On ambiguity-seeking behavior in finance models with smooth ambiguity

Dmitry Makarov, HSE University and ICEF

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Motivation

- We identify a methodological problem affecting (all?) finance papers (incl. in top journals) considering smooth ambiguity preferences
- The misconception concerns the **ambiguity aversion assumption**
- Theoretical works routinely make this assumption (typically without any discussion)
- Empirical works calibrating such models restrict ambiguity attitude parameter to stay in ambiguity aversion region
- **This paper:** “intuitive” ambiguity aversion assumption considered to be necessary whereas in fact it is only sufficient

Background: risk and ambiguity attitudes

- If investor knows that a stock return is distributed $N(\mu, \sigma^2)$, her decision on how much stock to buy is determined by her risk aversion.
- In reality, no one tells investor the true distribution; investors are **ambiguous** about the distribution
- "Smooth ambiguity" is a leading approach proposed by KMM to modelling behavior under ambiguity
- KMM utility function features two parameters: risk aversion and ambiguity aversion
- Relative to maxmin approach by Gilboa and Schmeidler: i) smooth function vs kinked function, and ii) chosen level of ambiguity aversion/seeking vs infinite ambiguity aversion

Why ambiguity aversion is an “intuitive” assumption?

- Researchers seem to think that what is true for risk seeking is mechanically true for ambiguity seeking
- In standard models with classical utility, risk aversion is necessary because risk seeking cannot be allowed
  - a risk-seeker has an infinite demand for risky stocks, non-existence of equilibrium
- It seems “intuitive” that the same logic should work for ambiguity—we should disallow ambiguity-seeking to rule out infinite demands for ambiguous stocks
- This mechanism seems so obvious that no paper has formally examined its validity
- Our paper is the first to do this, and we find that what everyone takes as given is in fact not true

Constructive messages

- *For theoretical researchers*: derive and report the admissible levels of ambiguity seeking behavior for which your model is well-defined
- *For empirical researchers*: when calibrating models, allow ambiguity attitude to be in the admissible range of ambiguity-seeking
- We should let data speak for themselves as to what ambiguity attitude is more empirically relevant, instead of the current approach of postulating ambiguity-aversion

Aren’t people ambiguity averse in reality?

- Documented share of ambiguity-seekers varies across studies:
    - 30% in Anantanasuwong et al. (2019)
    - 49%, 22%, or 35% depending on event likelihood in Dimmock et al. (2016b)
    - 16% in Cubitt et al. (2019)
    - 27% in Kelsey and le Roux (2018).
- Researchers increasingly argue that models “need to be extended beyond the common assumption of universal ambiguity aversion” (Dimmock et al. 2015)
- Share of ambiguity-seekers among stock investors is likely to be higher than in general population
    - ambiguity-averters are less likely to become investors (Bossaerts et al. 2010, Dimmock et al. 2016a, Dimmock et al. 2016b)

Smooth ambiguity literature focusing on ambiguity aversion

- Papers *disregarding* ambiguity seeking:
  Caskey (2009, RFS), Gollier (2011, ReStud), Jahan-Parvar and Liu (2014, RFS), Chen, Ju, and Miao (2014, RED), Backus, Ferriere, and Zin (2015, JME), Thimme and Volkert (2015), Guidolin and Liu (2016, JFQA), Collard, Mukerji, Sheppard, and Tallon (2018, QE), Altug, Collard, Akmakli, Mukerji, and Ozsoylev (2019), Miao, Wei, and Zhou (2019), and Liu and Zhang (2020).

Model: setting

- We consider Maccheroni et al. (ECMA, 2013) setting. They refer to ambiguity-aversion as “key condition for the paper”
- Risk free rate is r_f, two risky stocks with payoffs

$$X_1 = \mu_1 + \varepsilon_1, \quad X_2 = \mu_2 + \varepsilon_2,$$

$$\varepsilon_1 \sim N(0, \sigma_1^2),\ \varepsilon_2 \sim N(0, \sigma_2^2),\ cor(\varepsilon_1, \varepsilon_2) = \rho \in (-1, 1).$$

- Final wealth w of an investor is

$$w = \theta_1 X_1 + \theta_2 X_2 + (w_0 - \theta_1 P_1 - \theta_2 P_2) r_f,$$

w_0 is initial wealth, θ_i is holding and P_i price of stock i

Model: certainty equivalent

- Investor's utility function is

$$u(w) = -\exp(-\gamma_r w)$$

where $\gamma_r > 0$ is risk aversion

- Compute $E[u(w)]$, and the certainty equivalent $CE = u^{-1}(E[u(w)])$:

$$CE = \theta_1(\mu_1 - P_1 r_f) + \theta_2(\mu_2 - P_2 r_f) - \gamma_r(\theta_1^2\sigma_1^2 + \theta_2^2\sigma_2^2 + 2\theta_1\theta_2\sigma_1\sigma_2\rho)/2$$

Ambiguous stock 2

- Stock 2's mean payoff μ_2 is ambiguous, and the belief is that $\mu_2 \sim N(\mu_{2a}, \sigma_{2a}^2)$
- As per KMM, we introduce ambiguity-pertinent utility $v(CE)$

$$v(CE) = \begin{cases} -\exp(-\gamma_a CE)/\gamma_a, & \gamma_a \neq 0 \\ CE, & \gamma_a = 0 \end{cases},$$

- The overall objective function is

$$V(w) = v^{-1}(E_\mu v(u^{-1}[E_\varepsilon u(w)]))$$

Ambiguity attitudes

$$V(w) = E_\mu v(u^{-1}[E_\varepsilon u(w)])$$

- If $\gamma_a = \gamma_r$, investor is *ambiguity-neutral*
- If $\gamma_a > \gamma_r$, investor is *ambiguity-averse*
- If $\gamma_a < \gamma_r$ investor is *ambiguity-seeking*
- All papers we are aware of assume ambiguity-aversion, $\gamma_a > \gamma_r$. It has become “common wisdom” assumption

Computing value of objective function

- Straightforward calculations yield that the objective function is
$$V = CE - \gamma_a \theta_2^2 \sigma_{2a}^2 / 2$$

- Substituting CE and keeping terms depending on choice variables, we get objective function:

$$-\theta_1^2 \gamma_r \sigma_1^2 - \theta_2^2 (\gamma_r \sigma_2^2 + \gamma_a \sigma_{2a}^2) - 2\theta_1 \theta_2 \gamma_r \sigma_1 \sigma_2 \rho$$

Existence of unique maximum

- Bi-variate quadratic function has a unique maximum if the Hessian is negative definite: $-\gamma_r\sigma_1^2 < 0$ (satisfied) and

$$4\gamma_r\sigma_1^2(\gamma_r\sigma_2^2 + \gamma_a\sigma_{2a}^2) - 4\gamma_r^2\sigma_1^2\sigma_2^2\rho^2 > 0 \quad \Rightarrow$$
$$\gamma_a > -\gamma_r\sigma_2^2(1-\rho^2)/\sigma_{2a}^2$$

- This is the proper “key condition” ensuring that the portfolio problem is well-posed

- Region $\gamma_a \in (-\gamma_r\sigma_2^2(1-\rho^2)/\sigma_{2a},\ \gamma_r)$: ambiguity-seeking investor and yet the model is well-defined in this case!

Intuition

- Investor is ambiguity-seeking and risk averse
- Stocks are both risky and ambiguous
- Adding ambiguous stocks to a portfolio makes it more ambiguous, which investor likes, but also riskier, which investor dislikes
- If risk-aversion dominates ambiguity-seeking, investor has finite demand
- The above condition formally specifies when the risk aversion effect dominates

Another reason to assume ambiguity aversion

- Despite above, one may believe that assuming ambiguity aversion of representative investor is justified by empirical evidence
- She has to adequately “represent” individual traders who are ambiguity averse on average

- Our exercise:
  - consider a setting with multiple investors who are ambiguity-averse on average
  - compute equilibrium asset prices in this setting
  - find the utility of a single representative investor generating the same prices

Economy with two investors

- Two investors into above setting, with ambiguity attitudes $\gamma_{1a} = A_a - H_a$ and $\gamma_{2a} = A_a + H_a$
- Both have the same risk aversion γ_r, on average are ambiguity averse $A_a > \gamma_r$, and H_a is heterogeneity of ambiguity attitudes
- Representative investor: a single investor with (endogenous) risk- and ambiguity-attitudes γ_r^{II} and γ_a^{II}
- γ_r^{II} and γ_a^{II} are computed so that the stock prices in the representative-investor economy coincide with those in Economy-II

Representative investor's ambiguity attitude

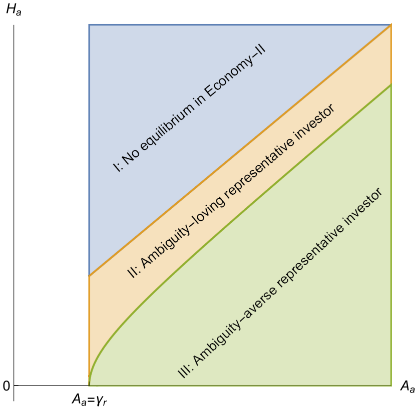

Figure: Attitude towards ambiguity of the representative investor depending on the attitudes of the individual investors, as measured by $A_a > \gamma_r$ (average ambiguity-aversion) and $H_a > 0$ (dispersion of ambiguity-aversion).

Intuition

- Ambiguity-seeker finds ambiguous stocks more attractive and so holds a larger position in them
- Therefore, her positive attitude towards ambiguity plays a dominant role in forming ambiguity preference of representative investor
- This mechanism is reminiscent of that in models with risk aversion heterogeneity
- Risky stocks are mostly held by more risk-tolerant traders, generating interesting implications (Chan and Kogan (2002), Bhamra and Uppal (2014), Garleanu and Panageas (2015)).

Another reason for the assumption

- When calibrating representative-investor models to data, the investor is found to be ambiguity-averse (Ju and Miao 2012)
- Does this mean that individual traders, who she "represents", are on average ambiguity-averse?
- I consider a model with limited stock market participation, and find this is **not** the case

Conclusion

- Existing finance models automatically assume that investors are ambiguity-averse, despite growing evidence of ambiguity–seeking behavior
- Our paper analyzes three main justifications behind the approach in the literature
- We find that imposing ambiguity-aversion is neither a technical requirement for models to be well-defined nor does it directly follows from empirical evidence
- Researchers should characterize admissible levels of ambiguity-seeking in their models, and then let data speak for themselves as to what ambiguity attitude is more empirically relevant

"True" economy with limited participation

- Two dates, $t = 1$ and $t = 2$
- Two assets: a risky stock in supply $x > 0$ and a bond in zero supply paying the (endogenous) rate of return r_f
- Date-2 dividend is distributed normally:

$$X_2 \sim N(\mu, \sigma^2).$$

  and its mean is ambiguous, $\mu \sim N(\mu_a, \sigma_a^2)$.

- The "true" economy is populated by two individual traders: stockholder and non-stockholder

"True" economy: stockholder and non-stockholder

- Non-stockholder invests only in the bond and so is not exposed to ambiguity; she can be taken to be ambiguity-neutral solving the problem

$$\max_{c_1^{ns}} \quad u\left(c_1^{ns}\right) + \beta u\left(c_2^{ns}\right),$$

- Stockholder can invest in both assets and so faces ambiguity to which she is sensitive; her maximization problem is:

$$\max_{c_1^{s},\theta} \quad u\left(c_1^{s}\right) + \beta \phi^{-1}\left(E_\mu[\phi\left(E_{X_2} u(c_2^s)\right)]\right),$$

  where θ is stock investment

- Stockholder is, by assumption, *ambiguity-seeking*. Therefore, by construction ambiguity-seeking is "empirically relevant" behavior

Ambiguity attitudes in the “true” multiple-agent and hypothetical single-agent economies

- Exercise:
- 1) Compute the equilibrium stock price and risk free rate in the true economy with limited participation
- 2) Compute the same quantities in a hypothetical representative-investor economy with full participation
- 3) Calibrate the two utility parameters of the representative investor to match the prices in the two economies
- 4) If the representative investor may be ambiguity-averse, prevailing ambiguity attitudes in the “true” and hypothetical economies are opposite

Numerical analysis: ambiguity attitudes of actual traders vs of representative investor

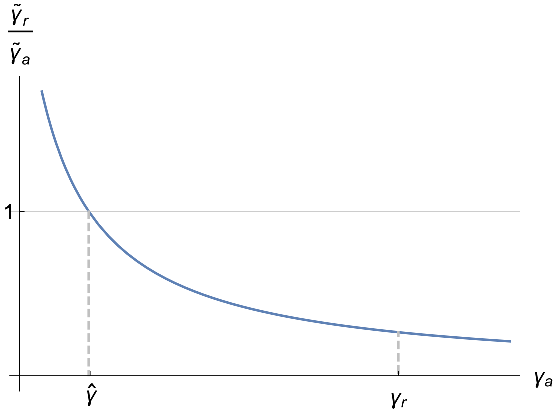

Figure: The Figure plots the ratio $\tilde{\gamma}_r/\tilde{\gamma}_a$ for the representative investor as a function of the stockholder's ambiguity attitude γ_a. The plot is above (below) the thin grey line when the representative investor is ambiguity-seeking (averse).

Numerical analysis, cont'd

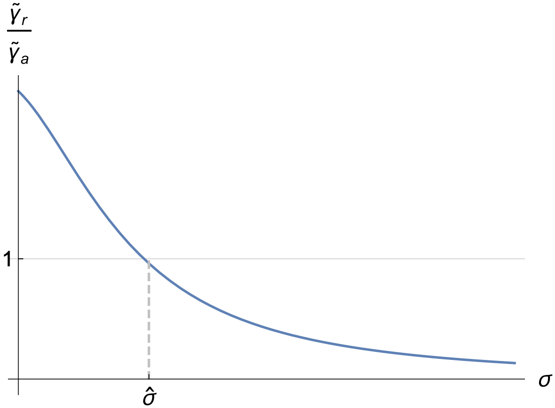

Figure: The Figure plots the ratio $\tilde{\gamma}_r/\tilde{\gamma}_a$ for the representative investor as a function of the dividend volatility. The plot is above (below) the thin grey line the representative investor is ambiguity-seeking (averse).

Numerical analysis, cont'd

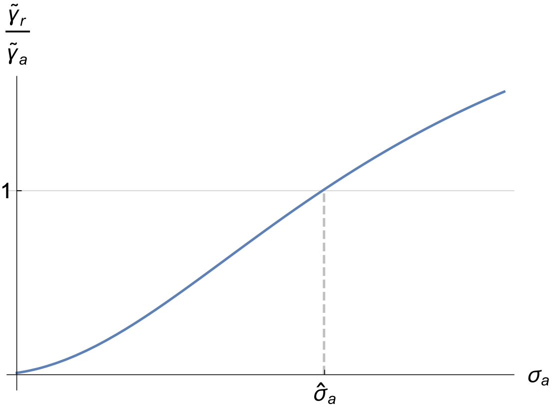

Figure: The Figure plots the ratio $\tilde{\gamma}_r/\tilde{\gamma}_a$ for the representative investor as a function of the level of ambiguity about the dividend. The plot is above (below) the thin grey line the representative investor is ambiguity-seeking (averse).