Order routing and market quality: Who benefits from internalization?

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  - Transaction costs are not increased by the introduction of PFOF (Battalio (1997)) and internalizing dealers (Battalio (1997)).

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- No: the retail order flow can predict future returns. (Kaniel et al. (2008), Barber et al. (2008), Kaniel et al. (2012), Kelley and Tetlock (2013), Fong et al. (2014), Barrot et al. (2016), Boehmer et al. (2021))

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- Is liquidity affected by the design?

Market structure

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- Noisy/liquidity traders: the noise demand is given by $Z_t = \sigma B_t$.
- Informed investor: observes the noise demand and the fundamental value of the asset. She is risk-neutral, i.e. solves

$$\sup_{X \in \mathcal{A}(H)} \mathbb{E}^v \left[W_1^X \right] = \sup_{X \in \mathcal{A}(H)} \mathbb{E}^v \left[(V - P_1) X_1 + \int_0^1 X_s dP_s \right],$$

where $\mathbb{E}^v$ is the expectation using the probability measure of the insider who is given the realisation $V = v$.

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  - And are either:
    a) perfectly competitive agents that form a continuum of mass one and take prices P as given,or
    b) market makers who compete in a Bertrand fashion for the net demand of the risky asset. The number of market makers is assumed to be finite.

Theorem (Competitive agents equilibrium)

There exists an equilibrium and the market characteristics are:

$$P_t^* = \mu + \int_0^t \lambda^* dY_s^* \text{ with} \tag{1}$$

$$\lambda^* = \frac{\gamma}{\sigma}\frac{1}{2}(\rho_M + \sqrt{\rho_M^2 + 4}) = \frac{\gamma}{\sigma}\lambda_r = \lambda_K \lambda_r, \tag{2}$$

where $\rho_M = \rho\gamma\sigma$. *Total demand:*

$$dY_t^* = \sigma dB_t + \frac{\frac{V-\mu}{\lambda^*} - Y_t^*}{1-t} dt, \quad \text{insider's view,}$$

$$dY_t^* = \sigma d\beta_t^* - \frac{\lambda_r \rho_M Y_t^*}{1 + \lambda_r \rho_M (1-t)} dt, \quad \text{competitive agents' view.}$$

The expected utility is:

$$\gamma\sigma\sqrt{1 + \frac{\rho_M^2}{4}} \text{ for insider and } 1 - \lambda_r e^{-\frac{\lambda_r \rho_M}{2}} \text{ for CA} \tag{3}$$

First implications of equilibrium

- When insider is present, net demand flow has a drift in its own filtration:

$$dY_t^* = \sigma d\beta_t^* - \frac{\lambda_r \rho_M Y_t^*}{1 + \lambda_r \rho_M (1-t)} dt.$$

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Definition of Equilibrium

An equilibrium is a pair $((\Lambda^*, \Phi^*), X^*)$ satisfying the following conditions:

- (Λ^*, Φ^*), and X^* are admissible.
- X^* is optimal given (Λ^*, Φ^*), i.e.

$$\sup_X \mathbb{E}^v \left[\int_0^1 X_t dP_t + X_1(V - P_1) \right],$$

where $P_t^* = \phi(t) + \int_0^t \lambda^*(s) d(Z_s + X_s)$, $\lambda^*(t) = \min_i \lambda^{i,*}(t)$.

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  where $P_t^* = \phi(t) + \int_0^t \lambda^*(s) d(Z_s + X_s)$, $\lambda^*(t) = \min_i \lambda^{i,*}(t)$.
- For any market maker the deviation is sub-optimal, assuming that the insider's strategy is optimal for the new quote.

Theorem

There exist two symmetric equilibria with market characteristics:

$$P_t^{*,i} = \phi_i^* + \lambda^* Y_t^{*,i}, \text{ with } \phi_i^* = \mu - \frac{(-1)^i}{\sigma\rho}\sqrt{\rho_M^2 - \frac{2\rho_M \log(\lambda_r)}{\lambda_r}} \quad (4)$$

and λ_r, λ^* *given by (2). The total demand is*

$$dY_t^{*,i} = \left(\frac{V-\phi_i^*}{\lambda^*} - \frac{\alpha_{1,i}(t)}{\rho\lambda^*} + b_i(t)\sigma^2(1-t) - Y_t^{*,i}\right)\frac{dt}{1-t} + \sigma dB_t,$$

$$dY_t^{*,i} = \sigma^2(a(t)Y_t^{*,i} + b_i(t))dt + \sigma d\beta_t, \text{ where}$$

$\sigma^2 a(t) = -\frac{\lambda_r \rho_M}{1+\lambda_r\rho_M(1-t)}$. *The ex-ante insider's profit is given by*

$$\frac{(\phi_i^* - \mu)^2}{2\lambda^*} + \gamma\sigma\sqrt{1 + \frac{\rho_M^2}{4}}. \quad (5)$$

Insider's profits

Table: Expected profit of insider/strategic trader

	CA	MM	Additional value
Insider trader	$\gamma\sigma\sqrt{1+\frac{\rho_M^2}{4}}$	$\gamma\sigma\pi(\rho_M)$	$\gamma\sigma\Delta(\rho_M)$
Strategic trader	$\gamma\sigma\frac{\rho_M}{4}$	$\gamma\sigma\frac{\rho_M}{3}$	$\gamma\sigma\frac{\rho_M}{12}$
Normalized value of information (NVI)	$\sqrt{1+\frac{\rho_M^2}{4}}-\frac{\rho_M}{4}$	$v(\rho_M)$	

In above $\Delta(x) := \frac{\Delta_0(u(x))}{2x}$, $\Delta_0(x) := x\log x - x + 1$, $u(x) := 1 - \frac{2}{1+\sqrt{1+\frac{4}{x^2}}}$,

$$v(x) := \frac{u(x)\log u(x) + \frac{1+7u(x)-8u^2(x)}{3u(x)}}{2x}, \text{ and } \pi(x) := \frac{x}{3} + v(x).$$

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$\Delta_0(x) \geq 0$ for $x \in [0,1] \Rightarrow$ insider's profit is higher in MM market.

Comparison of NVI in CA market and MM market.

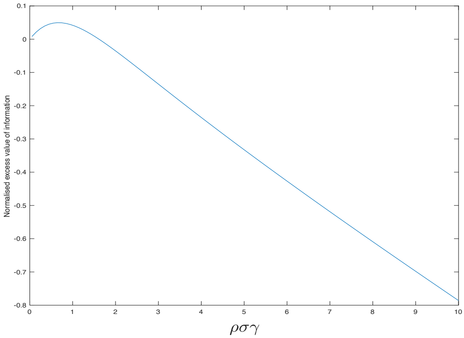

Figure: The difference between the normalized value of information in the market makers and competitive agents equilibrium as a function of ρ_M.

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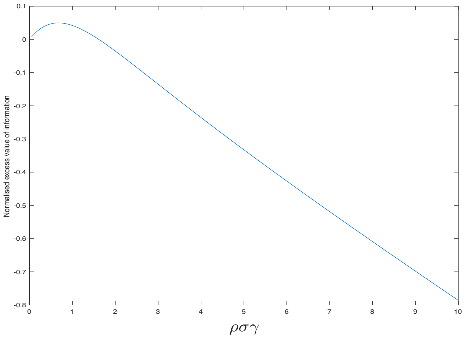

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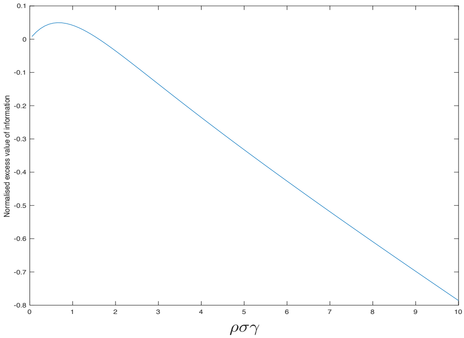

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- $\Rightarrow$ Information is more valuable in MM market.
- $\Rightarrow$ MM retail order flow is more likely to be informed.

Normalized value of information

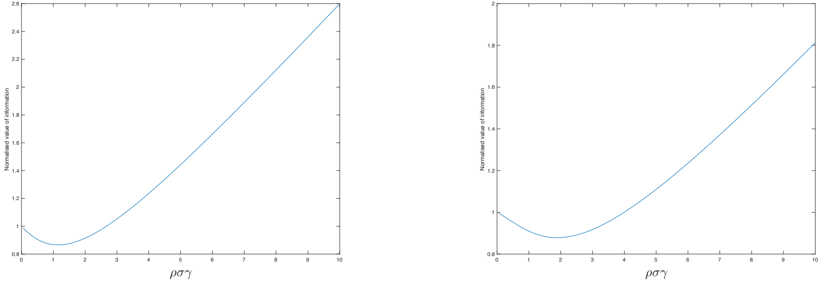

Figure: Normalized value of information in competitive agents equilibrium is reported in the left pane and the right plot illustrates corresponding value for the market makers equilibrium.

⇒ Risk aversion of liquidity providers disincentivise acquisition of private information.

What about noisy traders?

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The expected profit of the noisy traders is given by

$$\mathbb{E}\left[\sigma \int_0^T B_t dP_t\right] = \sigma \mathbb{E}\left[B_1 V - \int_0^1 P_t dB_t - \lambda^* \sigma\right] = -\lambda^* \sigma^2,$$

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- $\Rightarrow$ Noisy traders are indifferent.
- $\Rightarrow$ The profits of the competitive agents above the zero-utility level are passed to the strategic traders in MM equilibrium.
- Curious fact: the percent increase in the profit between CA and MM is higher for uninformed strategic trader.

Market characteristics are stable:

- **The market depth** is constant, as in Kyle, and equal to $\frac{1}{\lambda_K \lambda_r}$, where $\lambda_r = \frac{\rho_M}{2} + \sqrt{\frac{\rho_M^2}{4} + 1}$.

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$$\Sigma(t) = \mathrm{Var}(V|\mathcal{F}_t^M) = \Sigma_K(t) \frac{2 + \rho_M(1 + \sqrt{\rho_M^2 + 4})}{2 + \rho_M(1 + \sqrt{\rho_M^2 + 4})(1 - t)}.$$

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- **Momentum** is defined as

$$M(s) := \lim_{\substack{t-s=\varepsilon \\ u-s=\varepsilon \\ \varepsilon \to 0}} \frac{\mathrm{Cov}\left(P_t - P_s, P_u - P_t\right)}{\sqrt{\mathrm{Var}(P_t - P_s)\mathrm{Var}(P_u - P_t)}\varepsilon}$$

$$= -\frac{2}{1 + \sqrt{1 + \frac{4}{\rho_M^2}} - 2s}$$

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- Both liquidity and efficiency worsen with higher ρ_M and price reversal exacerbates.