Accountable Voting

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1. Motivation

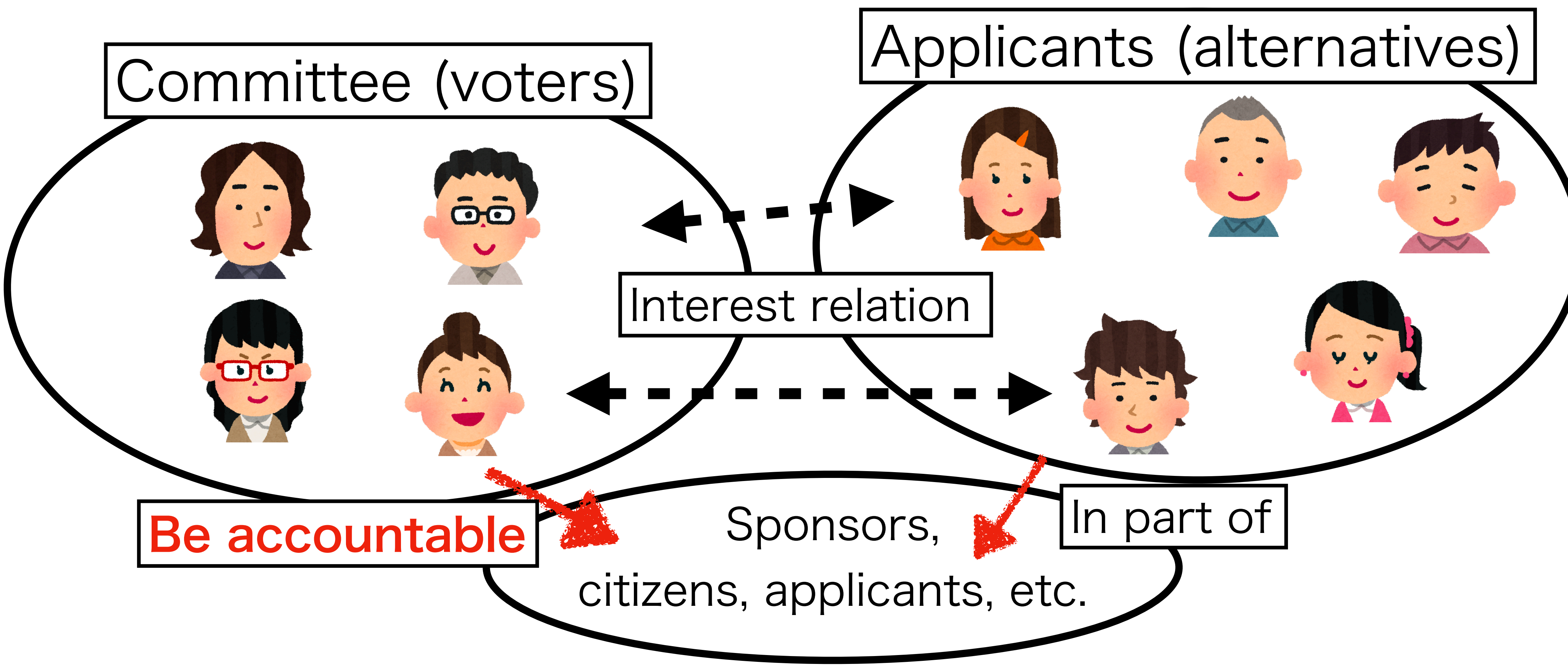
- The original problem is based on one of the authors' experience.
- 8 voters needed to rank 20+ applicants for fundings.
  Final budget was not determined, so ranking needed.
- Voter 1 had "an interest in" applicant **x**, and voter 2 had "an interest in" applicant **y**.
  (Co-author or former advisee)
- The rule: the voters with interests must leave the room while others discuss the evaluation of the "related" applicants.
  (Considering "accountability")
- **Question: How can the committee rank all applicants? What is a "desirable" voting rule in such a situation?**
- All of the voters did not know the answer.

Accountability Requirements

When some voters have interests in applicants, the committee should be accountable to sponsors, citizens, applicants, etc.

To this end, the committee should adapt a voting rule that

- eliminates biased opinions (as in the original problem),
- deters a power-game-type competition which would favor applicants who are good at increasing the number of interested voters.



Example

<u>An extension of the plurality rule</u>:
the related voters cannot vote their related applicants, i.e., voter 2 cannot vote **y** and voters 3-5 cannot vote **x**.

	1	2	3	4	5
1st	x	(y)	(x)	(x)	(x)
2nd	y	x	y	y	y
3rd	z	z	z	z	z

By the modified plurality rule,
x gets 2 votes, **y** gets 3 votes, and **z** gets 0 votes.

The plurality ranking: $y \succ x \succ z$.

However, **x** is **unanimously** supported by all unrelated voters.

How about averaging?

Consider averaging of votes by the number of unrelated voters.

	1	2	3	4	5
1st	x	(y)	(x)	(x)	(x)
2nd	y	x	y	y	y
3rd	z	z	z	z	z

By the averaging plurality rule,

x gets 2/2 points, **y** gets 3/4 points, and **z** gets 0/5 points.

The social ranking: $x \succ y \succ z$.

How about averaging?

Consider averaging of votes by the number of “eligible” voters.

	1	2	3	4	5
1st	x	(y)	(x)	(x)	(x)
2nd	(y)	x	y	y	y
3rd	z	z	z	z	z

By the averaging plurality rule,
x gets 2/2 points, **y** gets 3/4 points, and **z** gets 0/5 points.

The social ranking: $x \succ y \succ z$.

However, if **y** forms a new interest relation with voter 1,

the social ranking is $y \sim x \succ z$.

Applicant **y** is better off by such a “**power-game-type activity.**”

- In our framework, namely, **voting with interest structures**, the plurality rule does not satisfy an extended notion of **Unanimity.**

- Moreover, averaging of the votes may give advantages to applicants who are good at the power games.

In this paper, we introduce

- **two accountability axioms.**
- **impossibility theorems** among the accountability axioms and the extensions of some standard axioms in voting theory.
- **two directions for remedies**:
  - weakening of an accountability axiom
  - restrictions of the domain

2. Model

- $N = \{1, 2, \ldots, n\}$: set of voters
- $X = \{x_1, x_2, \ldots, x_m\}$: set of applicants/alternatives
- $\mathscr{P}$: set of strict orders on X
- $\mathscr{R}$: set of weak orders on X

- interest structure $A \in \{0,1\}^{n \times m}$,

  $a_{ij} = 1$: voter i has no interest in (can vote on) applicant x_j

  $a_{ij} = 0$: voter i has an interest in (cannot vote on) applicant x_j

- $A(i) := \{x_j \in X \mid a_{ij} = 1\}$: set of applicants who has no interest in i

  We call this an **admissible set for** i.

E.g., interest structure A

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{array}{c}
\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\
\begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}
\end{array}
$$

Voter 1 has no interest in x_1 and x_3.

Voter 1 has an interest in x_2.

$A(1) = \{x_1, x_3\}, \; A(5) = X$

Assumption (Non-trivial interest structure)

An interest structure A satisfies **non-triviality** if

$$\forall x, y \in X (x \neq y),\ \exists i \in N, \{x, y\} \subset A(i)\ .$$

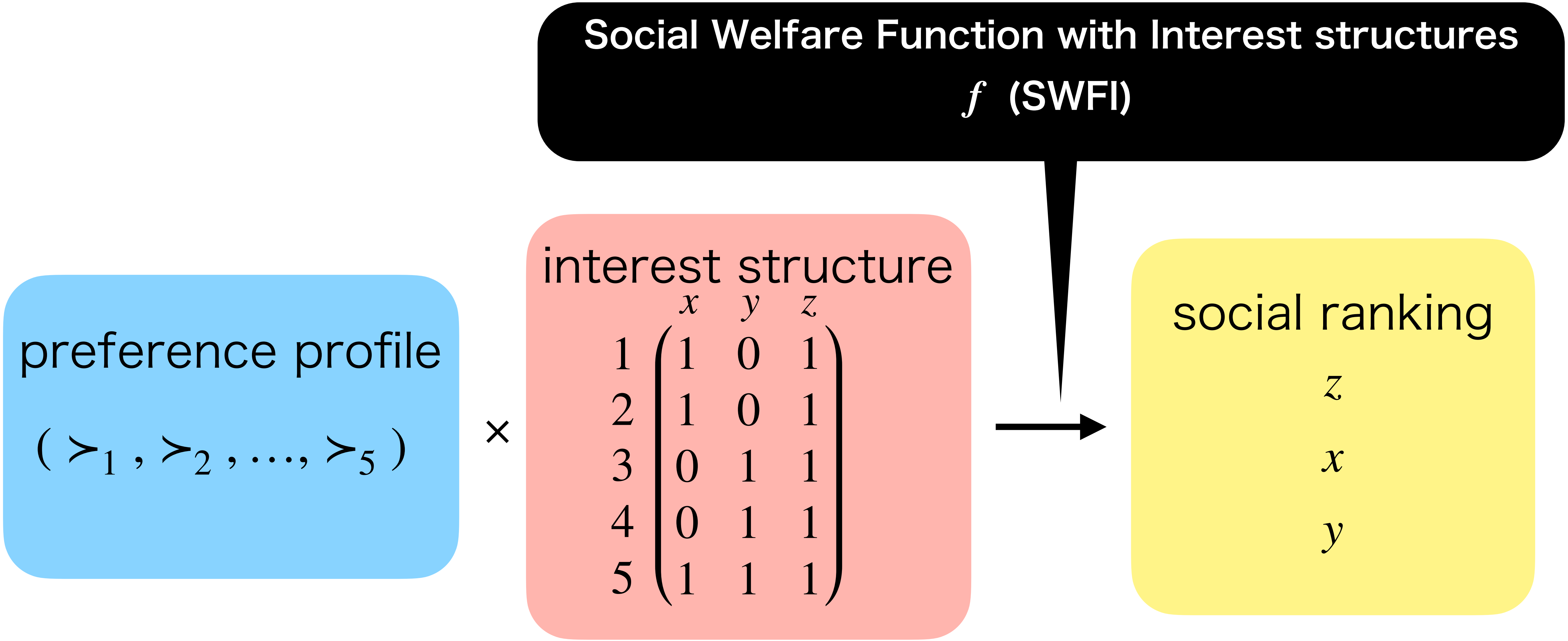
If non-triviality does not hold, voting problems become trivial.

This condition requires the minimum information to compare x and y without worrying about conflict of interests.

- $\mathscr{A} \subset \{0,1\}^{n \times m}$: the set of all non-trivial interest structures

  We only consider non-trivial interest structures.

- $f : \mathscr{L}^n \times \mathscr{A} \to \mathscr{R}$: social welfare function considering interest structures (**SWFI**)



3. Desirable Properties

- We formulate two novel accountability properties.

  1. Interest-Exclusion

  2. No-Power-Game property

  Our basic idea: applicants should not be favored by the existence of their interested voters.

- We introduce two democratic properties.
  (extensions of the standard properties in voting theory).

  1. A-Unanimity

  2. Condorcet-Loser Criterion (which is often used to criticize the plurality rule)

Interest-Exclusion:

No matter how i ranks her "related applicants," that alone should not affect the social ranking.

i's evaluations on $A(i)$ are the same

$\succ_i$

	i
1st	z
2nd	(x)
3rd	y
4th	w

$\succ'_i$

	i
1st	(x)
2nd	z
3rd	y
4th	w

Reduced ranking

	i
1st	z
2nd	y
3rd	w

$$f(\succ_i, \succ_{-i}, A) = f(\succ'_i, \succ_{-i}, A)$$

The social ranking is the same

Formal Definition (1)

For each $A \in \mathscr{A}, \succ_i \in \mathscr{P}$, define the set of all strict orders that coincide with $\succ_i$ on $A(i)$,

$$\mathscr{P}(\succ_i, A) := \{\succ_i' \in \mathscr{P} \mid x \succ_i y \iff x \succ_i' y \quad \forall x, y \in A(i)\}.$$

- **Interest-Exclusion**
  An SWFI f satisfies **Interest-Exclusion** if for any

  $A \in \mathscr{A}, \boldsymbol{\succ} \in \mathscr{P}^N, i \in N,$ and any $\succ_i' \in \mathscr{P}(\succ_i, A)$,

  $$f(\succ_i, \boldsymbol{\succ}_{-i}, A) = f(\succ_i', \boldsymbol{\succ}_{-i}, A).$$

No matter how i ranks the "related applicant" $x \in X \backslash A(i)$, that alone should not affect the social ranking.

No-Power-Game property :

If a voter changes the status from neutral to an interest party of x, then x's social ranking should not improve.

Voter 1 becomes x's interest voter

$(\succ, A)$

	1	2	3
1st	(y)	(x)	y
2nd	x	y	x
3rd	z	z	z

$(\succ, A')$

	1	2	3
1st	(y)	(x)	y
2nd	(x)	y	x
3rd	z	z	z

x's social ranking does not increase

(All applicants can focus on the correct effort to improve own qualities.)

Formal Definition (2)

- **No-Power-Game property**

  An SWFI f satisfies **the No-Power-Game property** if $\forall A, A' \in \mathscr{A}$, and $\succ \in \mathscr{P}^N$,

  if $\left[\exists i \in N,\ \underline{A(i) \setminus \{x\} = A'(i)}\ \text{ and }\ A(j) = A'(j)\ \ \forall j \neq i\right]$, then

  i becomes an interested voter of x

  $$\underline{y\, f(\succ, A)x \implies y\, f(\succ, A')x \text{ and}}$$

  $$\underline{\left[y\, f(\succ, A)x \wedge \neg x\, f(\succ, A)y\right] \implies \left[y\, f(\succ, A')x \wedge \neg x\, f(\succ, A')y\right].}$$

  x's ranking does not increase

- Adding one more interested voter for x does not give advantage to x.
- All applicants can focus on the correct effort to improve own quality.

Accountability Properties

- **<u>Additional merits of the accountability axioms.</u>**
  - Applicants are not disadvantaged even if they do not have related voters (**Interest-Exclusion**)
  - Applicants do not need to make efforts in influential activities to increase related voters **(No-Power-Game**)
  - Good for the prize-givers or sponsors: Transparency of the prize-giving system
  - Voters and related applicants can keep good relationships regardless of the outcome

A-Unanimity :

If everyone who can vote on x ranks it highest among her admissible set, then x should be socially best.

Unrelated voters rank x top of $A(i)$

$(\succ, A)$

	1	2	3	4	5
1st	x	(y)	(x)	(z)	(x)
2nd	y	x	y	x	(y)
3rd	z	z	z	y	z

$$\text{top}(f(\succ, A)) = \{x\}$$

x is at the top of the social ranking

Formal Definition (3)

- For each $\succ_i \in \mathscr{L}$ and $A \in \mathscr{A}$, let $\mathrm{top}(\succ_i, A)$ be the top applicant among $A(i)$ under $\succ_i$.

- For each social ranking $f(\succ, A) \in \mathscr{R}$, define

$$\mathrm{top}(f(\succ, A)) := \{x \in X \,|\, x\, f(\succ, A)\ y \quad \forall y \in X\}.$$

- **A-Unanimity**

  An SWFI f satisfies **A-Unanimity** if for any $A \in \mathscr{A}$, $\succ \in \mathscr{L}^N$,

$$\left[\ \mathrm{top}(\succ_i, A) = x \quad \forall i \in N \text{ with } x \in A(i)\ \right] \implies \mathrm{top}(f(\succ, A)) = \{x\}.$$

- Everyone who can vote on x ranks it highest, then x is socially best.

- If $A = \mathbf{1}$, this axiom coincides with the standard unanimity axiom.

Condorcet-Loser Criterion:

- The **Condorcet-Loser** is an applicant beaten by any other applicants in pairwise majority comparisons among $A(i)$'s.
- The social ranking should not rank the Condorcet Loser at the top.

z is the Condorcet-Loser

$(\succ, A)$

	1	2	3	4	5
1st	(y)	(y)	(x)	(x)	z
2nd	x	x	y	y	x
3rd	z	z	z	z	y

- x (2) vs. z (1)→ z is beaten
- y (2) vs. z (1)→ z is beaten

$$z \notin \text{top}(f(\succ, A)).$$

z is not the top of the social ranking

Formal Definition (4)

- An applicant $x \in X$ is **the Condorcet Loser** at $(A, \succ)$ if for any $y \in X \backslash \{x\}$,

  $|\{i \in N : y \succ_i x$ and $\{x, y\} \subset A(i)\}| > |\{i \in N : x \succ_i y$ and $\{x, y\} \subset A(i)\}|$.

- The Condorcet Loser is an applicant beaten by any other applicants in pairwise majority comparisons.

- **Condorcet-Loser Criterion**
  An SWFI f satisfies **Condorcet-Loser Criterion** if for any $A \in \mathscr{A}$ and $\succ \in \mathscr{P}^N$, if x is the Condorcet-Loser at $(A, \succ)$, then

  $$x \notin \mathrm{top}(f(\succ, A)).$$

Summary of Axioms

We have introduced the following four desirable properties.

1. **Interest-Exclusion**

2. **No-Power-Game property**

3. **A-Unanimity**

4. **Condorcet-Loser Criterion**

Remark:
Interest-Exclusion and the No-PG are compatible,
e.g., the modified plurality rule satisfies the both axioms.

4.1 Impossibility Results (1)

We discover that accountability and democracy axioms are hard to sustain jointly.

- **Theorem 1**
  There is no SWFI that satisfies the No-Power-Game property and A-Unanimity.

We must give up the No-Power-Game property or A-Unanimity.

4.1 Impossibility Results (2)

- A much weaker axiom than A-Unanimity is still not compatible with the two accountability axioms and Condorcet-Loser Criterion.

- **Theorem 2**
  There is no SWFI that satisfies Interest-exclusion, the No-Power-Game property, Condorcet-Loser Criterion, and Non-Restrictedness.

- Non-Restrictedness: (Weaker than A-unanimity)
  For any $A \in \mathscr{A}$ and $x \in X$, there exists $\succ \in \mathscr{P}^N$ such that

$$x \in \mathrm{top}(f(\succ, A)).$$

- Since some committees consider Interest-Exclusion as a fundamental requirement, we consider weakening of the No-power-game property.

Weak No-Power-Game property :

If a voter <u>who puts x at the top</u> changes the status from neutral to an interest party of x, then x's social ranking should not improve.

Voter 1 ranks x at the top
and becomes x's interest voter

$(\succ, A)$

	1	2	3
1st	x	(x)	y
2nd	y	y	x
3rd	z	z	z

$(\succ, A')$

	1	2	3
1st	(x)	(x)	y
2nd	y	y	x
3rd	z	z	z

x's social ranking does not increase

- x would more easily form a new relation with a voter who ranks her at the top.
- However, it does not benefit x.

4.2 Possibility Results

- **Proposition 5**

  There exists an SWFI that satisfies

  - **Interest-Exclusion**,
  - **A-Unanimity,**
  - **Condorcet-Loser Criterion**,
  - **Weak No-Power-Game property.**

- We have investigated a wide variety of concrete SWFI's that satisfy the above four axioms.

- **Here, we introduce a simple extension of the Borda rule, defined based on "net-winnings" in pairwise majority comparisons.**

How to calculate Net-winnings

- **Step 1**. Make a “tournament table” of pairwise majority comparisons.

Original table

	1	2	3	4	5
1st	(y)	(y)	(x)	(x)	z
2nd	x	x	y	y	x
3rd	z	z	z	z	y

step1. tournament table

	x	y	z
x	—	1	2
y	0	—	2
z	1	1	—

- **Step 2**. Calculate "# of wins - # of losses" for each applicant.

step1. tournament table

	x	y	z
x	—	1	2
y	0	—	2
z	1	1	—

Step 2. net winnings

	Net winnings = # of wins - # of losses
x	3-1=2
y	2-2=0
z	2-4=−2

The Winning Rate Rule

- An SWFI that compares net-winnings alone does not satisfy A-Unanimity (Applicants with many neutral voters may win).

- We take an average with respect to the number of comparisons between admissible applicants.

- The **winning rate rule** is an SWFI defined as follows:

$$x\ f(\succ, A)\ y \iff \frac{\text{net winnings of } x}{\text{(sum of wins and losses)}} \geq \frac{\text{net winnings of } y}{\text{(sum of wins and losses)}}.$$

- The **winning rate rule** satisfies **Interest-Exclusion, Condorcet-Loser Criterion, A-Unanimity, and the Weak No-Power-Game property.**

Domain restrictions (1)

- A Possible cause of the impossibility results would be the very large size of the interest structure domain $\mathscr{A}$.

> - **Non-trivial interest structure (revisited)**
>
>   An interest structure A satisfies **non-triviality** if
>
> $$\forall x, y \in X (x \neq y),\ \exists i \in N, \{x, y\} \subset A(i)\,.$$

- In many situations, the voter configuration could be controlled, so we could restrict the (interest-structure) domain of SWFI's.

> - For each $k = 1,2,\ldots,n,$ define
>
> $$\mathscr{A}(k) = \left\{ A \in \{0,1\}^{n \times m} : \forall x, y\ (x \neq y),\ \underbrace{\left| \{ i \in N : \{x, y\} \subset A(i) \} \right| \geq k}_{\text{x and y are admissible for at least k voters}} \right\}.$$
>
> x and y are admissible for at least k voters

Domain restrictions (2)

- All axioms can be adapted to $\mathscr{A}(k)$.

- Let the domain of SWFI be $\mathscr{L}^n \times \mathscr{A}(k)$, and we investigate if the impossibility result in Theorem 1 holds for each k.

- **<u>Proposition 2 (Theorem 1').</u>**

  For each $k \leq [n/2]$, there exists **no** SWFI $f: \mathscr{L}^n \times \mathscr{A}(k) \to \mathscr{R}$ that satisfies A-Unanimity and the No-Power-Game property.

- **<u>Proposition 3 (Theorem 1').</u>**

  When $k \geq [n/2] + 1$, there exists an SWFI $f: \mathscr{L}^n \times \mathscr{A}(k) \to \mathscr{R}$ that satisfies A-Unanimity and the No-Power-Game property. For example, the modified plurality-rule satisfies both of them.

Recommendation.

- **Large domain case**
  (when we allow any non-trivial interest structures)
  - **We recommend the wining rate rule.**
- **Restricted domain case**
  (when the committee can be appropriately structured)
  - **The modified plurality rule is a nice option.**
- If you become a member of a funding committee, and there are some interests, then please try to use the winning rate rule or the modified plurality rule.

5. Conclusion (1)

- We introduce the "voting with interest structures" model
    - A new interesting problem in social choice theory
- We formulate 4 properties in our model
    - **Interest-Exclusion, No-PG property, A-Unanimity, and Condorcet-Loser Criterion**
    - Interest-Exclusion and No-Power-Game are novel properties, formulations of "accountability" requirements

5. Conclusion (2)

- We prove two impossibility theorems.
  - telling us “bounds” of possibility of desirable voting rules.
- We consider the Weak No-PG property and show that **the winning rate rule** satisfies it and a set of desirable axioms.
- We consider restrictions of the domain of interest structures and show that an impossibility result is resolved.
- **Future research: more general interest structures, characterization of the winning rate rule.**
- Thank you for your attention!

Related Literature

- **Peer Selection:**
  Ng and Sun (2003), Ohseto (2012), Holzman and Moulin (2013), Aziz et al. (2019), Alcalde-Unzu et al (2022), etc.

- **Aggregation of Biased opinions:**
  Amoros (2011; 2020), Adachi (2014), etc.

- **Configuration of the voters:**
  Amros (2013), Yadav (2016), Bloch and Olckers (2022), etc.