A Measure of Behavioral Heterogeneity

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Heterogeneity

- Heterogeneity is the rule rather than the exception
- The aim is to move from the mere **observation** of heterogeneity, to its **quantification**
- This, in turn:
  - will allow the systematic study of the driving forces of heterogeneity, and
  - may be instrumental in a number of settings: prediction exercises, welfare analysis, assessment of the representative agent approach, etc.

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- Relevant, for instance, in welfare analysis:
  - If mostly **inter-personal** variability $\Rightarrow$ Classical welfare tools
  - If mostly **intra-personal** variability $\Rightarrow$ Need to borrow from the growing literature on behavioral welfare analysis

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   - It evaluates the probability that, over a sampled menu, the sampled choices of two sampled individuals differ

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1. Propose and study a **novel, choice-based, measure of behavioral heterogeneity**
    - It evaluates the probability that, over a sampled menu, the sampled choices of two sampled individuals differ
2. Provide **axiomatic foundations**
3. Study the comparative statics of inter- and intra-personal heterogeneity

Related Literature

- **Diversity as the probability that two random extractions produce different outcomes**: Greenberg (1956, Linguistics), Lieberson (1969, Sociology), Leonhardt (1997, Quantum Mechanics), Rényi or collision entropy (Statistics), Ely, Frankel and Kamenica (2015, Information Economics), Herfindahl-Hirschman index (Industrial Organization)
- Inter-personal variability in the **measurement of polarization and seggregation**: Esteban and Ray (1994, polarization), Frankel and Volij (2011, school segregation), Baldiga and Green (2013, consensus and aggregation), Gentzkow, Shapiro and Taddy (2019, political predictability), and Bertrand and Kamenika (2023, cultural distance)
- **Random utility models** describing the behavior of individuals and populations. E.g., **mixed-logit**, where a distribution of individual logit behaviors is entertained (Train, 2009)

We contribute by:

1. focusing on choice behavior, which involves a number of overlapping situations (i.e., choices from not just one, but different menus),
2. proposing an overall measure of heterogeneity that applies to settings where there are two layers of heterogeneity, inter- and intra-personal, and
3. by providing axiomatic foundations

1. The Measure

Setting

Let $\mathcal{P}$ be the collection of preferences (linear orders) over a finite set of alternatives X

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- An individual ψ is a random utility model (RUM)
  - ψ is a probability distribution on $\mathcal{P}$
  - $\rho_\psi(a, A) = \sum_P \psi(P) \cdot \mathbb{I}_{[a=m(A,P)]}$

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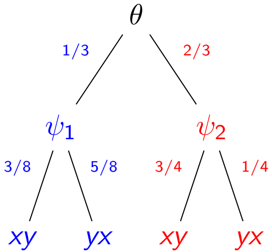
- A population θ is a (finite) distribution over the space of individuals
  - $\theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_1, \psi_2, \ldots, \psi_m]$

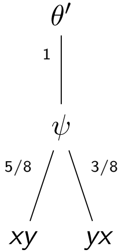
Example 1

Population θ has both inter- and intra-personal variation



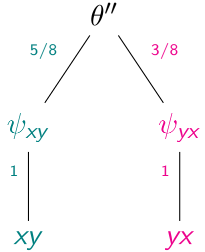
Example 2

Population θ' has only intra-personal variation. It belongs to the class Θ^{hom} of homogeneous populations

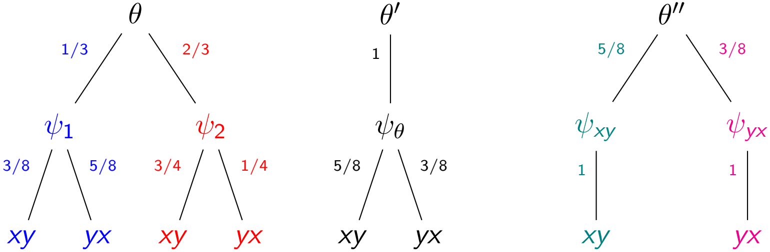


Example 3

Population θ'' has only inter-personal variation. It belongs to the class Θ^D of populations formed by deterministic individuals



Examples 1, 2 and 3



The Measure: Choice Heterogeneity CH

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$$\mathsf{CH}_\lambda(\theta) = \sum_A \lambda(A) \sum_i \theta_i \sum_j \theta_j \sum_a \rho_{\psi_i}(a, A)(1 - \rho_{\psi_j}(a, A))$$

- Choice heterogeneity is the probability that, over a sampled menu, the sampled choices of two sampled individuals differ

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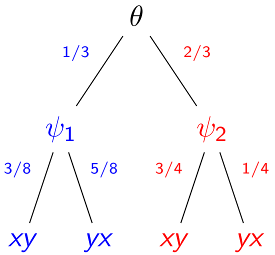

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$\lambda(\{x, y\}) = 1$

$$\mathrm{CH}_\lambda(\theta) = \frac{1}{3}\frac{1}{3}\left(\frac{3}{8}\frac{5}{8}+\frac{5}{8}\frac{3}{8}\right)+\frac{1}{3}\frac{2}{3}\left(\frac{3}{8}\frac{1}{4}+\frac{5}{8}\frac{3}{4}\right)+\frac{2}{3}\frac{1}{3}\left(\frac{3}{4}\frac{5}{8}+\frac{1}{4}\frac{3}{8}\right)+\frac{2}{3}\frac{2}{3}\left(\frac{3}{4}\frac{1}{4}+\frac{1}{4}\frac{3}{4}\right) = \frac{15}{32}$$

2. Features of CH_λ

- Aggregate data
- A matrix representation
- A Euclidean representation
- Inter- and Intra-personal heterogeneity

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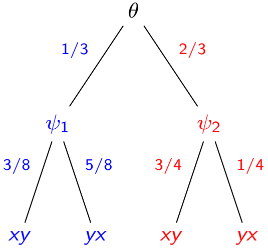
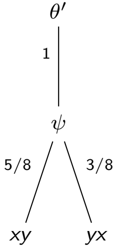
Proposition. $\mathsf{CH}_\lambda(\theta) = \mathsf{CH}_\lambda([1; \psi_\theta])$

Examples 1 and 2





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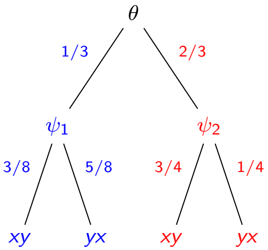

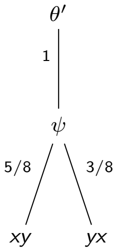

- $\psi_\theta(xy) = \frac{1}{3}\frac{3}{8} + \frac{2}{3}\frac{3}{4} = \frac{5}{8}$

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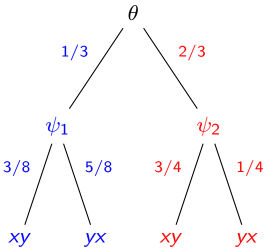

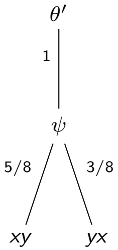

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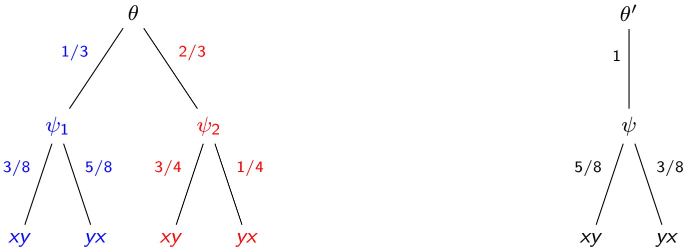

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- $\mathrm{CH}_\lambda(\theta') = \frac{5}{8}\frac{3}{8} + \frac{3}{8}\frac{5}{8} = \frac{15}{32} = \mathrm{CH}_\lambda(\theta)$

Matrix representation

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  - (i, j)-entry: $2 \cdot \mathrm{CH}_\lambda([\frac{1}{2}, \frac{1}{2}; \psi_{P_i}, \psi_{P_j}])$

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Proposition. $\mathrm{CH}_\lambda(\theta) = \psi_\theta \; \mathcal{C}_\lambda \; {\psi_\theta}^\top$.

Euclidean representation

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- λ-Euclidean distance between individuals ψ and ψ':
  $d_\lambda(\rho_\psi, \rho_{\psi'}) = \sum_A \lambda(A) \sum_a [\rho_\psi(a, A) - \rho_{\psi'}(a, A)]^2$

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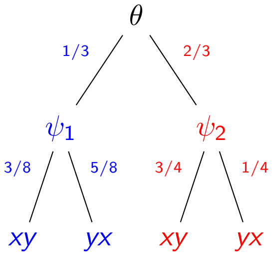

$\lambda(\{x, y\}) = 1$

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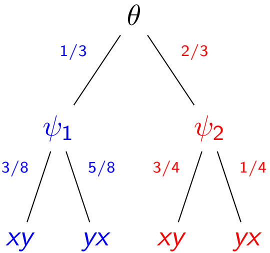

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$\mathrm{CH}_\lambda(\theta) = \beta_\lambda - d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_{\mathcal{U}}}) = \frac{1}{2} - [(\frac{5}{8} - \frac{1}{2})^2 + (\frac{3}{8} - \frac{1}{2})^2] = \frac{15}{32}$

Inter- and Intra-personal heterogeneity

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Proposition.

$$\mathrm{CH}_\lambda(\theta) = \sum_i \theta_i [\beta_\lambda - d_\lambda(\rho_{\psi_i}, \rho_{\psi_{\mathcal{U}}})] + \sum_i \theta_i \sum_{i<j} \theta_j \; d_\lambda(\rho_{\psi_i}, \rho_{\psi_j})$$

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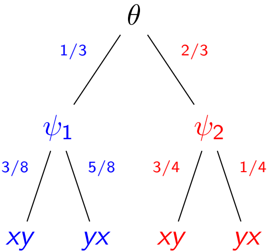

$$d_\lambda(\rho_{\psi_1}, \rho_{\psi_{\mathcal{U}}}) = (\frac{3}{8} - \frac{1}{2})^2 + (\frac{5}{8} - \frac{1}{2})^2 = \frac{1}{32}$$

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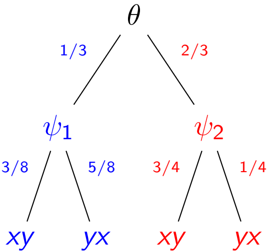

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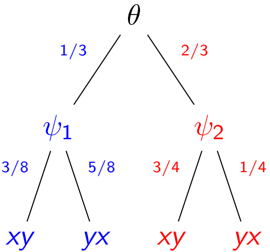

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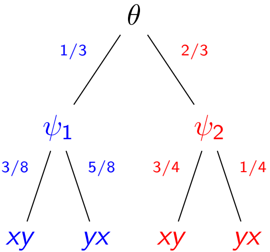

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$$\begin{aligned} \mathrm{CH}_\lambda(\theta) &= \theta_1[\beta_\lambda - d_\lambda(\rho_{\psi_1}, \rho_{\psi_\mathcal{U}})] + \theta_2[\beta_\lambda - d_\lambda(\rho_{\psi_2}, \rho_{\psi_\mathcal{U}})] + \theta_1\theta_2 d_\lambda(\rho_{\psi_1}, \rho_{\psi_2}) \\ &= \tfrac{1}{3}(\tfrac{1}{2} - \tfrac{1}{32}) + \tfrac{2}{3}(\tfrac{1}{2} - \tfrac{4}{32}) + \tfrac{1}{3}\tfrac{2}{3}\tfrac{9}{32} = \tfrac{15}{32} \end{aligned}$$

3. Axiomatic characterization

- Reduction
- Decomposition
- Monotonicity

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- Decomposition
- Monotonicity
  - H : $\Theta \to \mathbb{R}_+$, such that $H(\theta) = 0$ if and only if $\theta \in \Theta^D \cap \Theta^{hom}$

Reduction

Reduction. $\mathsf{H}(\theta) = \mathsf{H}([1; \psi_\theta])$.

Decomposition

- $\theta = [\theta_1, \theta_2, \ldots, \theta_m; \psi_{P_1}, \psi_{P_2}, \ldots, \psi_{P_m}] \in \Theta^D$

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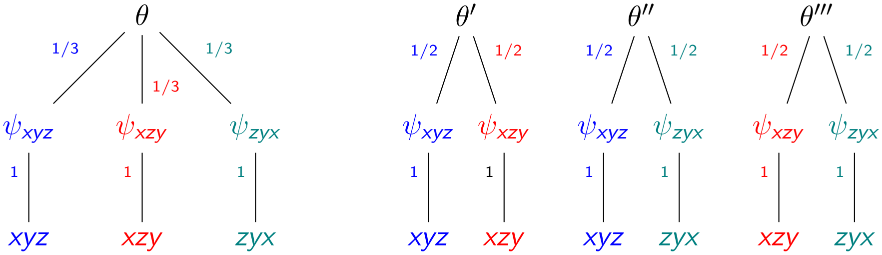
Decomposition. For every $\theta \in \Theta^D$,

$$\mathsf{H}(\theta) = \sum_{i<j} (\theta_i + \theta_j)^2 \; \mathsf{H}([\frac{\theta_i}{\theta_i + \theta_j}, \frac{\theta_j}{\theta_i + \theta_j}; \psi_{P_i}, \psi_{P_j}]).$$

Example 4



$$\mathrm{H}(\theta) = (\frac{2}{3})^2\ \mathrm{H}(\theta') + (\frac{2}{3})^2\ \mathrm{H}(\theta'') + (\frac{2}{3})^2\ \mathrm{H}(\theta''')$$

Monotonicity

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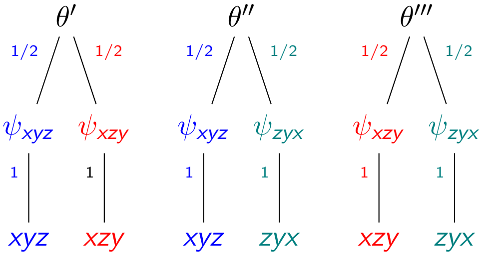
Monotonicity. If $N = N'$ and $\Delta_A(C) \geq \Delta_A(C')$ for every A, then $\frac{\sum_n \mathrm{H}([\frac{1}{2}, \frac{1}{2}; \psi_{P^n}, \psi_{Q^n}])}{N} \geq \frac{\sum_{n'} \mathrm{H}([\frac{1}{2}, \frac{1}{2}; \psi_{P'^{n'}}, \psi_{Q'^{n'}}])}{N}$

Example 4



- $C = \{\theta', \theta'', \theta'''\}$
- $\Delta_{\{x,y,z\}}(C) = \Delta_{\{x,y\}}(C) = \Delta_{\{x,z\}}(C) = \Delta_{\{y,z\}}(C) = 2$

Characterization

Theorem. H satisfies Reduction, Decomposition and Monotonicity if and only if there exists a probability distribution λ on $\mathcal{A}$ and $k > 0$ such that $\mathsf{H} = k \cdot \mathsf{CH}_\lambda$

Intuition

- Reduction: from θ to $[1, \psi_\theta]$, and from here to the deterministic population θ^d that assigns the same probability to every preference as the representative agent ψ_θ

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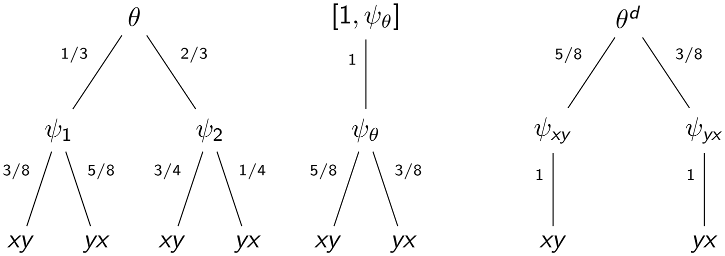

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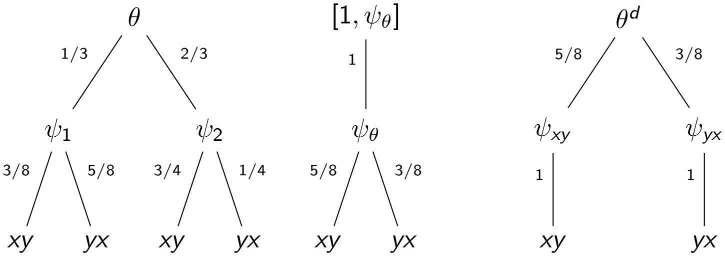

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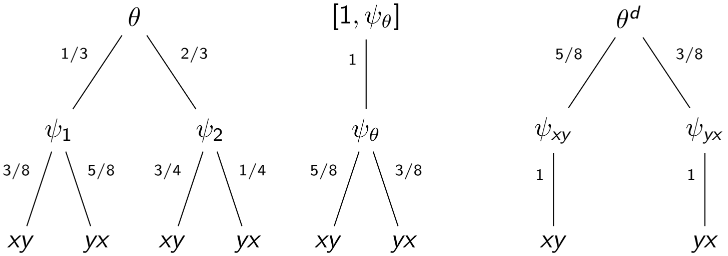

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- $\mathrm{H}([1 - \gamma, \gamma; \psi_P, \psi_Q]) = 4\gamma(1 - \gamma)\mathrm{H}([\frac{1}{2}, \frac{1}{2}; \psi_P, \psi_Q])$.

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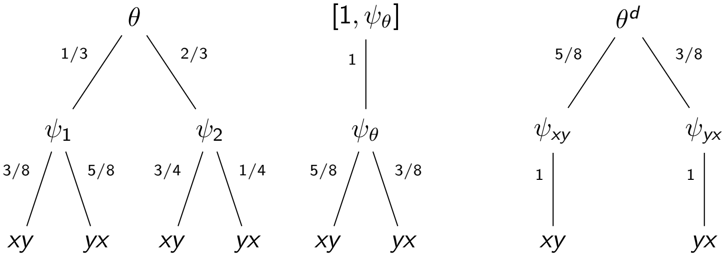

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- Identify the contribution to heterogeneity of each menu A by constructing collections of couples for which the Δ-vectors differ only in menu A, and apply Monotonicity

4. Comparative statics: Intra- and inter-personal heterogeneity

$$\mathsf{CH}_\lambda(\theta) = \sum_i \theta_i [\beta_\lambda - d_\lambda(\rho_{\psi_i}, \rho_{\psi_{\mathcal{U}}})] + \sum_i \theta_i \sum_{i<j} \theta_j \; d_\lambda(\rho_{\psi_i}, \rho_{\psi_j})$$

Intra-Personal Heterogeneity

- *P*-central individual ψ: there is $P \in \mathcal{P}$ such that xPy and $\{x, y\} \subseteq A$ implies $\rho_\psi(x, A) \geq \rho_\psi(y, A)$

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- ψ_1 and ψ_2 are P-central individuals. We say, ψ_2 is a decentralization of ψ_1, if there is $\epsilon > 0$ and preferences Q_1, Q_2 such that:
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Proposition. If ψ_2 is a sequential decentralization of ψ_1, then $d_\lambda(\rho_{\psi_1}, \rho_{\psi_\mathcal{U}}) \geq d_\lambda(\rho_{\psi_2}, \rho_{\psi_\mathcal{U}})$.

Intra-Personal Heterogeneity: Luce

- $u : X \to \mathbb{R}_{++}$, and let, w.l.o.g., $\sum_{x \in X} u(x) = 1$
- $\rho_u(a, A) = \frac{u(a)}{\sum_{b \in A} u(b)}$

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Proposition. Suppose that $u_1(x_1) \geq \cdots \geq u_1(x_n)$ and $u_2(x_1) \geq \cdots \geq u_2(x_n)$. If $\frac{u_2(x_j)}{u_2(x_i)} \geq \frac{u_1(x_j)}{u_1(x_i)}$ for every $i < j$, then $d_\lambda(\rho_{\psi_{u_1}}, \rho_{\psi_{\mathcal{U}}}) \geq d_\lambda(\rho_{\psi_{u_2}}, \rho_{\psi_{\mathcal{U}}})$.

Example

- $X = \{x, y, z\}$
  - $u_1 = (u_1(x), u_1(y), u_1(z)) = (3/6, 2/6, 1/6)$
  - $u_2 = (u_2(x), u_2(y), u_2(z)) = (4/9, 3/9, 2/9)$

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  - The monotone-likelihood ratio applies to u_1 and u_2 and hence individual 1 has more intra-personal heterogeneity

Inter-Personal Heterogeneity

Corollary. For every $\alpha \in [0, 1]$,

$$\mathsf{CH}_\lambda(\alpha\theta + (1-\alpha)\theta') = \alpha\mathsf{CH}_\lambda(\theta) + (1-\alpha)\mathsf{CH}_\lambda(\theta') + \alpha(1-\alpha)d_\lambda(\rho_{\psi_\theta}, \rho_{\psi_{\theta'}}).$$

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2. Correlated choices: state-dependent preferences
   - S (a common set of states) and μ (a common probability distribution on S)
   - $f_i : S \to \mathcal{P}$
   - $\text{CH}_\lambda^S = \sum_A \lambda(A) \sum_s \mu(s) \sum_i \theta_i \sum_j \theta_j \mathbb{I}_{[m(A,f_i(s)) \neq m(A,f_j(s))]}$

In this paper:

1. We propose a novel, choice-based, measure of behavioral heterogeneity
2. We provide axiomatic foundations for our measure
3. We obtain a decomposition into inter- and intra-personal heterogeneity

Thank you!!