Information Provision in Competitive Search Equilibrium

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Abstract

In this paper we study information provision by firms within a model of competitive search. A job gives workers utility through the wage the worker receives and through non-pecuniary aspects of the job, which we refer to as "amenities". This amenity level is match specific, and unknown to both workers and firms. However, a firms may provide an informative signal about its level. The firms advertise the precision level of the signal they will provide, as well as the wages they will pay the workers. With fully flexible wages, the firms always commit to disclose all available information. In the presence of minimum wages, firms may restrict the amount of information provided to the workers. In this "information rationing regime", a worker's share of the match surplus is given by a modified Hosios condition, and the sharing rule is constrained efficient in a well defined sense. Furthermore, wages and hence the unemployment rate tend to respond more to macroeconomic shocks in the constrained than in the unconstrained equilibrium. We also discuss possible extensions of the model.
Key words: Competitive search equilibrium, Bayesian learning, efficiency, unemployment, amenities.
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1 Introduction

An important aspect of the trading process is to allocate the “right buyers” to the “right sellers”, or in a labour context to get the “right person to the right job”. In the presence of search frictions this may be a costly and time-consuming task. Workers may have different preferences for non-pecuniary aspects of the job (which we somewhat imprecisely refer to as “amenities), or a worker’s productivity may differ between jobs and tasks (match specific productivities), see Jovanovic (1979) or Pissarides (2000). In the retail market, buyers may value the goods at offer differently, like in Wolinsky (1986).

In this paper we assume that the “quality” of the match is unobservable to both parties. However, one of the parties – say the firm – may send an informative signal about the match quality to a potential trading partner (the worker). As in Kamenica and Gentzkow (2011) the sender of the signal governs the precision of the signal, and can commit to its level of precision.

In our model the trading agents are denoted firms and workers. Firms are capacity constrained, and can only hire a limited number of workers normalized to 1. Due to search frictions, the agents do not match immediately (in fact our model is a one-shot model, implying that not all agents trade). Our model is thus within the Diamond-Mortensen-Pissarides tradition, see Pissarides (2000) for an overview of such models. Although our reference market is the labour market, our model is also relevant for the housing market, in which both a buyer and a seller typically trade one house.

When workers apply for jobs, we assume that they have limited knowledge about the level of amenities the job provides. The amenities capture all aspects of the job that a worker has preferences over in addition to the wage, like how much the worker will enjoy the job tasks, the work environment, and potentially the amount of on-the-job learning. It is well documented that amenities are important when workers make their job acceptance decisions, see for instance Sorkin (2018) and Taber and Vejlin (2020). We assume that workers and firms are symmetrically informed about these non-wage aspects of the job.

In our model, search is competitive. Firms with vacancies commit to and advertise contracts, and workers apply to one of them. In the main part of our analysis the advertised contract specifies both the amount of information that will be revealed about the amenity level of the job and the wage paid. In equilibrium, workers are indifferent between applying to any of the job openings that attract applicants. Hence a firm that offers a more favourable contract has a higher probability of filling its vacancy.

If a worker and a firm team up, the signal about the amenity level is realized, and based on the signal the worker decides whether to accept the job or not. The worker will accept the job if the expected value of doing so, contingent on the information provided, is higher than of staying unemployed (or, if the model were set in continuous time, to continue searching). At this *ex post* stage, it will be in the firm's interest to provide the worker with an amount of information that makes her just willing to accept the job if the information is favourable, as this maximizes the probability that the worker will accept the job.

Our first finding is that if the firms are able to commit to the amount of information they will provide and free to set wages, they will still commit to disclose all available information, so the workers make fully informed choices. Hence, even though it is in the firm's *ex post* interest to restrict the information given to the worker, the firm commits *ex ante* to full disclosure. The reason is that more information always increases the expected total surplus associated with a meeting. Hence the gain to the worker of obtaining more information is greater than the cost to the firm. Transferring rents to the worker by increasing the wage, by contrast, does not increase the match surplus, so the costs to the firm is one to one with the gains to the worker. Therefore it is optimal for the firm to commit to full information disclosure, and secure profits by holding back on wages. We refer to this as the full information regime. The profit-maximizing sharing rule satisfies the Hosios condition, stating that the share of the surplus that is allocated to the worker is equal to the absolute value of the elasticity of the probability that the firm's vacancy is filled with respect to the tightness in the market (ratio of firms to workers).

Next, we assume that there is a lower bound $w^{\min}$ below which the firm cannot set the wage. $w^{\min}$ may reflect a legally binding minimum wage, or a wage set through collective agreements at a level above the profit-maximizing level. We show that if $w^{\min}$ is set sufficiently high, the firms will provide less than full information to the workers in order to extract rents. Within this "information rationing" regime, we analyse how much information the firms will disclose. We show that the resulting equilibrium allocation satisfies a "modified Hosios condition", with the share of the surplus that is allocated to the worker being adjusted upwards compared with the share in the unconstrained equilibrium in which firms are free to set wages. The magnitude of the adjustment depends on the elasticity of the match surplus with respect to the precision of the signal properly defined. Our modified Hosios condition resembles the modified Hosios condition in Moen and Rosen (2011). Our results also suggest that the unemployment rate will be more responsive to productivity- and unemployment benefit shocks in the information rationing regime than in the full information regime.

We also analyse the welfare properties of the equilibrium allocation in the information rationing regime. Obviously, welfare is lower when there are binding restrictions on the contract set the firms can choose from, as the unconstrained equilibrium is efficient. We therefore investigate to what extent the equilibrium allocation is *constrained efficient*. We say that the level of information provided is constrained efficient if a benevolent planner, who could choose the precision of the signal but make no other choices, would make the same choices as the firms. We show that the market solution is indeed constrained efficient in this sense.

Related literature: Following the seminal paper by Kamenica and Gentzkow (2011), a series of papers have been written on the topic of Bayesian persuasion. For instance, Gentzkow and Kamenica (2016) analyse competition and persuasion, but in a market without search frictions. There only exists a few papers that relate Bayesian persuasion to search. Board and Lu (2018) analyse a model in which buyers search sequentially and firms decide how much information to release to the consumers. Dogan and Hu (2022) also analyse optimal information in a sequential search. Au and Whitmeyer (2023) study optimal information disclosure with directed search, as we do in the present paper.

In none of the mentioned papers firms are capacity constrained, which is key in competitive search equilibrium as it is defined in Moen (1997) and Acemoglu and Shimer (1999), see Wright, Kircher, Julien, and Guerrieri (2021) for a survey. As already alluded to, our model has similarities with competitve search models with asymmetric information, see Moen and Rosen (2011) and Guerrieri, Shimer, and Wright (2010). Capacity constraints on firms severely reduce the competitive pressure in the market. This increases the firms' scope for strategic manipulation of the information they provide to the workers.

2 Model

We set up a one-shot model of the labour market. A worker searching for a job sends off one application to one job. If the application results in a job offer which the worker accepts, she becomes employed, if not she becomes unemployed. In future work we will extend the model to a dynamic framework, and expect that the main results derived in this paper still will go through.[1]

There is a measure v of firms with one vacancy each and a measure u of buyers in the economy. Both workers and firms are risk neutral. Firms are capacity constrained and can hire at most one worker. The productivity of a worker is $y > 0$, and the value of unemployment is b, $0 < b < y$, representing the value of home production and leisure, not transfers.

Search is competitive. Hence the labour market may potentially be divided into submarkets in which firms offer different contracts. As all firms are equal, all agents search in the same submarkets in equilibrium. The probability that a worker finds a job is $p(\theta)$, and the probability that a vacancy is filled is $q(\theta)$, where $\theta = v/u$ is the tightness in the market. Since each match involves one worker and one firm, it follows that $p(\theta) = \theta q(\theta)$. We assume that $p(\theta)$ is increasing and $q(\theta)$ is decreasing in θ, and require that $\lim_{\theta\to\infty} p(\theta) = \lim_{\theta\to 0} q(\theta) = 1$ and that $\lim_{\theta\to\infty} q(\theta) = \lim_{\theta\to 0} p(\theta) = 0$. In addition we assume that of $\eta = |q'(\theta)\theta/q|$ is (weakly) increasing in θ. This is a weak assumption, also made in Moen (1997). A worker/firm is matched with at most one firm/worker. To ensure an internal solution we also assume that $\lim_{\theta\to 0} \eta(\theta) = \lim_{\theta\to\infty} 1 - \eta(\theta) = 0$

The measure of workers in the economy is fixed and normalized to 1. The measure of firms with vacancies is endogenous and determined by a free entry conditions, as firms can enter the economy at cost k.

When a worker and a firm meet, the firm releases a signal about the amenity level in that job, and the worker decides whether or not she will accept the job. If the contract is accepted, it cannot be renegotiated.

We assume that the amenity level of a job, denoted z, is either z_l or z_h. The prior

[1] In theoretical work it is common to model the labour market as a one-shot game, see for instance Heggedal, Moen, and Preugschat (2017) and Cai, Gautier, and Wolthoff (2022). A difference between a one-shot model and a dynamic model is that in the latter, the reservation value of a worker when evaluating a job offer would be the continuation value of search, which includes the value of future job offers. In a one-shot model by contrast, the reservation value is the unemployment benefit.

beliefs that that $z = z_h$ is denoted μ, the same for both the worker and the firm.

The firm can costlessly set up a and commit to a signalling mechanism that generates information about z, which both the worker and the firm observe. Without loss of generality we restrict our-self to "straightforward signals", which give recommendations as to which action the worker should take. The action space is $a \in \{0, 1\}$, where 0 denotes "reject the job" and 1 "accept the job. Let $m(a)$ denote the posterior probability that $z = z_h$ given a, $a \in \{0, 1\}$. It follows easily that it is optimal to set $m(0) = 0$. Write $m = m(1)$. For the worker to follow the recommendation we must have that

$$w + mz_h + (1 - m)z_l \geq b \tag{1}$$

where w is the wage.

Let $\kappa(m)$ denote the probability that the recommendation $a = 1$ materializes. For the signal to be Bayes plausible (adher to Bayes law) we must have that $\kappa = \mu/m$. If $m = \mu$, the signal is uninformative. If $m = 1$, the signal is fully informative and all information is revealed. Hence we can interpret m as a measure of the information provided by the firm to the worker.

In the main part of the paper the firms advertise contracts (m, w). We assume that there is a lower bound on w, $w^{\min}$, which may or may not bind. We may think of $w^{\min}$ as a minimum wage, or a wage that is negotiated at a sectoral level and which may be higher than the equilibrium wage.

To summarise, the timing of the model is as follows:

1. The firms advertise contracts (m, w), where $m \in [\mu, 1]$.
2. After observing the contracts offered, each worker decides which firm to apply to.
3. The number of matches is determined by the matching function $x(u, v)$.
4. In each match, the signal provided by the firm is realized.
5. Based on the signal, the worker decides whether she will accept the job or not.
6. If the worker accepts the job, she is employed at the advertised wage. If not she stays unemployed while the job slot remains vacant.

We assume that $y + z^h > b > y + z_l$. The expected utility of being matched with a firm offering a contract (m, w) is thus (provided that the worker accepts the job if $a = 1$)

$$\begin{aligned} u(w,m) &= \kappa(m)\left(w + mz_h + (1-m)z_l\right) + (1-\kappa)b \\ &= \mu\left(z_h - z_l - \frac{b - z_l - w}{m}\right) + b \end{aligned} \tag{2}$$

where we have used that $\kappa = \mu/m$. Since $w < y < b - z_l$, $u(w, m)$ is strictly increasing in m.

The expected profit for the firm when meeting a worker is

$$\pi = \kappa(y - w) = \frac{\mu}{m}(y - w) \tag{3}$$

Hence the profit of the firm is decreasing in m provided that (1) is satisfied. Let S denote the expected joint surplus of a match, $S = \pi + u - b$ (the joint income when the worker and the firm are together less the joint income if they are not). It follows that

$$S = \pi + u - b = \mu(z_h - z_l - \frac{b - z_l - y}{m}) \tag{4}$$

Given our assumption that $b > y + z_l$, it follows that the match surplus is increasing in m.[2]

3 Equilibrium

Firms advertise wage contracts (m, w) so as to maximize profits given a set of constraints. As is standard in competitive search, the probability that a firm attracts an applicant depends on the attractiveness of the contract it offers. Let U be the *ex ante* expected utility of an unemployed worker. It follows that

[2] If $y+z_l > b$, the match surplus is always positive, and information about match quality is irrelevant for the joint surplus.

$$\begin{aligned} U &= p(\theta)u(w,m) + (1-p(\theta))b \\ &= p(\theta)\mu\left(z_h - z_l - \frac{b - z_l - w}{m}\right) + b \end{aligned}$$

The expected profit of a firm can be written as $\Pi = q(\theta)\pi$. Competitive search equilibrium is a contract (m^*, w^*), a market tightness θ^*, and an npv utility U^* such that

1. (m^*, w^*), maximizes $\Pi = q(\theta)\frac{\mu}{m}(y - w)$ subject to the following constraints:
   (a) $p(\theta)u(w,m) + (1-p(\theta))b = U^*$.
   (b) $m \in [\mu, 1]$
   (c) $w \geq w^{\min}$
2. Zero profit, that is

$$\Pi = \frac{\mu}{m}q(\theta^*)(y - w^*) = k \tag{5}$$

   where k is the entry cost.

3.1 A detour: m determined *ex post*

Before we proceed, let us discuss equilibrium information provision when firms cannot commit to m, but set m *ex post* after being matched with the worker. Since a firm's *ex post* profits are decreasing in m, the firm will set m as low as possible given (1). It follows that the *ex post* optimal m is given by

$$m = \max[\hat{m}, \mu]$$

where $\hat{m}$ satisfies (1) with equality;

$$\hat{m} = \frac{b - (w + z_l)}{z_h - z_l}$$

At $m = \hat{m}$, the worker only receives her reservation utility b. In order to attract workers, firms may increase the wage. However, the highest wage the firm can commit to is $w = y$. Hence the firm can only give the worker a utility strictly higher than b if if

$$y + \mu z_h + (1 - \mu) z_l > b$$

If this inequality is satisfied, workers are always hired in equilibrium. Furthermore, equilibrium wages and tightness will be as in the full-information equilibrium with total output (including amenities) given by $\tilde{y} = y + \mu z_h + (1 - \mu) z_l$ and the surplus split according to the Hosios condition (see below). If the inequality is not satisfied, the workers receive their outside option, i.e., the Diamond paradox applies. If there is a strictly positive (but arbitrarily small) search cost, no workers will search and employment is zero.

3.2 First order conditions

We now assume that m is advertised by the firms. Without loss of generality we assume that (1) is always satisfied. If not, a worker will gain nothing from sending an application to the firm in question, as she will never accept the job offer anyway. We will assume that the constraint $m \geq \mu$ does not bind, and then check afterwards. The Lagrangian associated with the optimal contracting problem then reads

$$\begin{aligned} L &= q(\theta)\frac{\mu}{m}(y - w) + \lambda \left(p(\theta)\mu \left(z_h - z_l - \frac{b - z_l - w}{m} \right) + b - U^* \right) \\ &- \rho(m - 1) - \omega(w^{min} - w) \end{aligned}$$

where ρ and ω are greater than or equal to zero with complementary slackness ($\rho(m - 1) = 0$ and $\omega(w^{\min} - w) = 0$). The first order conditions wrt θ, w and m are given by

$$\eta(y - w) = \lambda\theta(1 - \eta)\left(m(z_h - z_l) - (b - z_l - w)\right) \quad (6)$$

$$q(\theta) = \lambda p(\theta) - \omega m/\mu \quad (7)$$

$$y - w = \lambda\theta(b - z_l - w) - \rho\frac{m^2}{q(\theta)\mu} \quad (8)$$

where $\eta = |q'(\theta)\theta/q|$ as above. Equations (6)-(8) and the constraints determine θ, w, and m, and hence also Π, for a given U, $\Pi = \Pi(U)$, $\Pi'(U) < 0$. The zero profit condition then determines U^*.

It is sometimes more convenient to use a slightly modified version of (6). If we substitute in $\pi = (y - w)\frac{\mu}{m}$ and $u - b = \mu\left(z_h - z_l - \frac{b-w-z_l}{m}\right)$ into (6), we get that

$$\eta\pi = (1-\eta)\lambda\theta(u-b) \tag{9}$$

We note that this equation resembles the Hosios sharing rule, saying that an optimal allocation is obtained when the surplus is split between the worker and the firm in accordance with the Nash sharing rule, with a share η of the surplus being allocated to the worker and the rest to the firm. The only difference is the factor $\theta\lambda$, reflecting the profit-utility "exchange rate", that is, the weight of income to the worker relative to income to the firm.

3.3 Unconstrained wages

We first assume that $w^{\min}$ is so low that the wage constraint certainly does not bind (for instance that $w^{\min} = b$). In this case $\omega = 0$. From (7) we get that $\lambda = \theta^{-1}$, which inserted into (8) gives that $\frac{\rho m^2}{q(\theta)\mu} = b - y - z_l > 0$ by assumption. Hence $\rho > 0$. Together with the complementary slackness condition $\omega(m-1)$ this implies that $m = 1$.

Proposition 1. *Suppose firms are unconstrained when setting wages. Then the profit-maximising contract prescribes that $m = 1$. Hence the firms disclose all available information to the workers.*

To get intuition for the result, note that a firm can govern the expected payment to the worker (conditional on being matched) in two ways: by changing wages and and by changing the informativeness m of the signal. Increasing the salary will not influence total surplus given by (4), and hence reduces the expected *ex post* profits of the firm one to one with the increase in worker income. Increasing m (if less than one initially) in contrast will increase the surplus S, and hence reduce profits less than one to one with the increase in worker income. Hence as long as there are no constraints on w, the firm will set m as high as possible, and thus set $m = 1$ and disclose all information.

With $\lambda = \theta^{-1}$, the "exchange rate" $\lambda\theta = 1$. Inserting this into (9) gives that $\eta\pi = (1-\eta)(u-b)$. Substituting in for S from (4) gives that

$$u = b + \eta S \tag{10}$$

Hence the *ex post* surplus is divided according to the Hosios condition, stating that the share of the surplus that goes to the worker should be equal to η.

Reorganizing (10) and including the zero profit condition allow us to characterize the equilibrium as follows:

Lemma 1. *With flexible wages and full information disclosure ($m = 1$), the equilibrium satisfies the equations*

$$\begin{aligned} U &= b + p(\theta)\eta S \\ k &= q(\theta)(1-\eta)S \\ S &= \mu\left(y + z_h - b\right)) \end{aligned}$$

This corresponds to the standard competitive search equilibrium with amenities. It follows readily that[3]

$$w = (1-\eta)(b - z_h) + \eta y.$$

Note that z_h is part of the match surplus, of which the firm gets a share $1-\eta$. Since the amenities are consumed by the worker, the firm gets its share through a lower wage.

3.4 Constrained wages

We now assume that the constraint on wages binds, so that $w^{\min}$ is strictly higher than the unconstrained equilibrium wage which we refer to as w^{uc}.

We want to identify a wage $\bar{w}$ such that $m = 1$ and $\rho = 0$ at $w = \bar{w}$. Hence $\bar{w}$ (if it exists and is unique) is such that the firm sets $m = 1$ if $w^{\min} \leq \bar{w}$ and $m < 1$ if $w^{\min} > \bar{w}$. In the appendix we show the following proposition

Proposition 2. *There is a unique value $\bar{w}$ such that firms provide less than full information whenever $w^{\min} \geq \bar{w}$.*

The intuition for the result is straight-forward: A firm may extract rents from the worker *ex post* (when a meeting takes place) by reducing the wage or by reducing the information it provides to the worker. Extracting rents by reducing wages is more

[3] To see this, note that $u - b = \mu(z_h + w - b)$. Furthermore, $u - b = \eta S$. This gives the expression for w.

efficient, as it does not reduce the match surplus as reduction in information does. Hence if that option is available, the firms will always implement the optimal *ex post* division of rent between itself and the worker by adjusting the wage w. However, if that option is removed, and firms are forced to set a wage sufficiently much above w^{uc}, the firm will find it in its interest to increase its share of the *ex post* rent by reducing the information available to the worker.

To what extent the constraint $m \geq \mu$ is always satisfied depends on parameter values. If $u < b$ for $m = \mu$, then m cannot be equal to μ in equilibrium beacause workers then would reject the job. If $S(\mu) > 0$, we cannot rule out a priori that the $m \geq \mu$ constraint binds in equilibrium. In particular, if $b - w^{\min} - z_l$ is close to zero, the cost of extracting rents from the worker by reducing m is low, and it may well be that $m = \mu$ in optimum. Below we assume that the constraint does not bind.

3.5 Endogenous information provision and the modified Hosios condition

Suppose now that $w^{\min} > \bar{w}$. A firm's decision variable (in addition to entering the market in the first place) is m, and from proposition 2 we know that the firm will set $m < 1$.

In this case $\rho = 0$. It follows that we can write the first order condition (8) as

$$-\pi_m = \theta \lambda u_m \tag{11}$$

where $\pi_m = \frac{\partial \pi}{\partial m} = -\frac{\mu}{m^2}(y - w)$ and $u_m = \frac{\mu}{m^2}(b - w^{\max} - z_l)$. We may think of the lhs as the loss of profit to the firm from increasing m, and the right-hand side as the gain to the worker of increasing m multiplied by the exchange rate $\theta\lambda$. Using that $S_m = \pi_m + u_m$ and that $\pi_m = -\pi/m$ give that

$$\pi = \lambda\theta\left(\pi + S_m m\right) \tag{12}$$

which inserted into (9) gives that

$$\eta(\pi + S'_m m) = (1-\eta)(u-b) = (1-\eta)(S-\pi)$$

or

$$\pi = (1 - \eta - \gamma\eta)S = (1 - \beta^{eff})S(m) \tag{13}$$

where γ is the elasticity of S with respect to m, $\gamma = \frac{S'(m)m}{S} > 0$, and $\beta^{eff} \equiv \eta(1+\gamma)$. By substituting in $u - b = S - \pi$ we get that

$$u = b + \beta^{eff} S \tag{14}$$

Proposition 3. *Suppose* $w^{\min} > \bar{w}$ *defined in proposition 2. Then the firms set* $m < 1$. *The share of the surplus allocated to the worker is* $\beta^{eff} > \eta$, *where* $\beta^{eff} = \eta(1+\gamma)$. *This uniquely determine* m.

Uniqueness is proved in the appendix. If we compare (10) and (14), we see that the worker gets a larger share of the surplus in the constrained equilibrium than in the unconstrained equilibrium. On the other hand, the surplus S is lower in the unconstrained equilibrium, so it is an open question whether the *ex post* utility of the worker is higher or lower in the constrained economy. However, as will be clear below, the expected (*ex ante*) income of the unemployed worker will always be lower in the constrained economy.

The point is that if the floor on wages is set sufficiently high, the firms will start to extract rents from the workers by providing less information. However, this comes at cost, as reducing the information available to the worker reduces total surplus, it is not just a transfer as wages. The optimal sharing rule for the firm therefore gives a higher share of surplus to the worker than the optimal sharing rule when the wage is used to transfer rents between the agents.

The share of the surplus that the firm allocates to the worker, $\beta^{eff} = \eta(1+\gamma)$, depends positively on γ, the elasticity of the match surplus to m. This result resembles findings in Moen and Rosen (2011). Moen and Rosen show that if workers have private information regarding the quality of the match, and this leads to agency costs associated with inducing the workers to exert effort, the firms will respond by offering wage contracts which give a higher share of the surplus to the worker than the share prescribed by the Hosios condition.

Our next question is whether the information provided by the firms is socially efficient given the constraints on wages that firms face. To be more specific, suppose a benevolent planner who cares about the sum of the agents' *ex ante* expected incomes can overturn the firms' decisions concerning m, but no other decisions in the economy. In particular, when setting m the planner takes into account that the zero profit condition still holds and that her choice of m will influence the firms' entry decisions.

If the planner's solution coincides with the market solution regarding m, we say that the market solution is constrained efficient.

Proposition 4. *The market solution is constrained efficient.*

The proof is given in the appendix. Again note the similarities with the results in Moen and Rosen (2011). They also find that the sharing rule in competitive search equilibrium is constrained efficient, and refer to the constrained optimal sharing rule as the *modified Hosios condition*. Proposition (4) demonstrates that a similar result holds for information provisions by firms in the absence of wage competition.

Note that if the planer could finance vacancies by lump-sum taxation, the planner would set $m = 1$ and open as many vacancies as in the unconstrained equilibrium.

3.6 Comparative statics

Before we conclude, let us analyse how the share of surplus allocated to the worker and the unemployment rate depend on the parameters in the model. To that end, note that from the definition of S in (4) it follows that

$$\begin{aligned}\gamma = S'(m)m/S &= \frac{b-y-z_l}{m(z_h-z_l)-(b-y-z_l)} \\ &= \frac{m(z_h-z_l)}{m(z_h-z_l)-(b-y-z_l)} - 1\end{aligned}$$

which is decreasing in y and increasing in b (although of course γ also depends on the endogenous m). Hence, for a given m, γ is increasing in b and decreasing in y. Furthermore, γ is decreasing in m. An explicit expression for m in terms of exogenous variables is given in equation (19) in the appendix.

Consider first an increase in $w^{\min}$. This has no direct effect on γ. However, for a given sharing rule, a high wage implies a low m, and hence a high γ (see also the appendix for a formal proof). It follows that a higher wage to some extent is undone by a less informative contract, but not fully. As m decreases, the elasticity of the surplus with respect to m increases, and hence the share of the surplus allocated to the worker increases. This prevents the firm from fully undo the higher wages through less information provision.

Consider then a negative shock to y. This has a direct negative effect on the match surplus, and hence on the unemployment rate. In addition, for a given m, the firm absorbs the entire shock unless m falls. In the appendix we show that m only falls a response to changes in $\eta(\theta)$. Cet par, a reduction in y reduces θ. On intervals at which $\eta(\theta)$ is constant, m is constant, and the firm absorbs the entire shock (see appendix for details). If $\eta'(\theta) > 0$, workers absorb part of the shock through a lower m.

Then consider changes in b. An increase in b reduces the match surplus and *cet par* increases γ. However, such an increase will also tend to increase m, which will reduce the workers' share of the surplus. See the appendix for details.

In our one-shot model setup, the continuation value if not finding a job, b, is exogenous. In a dynamic model the continuation value is partly endogenous, as it also depends on the return to future search. This will influence the comparative statics results.

4 Conclusion and the way forward

In this paper we analyse a model of competitive search with persuasion. Firms advertise and commit to contracts prescribing wage levels and the amount of information the firm reveals about the job. Workers use this information when directing their search.

We show that if firms are free to set wages, they will disclose all available information, and use the wage to adjust the remuneration to the workers. If there is a lower bound on wages, and this bound is sufficiently high, firms will reduce the amount of information provided to the workers in order to obtain a higher share of the match surplus. In this constrained information regime, firms will adjust the information provision so that the surplus from the match is split according to a modified Hosios condition, which gives the workers a higher share of the surplus than in the unconstrained equilibrium. Still the constrained equilibrium is constrained efficient in a well defined sense. Finally we conjecture that the unemployment rate is more responsive to productivity shocks and shocks to the unemployment benefit level in the constrained equilibrium than in the unconstrained equilibrium. In future work we will explore this in more detail in a continuous-time version of the model, in which the outside options of the workers is determined endogenously by the gains from continued search and not (only) unemployment benefits.

In ongoing work we analyse a model in which firms and workers are imperfectly (but symmetrically) informed about the productivity of the match, which may be high or low. We assume that the matching process is as in the urn-ball model, in which the number of applicants a job receives is Poisson distributed with parameter λ, which depends on the wage the firm offers. A firm that receives applicants interview them in random order, at a cost c per interview (the first interview is free). If there is only one applicant, this worker will maximize the probability that she is hired by providing the firm with the smallest amount of information needed to induce the firm to hire her after a positive signal. If there are more than one applicant, the applicants compete for the jobs by offering more information. In this case we explore how the firms should structure the interviews in order to maximize profits. We conjecture that with sufficiently many application there will be full information revelation. We are particularly interested in the role of wages. Obtaining a longer queue of applicants not only increases the probability of attracting a worker, it also induces the workers to provide more information about the match quality.

The model in the present paper is simplified along several dimensions. As already mentioned, an obvious simplification is that it is a one-shot model, with exogenous continuation values. In addition the job acceptance decision is trivial, in the sense that the (full information) job acceptance decisions that maximize joint surplus and the worker's utility coincide. With a continuously distributed amenity distribution, and with wages set unconditional of the amenity level, the job acceptance decision of the worker will typically not maximize joint surplus. The reason is that the worker, when deciding whether to accept the job or not, will fail to take into account the profit that accrues to the firm if the job is accepted. In such a situation firms may improve efficiency by restricting the amount of information given to the workers in order to induce them to accept more job offers, and thereby improve welfare. We will explore this in future research.

References

Acemoglu, D., and R. Shimer (1999): "Holdups and efficiency with search frictions," *International Economic Review*, 40(4), 827–849.

Au, P. H., and M. Whitmeyer (2023): "Attraction versus Persuasion: Information Provision in Search Markets," *Journal of Political Economy*, 131(1), 000–000.

Board, S., and J. Lu (2018): "Competitive information disclosure in search markets," *Journal of Political Economy*, 126(5), 1965–2010.

Cai, X., P. Gautier, and R. Wolthoff (2022): "Meetings and mechanisms," *International Economic Review*.

Dogan, M., and J. Hu (2022): "Consumer search and optimal information," *The RAND Journal of Economics*, 53(2), 386–403.

Gentzkow, M., and E. Kamenica (2016): "Competition in persuasion," *The Review of Economic Studies*, 84(1), 300–322.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse Selection in Competitive Search Equilibrium," *Econometrica*, 78(6), 1823–1862.

Heggedal, T.-R., E. R. Moen, and E. Preugschat (2017): "Productivity spillovers through labor mobility in search equilibrium," *Journal of Economic Theory*, 169, 551–602.

Jovanovic, B. (1979): "Job matching and the theory of turnover," *Journal of political economy*, 87(5, Part 1), 972–990.

Kamenica, E., and M. Gentzkow (2011): "Bayesian persuasion," *American Economic Review*, 101(6), 2590–2615.

Moen, E., and A. Rosen (2011): "Incentives in Competitive Search Equilibrium," *Review of Economic Studies*, 78, 733–761.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2).

Pissarides, C. A. (2000): *Equilibrium unemployment theory*. MIT press.

Sorkin, I. (2018): "Ranking Firms Using Revealed Preference," *The Quarterly Journal of Economics*, 133(3), 1331–1393.

Taber, C., and R. Vejlin (2020): "Estimation of a Roy/Search/Compensating Differential Model of the Labor Market," *Econometrica*, 88(3), 1031–1069.

Wolinsky, A. (1986): "True monopolistic competition as a result of imperfect information," *The Quarterly Journal of Economics*, 101(3), 493–511.

Wright, R., P. Kircher, B. Julien, and V. Guerrieri (2021): "Directed search and competitive search equilibrium: A guided tour," *Journal of Economic Literature*, 59(1), 90–148.

Appendix

Proof of proposition 2

We want to characterize a value $\bar{w}$ with the property that the constraint $w \geq w^{\min}$ is marginally binding for $w^{\min} = \bar{w}$ at $m = 1$, i.e., such that $\rho = 0$ for $m = 1$. We repeat the equilibrium conditions with $\rho = 0$, $m = 1$, and $w = \bar{w}$:

$$\begin{aligned}
\eta(y - \bar{w}) &= \lambda\theta(1-\eta)\left(z_h + \bar{w} - b\right) && (15)\\
q(\theta) &= \lambda p(\theta) - \omega m/\mu && (16)\\
y - \bar{w} &= \lambda\theta(b - z_l - \bar{w}) && (17)
\end{aligned}$$

From (17) we get that

$$\lambda\theta = \frac{y - \bar{w}}{b - z_l - \bar{w}},$$

which substituted into (15) gives that

$$\frac{\eta(\theta)}{1-\eta(\theta)} = \frac{z_h + \bar{w} - b}{b - z_l - \bar{w}} \tag{18}$$

From the assumptions on y and the fact that $\bar{w} \leq y$ it follows that both the nominator and the denomiator are positive.

From the zero profit condition (5) it follows that $\mu q(\theta)(y - \bar{w}) = k$, which defines θ as a strictly decreasing function of $\bar{w}$, $\theta = \theta(\bar{w})$ on $\bar{w} \in [b - z_h, y]$. Plugged into (18) this implies that the left-hand side (lhs) of the equation is strictly decreasing in $\bar{w}$. Clearly, the right-hand side (rhs) is strictly increasing in $\bar{w}$. Hence if $\bar{w}$ exists it is unique.

To show existence, note that the zero profit condition implies that $\lim_{\bar{w}\to y^-} \theta(\bar{w}) = 0$, in which case $\eta \to 0$. Hence lhs $<$ rhs for $\bar{w} = y$. Furthermore, $\theta(b - z_h) < \infty$. Hence for $\bar{w} = b - z_h$, the lhs is stritly greater than zero and the rhs is zero. Existence follows.

More on the determinants of m

If $w^{\min} \geq \bar{w}$, equation (15) instead writes

$$\eta(y-\bar{w}) = \lambda\theta(1-\eta)\left(m(z_h - z_l) + w^{\max} + z_l - b\right) \tag{19}$$

Inserting from (17) then gives that

$$\frac{1}{1-\eta} = m\frac{z_h - z_l}{b - z_l - w^{\max}}$$

An increase in m reduces π and hence θ, and this reduces η and hence reduces the lhs of the equation. An increase in m increases the rhs. Hence the equation has a unique solution.

An increase in w^{max} reduces π and cet par reduces θ, which reduces the lhs. An increase in $w^{\max}$ increases the rhs. Hence an increase in $w^{\max}$ reduces m.

Consider then changes in b and y. We first assume that $\eta(\theta)$ is constant on the relevant interval. Since the rhs is independent of y, m is then independent of y. An increase in b reduces the rhs of (19), and hence increases m. Suppose then that $\eta'(\theta) > 0$. An increase in y will cet par increase θ and hence the lhs of (19), so m goes up. An increse in b in contrast will reduce θ and hence the lhs of (19), and this in isolation will tend to decrese m.

Proof that the constrained equilibrium is constrained efficient

The planner maximizes total income, given the constraint that the profit of the firm when matched is at most $\mu\frac{y-w^{\max}}{m} = \frac{d}{m}$, where $d \equiv \mu(y - w^{max})$. The total cost of creating vacancies (with u normalized to 1) is θk. Hence we must have that $\frac{k}{q} \leq \frac{d}{m}$ or $p(\theta)\frac{d}{m} \geq \theta k$. The Lagrangian to the planner's maximization problem thus reads

$$L = p(\theta)\left(z_h - z_l - \frac{b-y-z_l}{m}\right) + b - k\theta - \lambda(k\theta - \frac{d}{m}p(\theta)) \tag{20}$$

First order conditions (given that the constraint binds, which we know it does since $w^{\max} > \bar{w}$) for θ and m write

$$(1-\eta)S = (1+\lambda)\frac{k}{q} - (1-\eta)\lambda\frac{d}{m} \tag{21}$$

$$p(\theta)S_m = p(\theta)\lambda\frac{d}{m^2} \tag{22}$$

The second first order condition gives that $\lambda = \frac{S_m}{d/m^2}$. Recall that since the constraint binds, $\pi = k/q = d/m$. Inserted into the first first order condition this gives:

$$\begin{aligned} \pi &= (1-\eta)S - S_m m + (1-\eta)S_m m \\ &= (1-\eta(1+\gamma)) = (1-\beta^{eff})S \end{aligned} \tag{23}$$

Hence the share of the surplus allocated to the firm is $1-\beta^{eff}$, while the complimentary share is allocated to the worker. This proves the claim.