Learning from Exchange Rates and Foreign Exchange Interventions

Giacomo Candian  Pierre De Leo  Luca Gemmi

HEC Montreal  University of Maryland  HEC Lausanne

EEA-ESEM 2023
Foreign exchange interventions: Communication

- Foreign exchange interventions (FXIs) common across the world

1. Central bankers state that FXIs work through market expectations (Patel & Cavallino 19)

2. FXIs often opaque: not publicly announced or published with lag (Sarno & Taylor 01, Canales-Kriljenko 03, Adler et al. 21)

3. Exchange rates contain information about future fundamentals (Engel & West 05, Chahrour et al. 22)

This paper: Develop a dynamic GE model to:

- Formalize informational role of the exchange rate
- Study informational effects of public vs secret FX interventions
Foreign exchange interventions: Communication

• Foreign exchange interventions (FXIs) common across the world

• Important role of information in FX markets:
  1. Central bankers state that FXIs work through market expectations
     (Patel & Cavallino 19) Stated channels

  2. FXIs often opaque: not publicly announced or published with lag
     (Sarno & Taylor 01, Canales-Kriljenko 03, Adler et al. 21) Data on FXI

  3. Exchange rates contains information about future fundamentals
     (Engel & West 05, Chahrour et al. 22) CCDGV
Foreign exchange interventions: Communication

- Foreign exchange interventions (FXIs) common across the world

- Important role of information in FX markets:
  1. Central bankers state that FXIs work through market expectations (Patel & Cavallino 19)
  2. FXIs often opaque: not publicly announced or published with lag (Sarno & Taylor 01, Canales-Kriljenko 03, Adler et al. 21)
  3. Exchange rates contains information about future fundamentals (Engel & West 05, Chahrour et al. 22)

- This paper: Develop a dynamic GE model to:
  - Formalize informational role of the exchange rate
  - Study informational effects of public vs secret FX interventions
A macro model of the informational role of FX and FXI

A model segmented int’l asset markets + dispersed information
A macro model of the informational role of FX and FXI

A model segmented int’l asset markets + dispersed information

1. A novel informational role of FX in macro allocation
   - Agents use FX information to choose consumption and investment
   - FXI can alter information content of exchange rate

2. Communication matters:
   - Public FXI \neq Secret FXI
     - Public FXI is additional public signal to agents
     - Secret FXI can either increase or decrease the informativeness of FX

3. Optimal communication depends on expectations’ formation
   - Rational expectations: more information is desirable
     - Public FXI is optimal
   - Extrapolation: agents use information sub-optimally
     - Secret FXI can be optimal
A macro model of the informational role of FX and FXI

A model **segmented int’l asset markets** $+$ **dispersed information**

1. A novel **informational role** of FX in macro allocation
   - Agents use FX information to choose consumption and investment
   - FXI can alter information content of exchange rate

2. **Communication matters:** Public FXI $\neq$ Secret FXI
   - Public FXI is additional public signal to agents
   - Secret FXI can either increase or decrease the informativeness of FX
A macro model of the informational role of FX and FXI

A model segmented int’l asset markets + dispersed information

1. A novel informational role of FX in macro allocation
   ▶ Agents use FX information to choose consumption and investment
   ▶ FXI can alter information content of exchange rate

2. Communication matters: Public FXI ≠ Secret FXI
   ▶ Public FXI is additional public signal to agents
   ▶ Secret FXI can either increase or decrease the informativeness of FX

3. Optimal communication depends on expectations’ formation
   ▶ Rational expectations: more information is desirable → Public FXI is optimal
   ▶ Extrapolation: agents use information sub-optimally → Secret FXI can be optimal
Related literature

Empirics:

- Exchange rates & fundamentals: Evans & Lyons 02, Engel & West 05, Itskhoki & Mukhin 21, Chahrour et al. 22, Stavrakeva & Tang 20a, 20b, Goldberg & Krogstrup 23
- Survey expectations on macro variables and FX: Coibion & Gorodnichenko 15, Kohlhas & Walther 20, Bordalo, Gennaioli, Ma & Schleifer 20, Angeletos, Huo & Sastry 20, Candian & De Leo 23
- Empirical properties of FXI: Sarno & Taylor 01, Canales-Kriljenko 03, Kuersteiner et al. 18, Patel & Cavallino 19, Fratzscher et al. 19, Adler et al. 21

Theory:

- Learning from prices: Grossman 76, Kimbrough 83 84, Bacchetta & Van Wincoop 06, Amador & Weill 10, Gaballo & Galli 22
- PE theories of FXI under RE: Vitale 99 03, Fernholz 15
- GE models of FXI under FIRE: Gabaix & Maggiori 15, Amador et al. 19, Cavallino 19, Fanelli & Straub 21, Itskhoki & Mukhin 22
- FXI without RE: Iovino & Sergeyev 21
- CB communication on monetary policy: Angeletos & Sastry 20, Chahrour 14, Tang 15, Melosi 17, Kohlhas 20, Candian 21
SUMMARY OF MODEL

• Small open economy, two periods: \( t = [0, 1] \) | Real model
Summary of Model

- Small open economy, two periods: $t = [0, 1]$ | Real model
- Continuum of atomistic islands $i \in [0, 1]$ (Lucas 72)
Summary of Model

- Small open economy, two periods: \( t = [0, 1] \) | Real model

- Continuum of atomistic islands \( i \in [0, 1] \) (Lucas 72)

- Households can save in risk-free bonds \( b^i_1 \) and physical capital \( k^i_1 \)
Summary of Model

• Small open economy, two periods: \( t = [0, 1] \) | Real model

• Continuum of atomistic islands \( i \in [0, 1] \) (Lucas 72)

• Households can save in risk-free bonds \( b_1^i \) and physical capital \( k_1^i \)

• Non-tradable \((y_N)\): exogenous endowment | Tradable \((y_T, 1)\) technology:

\[
y_{T, 1}^{i, H} = a_1 + \alpha k_1^i
\]

\[
a_1 \sim N(0, \beta_a^{-1})
\]
Summary of Model

- Small open economy, two periods: \( t = [0, 1] \) | Real model

- Continuum of atomistic islands \( i \in [0, 1] \) (Lucas 72)

- Households can save in risk-free bonds \( b_i^1 \) and physical capital \( k_i^1 \)

- Non-tradable \( (y_N) \): exogenous endowment | Tradable \( (y_{T,1}^i) \) technology:
  \[
  y_{T,1}^i = a_1 + \alpha k_i^1
  \]
  \[a_1 \sim \mathcal{N}(0, \beta_a^{-1})\]

- Segmented int’l markets | **Financiers** in island \( i \) are subject to position limits
  \[
  r_i^0 - r_0^* - (E_0 q_1^i - q_0^i) = \Gamma \left(n_1^i + f_1^* - b_1^*\right)
  \]
  \[
  \int_i n_1^i \, di = n_1^* \sim \mathcal{N}(0, \beta_n^{-1})
  \]
Equilibrium exchange rate

- Equilibrium aggregate real exchange rate:

\[ q_0 = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} (n_1^* + f_1^*) - \frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \bar{E}_0 a_1 \]

\[ \tilde{\theta}, \omega_1, \omega_2, \omega_3 > 0 \]
Equilibrium Exchange Rate

- Equilibrium aggregate real exchange rate:

\[ q_0 = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta} \omega_1 + \omega_3} (n_1^* + f_1^*) - \frac{\omega_2}{\Gamma \tilde{\theta} \omega_1 + \omega_3} \bar{E}_0 a_1 \]

\[ \tilde{\theta}, \omega_1, \omega_2, \omega_3 > 0 \]

- Exchange rate in frictionless benchmark:

\[ q_0^F = -\frac{\omega_2}{\omega_3} a_1 \]

\( \Gamma = 0, \text{FIRE} \)
**Equilibrium exchange rate**

- Equilibrium *aggregate* real exchange rate:

\[
q_0 = \frac{\Gamma \omega_1}{\tilde{\theta} \omega_1 + \omega_3}(n_1^* + f_1^*) - \frac{\omega_2}{\tilde{\theta} \omega_1 + \omega_3} \bar{E}_0 a_1 \\
\tilde{\theta}, \omega_1, \omega_2, \omega_3 > 0
\]

- Exchange rate in frictionless benchmark: \((\Gamma = 0, \text{FIRE})\)

\[
q_0^F = -\frac{\omega_2}{\omega_3} a_1
\]

- Deviations of \(q_0\) from \(q_0^F\):

\[
q_0 - q_0^F = \frac{\Gamma \omega_1}{\tilde{\theta} \omega_1 + \omega_3} \left[(n_1^* + f_1^*) + \frac{\tilde{\theta} \omega_2}{\omega_3} a_1\right] - \frac{\omega_2}{\tilde{\theta} \omega_1 + \omega_3} \left(\bar{E}_0 a_1 - a_1\right)
\]

**Intermediation wedge** \(\frac{\Gamma \omega_1}{\tilde{\theta} \omega_1 + \omega_3} \left(n_1^* + f_1^*\right)\)

**Belief wedge** \(-\frac{\omega_2}{\tilde{\theta} \omega_1 + \omega_3} \left(\bar{E}_0 a_1 - a_1\right)\)
Information structure  

**Dispersed information**

- HHs receive a private signal $v^i$ about $a_1$
  
  $$v^i = a_1 + \epsilon^i \quad \epsilon^i \sim N(0, \beta_v^{-1}), \quad \text{w/ prior } a_1 \sim N(0, \beta_a^{-1})$$

- Agents observe $q_0$ as they share same currency

**Over-extrapolation**

- HH over-extrapolate new information compared to RE
  
  $$E_{0}^i a_1 = (1 + \delta)(E_{0}^{i,RE} a_1)$$
Learning from the exchange rate (Laissez faire)

• Guess a linear solution for the exchange rate → $q_0 = \lambda_a a_1 + \lambda_n n^*_1$
Learning from the exchange rate (Laissez faire)

• Guess a linear solution for the exchange rate → $q_0 = \lambda_a a_1 + \lambda_n n_1^*$

• Each island $i$ has 3 sources of information about $a_1$
  1. Prior: $a_1 \sim N(0, \beta_a^{-1})$
  2. Private signal: $v^i = a_1 + \epsilon^i$  $\epsilon^i \sim N(0, \beta_v^{-1})$
  3. Public signal: $\frac{q_0}{\lambda_a} = a_1 + \frac{\lambda_n}{\lambda_a} n_1^*$  $\frac{\lambda_n}{\lambda_a} n_1^* \sim N(0, \beta_q^{-1})$,  $\beta_q \equiv \frac{\lambda_a^2}{\lambda_n^2} \beta_n$
Learning from the exchange rate (Laissez faire)

- Guess a linear solution for the exchange rate: \( q_0 = \lambda_a a_1 + \lambda_n n_1^* \)

- Each island \( i \) has 3 sources of information about \( a_1 \)
  1. Prior: \( a_1 \sim N(0, \beta_a^{-1}) \)
  2. Private signal: \( v^i = a_1 + \epsilon^i \) \( \epsilon^i \sim N(0, \beta_v^{-1}) \)
  3. Public signal: \( \frac{q_0}{\lambda_a} = a_1 + \frac{\lambda_n}{\lambda_a} n_1^* \) \( \frac{\lambda_n}{\lambda_a} n_1^* \sim N(0, \beta_q^{-1}) \), \( \beta_q = \frac{\lambda_a^2}{\lambda_n^2} \beta_n \)

- Each agent \( i \) forms the posterior:
  \[ E_0^i a_1 = (1 + \delta) \frac{\beta_v v^i + \beta_q q_0}{\beta_a + \beta_v + \beta_q} \]

- If \( \delta > 0 \) agents over-extrapolate relative to RE

Bordalo et al. 20
Learning from the exchange rate (Laissez faire)

• Guess a linear solution for the exchange rate → \( q_0 = \lambda_a a_1 + \lambda_n n_1^* \)

• Each island \( i \) has 3 sources of information about \( a_1 \)
  1. Prior: \( a_1 \sim N(0, \beta_a^{-1}) \)
  2. Private signal: \( v^i = a_1 + \epsilon^i \quad \epsilon^i \sim N(0, \beta_v^{-1}) \)
  3. Public signal: \( \frac{q_0}{\lambda_a} = a_1 + \frac{\lambda_n}{\lambda_a} n_1^* \quad \frac{\lambda_n}{\lambda_a} n_1^* \sim N(0, \beta_q^{-1}) \), \( \beta_q \equiv \frac{\lambda_a^2}{\lambda_n^2} \beta_n \)

• Each agent \( i \) forms the posterior:

\[
E_{0}^{i}a_1 = (1 + \delta) \frac{\beta_v v^i + \beta_q \frac{q_0}{\lambda_a}}{\beta_a + \beta_v + \beta_q} \quad \mathcal{I}_R = \frac{\beta_q}{\beta_a + \beta_v + \beta_q}
\]

• If \( \delta > 0 \) agents over-extrapolate relative to RE

Bordalo et al. 20

Definition of equilibrium
Uniqueness of equilibrium
INFORMATION CONTENT OF EXCHANGE RATE (Laissez faire)

Information content of $q_0 (I_R)$

![Graph showing the information content of $q_0 (I_R)$ with curves for 'No learning from FX' and 'Baseline ($\delta=0$)'.]
Noise trading shock \((n_1^*)\)

(Laissez faire)

- Full information economy
- Incomplete information economy
- Information content of exchange rate
- Parameterization

\[ I_R \]
\[ E_0a_1 \]
\[ q_0 \]

\[ c_0 \]
\[ k_1 \]
\[ c_1 \]

---

No learning from FX  Baseline \((\delta=0)\)
Noise trading shock ($n_1^*$) (Laissez faire)

Full information economy  Incomplete information economy  Information content of exchange rate  Parameterization
Foreign exchange interventions

• Central bank observes aggregates $\bar{E}[a_1], q_0 \rightarrow$ Fully informed

• FXI: central bank purchases foreign-currency bond $f_1^*$ according to:

$$f_1^* = \kappa_a a_1 + \kappa_n n_1^*$$

• Consider two limit cases

1. **Public FXI**: Agents perfectly observe FX intervention $f_1^*$

2. **Secret FXI**: Agents do not observe FX intervention $f_1^*$

Agents always know the central bank’s reaction function $(\kappa_a, \kappa_n)$
Suppose central bank adopts a public FX intervention, according to:

\[ f_1^* = \kappa a_1 + \kappa n_1^* \]

\[ \frac{f_1^*}{\kappa a} = a_1 + \frac{\kappa n}{\kappa a} n_1^* \quad \frac{q_0}{\lambda a} = a_1 + \frac{\lambda n}{\lambda a} n_1^* \]

\[ f_1^* \text{ becomes an additional public signal} \]

Two signals \((f_1^* \text{ and } q_0)\) and two shocks \((n_1^* \text{ and } a_1)\) ⇒ Full Information
Public FXI and wedges

\[ q_0 - q_0^F = \frac{\Gamma \omega_1}{\Gamma \tilde{\theta}_1 + \omega_3} \left[ (1 + \kappa_n) n_1^* + \left( \frac{\tilde{\theta} \omega_2}{\omega_3} + \kappa_a (1 + \delta) \right) a_1 \right] - \frac{\omega_2}{\Gamma \tilde{\theta}_1 + \omega_3} (\tilde{E}_0 a_1 - a_1) \]

Intermediation wedge

Belief wedge

- Public FXI can close intermediation wedge with \( \left( \kappa_a = -\frac{\tilde{\theta} \omega_2}{\omega_3 (1 + \delta)} \right), \kappa_n = -1 \)

- Implies full information: \( \tilde{E}_0 a_1 = (1 + \delta) a_1 \)
  - RE \((\delta = 0)\): it closes also belief wedge (a form of “divine coincidence”)
  - Extrapolation \((\delta \neq 0)\): belief wedge persists under full information
Suppose central bank adopts a secret FX intervention, according to

\[ f_1^* = \kappa_a a_1 + \kappa_n n_1^* \]

\[ f_1^* \] alters stochastic properties of

\[ q_0 = \lambda_a a_1 + \lambda_n n_1^* \]

The endogenous precision of \( q_0 \) as a public signal is proportional to:

\[
\left( \frac{\lambda_a}{\lambda_n} \right)^2 = \left[ \frac{\omega_2 - \Gamma \omega_1 \tilde{\kappa}_a}{\Gamma \omega_1 (1 + \tilde{\kappa}_n)} (1 + \delta) \frac{\beta_v}{\beta_a + \beta_v + \Lambda^2 \beta_n} \right]^2
\]
SECRET FOREIGN EXCHANGE INTERVENTIONS

• Suppose central bank adopts a secret FX intervention, according to

\[ f_1^* = \kappa_a a_1 + \kappa_n n_1^* \]

• \( f_1^* \) alters stochastic properties of \( q_0 = \lambda_a a_1 + \lambda_n n_1^* \)

• The endogenous precision of \( q_0 \) as a public signal is proportional to:

\[
\left( \frac{\lambda_a}{\lambda_n} \right)^2 = \left[ \frac{\omega_2 - \Gamma \omega_1 \tilde{\kappa}_a}{\Gamma \omega_1 (1 + \tilde{\kappa}_n)} (1 + \delta) \frac{\beta_v}{\beta_a + \beta_v + \Lambda^2 \beta_n} \right]^2
\]

• \( \tilde{\kappa}_n \to -1 \) offset noise traders, \( \lambda_n \to 0 \) \( \Rightarrow \) higher information \( \mathcal{I}_R \uparrow \)

\( \leftrightarrow \) \( \tilde{\kappa}_n = -1, q_0 = \lambda_a a_1 \) perfectly informative, \( \mathcal{I}_R \to \infty \) \( \Rightarrow \) Full Information
SECRET FOREIGN EXCHANGE INTERVENTIONS

• Suppose central bank adopts a secret FX intervention, according to

\[ f_1^* = \kappa_a a_1 + \kappa_n n_1^* \]

• \( f_1^* \) alters stochastic properties of \( q_0 = \lambda_a a_1 + \lambda_n n_1^* \)

• The endogenous precision of \( q_0 \) as a public signal is proportional to:

\[
\left( \frac{\lambda_a}{\lambda_n} \right)^2 = \left[ \frac{\omega_2 - \Gamma \omega_1 \tilde{\kappa}_a}{\Gamma \omega_1 (1 + \tilde{\kappa}_n)} (1 + \delta) \frac{\beta_v}{\beta_a + \beta_v + \Lambda^2 \beta_n} \right]^2
\]

• \( \tilde{\kappa}_n \to -1 \) offset noise traders, \( \lambda_n \to 0 \) ⇒ higher information \( I_R \uparrow \)

\[ \iff \tilde{\kappa}_n = -1, \ q_0 = \lambda_a a_1 \text{ perfectly informative, } I_R \to \infty \Rightarrow \text{ Full Information} \]

• \( \tilde{\kappa}_a \to \frac{\omega_2}{\Gamma \omega_1} \) offset fundamental shock \( \lambda_a \to 0 \) ⇒ lower information \( I_R \downarrow \)

\[ \iff \kappa_a = \frac{\omega_2}{\Gamma \omega_1}, \ q_0 = \lambda_n n_1^* \text{ perfectly uninformative, } I_R \to 0 \]
SECRET FXI AND WEDGES

\[ q_0 - q_0^F = \frac{\Gamma \omega_1}{\Gamma \bar{\theta} \omega_1 + \omega_3} \left[ (1 + \kappa_n) n_1^* + \left( \frac{\bar{\theta} \omega_2}{\omega_3} + \kappa_a (1 + \delta) \right) a_1 \right] - \frac{\omega_2}{\Gamma \bar{\theta} \omega_1 + \omega_3} \left( \bar{E}_0 a_1 - a_1 \right) \]

- **Intermediation wedge**
  - \( (1 + \kappa_n) n_1^* + \left( \frac{\bar{\theta} \omega_2}{\omega_3} + \kappa_a (1 + \delta) \right) a_1 \)
- **Belief wedge**
  - \( \frac{\omega_2}{\Gamma \bar{\theta} \omega_1 + \omega_3} \left( \bar{E}_0 a_1 - a_1 \right) \)

- **Rational expectations** \((\delta = 0)\)
  - Offsetting noise traders \(\kappa_n = -1\) ⇒ endogenous full information, close belief wedge
  - Then close the intermediation wedge with \(\kappa_a < 0\) ⇒ same “divine coincidence”
SECRET FXI AND WEDGES

\[ q_0 - q_0^F = \frac{\Gamma \omega_1}{\Gamma \theta_1 + \omega_3} \left[ (1 + \kappa_n) n_1^* + \left( \frac{\theta \omega_2}{\omega_3} + \kappa_a (1 + \delta) \right) a_1 \right] - \frac{\omega_2}{\Gamma \theta_1 + \omega_3} \left( \tilde{E}_0 a_1 - a_1 \right) \]

Intermediation wedge

Belief wedge

- **Rational expectations** \((\delta = 0)\)
  - Offsetting noise traders \(\kappa_n = -1\) \(\Rightarrow\) endogenous full information, close belief wedge
  - Then close the intermediation wedge with \(\kappa_a < 0\) \(\Rightarrow\) same “divine coincidence”

- **Extrapolation** \((\delta > 0)\)
  - if \(\delta > \hat{\delta}\), central bank needs to lower info to reduce belief wedge
  - Offset noise traders \(\kappa_n \rightarrow -1\) to lower intermediation wedge, but
  - Offset fundamental \(\kappa_a > 0\) to lower information and belief wedge

⇒ **Tradeoff** between intermediation and belief wedge \(\rightarrow\) can’t close both
Central bank decide optimal $\kappa_a, \kappa_n$ to maximize welfare
Conclusions

• Formalize the informational role of FX in macro allocation in SOE

• The information channel of FXI depend on communication:
  1. Public FXI akin to public announcement
  2. Secret FXI tool to affect exchange rate informativeness

• Rationalizes a signaling channel of FXI as well as the opaqueness in many central banks’ practices.
Appendix
**Final good aggregator**

- Consumption and period-1 capital are composites of tradable and non-tradable goods:
  \[ C_0^i + K_1^i = G(Y_N, Y_{T,0}^i), \quad C_1^i = G(Y_N, Y_{T,1}^i) \]

where \( G(Y_N, Y_T) = \left[ (1 - \gamma) \frac{1}{\theta} Y_N^{\frac{\theta-1}{\theta}} + \gamma \frac{1}{\theta} Y_T^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \) is homogenous of degree 1.

- \( \theta \) denotes the elasticity of substitution between tradable and non-tradable goods in the production of final goods
- \( \gamma \) is related to the share of tradable goods in the final composite good
- \( Y_{T,t}^i \) represents domestic absorption of the tradable good, which is the sum (difference) of production and imports from (exports to) the rest of the world
  \[ Y_{T,t}^i = Y_{T,t}^{i,H} + Y_{T,t}^{i,F} \]
- We assume that each island trades with the rest of the world but not with other islands to avoid full information revelation by inter-island interactions.
Demand for tradables:
\[ q_0^i = -\frac{1-\gamma}{\theta} y_{T,0} \quad q_1^i = -\frac{1-\gamma}{\theta} y_{T,1} \]

Modified UIP condition:
\[ r_0^i = E_0^i q_1^i - q_0^i + \frac{\bar{\gamma}(1+\phi)}{\beta} y_{T,0} + \tilde{r} n_1^* + \tilde{r} f_1^* \]
\[ r_1^i = \sigma \gamma E_0^i y_{T,1} - (\sigma \gamma)(1 + \phi)y_{T,0} + \sigma \phi k_1^i \]
\[ \frac{(1+\phi)}{\beta} y_{T,0} = a_1 + \alpha k_1^i - y_{T,1} \]

Country budget constraint:
\[ (1+\phi) \beta y_{T,0} = a_1 + \alpha k_1^i - y_{T,1} \]

Demand for capital:
\[ k_1^i = \frac{1}{1-\alpha} E_0^i q_1^i + \frac{1}{1-\alpha} E_0^i a_1 - \frac{1}{1-\alpha} r_0^i \]
Relative information content of exchange rate (Laissez faire)

**Definition (Relative information content of exchange rate)**

Define the relative information content of the exchange rate as its relative accuracy as a signal about the fundamental shock $a_1$ compared to prior and private signal. That is, the Bayesian weight on public signal: $I_R = \frac{\Lambda^2 \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}$.
## Parameterization (Laissez faire)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
<td>Standard</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>IES</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.3</td>
<td>Standard</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Trade openness</td>
<td>0.3</td>
<td>Standard</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Trade elasticity</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Slope of currency demand</td>
<td>3</td>
<td>Pandolfi &amp; Williams 19</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Std. dev. of $a_1$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Std. dev. of $n_1^*$</td>
<td>0.2</td>
<td>Chahrour et al. 22</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Std. dev. of $v^i$</td>
<td>TBD</td>
<td>Chahrour et al. 22</td>
</tr>
</tbody>
</table>
In the case of perfectly inaccurate private signals, $\beta^v \to 0$, the exchange rate coefficients equal $\lambda_a = 0$ and $\lambda_n = \frac{\Gamma \omega_1}{\Gamma \theta \omega_1 + \omega_3}$. The relative information content of the exchange rate is nil, i.e. $\mathcal{I}_R = 0$ and the overall posterior accuracy is nil, i.e. $D = 0$. 
Corollary (Full Information economy)

In the case of perfectly accurate private signals, $\beta_v \to \infty$, the exchange rate coefficients equal $\lambda_a = -\frac{\omega_2}{\Gamma_{\theta\omega_1+\omega_3}} (1 + \delta)$ and $\lambda_n = \frac{\Gamma_{\omega_1}}{\Gamma_{\theta\omega_1+\omega_3}}$. The relative information content of the exchange rate is nil, i.e. $I_R = 0$, while the overall posterior accuracy is infinite, i.e. $D \to \infty$. 
**Definition of equilibrium (Laissez-faire)**

**Definition (Market equilibrium with laissez-faire)**

Given shocks realization \( \{a_1, n_1^*\} \) and agents’ prior and signals \( \{v^i, q_0\}_{i \in [0,1]} \), a symmetric linear market equilibrium is defined as

- an allocation \( \{c^i_0, c^i_1, k^i_1, y^i_{T,0}, y^i_{T,1}, b^i_1^*, d^i_1^*\}_{i \in [0,1]} \)
- a vector of local prices \( \{q^i_0, r^i_0\}_{i \in [0,1]} \)
- A aggregate real exchange rate as a linear function of the states \( q_0 = \lambda_a a_1 + \lambda_n n_1^* \)

solving equations (??)-(??) with expectations respecting (??) and (??).
Uniqueness of equilibrium

Proposition

Let $\Lambda \equiv \frac{\lambda_a}{\lambda_n}$. The symmetric linear market equilibrium is unique and the exchange rate is described by (1) with coefficients

$$
\lambda_a = -\frac{\omega_2}{\Gamma \omega_1 + \omega_3} (1 + \delta) \frac{\beta_v + \Lambda^2 \beta_n}{\beta_a + \beta_v + \Lambda^2 \beta_n}
$$

$$
\lambda_n = \frac{\Gamma \omega_1}{\Gamma \omega_1 + \omega_3} \frac{\beta_v + \Lambda^2 \beta_n}{\beta_v}
$$

(1)

where $\Lambda^2$ is unique and implicitly defined by

$$
\Lambda^2 = \left( \frac{\omega_2}{\Gamma \omega_1} \right)^2 (1 + \delta)^2 \frac{\beta_v^2}{(\beta_a + \beta_v + \Lambda^2 \beta_n)^2}
$$

(2)

while the explicit solution of $\Lambda$ is reported in Appendix ??.

Proof.

See Appendix ??.
Proposition (Public discretionary FXI)

Suppose the central bank adopts a public discretionary FX intervention, i.e. $f^*_1 = \varepsilon^*_1$ and $\sigma^2_{\eta_f} \to 0$. A more volatile FX intervention does not affect the relative information content of the exchange rate $I_R$ nor the overall agents’ posterior accuracy about fundamental $D$. The equilibrium exchange rate is given by (??) with the same $\lambda_a$ and $\lambda_n$ as in the laissez-faire equilibrium (1).

Proof.
See Appendix ??.
Secret discretionary FXI

Proposition (Secret discretionary FXI)

Suppose the central bank adopts a secret discretionary FX intervention, i.e. \( f^*_1 = \varepsilon^*_1 \) and \( \sigma^2_{\eta_f} \to \infty \). A more volatile FX intervention decreases the relative information content of the exchange rate \( \mathcal{I}_R \) and agents’ posterior accuracy about fundamental \( D \). The equilibrium exchange rate is given by (??) with \( \lambda_a \) and \( \lambda_n \) described in Appendix ??.

Proof.
See Appendix ??.
**Discretionary FXI**

- Suppose the central bank adopts a “discretionary” FX intervention, according to:

  \[ f_1^* = \varepsilon_1^f \]

- Public FX interventions
  - FXI does not affect the relative information content of the exchange rate \( I_R \)
  - The equilibrium exchange rate features the same \( \lambda_a \) and \( \lambda_n \) as in laissez-faire
  - Discretionary FXI is uninformative on state of the economy

- Secret FX interventions
  - FXI decreases the information content of the exchange rate \( I_R \)
  - Discretionary FXI adds non-fundamental noise to the exchange rate \( q_0 \)
Central bank’s information
FINANCIERS’ PROBLEM (1)

- Continuum of risk-neutral financiers, $j \in [0, \infty)$, in each island $i$.
- Financiers hold a zero-capital portfolio in H and F bonds ($d_{j,1}, d_{j,1}^*$).
- Financier’s investment decisions s.t. two restrictions:
  - First, each intermediary is subject to a net open position limit $D > 0$.
  - Second, intermediaries face heterogeneous participation costs.
    Each intermediary $j$ active in the foreign bond market at $t$ is obliged to pay a participation cost of exactly $j$ per unit of FC invested.
- Intermediary $j$ in island $i$ optimally invests $\frac{d_{j,1}^*}{R_0^*}$ in F bonds:

$$
\max_{\frac{d_{j,1}^*}{R_0^*} \in [-D, D]} \frac{d_{j,1}^*}{R_0^*} E_0^i \left( \tilde{R}_1^i \right) - j \left| \frac{d_{j,1}^*}{R_0^*} \right|,
$$

where $\tilde{R}_1^i$ is the return on one foreign-currency unit holding expressed in foreign currency: $\tilde{R}_1^i \equiv R_0^* - R_0^i Q_0^i Q_1^i$. 

**Financiers’ problem (2)**

- Intermediary $j$’s expected cash flow conditional on investing is $D \left| E^i_0 \left( \tilde{R}^i_1 \right) \right|$ while participation costs are $jD$.

- Investing is optimal for all intermediaries $j \in [0, \bar{j}]$, with the marginal active intermediary $\bar{j}$ given by $\bar{j} = \left| E^i_0 \left( \tilde{R}^i_1 \right) \right|$.

- The aggregate investment volume is then

  \[
  \frac{D^i_1^\ast}{R_0^\ast} = \bar{j}D \ \text{sign} \left\{ E^i_0 \left( \tilde{R}^i_1 \right) \right\}.
  \]

- Defining $\Gamma \equiv D^{-1}$ and substituting out $\bar{j}$, we obtain the total demand for foreign-currency bonds in island $i$, $D^i_1^\ast = \int d^i_{j,1}^\ast dj$:

  \[
  \frac{D^i_1^\ast}{R_0^\ast} = \frac{1}{\Gamma} E^i_0 \left( R_0^\ast - R^i_0 \frac{Q^i_0}{Q^i_1} \right)
  \]
Financiers’ problem (3)

• Zero-capital portfolio of each financier implies:

\[
\frac{D_1^i}{R_i^0} + Q_0^i \frac{D_1^{i*}}{R_0^{i*}} = 0.
\]

• Income from the carry trade of the financiers in island \(i\) is:

\[
\pi_1^{i,D^*} \equiv D_1^{i*} + \frac{D_1^i}{Q_i^i} = \cdots = \tilde{R}_1^i \frac{D_1^{i*}}{R_0^{i*}}.
\]

• Intermediaries’ demand for foreign bonds has a finite (semi-)elasticity to the expected excess return.

• Changes in home bond demand, e.g., induced by FX interventions, can indeed affect \(q_0\).

• \(Γ\) is a critical parameter.

• Participation costs constitute transfers to households in the H island economy | no extra cost terms enter the household’s budget constraint.
Concentration in currency markets

- Detailed data on risk taking in this international and opaque over-the-counter market are relatively scarce, which favors specialization and concentration. (Gabaix Maggiori, 15)

- Transaction volume data also portray a highly concentrated market: (Euromoney 14)
  - the top 10 banks accounted for 80 percent of all flows in 2014
  - with the top two banks (Citigroup and Deutsche Bank) accounting for 32 percent of all flows.

- Currency risk also accounts for a large fraction of their overall respective risk taking. (Deutsche Bank 13; Citigroup 13)
  - Regulatory filings reveal that currency risk accounted for 26–35 percent of total (stressed) value at risk at Deutsche Bank in 2013
  - and between 17 percent and 23 percent at Citigroup in the same period
Data on FXI (1/2)

- Information on FXI scant for most countries (only 16% of EMEs publish data.) → research on FXI has often relied on **coarse proxies**
  - typically, $\Delta$ in CB’s reserves or reserve flows from B-o-P statistics
- But coarse proxies of FXI are contaminated by
  1. valuation changes and investment income flows;
  2. CB’s FC transactions with residents & nonresidents that affect the amt of reserves but are not FXI (exchange of LC & FC assets).

- How to address:
  - Fratzscher et al. 19 AEJ:Macro: Confidential data from 33 central banks (includes secret FXI)
  - Identifying FXI via news reports: New data
  - Adler et al. 21: Official FXI data from reports + Proxy FXI data
  - download from Rui Mano’s website
Data on FXI: Adler et al 21 (2/2)

- FXI: ‘any transaction changing central bank’s FC position’.
  1. active transactions (no valuation effects)
  2. transaction by CB (no other public sector entities)
  3. focus on FC position (no distinction sterilized v. unsterilized)
  4. no focus on stated intent (eg. reserve accumulation, etc...)
    include both spot & derivative market operations
- Adler et al 21 address shortcomings of coarse proxies using:
  > available info on composition of reserve assets → estimate valuation $\Delta$s
  + info on market rates & interest payment → estimate investment income
  + other adjustments to “vis-a-vis” proxies.

→ download from Rui Mano’s website
# Stated Channels (Patel & Cavallino 22)

Signalling remains most important channel of FX intervention

<table>
<thead>
<tr>
<th></th>
<th>As a percentage of respondents</th>
<th>Graph 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portfolio balance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Signalling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Signalling future monetary policy stance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Signalling future exchange rate and interventions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


1 Central banks which did not provide an answer for a channel category but did fill out at least one other category are assumed as “Rarely”.

Source: BIS surveys in 2012 and 2018.
How to model secret v. public interventions

• Central bank is fully informed about $a_1$

• FXI: central bank purchases foreign-currency bond $f_1^*$ according to:

$$f_1^* = \kappa_n n_1^* + \kappa_a a_1 + \varepsilon_{f_1}^* \quad \varepsilon_{f_1}^* \sim N(0, \sigma_{\eta f}^2)$$

• FXI is intermediated by financiers in each island:

$$f_{1i}^* = f_1^* + \eta_{fi} \quad \eta_{fi} \sim N(0, \sigma_{\eta f}^2)$$

Consider two limit cases

1. **Public FXI**: Agents perfectly observe FX intervention $\sigma_{\eta f}^2 \to 0$

2. **Secret FXI**: Agents do not observe FX intervention $\sigma_{\eta f}^2 \to \infty$

Agents always know the central bank’s reaction function $(\kappa_a, \kappa_n)$
CCDGV: Bivariate Regression

\[ \Delta_4 q_t = \alpha + \beta_0 \Delta_4 TFP_t + \sum_{k=1}^{h} \beta_{-k}^{\text{lag}} (\Delta_4 TFP_{t-k}) + \sum_{k=1}^{h} \beta_{k}^{\text{lead}} (\Delta_4 TFP_{t+k}) + \varepsilon_t \]
MODEL ENVIRONMENT

- Two periods: \( t = [0, 1] \)
- Real model | Monetary policy in SOE sets \( P_t = 1 \ \forall t \)
- Continuum of atomistic islands \( i \in [0, 1] \) in SOE

- **Households** in island \( i \)

\[
\max_{C^i_0, B^i_1, K^i_1, C^i_1} \quad \frac{C^i_0^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_0 \left( \frac{C^i_1^{1-\sigma}}{1-\sigma} \right) \quad \text{s.t.} \]

\[
C^i_0 + K^i_1 + \frac{B^i_1}{R^i_0} = P^i_{N,0} Y_N + Q^i_0 Y^i_{T,0} + T^i_0, \quad C^i_1 = B^i_1 + P^i_{N,1} Y_N + Q^i_1 Y^i_{T,1} + T^i_1.
\]

- Resource constraint:

\[
C^i_0 + K^i_1 = \left[ (1 - \gamma)^{\frac{1}{\theta}} Y_N^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} Y^i_{T,1}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}
\]
Model environment

- **Firms** in island $i$
  - Non-tradable ($Y_N$): exogenous, constant endowment
  - Tradable ($Y_{T,1}^i$) produced with:
    \[
    Y_{T,1}^i = A_1 K_1^{i\alpha} \quad \ln(A_1) \equiv a_1 \sim N(0, \beta_{a}^{-1})
    \]

- Island’s budget constraint:
  \[
  \frac{B_1^i}{R_0^i} = Q_0^i(Y_{T,0}^i - Y_{T,0}^i) + T_0^i
  \]
Financial sector

Island $i$

F Households

$N X_1^i$

H Households

Financiers in island $i$ are subject to position limits.

Financiers' problem:

$$r_i - r^\ast = 0 - r^\ast - (E_i^0 q^i_1 - q^i_0) = \Gamma (n_i^1^\ast + f_i^1^\ast - b_i^1^\ast) \int i n_i^1^\ast d_i = n^\ast_1 \sim N(0, \beta - 1 n)$$

↪ $\Gamma \to 0$: no frictions;

$\Gamma \to \infty$: autarky.

Financiers owned by households in respective island.

FX market concentration

Island equilibrium
Financial sector

Island $i$

- F Households
- H Households
- Financiers

$NX_i^+\cdot B_1^*\cdot B_1^i\cdot Financiers$

- Financiers owned by households in respective island

FX market concentration

Island equilibrium

$\Gamma\rightarrow 0$: no frictions;
$\Gamma\rightarrow\infty$: autarky
Financial sector

Island $i$

F Households

H Households

Financiers

Noise Traders

$F_X^i$

$B_1^*$

$B_1^i$

$(N_1^i, N_1^i*)$

Financiers in island $i$ are subject to position limits.

$\mathbf{r}_i - \mathbf{r}_0^* - (E_i q_i^1 - q_i^0) = \Gamma(n_i^1* + f_i^1* - b_i^1*) \int_i n_i^1 d_i = n_i^* \sim N(0, \beta_{-1} n_i)$

$\Gamma \to 0$: no frictions; $\Gamma \to \infty$: autarky
Financial sector

- F Households
- H Households
- Financiers
- Noise Traders
- Government

Island $i$

$\text{Financiers in island } i \text{ are subject to position limits}$

$\text{Financiers' problem}$

$\Gamma \rightarrow 0: \text{no frictions;} \quad \Gamma \rightarrow \infty: \text{autarky}$

$\text{FX market concentration}$

$\text{Island equilibrium}$
Financial sector

- **Financiers** in island $i$ are subject to position limits

  $$r^i_0 - r^*_0 - (E^iq^i_1 - q^i_0) = \Gamma \left( n^i_1^* + f^i_1^* - b^i_1^* \right)$$

  $$\int_i n^i_1^* di = n^*_1 \sim N(0, \beta_n^{-1})$$

  $\Gamma \to 0$: no frictions; $\Gamma \to \infty$: autarky

- Financiers owned by households in respective island