Self-isolation under uncertainty

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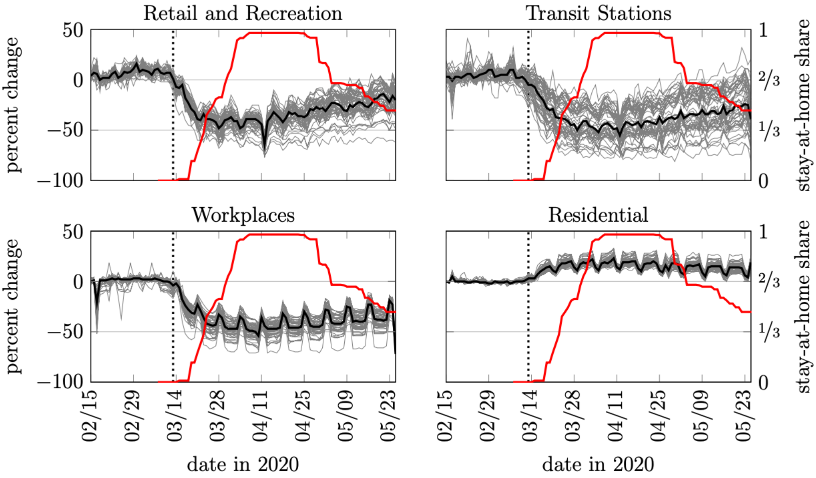
Introduction

Epidemiological models have revealed their usefulness to the general public during the COVID-19 epidemic.

- precious tools to predict the number of cases, to assess the risk of hospital saturation, to declare a lockdown...
- Difficulties:
  "These scenarios are made on the basis of incomplete data and uncertain assumptions.[...] changes in behavior are difficult to predict". (october 2021, Pasteur Institute)

Farboodi & al (2021) show that people responded to the WHO announcement in March 2020:



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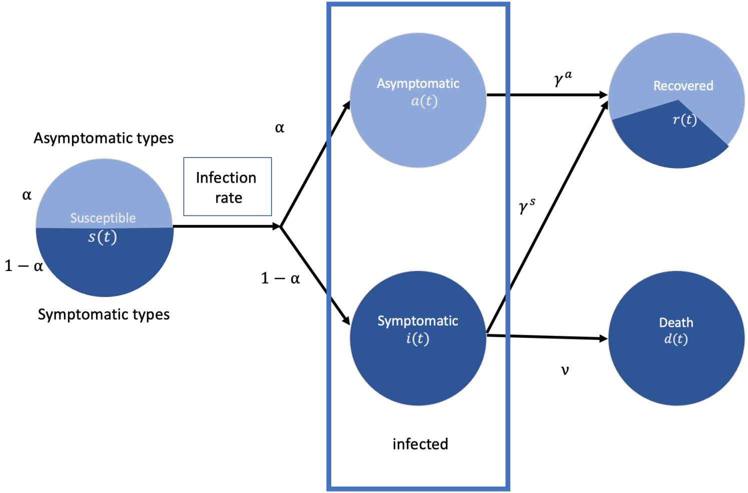
2. individuals are uncertain about the prevalence rate.

The model : Epidemiological part.


Asymptomatic types
α
Susceptible
s(t)
1 − α
Symptomatic types
Infection
rate
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1 − α
Asymptomatic
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Symptomatic
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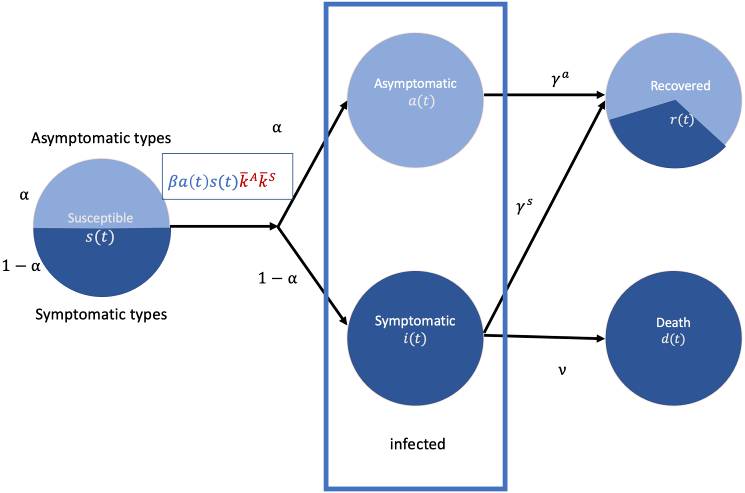
The average social interaction level of those who are Infected Asymptomatic is

$$\bar{k}^A(t) = \frac{1}{a(t)} \int_{i \in A(t)} k_i di$$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$$\dot{s}(t) = -\beta s(t)a(t)\bar{k}_S(t)\bar{k}_A(t) \tag{1}$$
$$\dot{a}(t) = \alpha\beta s(t)a(t)\bar{k}_S(t)\bar{k}_A(t) - \gamma^a a(t) \tag{2}$$
$$\dot{i}(t) = (1-\alpha)\beta s(t)a(t)\bar{k}_S(t)\bar{k}_A(t) - (\gamma^s + \nu)i(t) \tag{3}$$
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A state is a vector ω of initial conditions $C(0)$.

The model : Economic part.

What is unknown?

Individuals do not know:

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2. The epidemiological state ω :

   $\rightarrow \mu_i(t;\omega)$ is the belief at t that the true state is ω

What is observed?

Individuals only observe whether they have symptoms or not.

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Bayesian updating gives:

- $\dot{p}_i(t|\omega) = -(1 - p_i(t|\omega))p_i(t|\omega) \underbrace{k_i(t)\beta \bar{k}_A(t)a(t|\omega)}_{\text{Probability of being infected at } t.}$

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- $\mu_i(t;\omega) = \dfrac{\mu^0(\omega)/(1 - p_i(t|\omega))}{\sum_{\omega'} \mu^0(\omega')/(1 - p_i(t|\omega'))}$

Discounted payoffs

If an agent is infected at t and develops symptoms, then the game stops.

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$\rightarrow$ the continuation payoff is v_I.

Discounted payoffs

Given a discount factor $r > 0$ and a state ω, player i discounted payoff, conditional on ω is:

$$v_i(k_i, (k_j)_{j \neq i}, \omega) = \int_0^T e^{-rt} e^{-\int_0^t p_i(s,\omega) k_i(s) \beta \bar{k}_A(s) a(s,\omega) ds} \times$$

$$- \underbrace{(1 - k_i(t)) c_{SD}}_{\text{cost of social distancing}} + \underbrace{p_i(t,\omega) k_i(t) \beta \bar{k}_A(t) a(t)}_{\text{proba of developing symptoms}} \underbrace{v_I}_{\text{continuation if symptoms}} \quad dt$$

The expected discounted payoff is :

$$\sum_{\omega \in \Omega} \mu^0(\omega) v_i(k_i, (k_j)_{j \neq i}; \omega) \equiv E_{\mu^0}[v_i(k_i, (k_j)_{j \neq i}, .)]$$

s.t.

$$\dot{p}_i(t|\omega) = -(1 - p_i(t|\omega))p_i(t|\omega)k_i(t)\beta \bar{k}_A(t)a(t|\omega), \ \forall \omega$$

Equilibrium

Bellman equation

Let $p = \{p(.|\omega)\}_{\omega}$.

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$$rV(t,p) = V_t(t,p) - c_{SD} + \max_{k\in[0,1]} k\times$$

$$\Big(c_{SD} - \underbrace{\beta \sum_\omega \mu(t,\omega)p(t\mid\omega)\bar{k}_A(t)a(t\mid\omega)\Big(V(t,p) - v_I + \frac{1-p(t\mid\omega)}{\mu(t,\omega)}V_{p(t|\omega)}(t,p)}_{\text{expected net cost of social activity}}$$

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Proposition (Best response)

k_i^* *is a best response* $\iff$ $k_i^*(t)$

$$
\begin{cases}
= 1 & \textit{if } c_{SD} > \textit{expected cost of social interaction} \\
\in [0,1] & \textit{if } c_{SD} = \textit{expected cost of social interaction}, \\
= 0 & \textit{if } c_{SD} < \textit{expected cost of social interaction},
\end{cases} \tag{6}
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Proposition (The symmetric equilibrium)

Let

$$\hat{k}(t) = \frac{c_{SD}}{\sum_\omega \beta a(t|\omega)\mu(t|\omega)p(t|\omega)(\psi(t|\omega) - v_I)}$$

where:

$$\begin{cases} \dot{p}(t|\omega) = -p(t|\omega)(1 - p(t|\omega))\beta\hat{k}^2(t)a(t|\omega), \\ \dot{s}(t|\omega) = -\beta\hat{k}^2(t)s(t|\omega)a(t|\omega), \\ \dot{a}(t|\omega) = -\alpha\dot{s}(t|\omega) - \gamma^a(\omega)a(t|\omega), \\ \dot{\psi}(t|\omega) - r\psi(t|\omega) = \hat{k}^2(t)\beta a(t|\omega)(\psi(t|\omega) - v_I) + (1 - \hat{k}(t))c_{SD}, \\ \psi(T|\omega) = 0 \end{cases} \tag{7}$$

If $\hat{k}(t) \in (0,1)$, $\forall t$ *then the strategy profile* $k_i(t) = \hat{k}(t)$, $\forall i, t$ *is an equilibrium.*

Simulations

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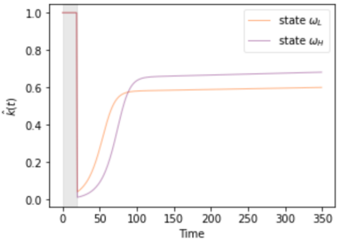
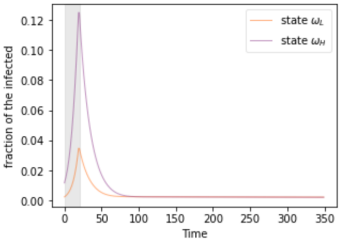
- State ω_L : $a(0) = 0.1/100$
- State ω_H : $a(0) = 0.5/100$

Simulations

Without uncertainty:





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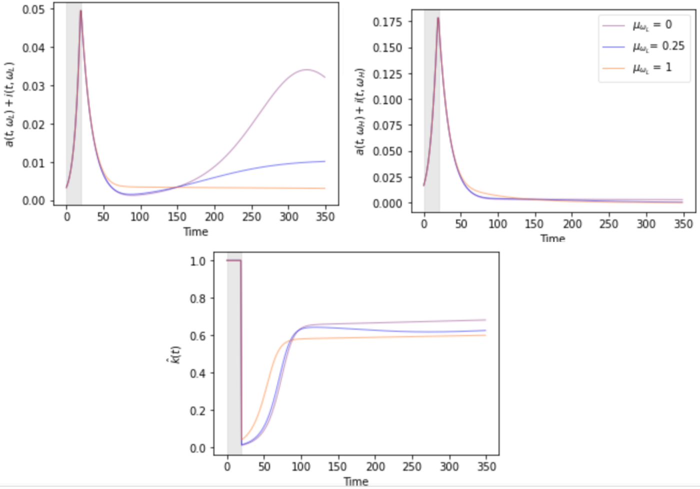

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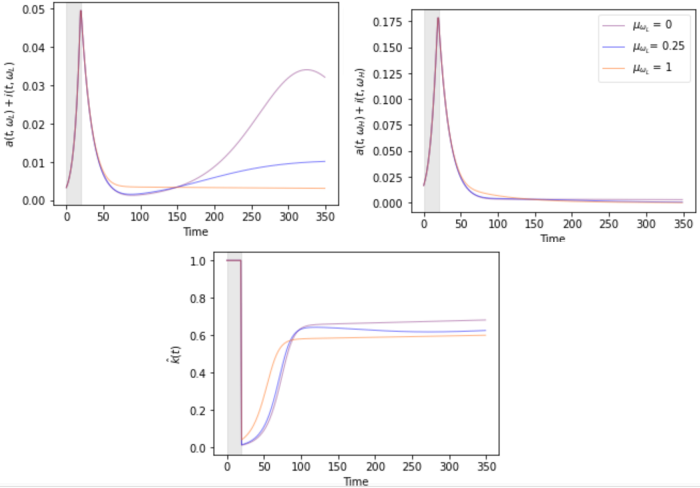
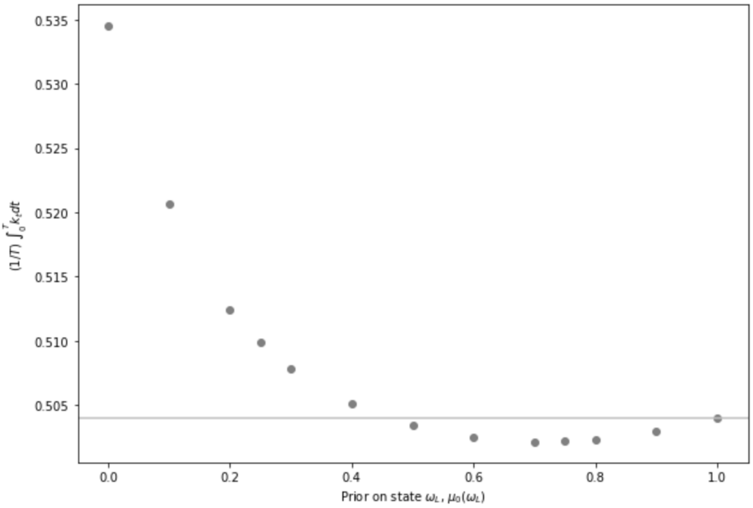


Result 1: a second wave may arise under uncertainty.

Simulations

Average social activity level: $(1/T)\int_0^T \hat{k}_t dt$.



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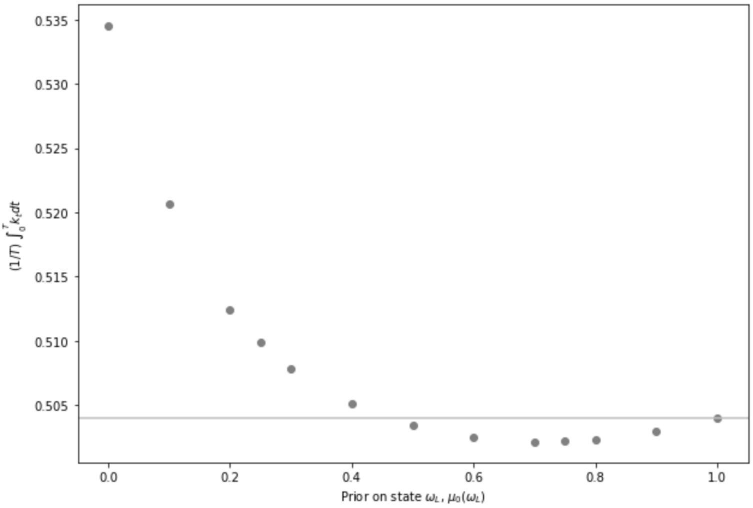

Result 2: average social activity may be lower under uncertainty.

Simulations

Information value in terms of deaths.

$$IVD_{\bar{\mu}} = - \underbrace{\bar{\mu}\, TD_1(\omega_L) + (1-\bar{\mu})\, TD_0(\omega_H)}_{\text{Ex-ante fraction of deaths without uncertainty}} + \underbrace{\bar{\mu}\, TD_{\bar{\mu}}(\omega_L) + (1-\bar{\mu})\, TD_{\bar{\mu}}(\omega_H)}_{\text{Expected fraction of deaths with uncertainty}}$$

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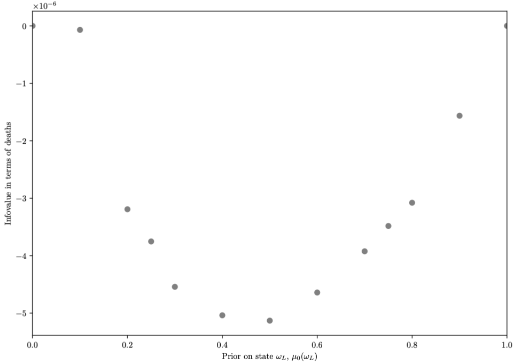

Result 3: Uncertainty can reduce the *ex-ante* number of deaths.

Simulations

Information value in terms of payoffs.

$$IVP_{\bar{\mu}} = \underbrace{\bar{\mu}v(\hat{k}_1 \mid \omega_L) + (1-\bar{\mu})v(\hat{k}_0 \mid \omega_H)}_{\text{Ex-ante payoff without uncertainty}} - \underbrace{\bar{\mu}v(\hat{k}_{\bar{\mu}} \mid \omega_L) - (1-\bar{\mu})v(\hat{k}_{\bar{\mu}} \mid \omega_H)}_{\text{Expected payoff with uncertainty}}$$

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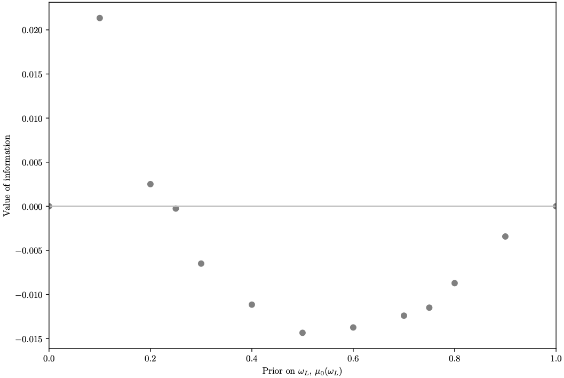

Result 4: Uncertainty can be welfare improving.

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  - Uncertainty may reduce the “ex-ante” expected number of deaths
  - Information value is negative for some priors.

Many thanks for your attention!