhao, 2018). In the following we mainly report for retirees with an *average* monthly pension and *high* financial wealth where the impact is generally larger. Appendix D provides detailed results (Figure D.1).

We find that a higher risk aversion increases insurance demand for males and females with *high* financial wealth, and we also find trade-offs of insurance for those with *low* financial wealth. When risk aversion increases from 2 to 9, for example, for male retirees with *high* financial wealth and an *average* monthly pension, annuity demand increases from 0% to 25%, critical illness insurance demand increases from 30% to 35%, and LTC insurance demand increases from 5% to 10%. These findings for risk aversion are in line with theoretical studies for annuities (e.g., Inkmann et al., 2011), critical illness insurance (Schendel 2014), and LTC insurance (Ameriks et al., 2020). However, Peijnenburg et al. (2017) showed that the optimal annuity demand from liquidised wealth in the U.S. stays at zero when risk aversion increased from 2 to 8. And Bommier et al. (2020) showed that more risk-averse retirees should purchase fewer annuities because of a positive value of life, and common life-cycle models predicting that annuity demand increases with risk aversion implicitly assuming a counterintuitive negative value of life due to the non-monotonicity of the EZW preferences (Bommier et al., 2017). Interestingly, in a similar setting but in the form of an online hypothetical experiment, Wan et al. (2023) show that risk aversion positively associates with critical illness and LTC insurance, but negatively with the annuity. For males with *low* financial wealth and a *low* monthly pension, the allocation to annuity and critical illness insurance increases from 53% to 73%, and 0% to 27%, respectively. However, the allocation to LTC insurance drops from 40% to 0%. The results for females are consistent.

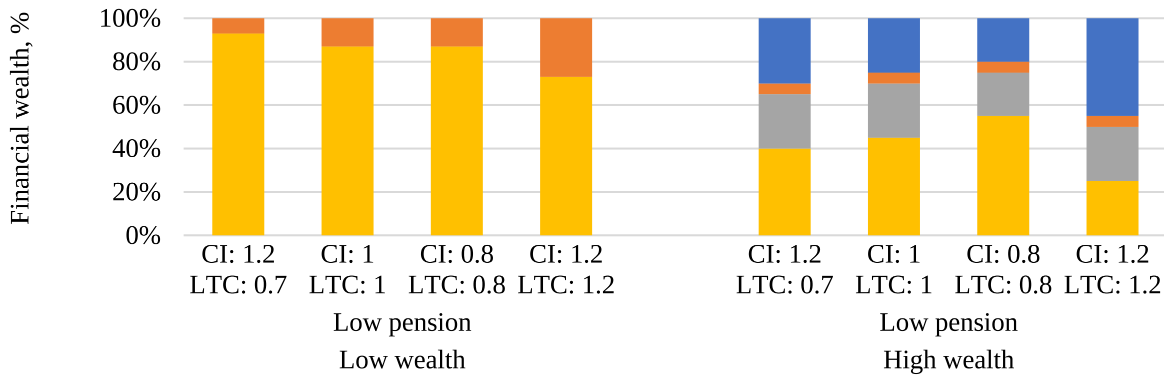
We find that a higher subjective discount factor only slightly increases the insurance demand. For example, when it increases from 0.96 to 0.999, for male retirees with an *average* pension and *high* wealth, the allocation to annuity increases from 0% to 10%, while the demand for critical illness insurance and LTC insurance remains at 30% and 5%, respectively. For females, the demand for critical illness insurance increases slightly from 25% to 30% while the demands for the other two insurance stay the same.

Regarding the strength of bequest motive, we find that the annuity demand decreases with it, while the demands for critical illness and LTC insurance are more stable. For example, for male retirees with an *average* pension and *high* wealth, annuity demand reduces from 30% to zero, and the demand for critical illness and LTC insurance decreases slightly.

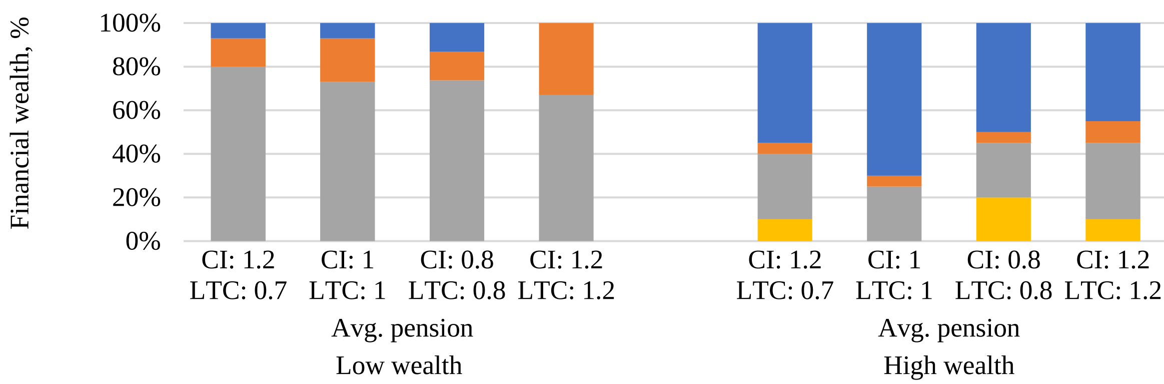
5.1.2. Health state-dependent utility

In this subsection, we test the impact of state-dependent utility on optimal portfolios. Note that for our benchmark results, we set the weight parameter for marginal utility of consumption to $\eta_{Ht=2}$ = 1.2 if the retiree is critically ill and to $\eta_{Ht=3}$ = 0.7 if she needs LTC. We explained in Section 3.2 that these numbers are selected based on the reduced-form estimates from Wang and Wang (2020), who find a higher marginal utility of consumption for those diagnosed with a critical illness but a lower one for those who were LTC dependent. However, studies in developed countries show mixed results on whether the marginal utility of consumption will be lower or higher if health deteriorates (e.g., Finkelstein et al., 2013). In the following, we test three assumptions about η_{Ht}:

independent with health, a lower and a higher marginal utility of consumption in the poor health states (critically ill, LTC dependent). We conduct the test for males and Figure 2 reports the results.



	Low pension, Low wealth: CI: 1.2 LTC: 0.7	CI: 1 LTC: 1	CI: 0.8 LTC: 0.8	CI: 1.2 LTC: 1.2	Low pension, High wealth: CI: 1.2 LTC: 0.7	CI: 1 LTC: 1	CI: 0.8 LTC: 0.8	CI: 1.2 LTC: 1.2
Savings account	0%	0%	0%	0%	30%	25%	20%	45%
LTCI	7%	13%	13%	27%	5%	5%	5%	5%
CII	0%	0%	0%	0%	25%	25%	20%	25%
Annuity	93%	87%	87%	73%	40%	45%	55%	25%
Welfare gain	65%	66%	94%	71%	32%	37%	0%	19%



	Avg. pension, Low wealth: CI: 1.2 LTC: 0.7	CI: 1 LTC: 1	CI: 0.8 LTC: 0.8	CI: 1.2 LTC: 1.2	Avg. pension, High wealth: CI: 1.2 LTC: 0.7	CI: 1 LTC: 1	CI: 0.8 LTC: 0.8	CI: 1.2 LTC: 1.2
Savings account	7%	7%	13%	0%	55%	70%	50%	45%
LTCI	13%	20%	13%	33%	5%	5%	5%	10%
CII	80%	73%	73%	67%	30%	25%	25%	35%
Annuity	0%	0%	0%	0%	10%	0%	20%	10%
Welfare gain	39%	61%	32%	0%	12%	14%	8%	32%

Figure 2: Optimal portfolio with state-dependent utility

Note: Results are for males at age 60 assuming three alternative assumptions for utility weights at the critical illness (CI) and the long-term care (LTC) states. Assumption 1: no state-dependent utility: weights are 1 in both CI and LTC states (CI: 1, LTC: 1). Assumption 2: utility weights are 0.8 in both CI and LTC states (CI:0.8, LTC: 0.8). Assumption 3: utility weights are 1.2 in both CI and LTC states (CI:1.2, LTC: 1.2). Welfare gain is on the secondary axis and markers have no colour. LTCI: long-term care insurance; CII: critical illness insurance.

We first assume that utility is not state-dependent by setting both η_{Ht} to 1 such that the marginal utility of consumption is the same in the healthy and the poor health states (Figure 2, Column 2). We find that, compared with the benchmark results (Figure 2, Column 1), the changes in portfolio allocation are mostly less than 10 percentage points, and the largest change occurs for retirees with *high* wealth and an *average* pension, where the annuity demand increases from 0% to 10%, and savings drop from 70% to 55%.

Second, we assume that η_{Ht} equals 0.8 in both poor health states such that the marginal utility of consumption is lower than in the healthy state (Figure 2, Column 3). Comparing our benchmark results, we find that the largest change of insurance demand occurs for the annuity, whereas its allocation increases from 40% to 55% for retirees with a *low* pension but *high* wealth. The changes in allocation for health-contingent insurance are less than 10 percentage points, regardless of the economic profiles.

Third, we assume that η_{Ht} equals 1.2 in the poor health states such that the marginal utility of consumption is higher than in the healthy state (Figure 2, Column 4). Comparing our benchmark results, we find that, for male retirees with a *low* pension and *low* wealth, the annuity demand drops from 93% to 73%, while LTC insurance demand increases from 7% to 27%. For those with an *average* pension and *low* wealth, the LTC insurance demand increases from 13% to 33% and critical illness insurance demand drops from 80% to 67%. For retirees with a *low* pension but *high* wealth, the annuity demand drops from 40% to 25% while the demand for health-contingent insurance is stable. For those with an *average* pension and *high* wealth, the annuity demand stays at 10%, while the demand for critical illness and LTC insurance increases from 30% to 35%, and from 5% to 10%, respectively.

Overall, the results presented in Figure 2 under alternative assumptions for state-dependent utility show broadly consistent results as presented in Section 4, with a larger impact for retirees with high wealth. The size of the impact is much larger than the findings for optimal annuity demand from Peijnenburg et al. (2017) for the U.S. A notable result is that annuity demand increases from zero to at least 10% while the savings decreases, for those with higher pension and retirement wealth when we consider state-dependent utility, suggesting a release of precautionary savings to purchase annuities. When the marginal utility of consumption increases from low to high (0.8 to 1.2) in poor health states, we find a decreased annuity demand for all economic profiles, while the demand for critical illness and LTC insurance together increases.

5.2. Health Transitions

Health transitions, such as mortality risks, are important for retirement planning. In this paper, we test the sensitivity of transitions between the critically ill and LTC states. This is because critical illness and LTC are key features in our model, and past studies regarding the transitions between these two health states are limited. In our benchmark results, because of data limitation, we assume that the age-specific transition probabilities from the critically ill to the LTC state are the same as that from the healthy state to the LTC state, and similarly for the transitions from the LTC to the critically ill state. We therefore test three additional assumptions for the transitions between the critically ill and LTC states to reflect a higher chance of transition from a poor state of health to another than that from a healthy to a poor state.

First, we assume that the transition from the critically ill state to the LTC state is twice that from the healthy state to the LTC state. Next, we assume that the transition from the LTC state to the critical illness state is twice that from the healthy state to the critical illness state. Last, we assume that the previous two assumptions hold simultaneously. The calibration procedure for the remaining health transition matrix is the same as in the benchmark.

Figure D.2 in Appendix D compares the optimal portfolios under the alternative health transitions with the benchmark results for male retirees. Overall, we find the impacts of the transition assumptions are small and moderate. The demand for LTC insurance for retirees with *low* wealth increases if the transition probability to the LTC state increases in the critical illness state, while it drops if the transition probability to the critically ill state increases. Increased probabilities of transition to critically ill or LTC dependent from the poor health states reduce the annuity demand for retirees with an *average* pension and *high* wealth. The welfare gains are generally robust with respect to the alternative assumptions of health transitions, although such gains for retirees with a low pension are more volatile.

5.3. Product pricing and bundled insurance

To generate our benchmark results in Section 4, we assumed that all insurance products are priced independently, following insurance pricing practices in China. For example, the annuity is priced according to the official mortality curve for pension business without explicitly considering the possibility of the beneficiary being critically ill or needing LTC. However, pricing retirement insurance products independently could make only the very healthy or unhealthy purchase their respective insurance, which in turn can result in a failing insurance market due to adverse selection (e.g., Finkelstein and Poterba, 2004; Braun et al., 2019). Also, as noted in Section 3.3, such a pricing approach would cause a mismatch of the health transitions used for pricing and in the life-cycle model, making the results less interpretable. Therefore, we also consider an approach to price the three insurance products 'jointly' rather than independently, to examine the impact of pricing on the optimal portfolio. To do so, we price the products based on the health transition matrix introduced in Section 3.2. We calculate the prices of the three insurance products as the expected present values of the sum of the discounted future insurance payouts based on 2,000,000 simulated health projections, and we add the same 15% loading to each insurance product. This pricing approach first allows us to predict the optimal insurance when health transitions for pricing and in the life-cycle model are the same. Doing so implies that the health status of the individual is assumed to be verifiable by the insurance company, and no residual subjective health information that can influence the demand for insurance. It also allows us to study the potential demand for a bundled longevity and health-contingent insurance product such that life annuities, critical illness insurance, and LTC insurance are three components. Furthermore, we use the same assumptions for health transitions introduced in Section 5.2 as a robust check. Figure D.3 in Appendix D shows the results for males.

Overall, compared with our benchmark results, whether insurance is priced independently or jointly shows a limited effect on optimal insurance demand, except for retirees with an *average* pension and *high* wealth, where the allocation to annuity increases from 10% to 35%. This is because the price of the annuity in a bundled insurance is much lower than that from pricing it independently according to the official mortality curve for pension business. Further, the results are robust with respect to the three alternative assumptions about health transitions in Section 5.2.[23] In all tests, the welfare gains are similar to the benchmark results.

[23] For the situation where the health transitions for pricing and for solving the life-cycle model are not the same, i.e., insurance companies do not know the full health information of the insured, we find similar results and are available upon request.

5.4. Consumption floor

Studies based on life-cycle models have shown that means-tested benefits, such as a minimum consumption floor, can affect demand for retirement insurance (e.g., Pashchenko, 2013), in particular for the low-income group. We test two assumptions for the consumption floor, the minimum (CNY 491) and maximum (CNY 1,170) of the official monthly government subsidy published in 2020 (Ministry of Civil Affairs, 2020), and compare the results with the benchmark results where we assumed the monthly average (CNY 687).

Similar to Pashchenko (2013), our results show that the subsidy has a large impact for retirees with a *low* pension and *low* wealth and little impact for other retirees because the subsidy amount is less relevant to the financial circumstances of these retirees. Figure D.4 in Appendix D shows that for retirees with a *low* pension and *low* wealth, when the subsidy amount increases from CNY 491 to CNY 1,170, the predicted demand for LTC insurance decreases while that for annuity increases. Also, this impact is substantially larger for females: Their allocation to LTC insurance drops from 67% to 0% and that for annuity increases from 33% to 100%, while for males this shift of allocation is 13 percentage points. These results contrast with the findings of Pashchenko (2013) who showed that annuity demand decreased with the consumption floor. However, we note that LTC insurance is not available in her model, while both annuities and LTC insurance provide an income and could substitute each other to an extent. Our model predicts that an increase in consumption floor could have opposite impacts on the demand for LTC insurance and that for annuities. This result highlights the importance of considering the interaction between means-tested benefits and the provision of retirement insurance when planning for relevant social policies.

6. Discussion and concluding remarks

Retirees in developing countries face risks of longevity, critical illness, and LTC. This paper developed a novel life-cycle model of retirement, with stochastic illness- and care-related costs and health state-dependent preference, to access the demand for annuities, critical illness and LTC insurance. This is the first study to study the demand for the three types of insurance as a portfolio choice. We calibrated the model to China and predicted the optimal portfolio for urban retirees.

6.1. Discussion of key results

First, in contrast to studies in the U.S. that predict a high annuity demand with or without health costs (e.g., Davidoff et al., 2005; Pang et al., 2010), we find a high annuity demand only for those with a low pension, while those with an average pension, have a substantially lower demand for annuities and a higher demand for critical illness insurance. The latter finding implies that health-related concerns alone can solve the ‘annuity puzzle’, and extends the literature on the role of precautionary savings due to health-related risks (e.g., De Nardi et al., 2010; Reichling and Smetters, 2015; Peijnenburg et al., 2017) by showing that individuals would purchase health-contingent insurance instead of an annuity. We are only able to find two life-cycle studies with critical illness insurance (Schendel 2014, Hambel 2020). Our results are consistent with theirs in that critical illness insurance demand rises with income and risk aversion.

Second, we predicted a relatively small but positive LTC insurance demand for all considered economic profiles and it was higher for females. At first glance, the relatively low demand for LTC insurance appears to be inconsistent with China’s effort to develop public LTC insurance programmes. The rationale behind our results is that wealthy retirees can rely on self-insurance and regular income from public and private pensions to build a buffer (e.g., Pang et al., 2010; Peijnenburg et al., 2017; Ameriks et al., 2020), while for the less wealthy retirees, the limited budget makes it optimal for them to focus on retirement income security or cover for catastrophic illness cost, as shown in Figure 1. As well, although the demand for LTC insurance is relatively small, it almost doubles for females, and the demand exists across all four economic profiles considered. The result relates to Ameriks et al. (2020) who find that almost more than half of older Americans across all income and wealth quintiles have a positive demand for LTC insurance. Such positive demand contrasts with that for critical illness insurance and annuity, which can be zero for certain economic profiles. Such findings reflect the unique role of LTC and are consistent with China's plan of providing basic LTC insurance. Also, our model does not consider the need of widowers, nor the increasing ageing population structure in the future, where the gap between the demand and supply of LTC services is expected to increase.

Third, we also contribute to the household finance literature with state-dependent utility (e.g., Viscusi and Evans, 1990; Finkelstein et al., 2013; Blundell et al., 2020) by showing that a higher marginal utility of consumption in the critical illness and LTC states increased the demand for their insurance but decreased the annuity demand. Also, we predicted a higher annuity demand for wealthier retirees when taking into account state-dependent utility, regardless of whether a higher or lower weight is assumed for consumption in poor health states. This result and our finding that bundling longevity and health-contingent insurance could increase annuity demand further contribute to the 'annuity puzzle' literature.

Overall, the optimal insurance was largely determined by a retiree's financial wealth and pension income, and the demand for the critical illness and LTC insurance was more stable than that for the annuities. The probabilities of being critically ill or LTC dependent in poor health states, pricing, and preferences show moderate and small impacts.

6.2. Implications

To the best of our knowledge, we are the first to derive the theoretical demand for the three types of insurance in China in a life-cycle framework. Our research has three institutional features reflecting a developing-country context, which make our study distinct and require caution when compared with studies for developed countries. First, we include the state of critically ill because of its high incidence and catastrophic financial consequences. Second, LTC is primarily informal. Although our model does not explicitly account for informal care, our calibration for LTC cost includes the financial cost of informal care, which is largely based on the number of hours of care-related services provided by relatives. As well, the payout from the LTC insurance could be used to pay for nurses or family members. This means that our predicted demand for LTC insurance can be interpreted as a demand to pay for care-related costs and services, including informal care. Last, our model does not include the stock market because the participation rate is less than 5% for the old in urban China.

However, although our model predicts substantial demand for retirement insurance, the three insurance products are not generally available for the old in China. Annuities are often embedded with many options for investment purposes and standard life annuities are rare. Long-term insurance products for critical illness and LTC almost do not exist for the old. In a related study of the stated demand for the same three insurance products, Wan et al. (2023) find that the most preferred portfolio includes insurance for half of the expected out-of-pocket costs for both critical illness and LTC, and an annuity. This indicates consistency between the stated and predicted demand for LTC insurance, a gap between the lower stated demand for critical illness insurance and our predicted demand, and a gap between the higher stated demand for the annuity and our predicted demand.[24]

As well, it is worth mentioning that our model derives the optimal insurance for an economically rational individual, which means that we do not consider behavioural aspects. Importantly, Wan et al. (2023) show that higher risk aversion is associated with a higher demand for critical illness and LTC insurance, but a lower demand for annuities. Similarly, they found that financial competence is positively associated with the demand for the two health-related insurance, but negatively associated with annuities. Other than that, framing, loss aversion, and mental accounting could all affect insurance demand (e.g., see a review from Benartzi et al., 2011).

Our findings also provide valuable insights for policymakers and insurers in other developing countries. First, it is more welfare improving to increase retirement income security and then focus on cover of catastrophic medical expenditures. This is plausible because sufficient medical treatments are necessary to live long enough to experience longevity and LTC risks. Second, it is important to consider the interaction between means-tested benefits and the provision of retirement insurance when planning policies for pensions and LTC. Third, bundling health-contingent and longevity insurance products can increase annuity demand, hence hybrid insurance products could be an important component in insurance markets. Fourth, the welfare gain with optimal insurance is much larger for financially more vulnerable retirees, and financial education could be especially valuable. Finally, our model is not limited to China and can be calibrated to other countries with less well-developed public insurance systems.

[24] Weighting our results to match the average wealth (CNY 456,000) and pension income (CNY 2,934) reported by the survey participants, the predicted optimal portfolio indicates that the average participants should choose a full cover for critical illness, a half cover for long-term care, and a very small allocation to the annuity.

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Appendix A. Health transition model

Following Yogo (2016) and Koijen et al. (2016), we use a Markov process to model the health evolution over time. We denote the 4×4 transition probability matrix at period t by P_t, where its element $\pi_t(i, j)$ denotes the transition probability from the health state $H_t = i$ in period t to the health state $H_{t+1} = j$ in period $t + 1$:

$$\pi_t (i, j) = \text{Prob} (H_{t+1} = j \mid H_t = i). \tag{3}$$

This stylised health model allows us to study the impact of three key risks faced by retirees: longevity, critical illness, and long-term care risk. In our main health transition model, we assume that there is no recovery to the healthy state from being critically ill or needing long-term care because there is limited data to estimate the recovery rates. We also assume that there are transitions between the critically ill state and the long-term care state. Since there is limited data to estimate such transitions, we assume that the transition probabilities between those states are the same as those from the healthy state to the critically ill or from the healthy to the long-term care state, respectively. We test the sensitivity of this assumption in Section 5.2.

The transition probabilities are calibrated separately for both males and females using official tables for mortality and critical illness and estimates for long-term care transitions based on the CHARLS data. We set the age-specific transition probability from healthy to critically ill, using the critical-illness incidence rates provided by CBIRC. We set the age-specific probability of death from the healthy state by an adjusted CBIRC mortality curve for the pension industry. The adjustment is made to exclude the deaths resulting from the critically ill or long-term care state, and we follow the insurance practice to calculate these adjusted mortality rates (Partner Re, 2016). Long-term care transitions are calibrated with the CHARLS data. We tested the ordered probit model used in Koijen et al. (2016) and Yogo (2016) and also tested ordered logit and multinomial logit models and the formulation with the complementary log-log link function. We selected a multinomial logit model based on the Akaike Information Criterion to estimate the long-term care transition rates. The technical details are provided in Appendix B.[25] Appendix A summarises the calibration method for each health transition in our model. A better approach would be to estimate the full transition model based on the same data. However, longitudinal surveys like the CHARLS do not provide sufficient information to accurately identify the critically ill state. Furthermore, the exposure times for the critically ill or long-term care state are inadequate to generate reliable estimates. Also, our model assumes that the health transitions are exogenous and do not vary according to economic variables, such as pension or wealth. We also note that the health transitions in the life-cycle model are not the same used for product pricing, which reflects pricing practices in the industry. We investigate the sensitivity of the assumptions for health transitions In Section 5.2, and we consider the case where health transitions used for insurance pricing and in the life-cycle model are the same in Section 5.3.

[25] These transition probabilities are period transition rates, the cohort transition rates are not available due to data limitations.

Table A.1. Calibration sources for the health transition probability matrix

From	To	Notation	Calibration source
Healthy	Healthy	$\pi_t(1,1)$	1-$\pi_t(1,2) - \pi_t(1,3) - \pi_t(1,4)$
	Critically ill	$\pi_t(1,2)$	CBIRC incidence rates
	Long-term care	$\pi_t(1,3)$	CHARLS estimates
	Death	$\pi_t(1,4)$	Adjusted from CBIRC mortality rates for pension business to exclude death from other states
Critically ill	Healthy	$\pi_t(2,1)$	0
	Critically ill	$\pi_t(2,2)$	1-$\pi_t(2,1) - \pi_t(2,3) - \pi_t(2,4)$
	Long-term care	$\pi_t(2,3)$	CHARLS estimates
	Death	$\pi_t(2,4)$	Adjusted CBIRC mortality rates, and incidence rates (and associated k_x values)
Long-term care	Healthy	$\pi_t(3,1)$	0
	Critically ill	$\pi_t(3,2)$	CBIRC incidence rates
	Long-term care	$\pi_t(3,3)$	1-$\pi_t(3,1) - \pi_t(3,2) - \pi_t(3,4)$
	Death	$\pi_t(3,4)$	CHARLS estimates
Death	Healthy	$\pi_t(4,1)$	0
	Critically ill	$\pi_t(4,2)$	0
	Long-term care	$\pi_t(4,3)$	0
	Death	$\pi_t(4,4)$	1

Notes: The China Banking and Insurance Regulatory Commission (CBIRC) provide the official mortality rates and diseases incidence rates (CBIRC, 2016, 2013). The CHARLS estimates are the long-term care related transitions estimated with the China Health and Retirement Longitudinal Survey (CHARLS) in 2011, 2013 and 2015 waves.

Appendix B. Insurance product pricing

In our model, the life annuity, critical illness insurance, and long-term care insurance can be purchased by male and female individuals at the age of 60 and 55 by making a one-off payment, respectively. We priced the three products in an actuarially fair way based on gender and age. We assumed a real discount rate of 1.5% for each year in the future due to a constant 3.5% nominal discount rate (the maximum discount rate allowed by the CBIRC) and a constant 2% inflation rate (approximately a ten-year average of the CPI during 2010-2019 in China).

We used the official mortality rates and disease incidence rates provided by the CBIRC to price life annuities and critical illness insurance products independently (CBIRC, 2016, 2013). For the life annuity, we used the mortality curves for the pension business for males (females) starting at age 60 (55). For the critical illness insurance, we used the incidence rate curves and the mortality curves for the health insurance business for 25 diseases for males (females) starting at age 60 (55). We note that insurance companies in China also use the mortality curves for pension business for a more defensive price.

For long-term care insurance, there is no industry health transition table. Therefore, we estimated the health transition rates based on data from the CHARLS survey. We used the three waves of CHARLS data in 2011, 2013, and 2015. A two-year transition, that is, from 2011 to 2013 or from 2013 to 2015, was observable at each age for both genders by the longitudinal survey design. As the sample size for certain transitions is limited, we pooled the first (2011-2013) and the second (2013-2015) transition data together, based on which we estimated a one-year transition at each age for each gender. We only used data for respondents in the starting years (2011 or 2013) that were at least 35 years old. We conducted sensitivity tests using a subset with ages between 45-84 and using aggregated data with a 10-year age group starting from 35-45. The impact on product pricing was immaterial. We excluded observations with missing information for ADL status or death information.

We defined four health states: Healthy, Fair (1-2 ADLs), Disabled (3 or more ADLs, long-term care insurance payable), and Dead. Different from the health states used in the life-cycle model, the inclusion of a Fair state here was to control the result such that the estimation of the transition Healthy-LTC was close to the insurance population. We allowed recovery from states Fair or Disabled to Healthy, and Dead was an absorbing state. We modelled the health transitions in a Markov framework. We use a multinomial logit model to estimate the relevant health transition probabilities. We have tested a probit model, which had been used to estimate the transition probabilities in a similar context in the U.S. by Yogo (2016) and Koijen et al. (2016), and we have tested ordered logit, probit, and cloglog models. In the end, the multinomial logit model has the best performance according to Akaike Information Criterion. The dependent variable was each respondent's health state observed in the follow-up wave (2013 or 2015), and the explanatory variables were the respondent's age and health state in the initial wave (2011 or 2013).

We predicted the two-year transition rates by gender from age 60 to age 104 based on the fitted multinomial logit model, and we closed the transition table at age 105. We calculated the one-year transition probability matrix at age *x* based on the Markov property with the following conversion formula:

$$\boldsymbol{P}_x^{2-\text{year}} = \boldsymbol{P}_x^{1-\text{year}} \times \boldsymbol{P}_{\boldsymbol{x}}^{1-\text{year}}, \tag{7}$$

where $\boldsymbol{P}_x^{1-\text{year}}$ is the probability of a 1-year transition at each state at age *x*.

The insured period was lifetime for all three products. However, for critical illness insurance, the contract ended if one payment was made, and for long-term care insurance, the payments would only be made when conditions with three or more ADLs were triggered. For simplicity and a cleaner interpretation, we assumed that the curves and estimated transitions rate used for pricing the three insurance products were unchanged in the future.

For bundled products, we used the calibrated health transition matrix (Appendix A, Table A.1) and set the post-illness mortality to be an average of the mortality table used for pension business and the mortality in the critically ill state to represent a pooled healthier population. We assumed the same 1.5% real discount rate and the 15% loading. 2,000,000 simulations are used to calculate the price for each insurance component.

Appendix C. Numerical solution

The original optimisation problem in our study (Equation 3 in Section 3.5) has four choice variables: consumption at each period c_t, allocations for annuity ω_a, critical illness insurance ω_c, and long-term care insurance ω_l. However, our problem has a nested structure. To use the EGM method (Carroll, 2006), we first solve the optimal consumption problem conditioning on exogenous insurance states, and then we conduct a grid search under budget constraints for the optimal insurance choice based on the already obtained policy functions in each exogenous insurance state. To see that, the Bellman equation in our optimisation problem is:

$$\begin{cases} V_t(M_t, H_t) = \max_{c_t,\omega_a,\omega_c,\omega_l} E_t\left(u(c_t, H_t) + \beta\left[\sum_{H_{t+1}=1}^{4} \pi_t(H_t, H_{t+1}) V_{t+1}(M_{t+1}, H_{t+1})\right]\right) \\ V_t(M_t, 4) = v(M_t) \end{cases}, \text{s.t.}$$

$$A_t = M_t + P_t + Annuity_t(\omega_a) + CII_t(\omega_c) + LTCI_t(\omega_l) - CostCI_t - CostLTC_t - c_t,$$

$$M_{t+1} = R \times A_t,$$

$$A_t > 0,$$

$$c_t \geq S,$$

$$\omega_a, \omega_c, \omega_l \geq 0,$$

$$\omega_a + \omega_c + \omega_l \leq 1.$$

The optimal solution to the above problem can be obtained by solving the sub-problems in each exogenous insurance state (ω_a, ω_c, ω_l) defined below and finding their maximum:

$$\begin{cases} V_t^{\omega_a,\omega_c,\omega_l}(M_t, H_t) = \max_{c_t,} E_t\left(u(c_t, H_t) + \beta\left[\sum_{H_{t+1}=1}^{4} \pi_t(H_t, H_{t+1}) V_{t+1}^{\omega_a,\omega_c,\omega_l}(M_{t+1}, H_{t+1})\right]\right) \\ V_t^{\omega_a,\omega_c,\omega_l}(M_t, 4) = v(M_t) \end{cases}, \text{s.t.}$$

$$A_t = M_t + P_t + Annuity_t(\omega_a) + CII_t(\omega_c) + LTCI_t(\omega_l) - CostCI_t - CostLTC_t - c_t,$$

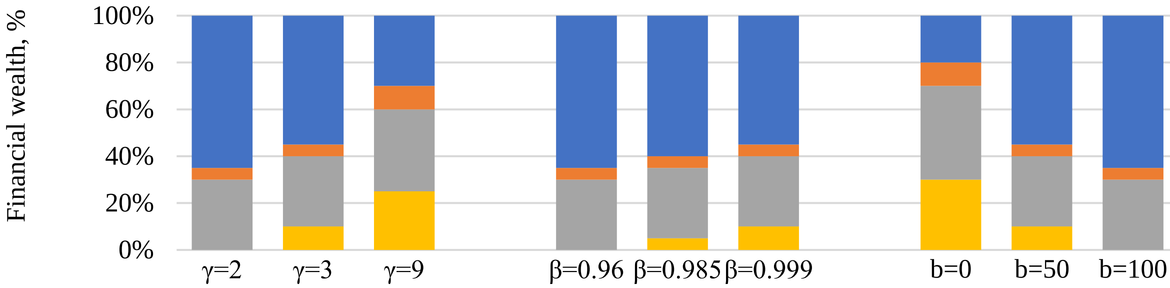
$$M_{t+1} = R * A_t,$$

$$A_t > 0,$$

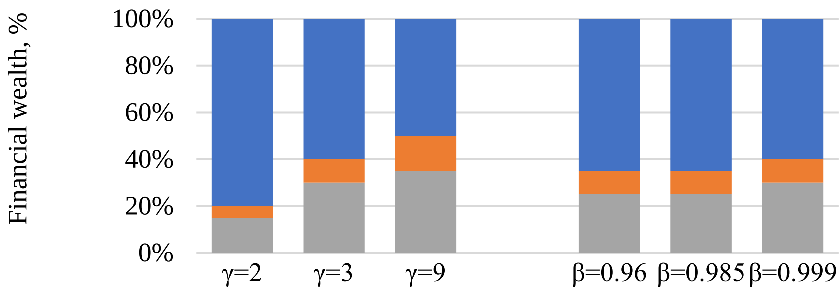
$$c_t \geq S.$$

We use a three-dimension grid to discretise the insurance amount from 0 to 100% of the initial wealth. We use a minimum amount of CNY 50,000 for allocation in the case of CNY 1 million financial wealth, and CNY 10,000 in the case of CNY 150,000 financial wealth. Next, we solve the sub-problem for each of the exogenous insurance states to derive the optimal consumption. After that, we use 10,000 simulations to project future scenarios for a retiree and calculate the realised lifetime utility of consumption for each simulation. The optimal insurance choice is the insurance state that yields the maximal average utility across all simulated scenarios and is within the budget constraint. This two-step approach essentially transforms the original problem with four control variables to #3-D-grid sub-problems where the standard 1-D EGM can be applied. We use an adaptive grid to focus on the most dedicated part that needs fine-tuning and test the size of the grid. We finally deploy the algorithm on the computing clusters provided by UNSW Katana to speed up.

Appendix D. Additional results



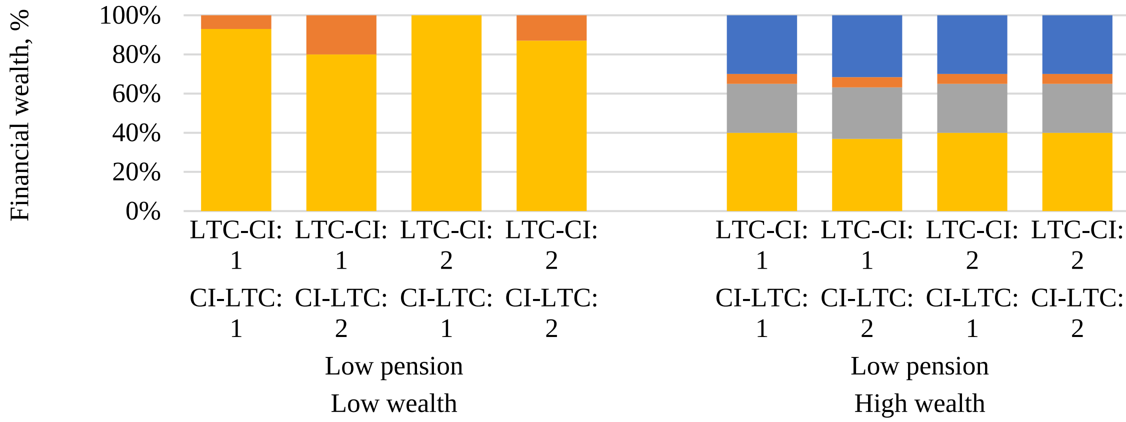
Males	Avg. pension High wealth			Avg. pension High wealth			Avg. pension High wealth		
	γ=2	γ=3	γ=9	β=0.96	β=0.985	β=0.999	b=0	b=50	b=100
Savings account	65%	55%	30%	65%	60%	55%	20%	55%	65%
LTCI	5%	5%	10%	5%	5%	5%	10%	5%	5%
CII	30%	30%	35%	30%	30%	30%	40%	30%	30%
Annuity	0%	10%	25%	0%	5%	10%	30%	10%	0%
Welfare gain	6%	12%	69%	11%	11%	12%	21%	12%	11%



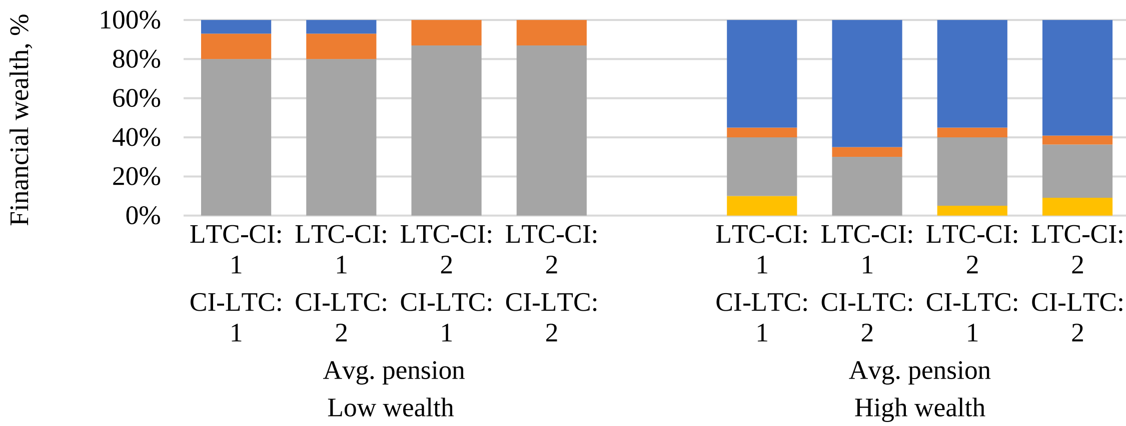
Females	Avg. pension High wealth			Avg. pension High wealth		
	γ=2	γ=3	γ=9	β=0.96	β=0.985	β=0.999
Savings account	80%	60%	50%	65%	65%	60%
LTCI	5%	10%	15%	10%	10%	10%
CII	15%	30%	35%	25%	25%	30%
Annuity	0%	0%	0%	0%	0%	0%
Welfare gain	1%	7%	72%	6%	7%	7%

Figure D.1. Optimal portfolio with alternative assumptions for risk aversion (γ), time preference (β), and bequest motives (b).

Note: Results are for males and females at age 60 and 55, respectively. A high wealth of CNY 1 million and an average pension income of CNY 3,000 are considered for the test of the above preferences parameters. Welfare gain is on the secondary axis and markers have no colour.



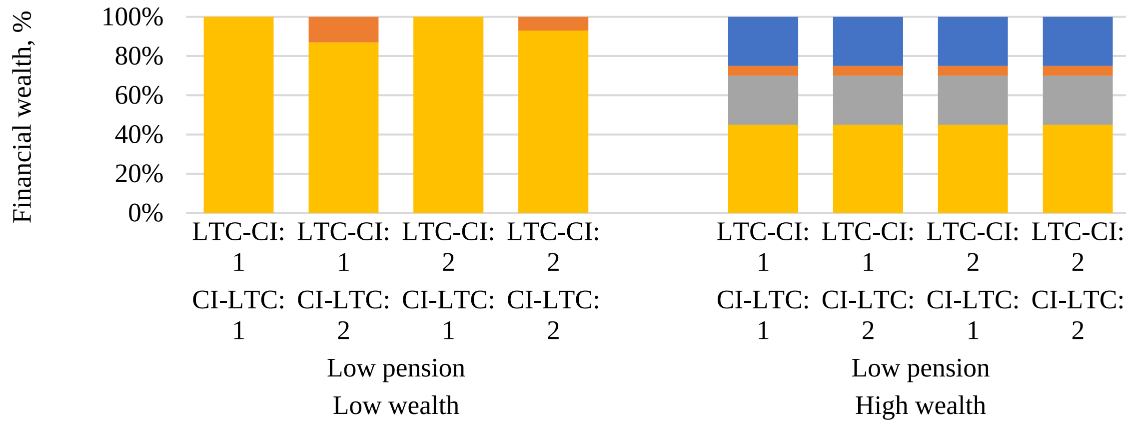
Savings account	0%	0%	0%	0%	30%	30%	30%	30%
LTCI	7%	20%	0%	13%	5%	5%	5%	5%
CII	0%	0%	0%	0%	25%	25%	25%	25%
Annuity	93%	80%	100%	87%	40%	35%	40%	40%
Welfare gain	65%	84%	59%	73%	32%	50%	25%	37%



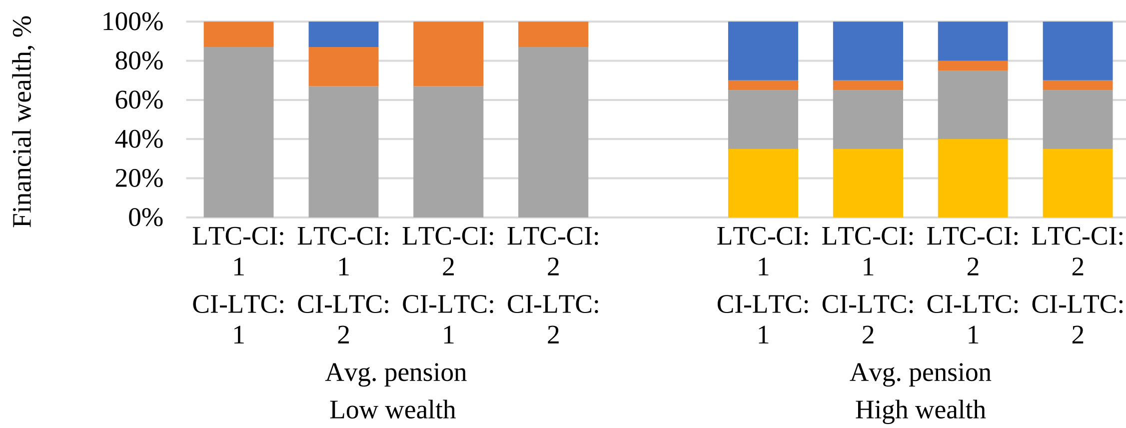
Savings account	7%	7%	0%	0%	55%	65%	55%	65%
LTCI	13%	13%	13%	13%	5%	5%	5%	5%
CII	80%	80%	87%	87%	30%	30%	35%	30%
Annuity	0%	0%	0%	0%	10%	0%	5%	10%
Welfare gain	39%	46%	45%	47%	12%	14%	12%	13%

Figure D.2. Optimal portfolio with alternative assumptions for health transition processes

Note: Results are for males at age 60 at specified wealth and pension profiles (Cases 1, 2, 3 and 4). For each case, the four different scenarios represent different assumptions for the health transition probabilities between the critical illness (CI) and the long-term care (LTC) states. For example, in the second column, the transition from the CI state to the LTC state is twice the transition from healthy to the LTC state (CI-LTC: 2), while the transition from the LTC state to the CI state is the same as in the benchmark case (LTC-CI: 1). Welfare gain is on the secondary axis and markers have no colour. LTCI: long-term care insurance; CII: critical illness insurance.



Savings account	0%	0%	0%	0%	25%	25%	25%	25%
LTCI	0%	13%	0%	7%	5%	5%	5%	5%
CII	0%	0%	0%	0%	25%	25%	25%	25%
Annuity	100%	87%	100%	93%	45%	45%	45%	45%
Welfare gain	>100%	>100%	93%	>100%	40%	59%	31%	44%



Savings account	0%	13%	0%	0%	30%	30%	20%	30%
LTCI	13%	20%	33%	13%	5%	5%	5%	5%
CII	87%	67%	67%	87%	30%	30%	35%	30%
Annuity	0%	0%	0%	0%	35%	35%	40%	35%
Welfare gain	23%	30%	23%	27%	15%	16%	15%	15%

Figure D.3. Optimal portfolio with insurance priced by a joint health transition matrix, evaluated under alternative assumptions for health transition processes.

Note: Results are for males at age 60 at specified wealth and pension profiles (Cases 1, 2, 3 and 4). The insurance products are priced by the same joint health transition matrix, such that the health transitions for pricing and in the life-cycle model are the same. For each case, the four different scenarios represent different assumptions for the health transition probabilities between the critical illness (CI) and the long-term care (LTC) states. For example, in the second column, the transition from the CI state to the LTC state is twice the transition from healthy to the LTC state (CI-LTC: 2), while the transition from the LTC state to the CI state is the same as in the benchmark case (LTC-CI: 1). Welfare gain is on the secondary axis and markers have no colour. LTCI: long-term care insurance; CII: critical illness insurance.

	Males Low	Males Average	Males High	Females Low	Females Average	Females High
Savings account	0%	0%	0%	0%	0%	0%
LTCI	13%	7%	0%	67%	33%	0%
CII	0%	0%	0%	0%	0%	0%
Annuity	87%	93%	100%	33%	67%	100%
Welfare gain	65%	65%	>100%	89%	89%	>100%

Figure D.4. Optimal portfolio with the amount of consumption floor set to three levels of monthly government subsidy: low (CNY 491), average (benchmark, CNY 687), and high (CNY 1,170).

Note: Results are for males at age 60 and females at age 55 with the specified wealth and pension profile (*low* pension and *low* wealth). We find no impact of subsidy on portfolio allocation for retirees with other wealth and pension profiles. Welfare gain is on the secondary axis and markers have no colour.