Negative Bubbles

Kosuke Aoki
University of Tokyo
Kalin Nikolov
European Central Bank

August 2023
Motivation

- Asset prices much more volatile than fundamentals
Motivation

- Asset prices much more volatile than fundamentals
- Theories of asset price bubbles explain this with possibly highly volatile deviations from fundamentals
Motivation

- Asset prices much more volatile than fundamentals
- Theories of asset price bubbles explain this with possibly highly volatile deviations from fundamentals
- But: the literature features only models with positive bubbles (Price $>$ Fundamentals)
Motivation

- Asset prices much more volatile than fundamentals
- Theories of asset price bubbles explain this with possibly highly volatile deviations from fundamentals
- But: the literature features only models with positive bubbles (Price > Fundamentals)
- In this paper
  - We show that negative bubbles (Firm price < Value of tangible assets) can also exist in a certain class of models
  - We characterize the macroeconomic and welfare effects of negative bubbles
Two main questions

- Under what conditions can negative bubbles exist?
Two main questions

- Under what conditions can negative bubbles exist?
- What are their real and welfare effects?
Q: Under what conditions can negative bubbles exist?
- A: Negative bubbles arise in economies with looser credit constraints
Findings

Q: Under what conditions can negative bubbles exist?
- A: Negative bubbles arise in economies with looser credit constraints

Q: What are the real and welfare effects of negative bubbles?
- A: Contractionary if credit constraints are not too loose
- A: Expansionary if credit constraints are very loose
- A: They always reduce welfare
- Simple neoclassical model with credit frictions on investment
  - Random investment opportunities financed with debt collateralized by firm value
  - Bubbles arise due to strong two-way feedbacks between firm profit/value and firm access to credit

Firm value can fall below value of capital because investment is assumed to be irreversible

Analytical results on existence and real impact of negative bubbles
Simple neoclassical model with credit frictions on investment
- Random investment opportunities financed with debt collateralized by firm value
- Bubbles arise due to strong two-way feedbacks between firm profit/value and firm access to credit
Negative bubbles due to self-fulfilling pessimism about firm value which reduces access to credit and hence profit
What we do

- Simple neoclassical model with credit frictions on investment
  - Random investment opportunities financed with debt collateralized by firm value
  - Bubbles arise due to strong two-way feedbacks between firm profit/value and firm access to credit
- Negative bubbles due to self-fulfilling pessimism about firm value which reduces access to credit and hence profit
- Firm value can fall below value of capital because investment is assumed to be irreversible
What we do

- Simple neoclassical model with credit frictions on investment
  - Random investment opportunities financed with debt collateralized by firm value
  - Bubbles arise due to strong two-way feedbacks between firm profit/value and firm access to credit
- Negative bubbles due to self-fulfilling pessimism about firm value which reduces access to credit and hence profit
- Firm value can fall below value of capital because investment is assumed to be irreversible
- Analytical results on existence and real impact of negative bubbles
  - Bubbles always positive (free disposal)
- Models of borrowing constraints that depend on firm market value (Gertler and Karadi (2011), Gertler and Kyotaki (2015))
- Bubbles due to borrowing constraints that depend on firm market value (Miao and Wang (2018) plus others)
  - Bubbles always positive (can sell tangible assets to other firms)
  - Bubbles always expansionary because they relax credit constraints
The Model
Households

- Preferences

\[ W_t = C_t + \beta W_{t+1}, \quad 0 < \beta < 1, \]

- Households entirely passive (for simplicity)
  - Supply 1 unit of labour inelastically
  - Constant real interest rate

\[ R_t = \beta^{-1} \]
Firms

- Own the capital stock
  - Can accumulate capital by buying new capital goods at price $q_t$
  - No possibility to accumulate financial assets (can be relaxed)
  - Capital is irreversible: resale value to other firms is zero
- Opportunity to produce new capital goods at unit cost (probability $\pi$)
  - Financed with rental income plus borrowing intraperiod from HHs
  - Borrowing collateralized by firm market value
- Residual paid out as dividend to the household
The Firm’s Problem

Value of the firm

\[ V(k_t) = \max_{m_t,i_t} r_t k_t - q_t m_t + \pi (q_t - 1) i_t + \beta V(k_{t+1}) \]

subject to:
- Capital law of motion (at firm level)
  \[ m_t = k_{t+1} - (1 - \delta) k_t \]
- Irreversibility constraint (at firm level)
  \[ m_t \geq 0 \]
- Budget constraint for investing firms
  \[ i_t = r_t k_t + d_t \]
- Borrowing constraint:
  \[ d_t \leq V(\lambda k_t) \]
- Limited liability:
  \[ V(k_t) \geq 0 \]
The Firm’s Problem: No frictions

- Solution when credit and irreversibility constraints do not bind
- Value function (guess and verify)

\[ V(k_t) = \phi_t k_t \]

- Capital price equal to replacement cost

\[ q_t = 1 \]

- FOC for capital

\[ 1 = \beta (r_{t+1} + 1 - \delta) \]

- Value of installed capital

\[ \phi_t = r_t + 1 - \delta \]
Value of the firm (with possible bubble component)

\[ V(k_t) = \phi_t k_t + b_t > 0 \]
The Firm’s Problem: Credit Frictions

- Value of the firm (with possible bubble component)
  \[ V(k_t) = \phi_t k_t + b_t > 0 \]

- Investment of firms with investment opportunity
  \[ i_t = r_t k_t + (\lambda \phi_t k_t + b_t) \]
The Firm’s Problem: Credit Frictions

- Value of the firm (with possible bubble component)
  \[ V(k_t) = \phi_t k_t + b_t > 0 \]

- Investment of firms with investment opportunity
  \[ i_t = r_t k_t + (\lambda \phi_t k_t + b_t) \]

- Value of installed capital:
  \[ \phi_t = r_t + q_t (1 - \delta) + \pi (q_t - 1) (r_t + \lambda \phi_t) \]
  - Collateral premium: \( \pi (q_t - 1) (r_t + \lambda \phi_t) \)
  - Creates a leverage multiplier that depends on \( \lambda \)
Value of the firm (with possible bubble component)

\[ V(k_t) = \phi_t k_t + b_t > 0 \]

Investment of firms with investment opportunity

\[ i_t = r_t k_t + (\lambda \phi_t k_t + b_t) \]

Value of installed capital:

\[ \phi_t = r_t + q_t (1 - \delta) + \pi (q_t - 1) (r_t + \lambda \phi_t) \]

- Collateral premium: \( \pi (q_t - 1) (r_t + \lambda \phi_t) \)
- Creates a leverage multiplier that depends on \( \lambda \)

Bubble value

\[ b_t = \pi (q_t - 1) b_t + \beta b_{t+1}. \]

- Bubble contains its own collateral premium: \( \pi (q_t - 1) b_t \)
Aggregate Equilibrium Conditions

Goods market clearing

\[ K_t^\alpha = C_t + I_t \]

Capital rental rate

\[ r_t = \alpha K_t^{\alpha - 1} \]

Investment under binding credit constraint

\[ I_t = \pi \left( (r_t + \lambda \beta^{-1} q_t) K_t + B_t \right). \]
Steady State Analysis
Negative and positive bubbles in the model

![Diagram showing positive and negative bubbles in a model with variables $q$, $q^B$, $q^N$, and $\lambda$. The diagram illustrates the constrained and unconstrained regions. Positive bubbles are shown for $\lambda^*$ to $\lambda^*$, and negative bubbles for $\lambda^*$ to $\lambda$.](image-url)
Bubbles: real impact

\[ r^N \]

\[ \beta^{-1} \]

\[ 0 \quad \lambda^* \quad \bar{\lambda} \quad \lambda^{**} \quad 1 \]

\[ r^B \]

expansionary \quad contractionary \quad expansionary

\[ B > 0 \quad B < 0 \quad B < 0 \]
Negative expansionary bubbles: intuition

- Value of installed capital:

\[ \phi_t = r_t + q_t (1 - \delta) + \pi (q_t - 1) (r_t + \lambda \phi_t) \]

- Negative bubbles reduce collateral and increase its value \((q_t, \phi_t \uparrow)\)

- Is this contractionary \((r_t \uparrow)\) or expansionary \((r_t \downarrow)\)?
  - Contractionary: \(\downarrow\) overall collateral \(\implies\) \(\downarrow K\)
  - Expansionary: \(q_t > 1 \implies\) collateral premium raises value of \(K \implies\) \(\uparrow K\)

- Collateral premium depends on \(\lambda\): dominates if \(\lambda > \lambda^{**}\)
Dynamic Analysis
Positive bubbles region ($\lambda \leq \lambda^*$)
- Unique saddle path to the bubbly equilibrium
- Multiple bubbly paths converging to the bubbleless equilibrium

Negative bubbles region ($\lambda > \lambda^*$)
- Multiple paths to the bubbly equilibrium
- No paths converging to the bubbleless equilibrium
Stability properties: intuition

- Bubble arbitrage equation implies that

\[ b_{t+1} - b_t = -\frac{\pi}{\beta} \left( q_t - q^{bss} \right) b_t \]

- Positive bubbles region \((\lambda \leq \lambda^*)\)
  - Bubble too small \(\Rightarrow\) collateral is scarce and \(q_t - q^{bss} > 0\)
  - Dividend too large and capital gain must be negative \((b_{t+1} - b_t < 0)\): bubble deflates towards bubbleless SS

- Negative bubbles region \((\lambda > \lambda^*)\)
  - Bubble too small (large negative) \(\Rightarrow\) collateral is scarce and \(q_t - q^{bss} > 0\)
  - Negative dividend too large and capital gain must be positive \((b_{t+1} - b_t > 0)\): bubble rises towards bubbly SS
Summary

- Neoclassical model with credit frictions
Summary

- Neoclassical model with credit frictions
- Bubbly solutions due to two-way interaction between firm value and credit constraints
  - Tight credit constraints: positive expansionary bubbles
  - Loose credit constraints: negative contractionary bubbles
  - Very loose credit constraints: negative expansionary bubbles

Dynamics around negative bubbly steady state very different
- Multiplicity of paths leading to the bubbly steady state; no path leading to bubbleless steady state
- In positive bubble region: unique saddle path to bubbly steady state; multiple paths to bubbleless steady state
Summary

- Neoclassical model with credit frictions
- Bubbly solutions due to two-way interaction between firm value and credit constraints
  - Tight credit constraints: positive expansionary bubbles
  - Loose credit constraints: negative contractionary bubbles
  - Very loose credit constraints: negative expansionary bubbles
- Dynamics around negative bubbly steady state very different
  - Multiplicity of paths leading to the bubbly steady state; no path leading to bubbleless steady state
  - In positive bubble region: unique saddle path to bubbly steady state; multiple paths to bubbleless steady state
Calibrated structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
<td>Capital share in GDP</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>Depreciation rate (quarterly)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.1</td>
<td>Prob of investment opportunity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
</tbody>
</table>
Transition to a negative contractionary bubble
Transition to a negative expansionary bubble
Transition to a bubbly economy with debt tax

- **Bubble**
  - Graph showing the transition to debt tax with a first-best ss.

- **Debt limit**
  - Graph showing the debt limit transition with a first-best ss.

- **Debt**
  - Graph comparing the transition to debt tax and first-best ss.

- **Capital price (q)**
  - Graph showing the capital price transition with a first-best ss.

- **Investment**
  - Graph showing the investment transition with a first-best ss.

- **Capital**
  - Graph comparing the capital transition to debt tax and first-best ss.

- **Output**
  - Graph showing the output transition with a first-best ss.

- **Consumption**
  - Graph showing the consumption transition with a first-best ss.

- **Welfare**
  - Graph comparing the welfare transition to debt tax and first-best ss.

Aoki, Nikolov ()

August 2023 25 / 25