Negative Bubbles

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August 2023

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- Theories of asset price bubbles explain this with possibly highly volatile deviations from fundamentals
- But: the literature features only models with positive bubbles (Price > Fundamentals)
- In this paper
  - We show that **negative bubbles** (Firm price < Value of tangible assets) can also exist in a certain class of models
  - We characterize the macroeconomic and welfare effects of negative bubbles

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Findings

- Q: Under what conditions can negative bubbles exist?
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- Q: Under what conditions can negative bubbles exist?
  - A: Negative bubbles arise in economies with looser credit constraints
- Q: What are the real and welfare effects of negative bubbles?
  - A: Contractionary if credit constraints are not too loose
  - A: Expansionary if credit constraints are very loose
  - A: They always reduce welfare

What we do

- Simple neoclassical model with credit frictions on investment
  - Random investment opportunities financed with debt collateralized by firm value
  - Bubbles arise due to strong two-way feedbacks between firm profit/value and firm access to credit

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- Firm value can fall below value of capital because investment is assumed to be irreversible
- Analytical results on existence and real impact of negative bubbles

Related literature

- Rational bubbles models (Tirole (1985), Farhi and Tirole (2015), Martin and Ventura (2015))
    - Bubbles always positive (free disposal)
- Models of borrowing constraints that depend on firm market value (Gertler and Karadi (2011), Gertler and Kyotaki (2015))
- Bubbles due to borrowing constraints that depend on firm market value (Miao and Wang (2018) plus others)
    - Bubbles always positive (can sell tangible assets to other firms)
    - Bubbles always expansionary because they relax credit constraints

The Model

Households

- Preferences

$$W_t = C_t + \beta W_{t+1}, \quad 0 < \beta < 1,$$

- Households entirely passive (for simplicity)
    - Supply 1 unit of labour inelastically
    - Constant real interest rate

$$R_t = \beta^{-1}$$

Firms

- Own the capital stock
  - Can accumulate capital by buying new capital goods at price q_t
  - No possibility to accumulate financial assets (can be relaxed)
  - Capital is irreversible: resale value to other firms is zero
- Opportunity to produce new capital goods at unit cost (probability π)
  - Financed with rental income plus borrowing intraperiod from HHs
  - Borrowing collateralized by firm market value
- Residual paid out as dividend to the household

The Firm's Problem

Value of the firm

$$V(k_t) = \max_{m_t, i_t} r_t k_t - q_t m_t + \pi (q_t - 1) i_t + \beta V(k_{t+1})$$

subject to:

- Capital law of motion (at firm level)

$$m_t = k_{t+1} - (1-\delta) k_t$$

- Irreversibility constraint (at firm level)

$$m_t \geqslant 0$$

- Budget constraint for investing firms

$$i_t = r_t k_t + d_t.$$

- Borrowing constraint:

$$d_t \leqslant V(\lambda k_t)$$

- Limited liability:

$$V(k_t) \geqslant 0$$

The Firm's Problem: No frictions

- Solution when credit and irreversibility constraints do not bind
- Value function (guess and verify)

$$V(k_t) = \phi_t k_t$$

- Capital price equal to replacement cost

$$q_t = 1$$

- FOC for capital

$$1 = \beta (r_{t+1} + 1 - \delta)$$

- Value of installed capital

$$\phi_t = r_t + 1 - \delta$$

The Firm's Problem: Credit Frictions

- Value of the firm (with possible bubble component)

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- Collateral premium: $\pi (q_t - 1)(r_t + \lambda \phi_t)$
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- Bubble value

$$b_t = \pi\left(q_t - 1\right) b_t + \beta b_{t+1}.$$

- Bubble contains its own collateral premium: $\pi\left(q_t - 1\right) b_t$

Aggregate Equilibrium Conditions

Goods market clearing

$$K_t^{\alpha} = C_t + I_t$$

Capital rental rate

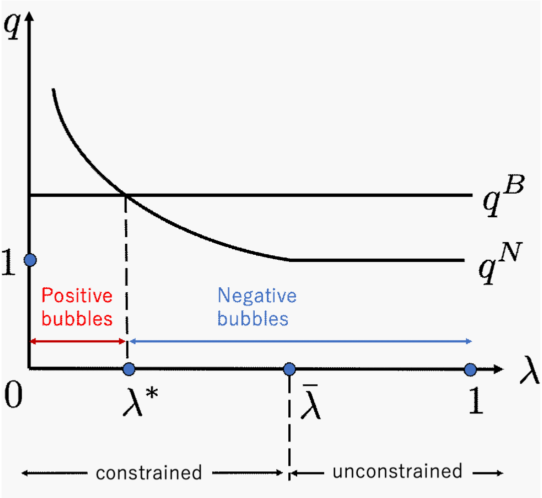
$$r_t = \alpha K_t^{\alpha - 1}$$

Investment under binding credit constraint

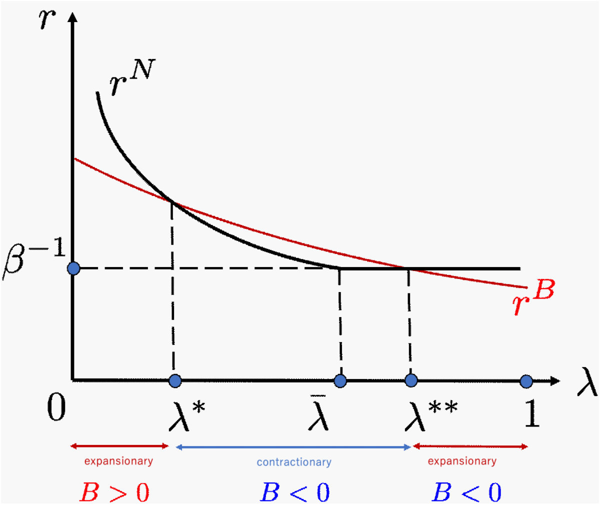
$$I_t = \pi \left(\left(r_t + \lambda \beta^{-1} q_t \right) K_t + B_t \right).$$

Steady State Analysis

Negative and positive bubbles in the model



Bubbles: real impact



Negative expansionary bubbles: intuition

- Value of installed capital:

$$\phi_t = r_t + q_t(1-\delta) + \pi\left(q_t - 1\right)\left(r_t + \lambda\phi_t\right)$$

- Negative bubbles reduce collateral and increase its value (q_t, $\phi_t \uparrow$)
- Is this contractionary ($r_t \uparrow$) or expansionary ($r_t \downarrow$)?
  - Contractionary: $\downarrow$ overall collateral $\Longrightarrow \downarrow K$
  - Expansionary: $q_t > 1 \Longrightarrow$ collateral premium raises value of $K \Longrightarrow \uparrow K$
- Collateral premium depends on λ: dominates if $\lambda > \lambda^{**}$

Dynamic Analysis

Bubbles: stability properties

- Positive bubbles region ($\lambda \leq \lambda^*$)
  - Unique saddle path to the bubbly equilibrium
  - Multiple bubbly paths converging to the bubbleless equilibrium
- Negative bubbles region ($\lambda > \lambda^*$)
  - Multiple paths to the bubbly equilibrium
  - No paths converging to the bubbleless equilibrium

Stability properties: intuition

- Bubble arbitrage equation implies that

$$b_{t+1} - b_t = -\frac{\pi}{\beta}\left(q_t - q^{bss}\right) b_t$$

- Positive bubbles region ($\lambda \leq \lambda^*$)
  - Bubble too small ==> collateral is scarce and $q_t - q^{bss} > 0$
  - Dividend too large and capital gain must be negative ($b_{t+1} - b_t < 0$): bubble deflates towards bubbleless SS
- Negative bubbles region ($\lambda > \lambda^*$)
  - Bubble too small (large negative) ==> collateral is scarce and $q_t - q^{bss} > 0$
  - Negative dividend too large and capital gain must be positive ($b_{t+1} - b_t > 0$): bubble rises towards bubbly SS

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  - Tight credit constraints: positive expansionary bubbles
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  - Very loose credit constraints: negative expansionary bubbles

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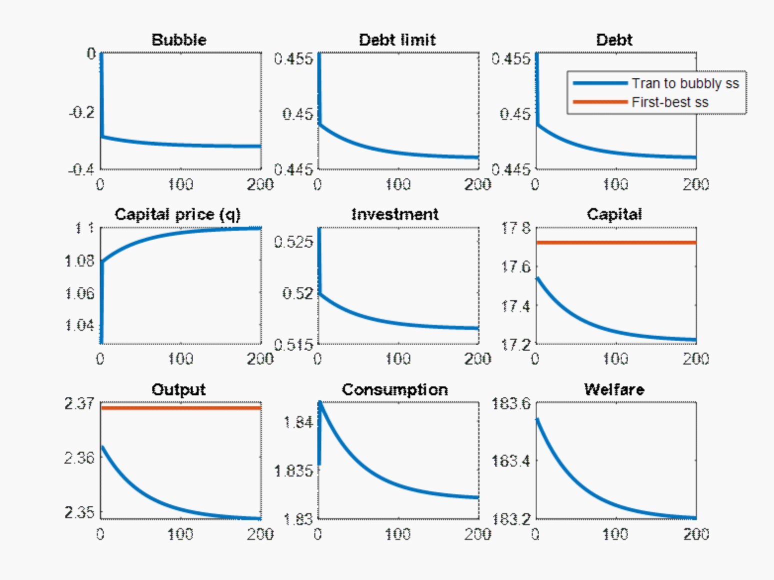
- Bubbly solutions due to two-way interaction between firm value and credit constraints
  - Tight credit constraints: positive expansionary bubbles
  - Loose credit constraints: negative contractionary bubbles
  - Very loose credit constraints: negative expansionary bubbles
- Dynamics around negative bubbly steady state very different
  - Multiplicity of paths leading to the bubbly steady state; no path leading to bubbleless steady state
  - In positive bubble region: unique saddle path to bubbly steady state; multiple paths to bubbleless steady state

Calibrated structural parameters

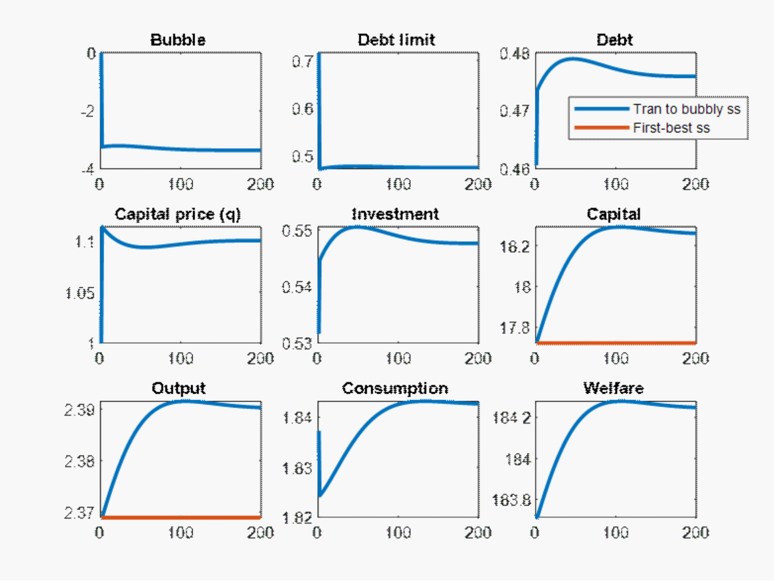
Calibrated Structural Parameters

Parameter	Value	Description
α	0.3	Capital share in GDP
δ	0.03	Depreciation rate (quarterly)
π	0.1	Prob of investment opportunity
β	0.99	Discount factor

Transition to a negative contractionary bubble



Transition to a negative expansionary bubble



Transition to a bubbly economy with debt tax