A Measure of Behavioral Heterogeneity

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Heterogeneity

- Heterogeneity is the rule rather than the exception
- The aim is to move from the mere observation of heterogeneity, to its quantification
- This, in turn:
  - will allow the systematic study of the driving forces of heterogeneity, and
  - may be instrumental in a number of settings: prediction exercises, welfare analysis, assessment of the representative agent approach, etc.
Two sources of behavioral heterogeneity:

1. **Inter-personal** variation
   - Variation of Tastes/Preferences across the population, and hence behaviors
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2. **Intra-personal** variation
   - The behavior of any given individual is also subject to variation
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2. **Intra-personal** variation
   - The behavior of any given individual is also subject to variation

▶ Relevant, for instance, in welfare analysis:

▶ If mostly **inter-personal** variability ⇒ Classical welfare tools
▶ If mostly **intra-personal** variability ⇒ Need to borrow from the growing literature on behavioral welfare analysis
In this paper we:

1. Propose and study a novel, choice-based, measure of behavioral heterogeneity
   - It evaluates the probability that, over a sampled menu, the sampled choices of two sampled individuals differ
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1. Propose and study a novel, choice-based, measure of behavioral heterogeneity
   - It evaluates the probability that, over a sampled menu, the sampled choices of two sampled individuals differ

2. Provide axiomatic foundations

3. Study the comparative statics of inter- and intra-personal heterogeneity
Related Literature

- **Diversity as the probability that two random extractions produce different outcomes**: Greenberg (1956, Linguistics), Lieberson (1969, Sociology), Leonhardt (1997, Quantum Mechanics), Rényi or collision entropy (Statistics), Ely, Frankel and Kamenica (2015, Information Economics), Herfindahl-Hirschman index (Industrial Organization)

- Inter-personal variability in the **measurement of polarization and segregation**: Esteban and Ray (1994, polarization), Frankel and Volij (2011, school segregation), Baldiga and Green (2013, consensus and aggregation), Gentzkow, Shapiro and Taddy (2019, political predictability), and Bertrand and Kamenika (2023, cultural distance)

- **Random utility models** describing the behavior of individuals and populations. E.g., **mixed-logit**, where a distribution of individual logit behaviors is entertained (Train, 2009)

We contribute by:

1. focusing on choice behavior, which involves a number of overlapping situations (i.e., choices from not just one, but different menus),
2. proposing an overall measure of heterogeneity that applies to settings where there are two layers of heterogeneity, inter- and intra-personal, and
3. by providing axiomatic foundations
1. The Measure
Setting

Let $\mathcal{P}$ be the collection of preferences (linear orders) over a finite set of alternatives $X$. 
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- An individual \( \psi \) is a random utility model (RUM)
  - \( \psi \) is a probability distribution on \( \mathcal{P} \)
  - \( \rho_\psi(a, A) = \sum_{P} \psi(P) \cdot \mathbb{I}[a=m(A, P)] \)
Setting

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- An individual $\psi$ is a random utility model (RUM)
  - $\psi$ is a probability distribution on $\mathcal{P}$
  - $\rho_\psi(a, A) = \sum_P \psi(P) \cdot \chi_{a=m(A, P)}$

- A population $\theta$ is a (finite) distribution over the space of individuals
  - $\theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_1, \psi_2, \ldots, \psi_m]$
Population $\theta$ has both inter- and intra-personal variation
Example 2

Population $\theta'$ has only intra-personal variation. It belongs to the class $\Theta^{hom}$ of homogeneous populations.
Example 3

Population $\theta''$ has only inter-personal variation. It belongs to the class $\Theta^D$ of populations formed by deterministic individuals.
Examples 1, 2 and 3
The Measure: Choice Heterogeneity CH

- $\lambda$: distribution over the possible menus of alternatives $A$
The Measure: Choice Heterogeneity CH

- \( \lambda \): distribution over the possible menus of alternatives \( A \)

\[
CH_\lambda(\theta) = \sum_A \lambda(A) \sum_i \theta_i \sum_j \theta_j \sum_a \rho_{\psi_i}(a, A)(1 - \rho_{\psi_j}(a, A))
\]

- Choice heterogeneity is the probability that, over a sampled menu, the sampled choices of two sampled individuals differ
Example 1

\[
\begin{align*}
\text{CH}_\lambda(\theta) &= \sum_A \lambda(A) \sum_i \theta_i \sum_j \theta_j \sum_a \rho_{\psi_i}(a, A)(1 - \rho_{\psi_j}(a, A)) \\
\lambda(\{x, y\}) &= 1 \\
\text{CH}_\lambda(\theta) &= \frac{1}{3} \left( \frac{3}{8} \frac{5}{8} + \frac{5}{8} \frac{3}{8} \right) + \frac{1}{3} \left( \frac{3}{8} \frac{1}{4} + \frac{5}{8} \frac{3}{4} \right) + \frac{2}{3} \left( \frac{3}{4} \frac{5}{8} + \frac{1}{4} \frac{3}{8} \right) + \frac{2}{3} \left( \frac{3}{4} \frac{1}{4} + \frac{1}{4} \frac{3}{4} \right) = \frac{15}{32}
\end{align*}
\]
2. Features of $\text{CH}_\lambda$

- Aggregate data
- A matrix representation
- A Euclidean representation
- Inter- and Intra-personal heterogeneity
Aggregate data

Question: what is the behavioral heterogeneity of $\theta$ when using aggregate choice data instead of panel data?
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Every population $\theta$ admits a representative agent $\psi_\theta$

$\psi_\theta = \sum_i \theta_i \psi_i$
Aggregate data

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- $[1; \psi_\theta]$
Aggregate data

- Question: what is the behavioral heterogeneity of $\theta$ when using aggregate choice data instead of panel data?
- Every population $\theta$ admits a representative agent $\psi_{\theta}$
  - $\psi_{\theta} = \sum_i \theta_i \psi_i$
- $[1; \psi_{\theta}]$

Proposition. $CH_\lambda(\theta) = CH_\lambda([1; \psi_{\theta}])$
Examples 1 and 2

\[ \theta \]

\[
\begin{align*}
\psi_1 &: \psi(\theta) = 3/8 + 3/8 = 5/8, \\
\psi_2 &: \psi(\theta) = 2/3 + 1/3 = 1/1 = 1.
\end{align*}
\]

• \( \psi_1 \): \( xy \), \( yx \), \( xy \), \( yx \)

\[ \psi \]

\[
\begin{align*}
\theta' &: \theta'(\psi) = 5/8 + 3/8 = 1.
\end{align*}
\]

• \( \theta' \): \( xy \), \( yx \)
Examples 1 and 2

$$\psi_\theta(xy) = \frac{1}{3} \cdot \frac{3}{8} + \frac{2}{3} \cdot \frac{3}{4} = \frac{5}{8}$$
Examples 1 and 2

\[
\begin{align*}
\psi \theta (xy) &= \frac{1}{3} \frac{3}{8} + \frac{2}{3} \frac{3}{4} = \frac{5}{8} = \psi(xy) \Rightarrow \theta' = [1; \psi_\theta] \\
\end{align*}
\]
Examples 1 and 2

\[ \psi_1 \] \[ \psi_2 \]

\[ \psi_{\theta}(xy) = \frac{1}{3} \frac{3}{8} + \frac{2}{3} \frac{3}{4} = \frac{5}{8} = \psi(xy) \Rightarrow \theta' = [1; \psi_\theta] \]

\[ \psi_{\theta'}(xy) = \frac{5}{8} \frac{3}{8} + \frac{3}{8} \frac{5}{8} = \frac{15}{32} = \psi_{\theta'} \]

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Matrix representation

Couple: \[
\begin{pmatrix}
1 & 2 \\
1 & 2
\end{pmatrix};
\psi_P, \psi_Q
\]

\(C_\lambda: \mid P \mid \times \mid P \mid -\text{matrix compiling (twice) the heterogeneity value}
\)

\((i,j)\)-entry: \(2 \cdot CH_\lambda(\begin{pmatrix}1 & 2 \\
1 & 2\end{pmatrix}; \psi_P_i, \psi_P_j)\)

\(C_\lambda\) is independent of the specific distribution \(\theta\) over the individuals, and hence independent of the population Proposition.

\(CH_\lambda(\theta) = \psi_{\theta} C_\lambda \psi_{\theta}^\top\).
Matrix representation

Couple: $\left[\frac{1}{2}, \frac{1}{2}; \psi_P, \psi_Q\right]$
Matrix representation

- Couple: $[\frac{1}{2}, \frac{1}{2}; \psi_P, \psi_Q]$

- $C_\lambda$: $|\mathcal{P}| \times |\mathcal{P}|$-matrix compiling (twice) the heterogeneity value of each possible couple
  - $(i, j)$-entry: $2 \cdot \text{CH}_\lambda([\frac{1}{2}, \frac{1}{2}; \psi_{P_i}, \psi_{P_j}])$

- $C_\lambda$ is independent of the specific distribution $\theta$ over the individuals, and hence independent of the population Proposition.
Matrix representation

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- $C_\lambda$ is independent of the specific distribution $\theta$ over the individuals, and hence independent of the population

Proposition. $\text{CH}_\lambda(\theta) = \psi_\theta \ C_\lambda \ \psi_\theta^\top$. 
Euclidean representation

λ-Euclidean distance between individuals ψ and ψ′:

\[ d_{\lambda}(\rho_\psi, \rho_{\psi'}') = \sum A_{\lambda}(A) \sum a\left[\rho_{\psi'}(a, A) - \rho_{\psi'}'(a, A)\right]^2 \]

β_{\lambda} = \sum A_{\lambda}(A) |A| - 1 |A|

ρ_{ψ_U}: choices of the (uniform) individual that assigns equal mass to all preferences

Proposition.

\[ CH_{\lambda}(\theta) = \beta_{\lambda} - d_{\lambda}(\rho_\psi \theta, \rho_{\psi_U}) = \max_{\psi' \in \Psi} d_{\lambda}(\rho_\psi, \rho_{\psi'}') - d_{\lambda}(\rho_\psi \theta, \rho_{\psi_U}) = d_{\lambda}(\rho_\psi P, \rho_{\psi_U}) - d_{\lambda}(\rho_\psi \theta, \rho_{\psi_U}) \text{ for every } P \in P. \]
Euclidean representation

- \( \lambda \)-Euclidean distance between individuals \( \psi \) and \( \psi' \):
  \[
d_{\lambda}(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2
  \]
Euclidean representation

- **$\lambda$-Euclidean distance** between individuals $\psi$ and $\psi'$:
  \[
  d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2
  \]

- **$\beta_\lambda$**:
  \[
  \beta_\lambda = \sum_A \lambda(A) \frac{|A| - 1}{|A|}
  \]
Euclidean representation

- **λ-Euclidean distance** between individuals $\psi$ and $\psi'$:
  \[
  d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2
  \]

- $\beta_\lambda = \sum_A \lambda(A) \frac{|A|-1}{|A|}$

- $\rho_{\psi U}$: choices of the (uniform) individual that assigns equal mass to all preferences
Euclidean representation

- **λ-Euclidean distance** between individuals $\psi$ and $\psi'$:
  \[
  d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a \left[ \rho_\psi(a, A) - \rho_{\psi'}(a, A) \right]^2
  \]

- **$\beta_\lambda$**:
  \[
  \beta_\lambda = \sum_A \lambda(A) \frac{|A|-1}{|A|}
  \]

- **$\rho_{\psi_U}$**: choices of the (uniform) individual that assigns equal mass to all preferences

**Proposition.**

\[\text{CH}_\lambda(\theta) = \beta_\lambda - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U})\]
Euclidean representation

- **λ-Euclidean distance** between individuals $\psi$ and $\psi'$:
  \[ d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2 \]

- \[ \beta_\lambda = \sum_A \lambda(A) \frac{|A| - 1}{|A|} \]

- $\rho_{\psi_U}$: choices of the (uniform) individual that assigns equal mass to all preferences

**Proposition.**

\[ CH_\lambda(\theta) = \beta_\lambda - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U}) \]

\[ = \max_{\psi \in \psi} d_\lambda(\rho_\psi, \rho_{\psi_U}) - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U}) \]
Euclidean representation

- **λ-Euclidean distance** between individuals $\psi$ and $\psi'$:
  \[
d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2
  \]

- $\beta_\lambda = \sum_A \lambda(A) \frac{|A|-1}{|A|}$

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**Proposition.**

\[
CH_\lambda(\theta) = \beta_\lambda - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U})
\]

\[
= \max_{\psi \in \Psi} d_\lambda(\rho_\psi, \rho_{\psi_U}) - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U})
\]

\[
= d_\lambda(\rho_{\psi_P}, \rho_{\psi_U}) - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U}) \text{ for every } P \in \mathcal{P}.
\]
Example 1

\[ \lambda \{x, y\} = 1 \]

\[ \beta_\lambda = \sum_A \lambda(A) \frac{n_A - 1}{n_A} = \frac{1}{2} \]

\[ \psi_u(xy) = \frac{1}{2} \text{ and recall } \psi_\theta(xy) = \frac{5}{8} \]
Example 1

\[ \lambda(\{x, y\}) = 1 \]

\[ \beta_\lambda = \sum_A \lambda(A) \frac{n_A - 1}{n_A} = \frac{1}{2} \]

\[ \psi_U(xy) = \frac{1}{2} \text{ and recall } \psi_\theta(xy) = \frac{5}{8} \]

\[ \text{CH}_\lambda(\theta) = \beta_\lambda - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_U}) = \frac{1}{2} - \left[ \left( \frac{5}{8} - \frac{1}{2} \right)^2 + \left( \frac{3}{8} - \frac{1}{2} \right)^2 \right] = \frac{15}{32} \]
Inter- and Intra-personal heterogeneity
Proposition.

\[ \text{CH}_\lambda(\theta) = \sum_i \theta_i [\beta_\lambda - d_\lambda(\rho_{\psi_i}, \rho_{\psi_U})] + \sum_i \theta_i \sum_{i<j} \theta_j \ d_\lambda(\rho_{\psi_i}, \rho_{\psi_j}) \]
Example 1

\[ d_\lambda (\rho_{\psi_1}, \rho_{\psi_2}) = (\frac{3}{8} - \frac{1}{2})^2 + (\frac{5}{8} - \frac{1}{2})^2 = \frac{1}{32} \]
Example 1

\[ d_\lambda(\rho_{\psi_1}, \rho_{\psi_U}) = \left(\frac{3}{8} - \frac{1}{2}\right)^2 + \left(\frac{5}{8} - \frac{1}{2}\right)^2 = \frac{1}{32} \]

\[ d_\lambda(\rho_{\psi_2}, \rho_{\psi_U}) = \left(\frac{3}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{4} - \frac{1}{2}\right)^2 = \frac{4}{32} \]
Example 1

\[ d_\lambda (\rho_{\psi_1}, \rho_{\psi_2}) = (\frac{3}{8} - \frac{1}{2})^2 + (\frac{5}{8} - \frac{1}{2})^2 = \frac{1}{32} \]

\[ d_\lambda (\rho_{\psi_2}, \rho_{\psi_1}) = (\frac{3}{4} - \frac{1}{2})^2 + (\frac{1}{4} - \frac{1}{2})^2 = \frac{4}{32} \]

\[ d_\lambda (\rho_{\psi_1}, \rho_{\psi_2}) = (\frac{3}{8} - \frac{3}{4})^2 + (\frac{5}{8} - \frac{1}{4})^2 = \frac{9}{32} \]
Example 1

\[
d_\lambda (\rho_{\psi_1}, \rho_{\psi_U}) = \left( \frac{3}{8} - \frac{1}{2} \right)^2 + \left( \frac{5}{8} - \frac{1}{2} \right)^2 = \frac{1}{32}
\]

\[
d_\lambda (\rho_{\psi_2}, \rho_{\psi_U}) = \left( \frac{3}{4} - \frac{1}{2} \right)^2 + \left( \frac{1}{4} - \frac{1}{2} \right)^2 = \frac{4}{32}
\]

\[
d_\lambda (\rho_{\psi_1}, \rho_{\psi_2}) = \left( \frac{3}{8} - \frac{3}{4} \right)^2 + \left( \frac{5}{8} - \frac{1}{4} \right)^2 = \frac{9}{32}
\]

\[
CH_\lambda (\theta) = \theta_1 [\beta_\lambda - d_\lambda (\rho_{\psi_1}, \rho_{\psi_U})] + \theta_2 [\beta_\lambda - d_\lambda (\rho_{\psi_2}, \rho_{\psi_U})] + \theta_1 \theta_2 d_\lambda (\rho_{\psi_1}, \rho_{\psi_2})
\]

\[
= \frac{1}{3} \left( \frac{1}{2} - \frac{1}{32} \right) + \frac{2}{3} \left( \frac{1}{2} - \frac{4}{32} \right) + \frac{1}{3} \frac{2}{3} \frac{9}{32} = \frac{15}{32}
\]
3. Axiomatic characterization

- Reduction
- Decomposition
- Monotonicity
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- Reduction
- Decomposition
- Monotonicity

- $H : \Theta \rightarrow \mathbb{R}_+$, such that $H(\theta) = 0$ if and only if $\theta \in \Theta^D \cap \Theta^{hom}$
Reduction. $H(\theta) = H([1; \psi\theta])$. 
Decomposition

\[ \theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_{P_1}, \psi_{P_2}, \ldots, \psi_{P_m}] \in \Theta^D \]
Decomposition

\[ \theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_{P_1}, \psi_{P_2}, \ldots, \psi_{P_m}] \in \Theta^D \]

\[ \left[ \frac{\theta_i}{\theta_i + \theta_j}, \frac{\theta_j}{\theta_i + \theta_j}; \psi_{P_i}, \psi_{P_j} \right] \]
Decomposition

\[ \theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_{P_1}, \psi_{P_2}, \ldots, \psi_{P_m}] \in \Theta^D \]

\[ \frac{\theta_i}{\theta_i + \theta_j}, \frac{\theta_j}{\theta_i + \theta_j}; \psi_{P_i}, \psi_{P_j} \]

**Decomposition.** For every \( \theta \in \Theta^D \),

\[
H(\theta) = \sum_{i<j} (\theta_i + \theta_j)^2 H\left[\frac{\theta_i}{\theta_i + \theta_j}, \frac{\theta_j}{\theta_i + \theta_j}; \psi_{P_i}, \psi_{P_j}\right].
\]
Example 4

\[ H(\theta) = \left(\frac{2}{3}\right)^2 H(\theta') + \left(\frac{2}{3}\right)^2 H(\theta'') + \left(\frac{2}{3}\right)^2 H(\theta''') \]
Monotonicity

\[ \left[ \frac{1}{2}, \frac{1}{2}; \psi_P^n, \psi_Q^n \right] \]
Monotonicity

\[ \begin{align*}
\quad & [\frac{1}{2}, \frac{1}{2}; \psi_{P^n}, \psi_{Q^n}] \\
\Rightarrow & \quad C = \{[\frac{1}{2}, \frac{1}{2}; \psi_{P^n}, \psi_{Q^n}]\}^N_{n=1}
\end{align*} \]
Monotonicity

- $\left[ \frac{1}{2}, \frac{1}{2}; \psi_{P_n}, \psi_{Q_n} \right]$  
- $C = \{ \left[ \frac{1}{2}, \frac{1}{2}; \psi_{P_n}, \psi_{Q_n} \right] \}_{n=1}^{N}$  
- $\Delta_A(C)$: number of couples in $C$ for which the two preferences involved disagree over $A$
Monotonicity

- $[\frac{1}{2}, \frac{1}{2}; \psi_{Pn}, \psi_{Qn}]$

- $C = \{[\frac{1}{2}, \frac{1}{2}; \psi_{Pn}, \psi_{Qn}]\}_{n=1}^{N}$

- $\Delta_A(C)$: number of couples in $C$ for which the two preferences involved disagree over $A$

Monotonicity. If $N = N'$ and $\Delta_A(C) \geq \Delta_A(C')$ for every $A$, then

$$\sum_n H([\frac{1}{2}, \frac{1}{2}; \psi_{Pn}, \psi_{Qn}]) \geq \sum_{n'} H([\frac{1}{2}, \frac{1}{2}; \psi_{P'n}, \psi_{Q'n}])$$
Example 4

\[ C = \{ \theta', \theta'', \theta''' \} \]

\[ \Delta\{x, y, z\}(C) = \Delta\{x, y\}(C) = \Delta\{x, z\}(C) = \Delta\{y, z\}(C) = 2 \]
Characterization

**Theorem.** $H$ satisfies Reduction, Decomposition and Monotonicity if and only if there exists a probability distribution $\lambda$ on $\mathcal{A}$ and $k > 0$ such that $H = k \cdot \text{CH}_\lambda$
Intuition

- **Reduction**: from $\theta$ to $[1, \psi_\theta]$, and from here to the deterministic population $\theta^d$ that assigns the same probability to every preference as the representative agent $\psi_\theta$
Intuition

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Intuition

▶ Reduction: from \( \theta \) to \([1, \psi_\theta]\), and from here to the deterministic population \( \theta^d \) that assigns the same probability to every preference as the representative agent \( \psi_\theta \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>([1, \psi_\theta])</th>
<th>( \theta^d )</th>
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<tbody>
<tr>
<td>( \frac{1}{3} )</td>
<td>( \psi_1 )</td>
<td>( \psi_{xy} )</td>
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<tr>
<td>( \frac{2}{3} )</td>
<td>( \psi_2 )</td>
<td>( \psi_{yx} )</td>
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▶ Decomposition: from \( \theta^d \) to populations of the form \([1 - \gamma, \gamma; \psi_P, \psi_Q]\)
Intuition

▶ Reduction: from $\theta$ to $[1, \psi_\theta]$, and from here to the deterministic population $\theta^d$ that assigns the same probability to every preference as the representative agent $\psi_\theta$.

▶ Decomposition: from $\theta^d$ to populations of the form $[1 - \gamma, \gamma; \psi_P, \psi_Q]$.

▶ $H([1 - \gamma, \gamma; \psi_P, \psi_Q]) = 4\gamma(1 - \gamma)H([\frac{1}{2}, \frac{1}{2}; \psi_P, \psi_Q])$. 
Intuition

▶ Reduction: from $\theta$ to $[1, \psi_\theta]$, and from here to the deterministic population $\theta^d$ that assigns the same probability to every preference as the representative agent $\psi_\theta$

![Tree Diagram]

▶ Decomposition: from $\theta^d$ to populations of the form $[1 - \gamma, \gamma; \psi_P, \psi_Q]$

$H([1 - \gamma, \gamma; \psi_P, \psi_Q]) = 4\gamma(1 - \gamma)H([\frac{1}{2}, \frac{1}{2}; \psi_P, \psi_Q])$.

▶ Identify the contribution to heterogeneity of each menu $A$ by constructing collections of couples for which the $\Delta$-vectors differ only in menu $A$, and apply Monotonicity
4. Comparative statics: Intra- and inter-personal heterogeneity

\[ CH_\lambda(\theta) = \sum_i \theta_i [\beta_\lambda - d_\lambda(\rho_{\psi_i}, \rho_{\psi_j})] + \sum_i \theta_i \sum_{i<j} \theta_j \ d_\lambda(\rho_{\psi_i}, \rho_{\psi_j}) \]
Intra-Personal Heterogeneity

- $P$-central individual $\psi$: there is $P \in \mathcal{P}$ such that $xPy$ and $\{x, y\} \subseteq A$ implies $\rho_\psi(x, A) \geq \rho_\psi(y, A)$
Intra-Personal Heterogeneity

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- \( \psi_1 \) and \( \psi_2 \) are \( P \)-central individuals. We say, \( \psi_2 \) is a decentralization of \( \psi_1 \), if there is \( \epsilon > 0 \) and preferences \( Q_1, Q_2 \) such that:
  1. \( \psi_2 = \psi_1 - \epsilon \psi Q_1 + \epsilon \psi Q_2 \) and
  2. \( xPy \) and \( xQ_2y \) imply \( xQ_1y \).
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2. \( xPy \) and \( xQ_2y \) imply \( xQ_1y \).

Proposition. If \( \psi_2 \) is a sequential decentralization of \( \psi_1 \), then \( d_\lambda(\rho_{\psi_1}, \rho_{\psi_U}) \geq d_\lambda(\rho_{\psi_2}, \rho_{\psi_U}) \).
Intra-Personal Heterogeneity: Luce

- $u : X \rightarrow \mathbb{R}_{++}$, and let, w.l.o.g., $\sum_{x \in X} u(x) = 1$

- $\rho_u(a, A) = \frac{u(a)}{\sum_{b \in A} u(b)}$
Intra-Personal Heterogeneity: Luce

\[ u : X \to \mathbb{R}_{++}, \text{ and let, w.l.o.g., } \sum_{x \in X} u(x) = 1 \]

\[ \rho_u(a, A) = \frac{u(a)}{\sum_{b \in A} u(b)} \]

**Proposition.** Suppose that \( u_1(x_1) \geq \cdots \geq u_1(x_n) \) and \( u_2(x_1) \geq \cdots \geq u_2(x_n) \). If \( \frac{u_2(x_j)}{u_2(x_i)} \geq \frac{u_1(x_j)}{u_1(x_i)} \) for every \( i < j \), then \( d_\lambda(\rho_{\psi u_1}, \rho_{\psi U}) \geq d_\lambda(\rho_{\psi u_2}, \rho_{\psi U}) \).
Example

\[ X = \{x, y, z\} \]

\[ u_1 = (u_1(x), u_1(y), u_1(z)) = (3/6, 2/6, 1/6) \]

\[ u_2 = (u_2(x), u_2(y), u_2(z)) = (4/9, 3/9, 2/9) \]
Example

- $X = \{x, y, z\}$
  - $u_1 = (u_1(x), u_1(y), u_1(z)) = (3/6, 2/6, 1/6)$
  - $u_2 = (u_2(x), u_2(y), u_2(z)) = (4/9, 3/9, 2/9)$
  - The monotone-likelihood ratio applies to $u_1$ and $u_2$ and hence individual 1 has more intra-personal heterogeneity
Corollary. For every $\alpha \in [0, 1],$

$$\text{CH}_\lambda(\alpha \theta + (1 - \alpha) \theta') = \alpha \text{CH}_\lambda(\theta) + (1 - \alpha) \text{CH}_\lambda(\theta') + \alpha(1 - \alpha) d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_{\theta'}}).$$
Final remarks:

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   - our measure $\text{CH}_\lambda$ can be used for populations of individuals described by any sort of stochastic choice function
   - our characterization strategy works for any stochastic model as long as:
     1.1 the domain of individual behaviors is convex
     1.2 it should be possible to link any menu to a pair of deterministic behaviors
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   ▶ $S$ (a common set of states) and $\mu$ (a common probability distribution on $S$)
   ▶ $f_i : S \rightarrow \mathcal{P}$
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2. Correlated choices: state-dependent preferences
   - $S$ (a common set of states) and $\mu$ (a common probability distribution on $S$)
   - $f_i : S \rightarrow \mathcal{P}$
   - $\text{CH}_\lambda^S = \sum_A \lambda(A) \sum_s \mu(s) \sum_i \theta_i \sum_j \theta_j \mathbb{I}[m(A, f_i(s)) \neq m(A, f_j(s))]$
In this paper:

1. We propose a novel, choice-based, measure of behavioral heterogeneity
2. We provide axiomatic foundations for our measure
3. We obtain a decomposition into inter- and intra-personal heterogeneity
Thank you!!