Unconditional Quantile Partial Effects via Conditional Quantile Regression

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Introduction

- (Conditional) Quantile Regression (CQR) is a general approach to estimate conditional quantile partial effects (CQPE), i.e., the effect of a covariate variable of interest (*ceteris paribus*) on the **conditional** quantile distribution of the outcome.

- CQR is a useful way to represent heterogeneity.

Introduction

- Unconditional Quantile Regression (UQR), proposed in a seminal paper by Firpo, Fortin, and Lemieux (2009) (FFL), has attracted interest in both applied and theoretical literatures.

- UQR provides a method to evaluate the impact of changes in the distribution of the explanatory variables on quantiles of the unconditional (marginal) distribution of the outcome variable.

- UQR leads to the unconditional quantile partial effect (UQPE), which refers to the effect of a covariate (*ceteris paribus*) on the **unconditional** quantiles of the outcome variable.

Introduction

- FFL propose several ways to estimate the UQPE.
- The most popular approach is the recentered influence function (RIF) regression method, commonly referred to as RIF regression. It is a two-step procedure, where in the first stage one estimates the RIF, and in the second step, a standard OLS regression of the RIF on covariates estimates the UQPE.
- While the method is appealing due to its simplicity, it relies on ability of the researcher to specify a regression equation for the influence function, a relatively abstract object.

Introduction

- This paper makes the following contributions.
- We obtain a new representation for the UQPE in terms of the CQPE. Following a result by FFL, we show that the the UQPE can be written as the conditional average of the CQPE effects, **given** the outcome variable (evaluated at the unconditional quantile).
- Based on the new representation, we establish how heterogeneity in CQPE propagates to heterogeneity in UQPE (a less studied topic).
- Finally, we suggest an alternative method for estimation of UQPE using simple CQR.

Related Literature

- Although the literature on applications of UQR methods is extensive, the literature on theoretical developments is relatively small.

- The seminal paper is Firpo, Fortin, and Lemieux (2009).

- Rothe (2010, 2012) propose some generalizations.

- Inoue, Li, and Xu (2021) tackle UQR in a two-sample problem, Sasaki, Ura, and Zhang (2022) develop high-dimensional UQR, and Martinez-Iriarte, Montes-Rojas, and Sun (2022) considers policies that affect the dispersion (in addition to a location shift).

(Conditional) Quantile Regression

- Consider a general model $Y = r(X, U)$, where $X = (X_1, X_2')'$.
- X_1 is the **target variable** of interest and is a scalar.
- X_2 is a $(d-1) \times 1$ vector consisting of other observable covariates.
- U consists of unobservables.

(Conditional) Quantile Regression

- Let $Q_Y[\eta|X_1, X_2]$ be the conditional η-quantile:

$$\Pr(Y \leq Q_Y[\eta|X_1, X_2]|X_1, X_2) = \eta.$$

- The typical object of study of CQR is the conditional quantile partial effect (CQPE):

$$CQPE_{X_1}(\eta, x) := \left. \frac{\partial Q_Y[\eta|X_1 = z, X_2 = x_2]}{\partial z} \right|_{z=x_1}.$$

- **Interpretation:** marginal effect of X_1 on the conditional quantiles of Y when $X_1 = x_1$ and $X_2 = x_2$.

- If $Q_Y[\eta|X_1, X_2] = \beta_0(\tau) + \beta_1(\eta)X_1 + X_2'\beta_2(\eta)$, then $CQPE_{X_1}(\eta, x) = \beta_1(\eta)$.

Unconditional Quantile Regression

- Consider the counterfactual outcome

$$Y_{\delta,X_1} = r(X_1 + \delta, X_2, U),$$

where δ captures a small location change in the variable X_1.

- Y_{δ,X_1} is the outcome we would observe if **every** individual receives an additional quantity δ of X_1.

- The unconditional quantile partial effect (UQPE) is defined as

$$UQPE_{X_1}(\tau) := \lim_{\delta \to 0} \frac{Q_{Y_{\delta,X_1}}[\tau] - Q_Y[\tau]}{\delta}.$$

CQPE vs UQPE

- The interpretations of $CQPE_{X_1}(\eta, x)$ and $UQPE_{X_1}(\tau)$ are different.
- The $CQPE_{X_1}(\eta, x)$ amounts to manipulating X_1 **locally** at x and evaluating a local impact on the η-conditional quantile of Y.
- The $UQPE_{X_1}(\tau)$ is obtained by what we may refer to as a **global change** in X_1 and evaluating the impact on the τ-unconditional quantile of Y.

UQPE in terms of CQPE

- Under mild conditions, the CQPE can be written as

$$CQPE_{X_1}(\eta, x) = -\frac{1}{f_{Y|X}(Q_Y[\eta|X=x]|x)} \frac{\partial F_{Y|X}(Q_Y[\eta|X=x]|z, x_2)}{\partial z}\bigg|_{z=x_1}.$$

- For the UQPE we have

$$UQPE_{X_1}(\tau) = -\frac{1}{f_Y(Q_Y[\tau])} \int \frac{\partial F_{Y|X}(Q_Y[\tau]|z, x_2)}{\partial z}\bigg|_{z=x_1} dF_X(x).$$

- Even if the conditional quantile is equal to the corresponding unconditional, that is, $Q_Y[\tau|X=x] = Q_Y[\tau]$, one is **not** able to recover $UQPE_{X_1}(\tau)$ from $CQPE_{X_1}(\tau, \cdot)$ by simply integrating the latter over X.

Matched Quantiles

- Consider the following matching map (introduced by FFL):

$$\xi_\tau(x) = \{\eta : Q_Y[\eta|X = x] = Q_Y[\tau]\}\,.$$

- The map $\xi_\tau(x) : (0, 1) \times \mathbb{R}^d \mapsto (0, 1)$ corresponds to the quantile index(es) in the conditional model, η, that produces the closest match with the unconditional quantiles τ for different values of x.

- If $\xi_\tau(x)$ is a singleton, we have that, for every x, $Q_Y[\xi_\tau(x)|X = x] = Q_Y[\tau]$.

UQPE in terms of CQPE

- The $CQPE_{X_1}$ evaluated at $\eta = \xi_\tau(x)$ is

$$CQPE_{X_1}(\xi_\tau(x), x) = -\frac{1}{f_{Y|X}(Q_Y[\tau]|x)} \frac{\partial F_{Y|X}(Q_Y[\tau]|z, x_2)}{\partial z}\bigg|_{z=x_1}.$$

- Except for $f_{Y|X}(Q_Y[\tau]|x)$, it looks like the integrand in $UQPE_{X_1}(\tau)$. Therefore

$$UQPE_{X_1}(\tau) = \int CQPE_{X_1}(\xi_\tau(x), x) \frac{f_{Y|X}(Q_Y[\tau]|x)}{f_Y(Q_Y[\tau])} dF_X(x).$$

- This weighted average representation result appears in FFL.

UQPE in terms of CQPE

- To obtain a new representation, the weights are rearranged as

$$\frac{f_{Y|X}(Q_Y[\tau]|x)}{f_Y(Q_Y[\tau])}f_X(x) = \frac{f_{Y,X}(Q_Y[\tau],x)}{f_Y(Q_Y[\tau])f_X(x)}f_X(x) = f_{X|Y}(x|Q_Y[\tau]).$$

- So we obtain a **reverse projection**

$$\begin{aligned} UQPE_{X_1}(\tau) &= \int CQPE_{X_1}(\xi_\tau(x),x)f_{X|Y}(x|Q_Y[\tau])dx \\ &= E[CQPE_{X_1}(\xi_\tau(X),X)|Y = Q_Y[\tau]]. \end{aligned}$$

η-heterogeneity vs. τ-heterogeneity

- **Question:** Heterogeneous CQPE $\overset{¿?}{\rightarrow}$ Heterogeneous UQPE.

- How does heterogeneity in the conditional effects across η-quantiles **propagate** to heterogeneity in unconditional effects across τ-quantiles?

- Define:

$$\eta\text{-heterogeneity} := \frac{\partial CQPE_{X_1}(\eta, x)}{\partial \eta},$$

and

$$\tau\text{-heterogeneity} := \frac{dUQPE_{X_1}(\tau)}{d\tau}.$$

η-heterogeneity vs. τ-heterogeneity

- Using the chain rule:

$$\frac{dUQPE_{X_1}(\tau)}{d\tau} = \int_{\mathcal{X}} \frac{\partial CQPE_{X_1}(\eta, x)}{\partial \eta}\bigg|_{\eta=\xi_\tau(x)} \frac{\partial \xi_\tau(x)}{\partial \tau} f_{X|Y}(x|Q_Y[\tau])dx$$
$$+ \frac{dQ_Y[\tau]}{d\tau} \int_{\mathcal{X}} CQPE_{X_1}(\xi_\tau(x), x) \frac{df_{X|Y}(x|y)}{dy}\bigg|_{y=Q_Y[\tau]} dx.$$

- The first term averages across the η-heterogeneity.
- In general, even if there is no η-heterogeneity, we may still have non-zero τ-heterogeneity through the second term.

Estimation

- Assume first that

$$Q_Y[\eta | X_1 = x_1, X_2 = x_2] = x_1\beta_1(\eta) + x_2'\beta_2(\eta) = x'\beta(\eta),$$

- Then $CQPE_{X_1}(\xi_\tau(x), x) = \beta_1(\xi_\tau(x))$.
- Therefore, $UQPE_{X_1}(\tau)$ has the convenient form

$$UQPE_{X_1}(\tau) = E\left[\beta_1(\xi_\tau(X)) | Y = Q_Y[\tau]\right].$$

- Our proposed estimator is a nonparametric regression of $\{\beta_1(\xi_\tau(x_i))\}_{i=1}^n$ on $\{y_i\}_{i=1}^n$ evaluated at $Q_Y[\tau]$.

Estimation

- To estimate $\beta_1(\xi_\tau(x_i))$ we first use CQR methods, and estimate $\beta(\eta)$ for a grid of m values of η's given by $\mathcal{H}_m = \{\epsilon < \eta_1 < \cdots < \eta_j < \cdots < \eta_m < 1 - \epsilon\}, \epsilon \in (0, \frac{1}{2})$.

- In the standard linear case we have that for a given value of η_j, and a sample $\{y_i, x_i\}_{i=1}^n$, we simply apply standard quantile regression methods as

$$(\hat{\beta}_1(\eta_j), \hat{\beta}_2(\eta_j)')' = \hat{\beta}(\eta_j) = \arg\min_b \frac{1}{n}\sum_{i=1}^n \rho_{\eta_j}(y_i - x_i'b),$$

  where $\rho_\tau(u) = u(\tau - 1[u < 0])$ is the check function.

- We also estimate the unconditional quantile $Q_Y[\tau]$ by

$$\hat{Q}_Y[\tau] = \arg\min_q \frac{1}{n}\sum_{i=1}^n \rho_\tau(y_i - q).$$

Estimation

- To find the matched coefficients $\hat{\beta}_1(\hat{\xi}_\tau(x_i))$, we employ the two previous estimates.
- Let

$$\hat{\xi}_\tau(x_i) = \begin{cases} \eta_1 \in \mathcal{H}_m & \text{if} \quad \hat{Q}_Y[\tau] < x_i'\hat{\beta}(\eta_1); \\ \eta_j \in \mathcal{H}_m & \text{if} \quad \left\{ x_i'\hat{\beta}(\eta_{j-1}) \leq \hat{Q}_Y[\tau] < x_i'\hat{\beta}(\eta_j) \right\} \text{ for } j = 2, ..., m; \\ 1 - \epsilon & \text{if} \quad x_i'\hat{\beta}(\eta_m) \leq \hat{Q}_Y[\tau], \end{cases}$$

for $i = 1, ..., n$.

- This is an inversion procedure (cf. Chernozhukov, Fernandez-Val, and Melly (2013)).

Estimation

- Finally, to estimate the $UQPE_{X_1}(\tau)$, we use a Nadaraya-Watson type-estimator:

$$\hat{E}\left[\hat{\beta}_1(\hat{\xi}_\tau(X))|Y = \hat{Q}_Y[\tau]\right] = \frac{\sum_{i=1}^n K_h(y_i - \hat{Q}_Y[\tau]) \cdot \hat{\beta}_1(\hat{\xi}_\tau(x_i))}{\sum_{i=1}^n K_h(y_i - \hat{Q}_Y[\tau])},$$

  where K_h is the rescaled kernel $K_h(u) := \frac{1}{h}K\left(\frac{u}{h}\right)$.

- This estimator avoids the curse of dimensionality: Y is the only regressor.

- However, the dimension of X matters for CQR and the matching function.

Estimation

- An alternative approach is just a linear regression of $\hat{\beta}_1(\hat{\xi}_\tau(X))$ on a constant and Y.
- The predicted fit at $Y = \hat{Q}_Y[\tau]$ is an easy-to-compute approximation to $UQPE_{X_1}(\tau)$.
- Yet another option is to do a local linear regression. Less bias?
- The estimator is $\hat{a}_{\tau,0} + \hat{a}_{\tau,1}\hat{Q}_Y[\tau]$, where $(\hat{a}_{\tau,0}, \hat{a}_{\tau,1})'$ solve

$$(\hat{a}_{\tau,0}, \hat{a}_{\tau,1})' = \arg\min_{a_{\tau,0}, a_{\tau,1}} \sum_{i=1}^{n} K_h(y_i - \hat{Q}_Y[\tau]) \left[\beta_1(\hat{\xi}_\tau(x_i)) - a_{\tau,0} - a_{\tau,1} \left(\frac{y_i - \hat{Q}_Y[\tau]}{h} \right) \right]^2 .$$

Asymptotic Theory

- The following assumptions are needed to establish that

$$\hat{\beta}_1(\hat{\xi}_\tau(x)) - \beta_1(\xi_\tau(x)) = O_p(n^{-1/2}).$$

- We work with i.i.d. data.

- $\{y_i, x_i\}_{i=1}^n$ is a random sample of i.i.d. observations with y_i a scalar and $x_i \in \mathbb{R}^d$.

Asymptotic Theory

1. The conditional quantiles are linear: $Q_Y[\eta|X=x] = x'\beta(\eta)$, $\eta \in [\epsilon, 1-\epsilon]$, $\epsilon \in (0, \frac{1}{2})$, with $X \in \mathbb{R}^d$ and $E|X| < \infty$.
2. For every x in the support of X, $f_{Y|X}(y|x)$ is bounded away from zero.
3. The conditional quantile regression estimators satisfy

$$\begin{aligned}\hat{\beta}(\eta) - \beta(\eta) &= E\left[f_{Y|X}(X'\beta(\eta)|X)XX'\right]^{-1} \frac{1}{n}\sum_{i=1}^{n}\left(\eta - \mathbb{1}\left\{y_i \le x_i'\beta(\eta)\right\}\right)x_i + o_p(n^{-1/2}) \\ &= \frac{1}{n}\sum_{i=1}^{n}\Psi_i(\eta) + o_p(n^{-1/2}),\end{aligned}$$

   uniformly in $\eta \in [\epsilon, 1-\epsilon]$, $\epsilon \in (0, \frac{1}{2})$, and $\eta \mapsto E\left[f_{Y|X}(X'\beta(\eta)|X)XX'\right]$ has uniformly bounded derivatives.

Asymptotic Theory

4. The unconditional quantile estimator satisfies

$$\hat{Q}_Y[\tau] - Q_Y[\tau] = f_Y(Q_Y[\tau])^{-1}\frac{1}{n}\sum_{i=1}^{n}(\tau - \mathbb{1}\{y_i \leq Q_Y[\tau]\})$$
$$= \frac{1}{n}\sum_{i=1}^{n}\psi_i(\tau) + o_p(n^{-1/2}).$$

5. The grid of quantiles $\{\epsilon < \eta_1 < \ldots < \eta_j < \ldots < \eta_m < 1-\epsilon\}$, $\epsilon \in (0, \frac{1}{2})$, satisfies $\Delta\eta = o(n^{-1/2})$ as $n \to \infty$ for $\Delta\eta := \eta_j - \eta_{j-1}$, $j = 2, ..., m$, and $\eta_1 = \epsilon$ and $\eta_m = 1-\epsilon$ for a small $\epsilon > 0$.

Asymptotic Theory

- Under the above assumptions, the CQR coefficient of X_1 evaluated at the random quantile $\hat{\xi}_\tau(x)$ satisfies $\hat{\beta}_1(\hat{\xi}_\tau(x)) - \beta_1(\xi_\tau(x)) = O_p(n^{-1/2})$ and can be represented as

$$\hat{\beta}_1(\hat{\xi}_\tau(x)) - \beta_1(\xi_\tau(x)) = \hat{\beta}_1(\xi_\tau(x)) - \beta_1(\xi_\tau(x)) + \dot{\beta}_1(\xi_\tau(x))(\hat{\xi}_\tau(x) - \xi_\tau(x)) + o_p(n^{-1/2}),$$

  where

$$\hat{\xi}_\tau(x) - \xi_\tau(x) = -\frac{1}{x'\dot{\beta}(\xi_\tau(x))}\frac{1}{n}\sum_{i=1}^{n} x'\Psi_i(\xi_\tau(x)) + \frac{1}{x'\dot{\beta}(\xi_\tau(x))}\frac{1}{n}\sum_{i=1}^{n}\psi_i(\tau) + o_p(n^{-1/2}).$$

- Here, $\dot{\beta}_1(\xi_\tau(x))$ is the β_1 component of the Jacobian vector $\dot{\beta}(\xi_\tau(x))$: the derivative of the map $\eta \mapsto \beta(\eta)$.

Asymptotic Theory

- Our parameter of interest given is

$$UQPE_{X_1}(\tau) = E\left[\beta_1(\xi_\tau(X))|Y = Q_Y[\tau]\right].$$

- We propose the following nonparametric estimator:

$$\widehat{UQPE}_{X_1}(\tau) = \hat{E}\left[\hat{\beta}_1(\hat{\xi}_\tau(X))|Y = \hat{Q}_Y[\tau]\right] = \frac{\sum_{i=1}^n K_h(y_i - \hat{Q}_Y[\tau]) \cdot \hat{\beta}_1(\hat{\xi}_\tau(x_i))}{\sum_{i=1}^n K_h(y_i - \hat{Q}_Y[\tau])}.$$

- The unfeasible (oracle) version is denoted by

$$\widetilde{UQPE}_{X_1}(\tau) = \hat{E}\left[\beta_1(\xi_\tau(X))|Y = Q_Y[\tau]\right] = \frac{\sum_{i=1}^n K_h(y_i - Q_Y[\tau]) \cdot \beta_1(\xi_\tau(x_i))}{\sum_{i=1}^n K_h(y_i - Q_Y[\tau])}.$$

Asymptotic Theory

1. $K(u)$ is a symmetric function around 0 that satisfies: (i) $\int K(u)du = 1$; (ii) For $r \geq 2$, $\int u^j K(u)du = 0$ when $j = 1, ..., r-1$, and $\int |u|^r K(u)du < \infty$; (iii) $\int K'(u)du = 0$; (iv) $u^j K(u) \to 0$ as $u \to \pm\infty$ for j=1,...,r+1; (v) $\sup_{u\in\mathbb{R}} |K'(u)| < \infty$ and $\sup_{u\in\mathbb{R}} |K''(u)| < \infty$; (vi) $\int K'(u)^2 du < \infty$ and $\int uK'(u)^2 du < \infty$.
2. (i) The density of Y is $r+1$ times continuously differentiable, with uniformly bounded derivatives; (ii) The joint density $f_{Y,X}(y,x)$ is $r+1$ times continuously differentiable, with uniformly bounded derivatives for every x in the support of X.
3. As $n \to \infty$, the bandwidth satisfies $h \propto n^{-a}$ with $1/(1+2r) \leq a < 1/2$.
4. The following approximation rate holds for $\hat{\xi}_\tau$:
$E\left[\left(n^{1/4}\left[\beta_1(e(X)) - \beta_1(\xi_\tau(X))\right]\right)^2\right]|_{e=\hat{\xi}_\tau} = o_p(1)$.

Asymptotic Theory

- Under the above assumptions, we obtain

$$\widehat{UQPE}_{X_1}(\tau) = \widetilde{UQPE}_{X_1}(\tau) + o_p(n^{-1/2}h^{-1/2}).$$

- The preliminary estimators of the CQR slopes, the matched quantiles and the unconditional quantile of Y vanish asymptotically because they converge at a parametric rate.

- The asymptotic distribution of the unfeasible estimator $\widetilde{UQPE}_{X_1}(\tau)$ is well-known and can be readily established.

Asymptotic Theory

- The following assumption is customary in order to apply the Lindeberg-Feller Central Limit Theorem: (i) For $U_\tau := \beta_1(\xi_\tau(X)) - E\left[\beta_1(\xi_\tau(X))|Y\right]$, and $\delta > 0$, $E[|U_\tau|^{2+\delta}|Y] < C < \infty$ a.s. for some C; (ii) $\int |K(u)|^{2+\delta} du < \infty$; (iii) The map $y \mapsto E\left[\beta_1(\xi_\tau(X))|Y = y\right]$ is $r+1$ times continuously differentiable, with uniformly bounded derivatives; (iv) The map $y \mapsto \sigma_\tau^2(y) := E[U_\tau^2|Y = y]$ is continuous.

- Under the above assumptions

$$\sqrt{nh}\left(\widehat{UQPE}_{X_1}(\tau) - UQPE_{X_1}(\tau)\right) \xrightarrow{d} N\left(0, \sigma_\tau^2(Q_Y[\tau]) f_Y(Q_Y[\tau])^{-1} \int K(u)^2 du\right).$$

Empirical Application

- We illustrate the UQPE estimator with an analysis of Engel's curves.
- The original concept corresponds to Ernst Engel who studied the European working class households consumption in the 19th century.
- Engel curves describe how household expenditures on particular goods and services depend on household income.
- An empirical result commonly referred to as "Engel's law" states that the poorer a family is, the larger the budget share it spends on food.

Empirical Application

- We apply this framework to food household expenditures in Argentina using the national survey of expenditures (Encuesta Nacional de Gasto de los Hogares, known as ENGHO 2017-2018).
- The ENGHO 2017-2018 surveys the households' living conditions in terms of their access to goods and services, as well as their income.
- The data contains information about household expenditures.
- About 21,547 households were randomly selected and participated on the survey.

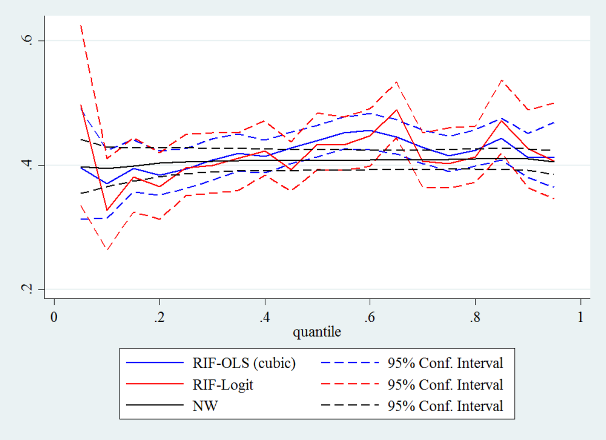
Table: Engel's curve for food expenditures.

	Quantile Partial Effect				
	10	**25**	**50**	**75**	**90**
Conditional distribution					
CQR	0.383***	0.407***	0.408***	0.408***	0.425***
	(0.000571)	(0.000422)	(0.000246)	(0.000278)	(0.000336)
Unconditional distribution					
RIF (linear model)	0.367***	0.388***	0.427***	0.396***	0.393***
	(0.0285)	(0.0170)	(0.0139)	(0.0130)	(0.0181)
RIF (quadratic model)	0.360***	0.383***	0.427***	0.403***	0.406***
	(0.0275)	(0.0166)	(0.0140)	(0.0129)	(0.0182)
RIF (cubic model)	0.370***	0.394***	0.440***	0.415***	0.412***
	(0.0279)	(0.0169)	(0.0143)	(0.0137)	(0.0183)
RIF (Logit)	0.327***	0.395***	0.434***	0.403***	0.427***
	(0.0373)	(0.0260)	(0.0234)	(0.0242)	(0.0316)
NW	0.395***	0.405***	0.408***	0.409***	0.410***
	(0.0166)	(0.0111)	(0.00851)	(0.00809)	(0.00870)
Observations	21,017	21,017	21,017	21,012	21,017

Notes: The CQR analysis corresponds to a regression of log food expenditures on log income. UQPE estimates the effect of a marginal change in log income on the unconditional distribution of log food expenditures. Standard errors in parentheses (analytical for CQR, bootstrap with 200 replications for RIF and NW). * indicates significance at 10 %, ** at 5 % and *** at 1 %.

Empirical Application

Figure: Engel's curves for food expenditures



Notes: UQPE NW (black), RIF-OLS (cubic polynomial, blue) and RIF-Logit (red) estimates together with 95% confidence intervals estimated using bootstrap with 200 replications.

Empirical Application

- The results for food expenditures show evidence that CQR coefficients are roughly constant across η, although mildly increasing.

- The proposed UQPE NW estimator is then also roughly constant across τ.

- The RIF estimates also have this pattern although they are estimated in a less precise manner.

- In all cases, the estimated effects can be interpreted as elasticities, implying that a 1% increase in income increase food consumption in less than 1%, about a 0.4.

Empirical Application

- Since the CQR coefficients are mildly increasing, the variation in the UQPE has to be coming from the variation in the density of X given $Y = Q_\tau[Y]$.

- As τ increases, for the UQPE to increase, higher CQR coefficients must be getting higher weight. This happens if the density of "income given food$= Q_\tau[\text{food}]$" is moving to the right.

- Observing this pattern in the results indicates that this shift happens relatively quickly: given food expenditure is getting higher, we expect income to become higher, but at a faster rate than the increase in food expenditure, so the share of food spending on income is falling.

Conclusion

- This paper considers the use of conditional quantile regression analysis to estimate unconditional quantile partial effects.
- The proposed methodology is based on an interesting byproduct of quantile analysis, that is, the unconditional effects can be recovered from conditional analysis.
- The current methodology can be extended to evaluate unconditional effects, starting from any initial consistent conditional estimation procedure.

Thank you!