Information and Authority in Multi-Divisional Organizations

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August 1, 2023

Abstract

This paper studies decision-making authority in an asymmetric two-divisional organization. The divisions differ in their payoff weights in the headquarters' payoff function and have access to different pieces of information. Information is either of common or of private interest and the respective signals can vary in precision. Signals can be communicated via cheap talk. While centralized decision making is often optimal for balanced payoff weights, delegating the decision rights to the more payoff-relevant division can dominate. Having access to better and more information can be more important than having a larger payoff weight, in that the better informed but less payoff-relevant division obtains decision authority.

JEL: D83, D82

Keywords: strategic communication, delegation, authority, noisy information, common and private interests

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This paper grew out of a joint project with Deszö Szalay and I thank him for his input. The paper has benefited from discussions with numerous colleagues. Many thanks to Andreas Blume, Wouter Dessein, Amanda Friedenberg, Hide Ichimura, Beatrice Michaeli, Ernesto Rivera Mora, Wojciech Olszewski, and Ina Taneva for helpful comments and suggestions.

1 Introduction

Decision-relevant pieces of information with varied importance are often dispersed within modern-day organizations. Optimal decision making requires that these pieces are aggregated – thus, they have to be communicated. Given that interests within an organization typically diverge at least to some extent, communication can be expected to be strategic (Crawford and Sobel; 1982). Taking this into account, one of the key questions standing in front of the headquarters is who the decision maker should be.

To fix ideas, consider a firm with production and sales divisions. The divisions jointly generate profit for the firm but their respective contributions can have different weights. The headquarters has to decide what quantity to produce. While the production division is endowed with information about manufacturing costs, the sales division not only has information about sales costs but also about market demand. All three pieces of information matter for the optimal decision of how much to produce, and they have to be communicated to whoever makes that decision. Now, should the decision rights be retained in the hands of the headquarters, because they are equipped with formal authority and can better communicate with the production and sales divisions? Or, should the rights be delegated to the manager of the division with the highest impact on generating the firm profits? Or, should the rights be delegated to the division manager with access to the most important information?

To address these questions, I construct a model which has the following features. A headquarters steers decision making in a two-divisional organization. There is one decision to be made. The state of nature is three-dimensional and all dimensions matter for the decision from the headquarters' point of view. One of the dimensions is relevant for both divisions (common interest state, in the example, market demand), the other two are relevant only for one division respectively (private interest states, in the example, individual costs). In particular, the divisions' payoffs are quadratic loss functions that depend on how well the decision matches the sum of the common interest state and the relevant private interest state. The headquarters' payoff is the weighted sum of the two divisional payoffs.

While division A_1 can only observe a signal about the own private interest state, division A_2 can observe two signals, one about the own private interest state and in addition a signal about the common interest state. The observed information can be shared with the headquarters and with the other division via strategic communication. The headquarters can choose between three different protocols of decision making: they can rely on communicating with the two divisions and make the decision themselves (centralization); or they can delegate the decision to one of the divisions, which then communicates with the other division before making the decision (delegation to A_1, delegation to A_2).

Intuitively, the payoff weights matter for the choice of the optimal protocol: the headquarters tends to assign decision authority to the division that contributes more to the joint payoff. Access to information, however, is a powerful source that – as is turns out – often overrules the advantage of a higher payoff weight. If division A_2 is very well informed (which depends on the quality of the signals), then the headquarters optimally delegates decision making to A_2 because a more informed decision is better even if it is biased. If information and payoff weights are more balanced, a loss in information transmission is unavoidable: any communication is biased because the informed divisions are self-interested. In this case, the headquarters prefers to make the decision themselves. This generates moderate losses in information transmission from both divisions, but no bias in the decision. Delegation, by contrast, generates higher losses in information transmission from one division and a bias in the decision created by the other division that makes the decision.

This intuition can more formally be described as follows. Recall that the headquarters' payoff is the weighted sum of the divisions' payoffs, where a weight of zero corresponds to H only caring about A_1 and a weight of one to H only caring about A_2; and all payoff weights in between are allowed. Proposition 1 shows that there exist two weight thresholds (which can attain zero or one) such that delegation to A_1 is optimal below the lower threshold, λ_1, when A_1 has a high payoff weight; delegation to A_2 is optimal above the higher threshold, λ_2, when A_2 has a high payoff weight; and centralization is optimal between the two thresholds. In other words, decision authority is linked to payoff relevance in a natural way.

Subsequent results then address how these thresholds and thus the allocation of decision authority changes as the information changes. The comparative statics results (Proposition 5) show that the forces are intuitive: An increase in A_2's private interest information and in common interest information imply more decision power for A_2, since the threshold λ_2 decreases. An increase in A_1's private component implies more decision power for A_1, since the threshold λ_1 increases.

When comparing the two possible asymmetries in payoff weights and in access to information, Propositions 2 and 3 show that the access to better information is more important than a larger payoff weight. Delegation to A_2 who observes two signals can be optimal for payoff weights that favor division A_1; this never holds for delegation to A_1 who only observes the own private interest signal. In the extreme case, in which the headquarters' payoff coincides with A_1's, delegation to A_2 is still optimal as long as A_1's private value information is not too important. Considering joint surplus maximization, Proposition 4 confirms the power of information: division A_1 never gets to decide; the tradeoff between centralization and delegation to A_2, however, depends on the value of A_1's information which is communicated to H under centralization, but disregarded by A_2 under delegation to A_2.

Section 5 adds the precision of information as a new instrument to the analysis. In particular, the headquarters can choose the amount of noise in the three signals.[1] By the nature of the payoff structure, optimal signals are either fully revealing of the state component or pure noise. Lemma 4 shows that optimally A_1's private signal and the common interest signal are perfectly precise. Interestingly, the headquarters can improve the transmission of the common interest component by turning A_2's private interest signal into pure noise. The reason is that the common interest signal alone – by its nature – can be communicated without impediments. It is A_2's private interest information that creates a bias in communication, which reduces the information that is transmitted. Proposition 6 shows that controlling the precision of the information effectively reduces the chances that A_2 makes the decision: When comparing the two delegation protocols, A_1 decides more often. When comparing it to centralization, H decides more often under endogenous signal precision than under exogenous.

[1]In Blume, Board and Kawamura (2007) exogenous noise can improve information transmission in cheap talk. In Deimen and Szalay (2019) the sender optimally chooses to be only partially informed to improve communication.

From a technical perspective, the paper offers a very tractable toolbox for studying information aggregation in large organizations or communication networks. The tractability stems from the distributional assumptions that the states are uncorrelated and follow a multivariate Laplace distribution. The separation into uncorrelated common and private interest components is to the best of my knowledge new to the communication literature. The Laplace distribution has convenient closure and linearity features, which has implications for the updating rule (Lemma 1). In particular, it allows updating from multiple different sources of information – here, noisy signals and cheap talk messages. As a consequence, the analysis can straightforwardly be extended to information aggregation of any finite number of signals and messages.

Related Literature. Dessein (2002) is the first study of the allocation of decision authority in Crawford and Sobel (1982)'s model of strategic communication with one sender and one receiver. He shows that whenever influential communication is possible, the receiver prefers to delegate decision-making to the sender. Even though delegation entails a loss of control, the informational loss under strategic communication is more severe. Harris and Raviv (2005) extend the analysis of Dessein (2002) to an organization in which both players have private information that they can communicate strategically to each other. They study the impact of private information and differences in preferences on the allocation of authority. While more information is favorable for delegation, the impact of the bias is ambiguous. In contrast to these two-player setups, this paper considers a larger organization and the effects of informational versus payoff-weight asymmetries on the allocation of decision authority.

Aghion and Tirole (1997) distinguish 'formal' from 'real' authority. Formal authority is the right to make the decision. Real authority is the effective control over the decision and determined by the available information. Both principal and agent can acquire information about different actions, one of them decides. In this paper, the divisions are the only ones with access to information, and (in the main model) there is no strategic decision of how much information to obtain. The information is then communicated strategically to the one who has the formal authority. The headquarters can choose between keeping formal authority or delegating the decision rights to one of the divisions. The headquarters optimally turns real authority into formal authority, in the sense that delegation to a division that is very well informed is optimal.

The paper relates to Deimen and Szalay (2019), which analyzes a two-player cheap talk game in a similar distributional environment. In particular, that paper compares delegation to communication in a setup in which information choice is endogenous.[2] Conflicts are endogenous as they depend on the information that the sender chooses to learn. Under this endogenous information protocol, communication can dominate delegation. In a similar setup with exogenous information, Deimen and Szalay (2023) compares delegation to communication with a focus on variation of the distributional environment. That paper uses different stochastic orders to rank distributions with respect to their payoff gains under communication. The focus here is on decision making in a larger organization, in which multiple sources of information feed into a single decision.

Alonso, Dessein and Matouschek (2008) and Rantakari (2008) analyze the allocation of decision-authority in a multi-divisional organization in which each division needs to take an action. In addition to the need to adapt to the respective state of the world, each division needs to coordinate their action with the other division.[3] The present paper abstains from coordination motives; instead, the multi-dimensional information needs to be merged into a one-dimensional decision.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes strategic communication. Section 4 derives cutoffs for the optimal decision protocol and then compares these cutoffs in different scenarios. Section 5 extends the analysis to endogenous signal precision. Section 6 concludes.

[2] Costly information acquisition is also endogenous in Argenziano, Severinov and Squintani (2016). Threatened by the decision maker's punishment (overt acquisition) or equilibrium beliefs (covert acquisition), the sender overinvests in information acquisition. As a consequence, communication performs better than delegation or information acquisition by the decision maker.

[3] Liu and Migrow (2022) analyzes uncertainty about relative division profits in that framework. The paper, however, focuses on verifiable information.

2 Model

There are two agents A_1 and A_2, that could represent divisional managers, and a principal/headquarters H. The players' payoffs depend on one common action to be taken and on the state of nature. Let $a \in \mathbb{R}$ be the action. The state of nature (x_C, x_1, x_2) is the realization of a random variable (X_C, X_1, X_2) that decomposes into a common interest component X_C and two private interest components X_1 and X_2 of A_1 and A_2, respectively. The players' payoffs are given by

$$\pi_1 (a, x_C, x_1) = \pi_1^* - (a - (x_C + x_1))^2$$

and

$$\pi_2 (a, x_C, x_2) = \pi_2^* - (a - (x_C + x_2))^2 ,$$

where π_1^* and π_2^* correspond to the maximum profits that can realize. Without loss of generality, assume $\pi_1^* = \pi_2^* = 0$. The headquarters takes the perspective of a social planner who cares about weighted joint surplus

$$\pi_H (a, x_C, x_1, x_2) = (1 - \lambda) \pi_1 (a, x_C, x_1) + \lambda \pi_2 (a, x_C, x_2) ,$$

with $\lambda \in [0, 1]$. The extreme cases of $\lambda = 1$ ($\lambda = 0$) correspond to a two player setup, as the interests of H and A_2 (A_1) coincide.

While the headquarters is assumed to be uninformed, A_1 and A_2 receive private information about the state of nature. In particular, A_1 privately observes one noisy signal $s_1 = x_1 + \varepsilon_1$ about A_1's private interest component, and A_2 privately observes two noisy signals $(s_C, s_2) = (x_C + \varepsilon_C, x_2 + \varepsilon_2)$ about the common interest component and A_2's private interest component. The vector $(\varepsilon_C, \varepsilon_1, \varepsilon_2)$ describes the realization of the noise variables. The state of the world is the random vector $\boldsymbol{Z} = (X_C, X_1, X_2, E_C, E_1, E_2)$ that follows a multivariate symmetric Laplace distribution $\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.[4] The mean vector $\boldsymbol{\mu}$ is normalized to zero, and the covariance matrix takes the form $\boldsymbol{\Sigma} = diag \left(\sigma_C^2, \sigma_1^2, \sigma_2^2, \sigma_{\varepsilon_C}^2, \sigma_{\varepsilon_1}^2, \sigma_{\varepsilon_2}^2 \right) \in \mathbb{R}_+^6$ as all state components are uncorrelated. The setup and the informational environment are common knowledge. Only the signal realizations are private information.

[4] Details about the Laplace distribution are given in the first paragraph in the appendix.

Before describing the structure of the game, some comments on the modeling choices are in order. There are two built-in asymmetries between A_1 and A_2. The divisions differ not only in their payoff relevance λ but also in the sources of information they have. Understanding the interaction of these asymmetries is a key part of the analysis that follows. The Laplace distribution not only features very tractable updating rules but also admits a closed-form expression of the equilibrium payoffs in a communication game (Deimen and Szalay; 2019). This allows for a transparent comparison of decision-making protocols and the tradeoffs at hand. The quadratic losses combined with the standardization to a zero mean capture the idea that realizations of the state represent 'shocks' that the organization needs to respond to. The signals indicate the deviations from the expectation, and the optimal action then is an adaptation to the state realization away from the 'status quo action' which is set to zero.

In the beginning of the game, the headquarters chooses between three protocols of decision-making: centralization, delegation to A_1, or delegation to A_2. The attention is restricted to these protocols as they seem natural for the question at hand and are standard in the literature.

Centralization. A_1 privately observes the signal realization $s_1 = x_1 + \varepsilon_1$, and A_2 privately observes $(s_C, s_2) = (x_C + \varepsilon_C, x_2 + \varepsilon_2)$. Then, A_1 and A_2 choose which messages to send to H. There is no commitment and no cost of sending messages – i.e., communication is modeled as cheap talk in the sense of Crawford and Sobel (1982). Formally, A_1's message strategy is a function $M_1 : \mathbb{R} \to \Delta M$ and A_2's message strategy is a function $M_2 : \mathbb{R} \times \mathbb{R} \to \Delta M$. The message space M is assumed to be sufficiently large. After observing the messages, H takes an action; H's action strategy is a function $\mathcal{A}_H : M \times M \to \mathbb{R}$.

Delegation to A_1. Given the privately observed signal realizations (s_C, s_2), A_2 chooses which message to send to A_1. Formally, A_2's strategy is a function $M_2' : \mathbb{R} \times \mathbb{R} \to \Delta M$. A_1's strategy is to choose an action as a function the own signal realization and A_2's message, $\mathcal{A}_1 : \mathbb{R} \times M \to \mathbb{R}$. As under centralization, communication is modeled as cheap talk.

Delegation to A_2. Given the privately observed signal realization s_1, A_1 chooses which message to send to A_2. Formally, A_1's strategy is a function $M_1' : \mathbb{R} \to \Delta M$.

A_2's strategy is to choose an action as a function the own signal realizations and A_1's message, $\mathcal{A}_2 : \mathbb{R} \times \mathbb{R} \times M \to \mathbb{R}$. As under centralization, communication is modeled as cheap talk.

The solution concept is Bayes Nash equilibrium.

3 Strategic communication

Understanding the flow of information and the gain from strategic communication in this setup involves multiple steps. Divisions first update their information about the optimal action given the signals their receive. A_2 combines the two-dimensional information into only one dimension because the action is one-dimensional. Under centralization, A_1 and A_2 communicate with H; under delegation, one division communicates with the other. Since the players prefer different actions, communication is shaped by disagreement. Naturally, the common interest component aligns interests whereas the private interest components increase the disagreement. As is standard in cheap talk, equilibria take a partitional form; instead of revealing their exact posterior means, A_1 and A_2 only communicate partition intervals that include their posterior means. The *gain from communication* – i.e., the reduction in uncertainty through equilibrium communication – finally, determines the payoffs.

The following lemma acts as the chore tool of the paper. It shows how to aggregate information provided from multiple sources. The impact of these information sources on the optimal action has a linear structure which keeps the analysis tractable.

Lemma 1 *Let $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_d)$ follow a multivariate symmetric Laplace distribution with dimension d, then*

(i) any linear combination $\sum_{i=1}^{d} \alpha_i Y_i$ is one-dimensional symmetric Laplace,

(ii) conditional expectations can be calculated for any $i, j, k \in \{1, \ldots, d\}$ as

$$\mathbb{E}\left[Y_i | Y_j = y_j, Y_k \in (\underline{y}_k, \overline{y}_k)\right] = \frac{cov(Y_i, Y_j)}{var(Y_j)} \cdot y_j + \frac{cov(Y_i, Y_k)}{var(Y_k)} \cdot \mathbb{E}\left[Y_k | Y_k \in (\underline{y}_k, \overline{y}_k)\right].$$

Part (ii) of the lemma states that the updating rules for the multivariate symmetric Laplace distribution are linear. Importantly, the conditional expectation is not constrained to conditioning on exact signals but (by the law of iterated expectations) also applies to conditioning on intervals which is common in cheap talk games. By part (i), the conditional expectation is again Laplace. As a consequence, these rules can directly be applied to any larger network structure in which multiple sources of information need to be aggregated into one action.

3.1 Merging signals into recommendations

When the divisions receive their signals, they update their beliefs about the optimal action from their perspective. Given the quadratic loss functions, the updated optimal actions are the posterior means given the signals. By Lemma 1, A_1's and A_2's posterior means are given by

$$\theta_1 := \mathbb{E}\left[X_C + X_1 | S_1 = s_1\right] = 0 + \frac{cov(X_1, S_1)}{var(S_1)} s_1 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_{\varepsilon_1}^2} s_1,$$
$$\theta_2 := \mathbb{E}\left[X_C + X_2 | \left(S_C, S_2\right) = (s_C, s_2)\right] = \frac{\sigma_C^2}{\sigma_C^2 + \sigma_{\varepsilon_C}^2} s_C + \frac{\sigma_2^2}{\sigma_2^2 + \sigma_{\varepsilon_2}^2} s_2.$$

Denote the associated random variables by Θ_1 and Θ_2. The posterior means are linear functions of the signals which are multiplied by the ratio of covariance over variance. The covariance measures the informativeness of the signal relative to the state variable of interest; the variance measures the overall (un)informativeness of the signal.

As only the posterior means θ_1, θ_2 matter for the optimal action choice, the signals s_1, s_2 can without loss be disregarded in the future analysis. Note that θ_1, θ_2 are the only sufficient statistic of the posterior distribution that interacts with the action a. More precisely, note that A_1's (A_2's) interim expected utility satisfies the single-crossing condition in the action a and θ_1 (θ_2).[5] Hence, all equilibria of the communication games are in terms of communication about θ_1, respectively θ_2, only.[6] Therefore, θ_1, θ_2 are sometimes called a sender's *type*.

[5]The interim expected utility of A_1 is, $\mathbb{E}[-\left(a - (X_C + X_1)\right)^2 | S_1 = s_1] = -a^2 + 2a\theta_1 - \mathbb{E}\left[X_C^2 + X_1^2 | S_1 = s_1\right]$. Likewise, for A_2, $\mathbb{E}[-\left(a - (X_C + X_2)\right)^2 | \left(S_C, S_2\right) = (s_C, s_2)] = -a^2 + 2a\theta_2 - \mathbb{E}[X_C^2 + X_2^2 | \left(S_C, S_2\right) = (s_C, s_2)]$.

[6]Note that merging the two signals s_C and s_2 is optimal and without loss only from A_2's perspective. The other players would prefer to learn separately about the common and A_2's private

The following notation will be convenient,

$$v_l := var(\mathbb{E}[X_l|S_l = s_l]) = \frac{\sigma_l^2}{\sigma_l^2 + \sigma_{\varepsilon_l}^2}\sigma_l^2, \quad l = C, 1, 2,$$

such that v_C represents the *amount of common interest information* and v_1 and v_2 the *amounts of private interest information* of A_1 and A_2, respectively. It is immediate, that the respective level of noise determines the value of each piece of information: for zero noise the value is maximal, for infinite noise it decreases to zero.

With this simplification, the variances of the random variables Θ_1 and Θ_2 can be written as $var(\Theta_1) = v_1$ and $var(\Theta_2) = v_C + v_2$. The covariances can be calculated using zero correlation, $cov\,(X_C, \Theta_2) = v_C$, and $cov\,(X_j, \Theta_j) = v_j$ for $j = 1, 2$.[7]

3.2 Conflicting interests

Since communication is about θ_1 and θ_2, the players form conditional expectations about the optimal actions given what can be inferred in equilibrium from the messages about θ_1 and θ_2 they receive. As standard in the cheap talk literature (by the single crossing property), equilibrium messages will reveal the partition element in which the state has realized.

Lemma 2 *The optimal actions under delegation to A_2, to A_1, and under centralization are*

$$\begin{aligned} a_2 &= \theta_2, \\ a_1 &= \theta_1 + \beta_{12} \cdot \mathbb{E}\left[\Theta_2|\Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right], \\ a_H &= \beta_{H1} \cdot \mathbb{E}\left[\Theta_1|\Theta_1 \in [\underline{\theta}_1, \overline{\theta}_1)\right] + \beta_{H2} \cdot \mathbb{E}\left[\Theta_2|\Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right], \end{aligned}$$

where

$$\beta_{H1} = (1-\lambda), \quad \beta_{H2} = \frac{v_C + \lambda v_2}{v_C + v_2}, \quad \beta_{12} = \frac{v_C}{v_C + v_2}.$$

component. This cannot be enforced as there is no commitment. A formal proof is given, e.g., in Deimen and Szalay (2019).

[7] For example, the covariance of H's optimal action and A_2's posterior Θ_2 is given by $cov\,(X_C + (1-\lambda)X_1 + \lambda X_2, \Theta_2) = v_C + \lambda v_2$, it measures the informational content of Θ_2 for H.

Under delegation to A_2, A_2 will take the optimal action based on the own information and will ignore A_1's message as it is uncorrelated to A_2's interests. Under delegation to A_1, A_1 updates on θ_1 based on the own private information and learns about the common interest component from A_2's message revealing that $\theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)$. Under centralization, A_1 and A_2 communicate to H by revealing that $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1)$ and that $\theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)$, respectively.

Note that $\beta \in [0, 1]$ for $\beta = \beta_{H1}, \beta_{H2}, \beta_{12}$. When receiving a message, not only the interval partition in which the state has realized is updated, but in addition the action is discounted by β towards the prior mean 0. In the terminology of the literature, senders have a *state-dependent bias* $(1 - \beta) \cdot \theta$. Note that the bias is uncertain as it depends on the realization of the state.[8] The linearity of the bias captures the idea that deviations from some 'status quo' (the mean) are more preferred by the sender than by the receiver: the sender wants to match the realization of the state, the receiver wants to take a lower action (either because of λ, or because common and private information are merged); the slope is determined by the ratio of covariance over variance which measures the relative relevance of the information from the receiver's perspective (covariance) to the sender's perspective (variance).

Note that β_{H2}, β_{12} are functions of the underlying noise. Both are decreasing in the noise in the common interest signal, $\sigma^2_{\varepsilon_C}$, and increasing in the noise in A_2's private interest signal, $\sigma^2_{\varepsilon_2}$. Intuitively, with more noise in the common interest signal interests become less aligned, while with more noise in A_2's private interest signal interests become more aligned. The idea of optimally choosing the amount of noise in the signals is analyzed in Section 5.

3.3 Equilibria and the gain from communication

As mentioned before, cheap talk equilibria take the standard partitional form:[9] due to single crossing, all equilibria are interval partitions of the respective sender type

[8] Li and Madarász (2008) and Li (2010) consider cheap talk with uncertain sender's bias.

[9] Since Θ_1 and Θ_2 are uncorrelated, the three communication games can be analyzed independently, each of them being one-dimensional. For a characterization and existence results of the one-dimensional game, see for example Deimen and Szalay (2019). For convenience, the results are summarized in this section.

spaces, Θ_j-spaces for $j = 1, 2$. Equilibria exist, are symmetric, and are essentially unique for any number $n \in \mathbb{N}$ of partition elements. Moreover, there exists a limit equilibrium with an accumulation point at zero and an infinite number of partition elements.

To be more precise, partitional equilibria are characterized by a sequence of *critical sender types*, $\boldsymbol{t}_j^n = \left(t_{j,i}^n\right)$, with $t_{j,i-1}^n < t_{j,i}^n$, $j = 1, 2$, and n relating to the number of actions that are induced in equilibrium. Sender types strictly within a partition interval, $\theta_j \in \left(t_{j,i-1}^n, t_{j,i}^n\right)$, pool by sending a message that indicates this interval. Upon receiving such a message, the conditional expectation is given by

$$\mu_{j,i}^n := \mathbb{E}\left[\Theta_j \middle| \Theta_j \in \left(t_{j,i-1}^n, t_{j,i}^n\right)\right], \qquad \text{for } i = 2, \ldots, n, j = 1, 2. \tag{1}$$

The optimal receiver response to such message is $\beta \cdot \mu_{j,i}^n$ for the respective $\beta \in \{\beta_{H1}, \beta_{H2}, \beta_{12}\}$. For the lowest and highest message the expressions are alike, $\mu_{j,1}^n := \mathbb{E}\left[\Theta_j \middle| \Theta_j < t_{j,1}^n\right]$ and $\mu_{j,n+1}^n := \mathbb{E}\left[\Theta_j \middle| \Theta_j > t_{j,n}^n\right]$, $j = 1, 2$. Quadratic loss functions imply that critical types, $t_{j,i}^n$, are indifferent between inducing the action in the interval below or above:

$$t_{j,i}^n - \beta \cdot \mu_{j,i}^n = \beta \cdot \mu_{j,i+1}^n - t_{j,i}^n, \qquad \text{for } i = 1, \ldots, n, j = 1, 2. \tag{2}$$

There is a vast multiplicity of equilibria in cheap talk games: As the meaning of a message is only determined in equilibrium, messages can be arbitrarily exchanged. Moreover, equilibria with different numbers n of partition elements exist. Keeping the type of equilibrium under communication fixed is crucial for a fair comparison between different forms of decision-making. One approach to this, which is often taken in the literature, is to focus on the equilibrium with the highest number of partition elements. The motivation is that all players unanimously prefer this equilibrium over any other equilibrium from an ex ante perspective.[10] Hence, limit equilibria with infinitely many partition elements are selected by this criterion. The remainder of the paper will focus on communication in a limit equilibrium.

To measure the gain from equilibrium communication, it is helpful to define the discrete random variables of the conditional expectations μ_1 and μ_2. These are

[10] See also Antić and Persico (2023) who provide a refinement that uniquely selects this ex-ante Pareto dominant equilibrium.

derived from the marginal distributions of Θ_1 and Θ_2 and have supports $\left\{\mu^{\infty}_{\theta_1,i}\right\}_{i=1}^{\infty}$ and $\left\{\mu^{\infty}_{\theta_2,i}\right\}_{i=1}^{\infty}$ determined by the respective equilibrium characterization $\boldsymbol{t}^{\infty}_j, j = 1,2$. By the law of total variance, the variance can be decomposed in explained variation plus expected residual variance, $var\left(\Theta_j\right) = var\left(\mathbb{E}[\Theta_j|\theta_j \in (t_{j,i-1}, t_{j,i})]\right) + \mathbb{E}[var(\Theta_j|\theta_j \in (t_{j,i-1}, t_{j,i}))]$. The typical focus is on the residual variance which entails the loss from communication. The focus here is on the counterpart, the explained variation $var\left(\mathbb{E}[\Theta_j|\theta_j \in (t_{j,i-1}, t_{j,i})]\right) = var\left(\mu_j\right)$, which measures the *gain from communication*. In limit equilibria, the gain from communication is given by[11]

$$var\left(\mu_j\right) = \frac{1}{2-\beta} var\left(\Theta_j\right), \tag{3}$$

where $\beta \in \{\beta_{H1}, \beta_{H2}, \beta_{12}\}$, $j = 1,2$. The fraction $\frac{1}{2-\beta}$ measures how much information can be strategically communicated: The fraction is increasing in β, and the maximal gain from communication is naturally reached for $\beta = 1$, when interests are perfectly aligned.

4 The optimal protocol for decision making

The objective is to maximize H's payoff, π_H, from an ex ante perspective. In other words, H maximizes the *expected payoff gain*, $\tilde{\pi}_H$, minus the *prior uncertainty*, $\sigma_C^2 + (1-\lambda)\sigma_1^2 + \lambda\sigma_2^2$. As the prior variances are constant across all decision protocols, it is sufficient for the comparison of the protocols to focus on the expected payoff gains.

In the first-best benchmark H can directly observe the signal realizations (s_C, s_1, s_2). In response, H takes the optimal first-best action $a_H^{fb} = \frac{\sigma_C^2}{\sigma_C^2+\sigma_{\varepsilon_C}^2} s_C + (1-\lambda)\frac{\sigma_1^2}{\sigma_1^2+\sigma_{\varepsilon_1}^2} s_1 + \lambda\frac{\sigma_2^2}{\sigma_2^2+\sigma_{\varepsilon_2}^2} s_2$. H's expected payoff gain in this first-best scenario can thus be calculated as

$$\tilde{\pi}_H^{fb} = v_C + (1-\lambda)^2 v_1 + \lambda^2 v_2.$$

By comparison, when information is decentralized, the optimal actions are given by $a_H = \beta_{H1}\hat{\mu}_1 + \beta_{H2}\hat{\mu}_2$ under centralization, $a_1 = \theta_1 + \beta_{12}\hat{\mu}_2$ under delegation to A_1,

[11]The expression is derived in Deimen and Szalay (2019), with notation $c = \beta$.

and $a_2 = \theta_2$ under delegation to A_2, where $\hat{\mu}_j$ is some realization of the conditional expectation μ_j for $j = 1, 2$. The expected payoff gains for H under the different decision protocol are as follows.

Lemma 3 *H's expected payoff gains under delegation to A_2, to A_1, and under centralization are*

$$\begin{aligned}
\tilde{\pi}_H^{del2} =& v_C + (2\lambda - 1)v_2, \\
\tilde{\pi}_H^{del1} =& \left(2\lambda\beta_{12} + (1 - 2\lambda)\beta_{12}^2\right)\frac{1}{2 - \beta_{12}}(v_C + v_2) + (1 - 2\lambda)v_1, \\
\tilde{\pi}_H^{cen} =& \beta_{H2}^2 \frac{1}{2 - \beta_{H2}}(v_C + v_2) + \beta_{H1}^2 \frac{1}{2 - \beta_{H1}} v_1.
\end{aligned}$$

To get an intuition for these expressions, consider first the payoff gain from delegation to A_2. The private interest information v_2 is directly observed by A_2 and is a gain to A_2 which counts with weight λ, but a loss from A_1's perspective which counts with weight $1 - \lambda$, adding up to $2\lambda - 1$. The common interest information v_C is also directly observed by A_2 and weighted by $\lambda + (1 - \lambda)$, thus $\tilde{\pi}_H^{del2}$ increases one to one with v_C.

Second, consider the value of delegating to A_1. The private interest information v_1 is directly observed and the payoffs weights are $-\lambda + (1 - \lambda)$. The common interest information has to be communicated from A_2 to A_1, which results in the fraction $\frac{1}{2-\beta_{12}}$ (see equation (3)). The factors together combine to $(1 - \lambda)\beta_{12}^2 - \lambda(\beta_{12}^2 - 2\beta_{12})$.

Third, the gain from centralization follows from H communicating with A_1 and A_2. Again, strategic communication reduces the values to the fractions $\frac{1}{2-\beta_{H2}}$ and $\frac{1}{2-\beta_{H1}}$, and the factors are β_{H2}^2 and β_{H1}^2.

Suppose that there is only a common but no private interest components, $v_1 = v_2 = 0$. In this case, $\beta_{12} = \beta_{H2} = 1$ and one can directly see from Lemma 3 that the payoffs from all three decision protocols coincide with the first-best payoff. If everyone agrees on the optimal action, information can be communicated without any impediments and it does not matter who makes the decision. For the remainder of the paper, thus assume

Assumption 1 *One private interest component is strictly positive, $v_1 > 0$ or $v_2 > 0$.*

The comparison of the three decision protocols yields the following characterization.

Proposition 1 *Given v_C, v_1, v_2, there exist λ_1, λ_2 with $0 \leq \lambda_1 \leq \lambda_2 \leq 1$ such that*

i) Delegation to A_1 is optimal if and only if $\lambda \in [0, \lambda_1]$ and $\lambda_2 > 0$.

ii) Centralization is optimal if and only if $\lambda \in [\lambda_1, \lambda_2] \cup \{1\}$ or ($\lambda = 0$ and $\lambda_2 > 0$).

iii) Delegation to A_2 is optimal if and only if $\lambda \in [\lambda_2, 1]$.

The proof first makes a pairwise comparison of the decision protocols, then determines the upper envelope as a function of λ. The key properties are that $\tilde{\pi}_H^{del2}$ and $\tilde{\pi}_H^{del2}$ are linear and $\tilde{\pi}_H^{cen}$ is convex in λ. Moreover, centralization coincides with delegation to A_1 in $\lambda = 0$ and with delegation to A_2 in $\lambda = 1$. The optimal decision protocol is characterized by the two intersections λ_1 and λ_2. These intersection points define intervals in which delegation to A_1, delegation to A_2, and centralization are optimal. For an illustration of the payoffs with equal values of information, $v_1 = v_2 = v_C = 1$, see Figure 1.

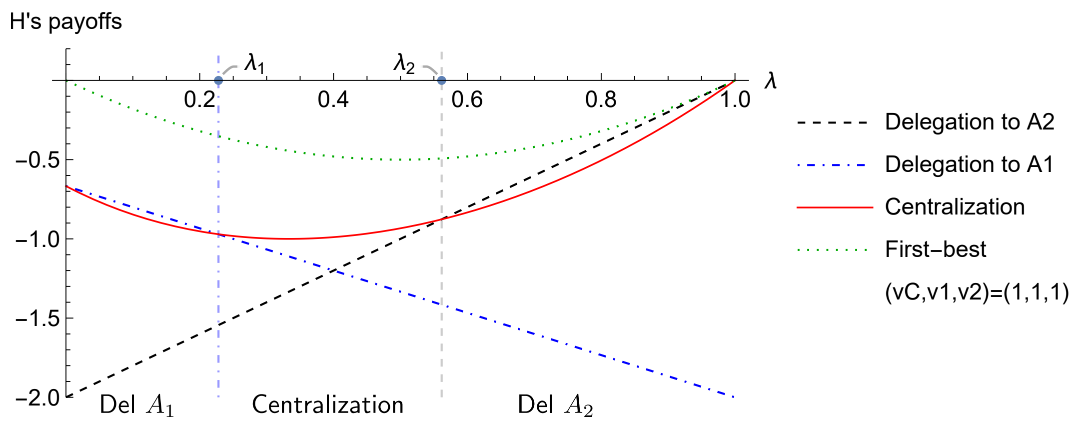

Figure 1: H's payoffs for equal information values, $v_C = v_1 = v_2 = 1$.

The setup captures two types of asymmetries between A_1 and A_2: with regard to H's payoffs, weight λ measures the relevance of A_2 relative to A_1; with regard to information, A_1 and A_2 are equipped with different amounts and pieces of information. It is not only the case that v_1 and v_2 can differ, but also that A_2 observes the common information v_C. To understand the impact of these different channels on authority, i.e., who makes the decision in the organization, it is convenient to look at these channels from different perspectives.

4.1 Comparing the two delegation alternatives

As a first comparison, suppose H has to delegate to either A_1 or A_2. Obviously, A_2 has the advantage of observing the common interest information in addition to the private interest information. This informational advantage can be translated into a critical weight λ^{del}, at which H is indifferent between delegation to A_1 and to A_2.

Proposition 2 *The solution to $\tilde{\pi}_H^{del1}(\lambda) = \tilde{\pi}_H^{del2}(\lambda)$ is given by*

$$\lambda^{del} = \frac{1}{2} - \frac{1}{2}\frac{v_2 v_C}{v_1(2v_2 + v_C) + 2v_2^2} \leq \frac{1}{2},$$

which is decreasing in v_C, increasing in v_1, and decreasing in v_2 if $v_1 \leq 2\frac{v_2^2}{v_C}$.

Since $\lambda^{del} \leq \frac{1}{2}$, A_2 has weakly more decision power than A_1. Naturally, as soon as A_2 also observes only one piece of information, i.e., $v_2 = 0$ or $v_C = 0$, the divisions are symmetric and $\lambda^{del} = \frac{1}{2}$. The signs of the derivatives of λ^{del} show how the quality of information impacts the comparison. In particular, more common interest information implies that A_2 gets more often to decide, λ^{del} is decreasing in common interest information v_C. If A_1 has more private interest information then A_1 gets more often to decide, λ^{del} is increasing in private interest information v_1. The impact of A_2's private interest information v_2 depends on the other available information. As long as v_2 is sufficiently large, λ^{del} is decreasing in v_2 and thus A_2's decision power is increasing in A_2's information. When v_1 and v_C are large relative to v_2 then λ^{del} is increasing in v_2 and thus A_1's decision power is increasing in v_2. Intuitively, since v_2 is relatively small it does not disturb the common interest information that is communicated from A_2 to A_1, and H prefers more information.

To summarize, having more information implies having more decision authority. Information is so powerful that it can overrule payoff asymmetries, the division with smaller payoff weight but more information can get to decide.

4.2 Only two players

Consider the two extreme payoff structures, for which H's interests either coincide with A_1 ($\lambda = 0$) or with A_2 ($\lambda = 1$). Note that by Proposition 1, it is never the case that A_1 decides for $\lambda = 1$, as v_1 does not matter for the optimal decision. By contrast, in a neighborhood around $\lambda = 1$ delegation to A_2 is optimal and even dominates centralization. This finding is reminiscent of the main result in Dessein (2002), who shows in a two player setup that for small conflicts of interests delegation is preferred over communication.

A more novel question is whether possessing relevant information can overrule the natural assignment of decision authority. In other words, when is it possible that A_2 decides, even though H shares A_1's interests.

Proposition 3 *If $\lambda = 0$, delegation to A_2 is optimal if and only if $v_1 \leq \frac{v_2(v_C - 2v_2)}{2v_2 + v_C}$.*

The common component v_C and A_2's private component v_2 need to be sufficiently important relative to A_1's private component v_1, for H to give up decision rights and delegate to A_2. Note that the condition is not generically satisfied. For example, for $v_1 = v_2$ it is never satisfied. However, for $v_1 = 0$ and $v_C > 2v_2$ it is always satisfied.

To summarize, it is indeed the case that the right amount of information can be more important in terms of decision rights than sharing identical preferences.

4.3 Joint surplus maximization

Under joint surplus maximization, both divisions are equally relevant from a payoff perspective, $\lambda = \frac{1}{2}$. In this case, it is only the informational advantage that determines who makes the decision.

Proposition 4 *If $\lambda = \frac{1}{2}$, then delegation to A_1 is always dominated; centralization is preferred to delegation to A_2 if and only if $v_1 \geq 3v_2 \frac{(2v_C - v_2)}{(3v_2 + 2v_C)}$.*

Being equally important in terms of payoff weights does not help A_1 to gain decision rights. In fact, A_1 never decides. The reason is that the gains and losses from private interest information exactly cancel out, so that access to v_C is the driving force. Since A_2 can better communicate with H than with A_1, either A_2 directly decides or centralization is the optimal protocol.

For the latter decision, the relative size of v_1 versus v_2 and v_C matters. Intuitively, whenever v_1 is sufficiently large, H cannot rely on A_2 to make a balanced decision because A_2 disregards v_1, and thus centralization is better. Similarly, when v_2 is very large, in particular very large relative to v_C, centralization is better because H's payoff under centralization is increasing in v_2 while H's payoff under delegation to A_2 is independent of v_2, it equals v_C. The reason is that any gain from private information to A_2 is a loss to A_1 of the same size. Figure 2 illustrates the boundary between delegation to A_2 and centralization for $v_C = 1$.

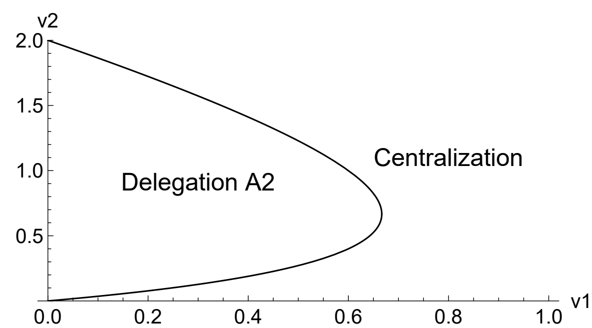

Figure 2: Comparison of centralization and delegation to A_2 for $v_C = 1$.

Note that communication under centralization is influential almost surely, i.e., H adjusts the optimal action given the messages from A_1 and A_2. Thus H can internalize both sources of information into the optimal decision. Moreover, when H decides, the decision is not biased. As a consequence, centralization beats delegation when private interest information is sufficiently large. Intuitively, in this case H rather controls the decision, as the interests are diverging and the bias in decision making becomes large.

4.4 The power of having common interest information

One key difference between A_1 and A_2 is that A_2 has direct access to the common interest information. To better understand the resulting decision power of observing this information, let v_C range from zero to infinity and see how the boundary λ_2 between centralization and delegation to A_2 changes.[12]

Proposition 5 *There is more delegation to A_2 in the infinite common interest environment than in the pure private interest environment, $\lambda_2^{C=\infty} \leq \lambda_2^{C=0}$. The values $\lambda_2^{C=0}, \lambda_2^{C=\infty}$ are increasing in v_1 and decreasing in v_2, in their respective ranges.*

Without common interest information, A_2's decision power is limited by the lower bound $\frac{2}{3}$. Delegation to A_2 is thus very limited. By contrast, for infinite common interest information there is no lower bound but delegation to A_2 can be optimal for any value of λ. Access to common interest information can thus overrule any payoff asymmetry in favor of A_1.

The value of λ_2 for infinite common interest information, $\lambda_2^{C=\infty}$, strongly depends on the relative sizes of private interest information v_2 versus v_1. Naturally, for large v_1, the cutoff $\lambda_2^{C=\infty}$ approaches one and the range of delegation to A_2 vanishes. For large v_2, the cutoff $\lambda_2^{C=\infty}$ approaches zero and A_2 always decides. For equal private interest information $v_1 = v_2$, the cutoff can be anywhere in $[0, 1]$.

Figure 3 illustrates the cutoffs $\lambda_1^{C=1000}, \lambda_2^{C=1000}$ for very large common interest information $v_C = 1000$ and variation in the relative sizes of private interest information. While A_1's private interest information is fixed at $v_1 = 1$, A_2's private interest information takes values $v_2 = 0.25, 1, 1.5$. For $v_2 = 0.25$, (left picture) all three decision protocols can be optimal and λ_1 and λ_2 are relatively equally spread out; for $v_2 = 1$, (central picture) both cutoffs meet exactly at zero and for $v_2 = 1.5$, (right picture) the only optimal decision protocol is delegation to A_2 for any composition λ of the organization.

To summarize, the access to common interest information is a powerful tool. However, if common interest information is abundant the relative amounts of private interest information determine the thresholds for delegation and as long as $v_2 < v_1$ all three decision protocols can be optimal.

[12]The range of possible values for λ_1 is the same for v_C equal to zero and equal to infinity. The threshold is derived in the proof of Proposition 5 and given by $\lambda_1 \in [0, \frac{1}{3}]$.

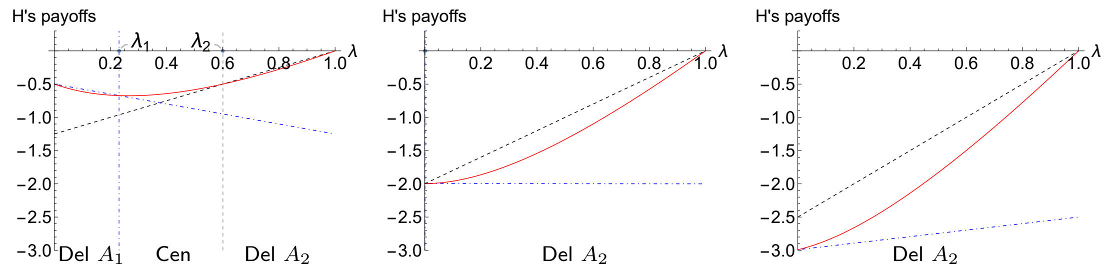

Figure 3: H's payoffs under delegation to A_1 (black, dashed), delegation to A_2 (blue, dash-dotted), and centralization (red, solid) for $v_C = 1000, v_1 = 1$, and $v_2 = 0.25$ (left panel), $v_2 = 1$ (central panel), and $v_2 = 1.5$ (right panel).

5 Endogenous quality of information

So far, the quality of the information available to A_1 and A_2 was assumed to be exogenously given. In particular, the amounts of noise $\sigma^2_{\varepsilon_1}$, $\sigma^2_{\varepsilon_2}$, and $\sigma^2_{\varepsilon_C}$ in the signals s_1, s_2, and s_C determined the amount of information v_1, v_2, and v_C available to A_1 and A_2, respectively. A natural extension of the previous analysis is to endogenize the precision of information by making $\sigma^2_{\varepsilon_1}$, $\sigma^2_{\varepsilon_2}$, $\sigma^2_{\varepsilon_C} \in \mathbb{R}_+ \cup \{\infty\}$ choice variables.

To keep the analysis comparable, the assumption is that there is no direct cost of information acquisition but only strategic costs. Moreover, to keep the analysis interesting, these choices are publicly observable. In other words, the players can commit to acquiring imperfectly precise signals.[13] Finally, as H has authority in the organization, H can choose how much information A_1 and A_2 observe. Of course H can delegate this choice to A_1 and A_2 respectively, so that they are allowed to choose the precision of their signals themselves.[14]

Note, first, that if information acquisition is delegated to A_1 and A_2 they acquire

[13] With no cost of information acquisition, A_1 and A_2 are at least weakly better of with perfectly precise signals. Under covert information acquisition, everyone would thus rightly assume A_1 and A_2 to be perfectly informed in their respective domains.

[14] In the communication subgames, there are possible threats to deviate to a babbling equilibrium if the acquired information is not ideal from a player's perspective. These threats to not use available information seem difficult to enforce within an organization. Therefore, the focus is on equilibria with the most informative communication on and off path.

perfectly precise information.[15] Second, note that H can replicate any choice of precision that A_1 and A_2 make. Therefore, H is at least weakly better off keeping authority over information acquisition. As it turns out, it can be optimal from H's perspective to leave noise in the private interest signals to reduce conflicts and improve information transmission. Thus, H can strictly gain from choosing the quality of information in the organization.

The optimal choice of the noise variances depends on the institution. A signal s_l for $l = 1, 2, C$ is called *noise free* if $\sigma^2_{\varepsilon_l} = 0$ and *pure noise* if $\sigma^2_{\varepsilon_l} = \infty$.

Lemma 4 *(i) For all institutions, optimally, the common interest signal is noise free.*

(ii) Optimally, A_1's private interest signal is noise free for centralization; for delegation to A_1 it is noise free if $\lambda < \frac{1}{2}$ and pure noise if $\lambda > \frac{1}{2}$; it is irrelevant for delegation to A_2.

(iii) Optimally, A_2's private interest signal is pure noise if $\lambda < \hat{\lambda}$ and noise free if $\lambda > \hat{\lambda}$, where $\hat{\lambda}^{del1} = 1$, $\hat{\lambda}^{del2} = \frac{1}{2}$, and $\hat{\lambda}^{cen} = \frac{2v_2+3v_C-\sqrt{(2v_2)^2+4v_2v_C+(3v_C)^2}}{2v_2}$.

In all cases, the optimal choices of the quality of the signals are either perfectly informative or totally uninformative. Regardless of which institution is chosen, common interest information is observed perfectly. Intuitively, there is no conflict with respect to common interest information and common interest information even reduces the conflict in communication with A_2. The precision of the private interest signals needs to be analyzed for each institution separately. Under any delegation protocol, private interest information is fully revealed whenever the payoff weight is in favor of the deciding party, and fully suppressed otherwise. Intuitively, revealing information that is not aligned with the main interest of H involves a tradeoff. While it is a gain for one division, it is a cost for the other division. The quadratic loss functions then imply a bang bang solution, with either full or no information.

Another interesting tradeoff arises with respect to A_2's private interest information. Better information seems more valuable, but whenever A_2 observes private

[15] This is obvious under a covert choice of signal precision. For an overt choice of signal precision an argument is given, for example, in Deimen and Szalay (2019).

interest information, A_2 not only garbles that information with the common interest information but also the conflict in communication increases. There exists a cutoff $\hat{\lambda}$ such that a noisy free signal s_2 is optimal above the cutoff and pure noise is optimal below. The cutoff differs for the three institutions. While it is maximal for delegation to A_1, it takes lower values for centralization and delegation to A_2. This is intuitive, since A_2's private interest information is pure noise from A_1's perspective but valuable information from H's perspective.

What is the additional decision power that arises from being able to manipulate the information? First, just compare the two delegation alternatives, under an optimal choice of information quality.

Lemma 5 *Under optimal endogenous signal precision, delegation to A_1 is preferred over delegation to A_2 if and only if $\lambda \leq \frac{1}{2}$.*

The additional tool of choosing the precision of the signals results in substantially more decision power for A_1. The original informational advantage of A_2 is mitigated by the optimal informational choice. The key force is that whenever A_1 decides, A_2's private information is suppressed such that the common interest information can be communicated without impediments.

The picture changes, when taking centralization into account. The comparison of the two delegation protocols indicates that A_2 looses decision power when information is controlled by H. To give A_2 the best chances and thus to derive an upper bound on A_2's decision power, consider the case of infinite common interest information.

Proposition 6 *Under optimal endogenous signal precision, and for infinite common interest information, $v_C = \infty$, there exist $\lambda_1^{endo}, \lambda_2^{endo}$ with $0 \leq \lambda_1^{endo} \leq \lambda_2^{endo} \leq 1$ such that delegation to A_1 is optimal for $\lambda \leq \lambda_1^{endo}$, centralization is optimal for $\lambda \in [\lambda_1^{endo}, \lambda_2^{endo}]$, and delegation to A_2 is optimal for $\lambda \geq \lambda_2^{endo}$. Moreover, $\lambda_1^{endo} = \lambda_1^{C=\infty}$ and $\lambda_2^{endo} \geq \lambda_2^{C=\infty}$.*

The comparison between delegation to A_1 and centralization is simple since under both institutions it is optimal to suppress A_2's private interest information and to give noise free information to A_1. As a consequence, the optimal cutoff $\lambda_1^{endo} = \frac{1}{3}$ is equal to the upper bound of the exogenous cutoff $\lambda_1^{C=\infty}$. Hence, the tradeoff between delegation to A_1 and centralization is unchanged. In other words, H cannot gain relative to A_1 by choosing the signal precision.

For the comparison between delegation to A_2 and centralization, by Lemma 4, two scenarios have to be considered. For sufficiently high levels of λ, $\lambda \geq \hat{\lambda}^{cen}$, the comparison reduces to the comparison for exogenous signal precision. For intermediate values of λ, $\frac{1}{2} \leq \lambda < \hat{\lambda}^{cen}$, the underlying information under delegation to A_2 differs from that under centralization. Hence a new threshold $\check{\lambda}_2^{endo}$ has to be calculated, which turns out to lie above the threshold for exogenous signal precision. In other words, there is more centralization in comparison to delegation to A_2 under endogenous signal precision than under exogenous signal precision. Choosing the quality of the information that A_2 observes gives H additional decision power in the sense of a larger range of payoff weights for which centralization is optimal.

6 Conclusions

This paper studies communication and the allocation of decision authority in a two-divisional organization. Multiple pieces of information are dispersed in the organization. They are either of common or of private interest to the different players and have to be aggregated into a one-dimensional decision. This creates a correlation of interests that makes communication strategic.

The paper shows that having access to information plays an important role for the allocation of decision authority: it is often the better informed player who decides than the one who contributes a larger share to the joint payoff (in case that these two differ). If payoff weights and access to information are relatively balanced, centralization is the optimal protocol.

The paper provides a tractable framework for analyzing strategic communication in multi-divisional organizations. The environment is flexible enough to be extended to larger organizational structures or communication networks. This is left for future research.

A Appendix

Laplace distribution. (See, e.g., Kotz, Kozubowski and Podgórski; 2001)

For the one dimensional Laplace distribution with mean zero and variance σ^2, the density is $f(t) = \frac{1}{\sqrt{2}\sigma}\exp(-\frac{\sqrt{2}\,|t|}{\sigma})$, and the characteristic function is of the form $\psi(t) = \frac{1}{1+\frac{1}{2}\sigma^2 t^2}$, for $t \in \mathbb{R}$. For the multivariate symmetric Laplace distribution, the characteristic function is of the form $\Psi(\boldsymbol{t}) = \frac{1}{1+\frac{1}{2}\boldsymbol{t'\Sigma t}}$, for $\boldsymbol{t} \in \mathbb{R}^d$. The distribution is thus elliptically contoured. For an illustration of the one-dimensional density, see Figure A.1.

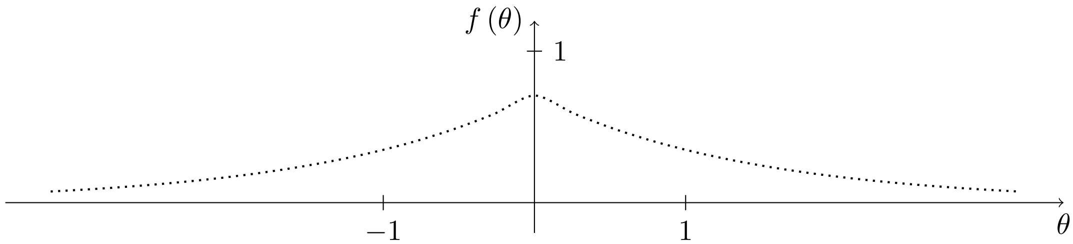

Figure A.1: Density of the Laplace distribution.

Lemma A.1 *Let $\boldsymbol{Z}$ follow an elliptically contoured symmetric distribution, $\boldsymbol{Z} \sim EC_d(\boldsymbol{\mu}, \boldsymbol{\Sigma}, \phi)$, where d is the dimension of $\boldsymbol{Z}$, $\boldsymbol{\mu}$ is the mean vector, $\boldsymbol{\Sigma}$ is the covariance matrix with rank$(\boldsymbol{\Sigma}) = k$, and ϕ is the characteristic generator. Further let*

$$\boldsymbol{Z} = (\boldsymbol{Z_1}, \boldsymbol{Z_2}), \quad \boldsymbol{\mu} = (\boldsymbol{\mu_1}, \boldsymbol{\mu_2}), \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma_{11}} & \boldsymbol{\Sigma_{12}} \\ \boldsymbol{\Sigma_{21}} & \boldsymbol{\Sigma_{22}} \end{pmatrix},$$

where the dimensions of $\boldsymbol{Z_1}$, $\boldsymbol{\mu_1}$, and $\boldsymbol{\Sigma_{11}}$ are $m \times 1$, $m \times 1$, and $m \times m$ for $0 < m < d$, respectively.

i) The variable $(\boldsymbol{Z_1}|\boldsymbol{Z_2} = \boldsymbol{z_2})$ follows an elliptically contoured symmetric distribution with mean[16]

$$\boldsymbol{\mu_1} + \boldsymbol{\Sigma_{12}}\boldsymbol{\Sigma_{22}^{-1}}(\boldsymbol{z_2} - \boldsymbol{\mu_2})$$

[16]The generalized inverse of a matrix is defined the standard way.

and covariance matrix

$$\mathbf{\Sigma_{11}} - \mathbf{\Sigma_{12}}\mathbf{\Sigma_{22}^{-1}}\mathbf{\Sigma_{21}}.$$

ii) Let $\boldsymbol{A}$ be an $d \times l$ matrix and $\boldsymbol{b}$ be an $l \times 1$ vector. Then

$$\boldsymbol{b} + \boldsymbol{A}'\boldsymbol{Z} \sim EC_l\left(\boldsymbol{b} + \boldsymbol{A}'\boldsymbol{Z}, \boldsymbol{A}'\boldsymbol{Z}\boldsymbol{A}, \phi\right).$$

Proof of Lemma A.1. i) Fang, Kotz and Ng (1990) Theorem 2.18.
ii) Fang, Kotz and Ng (1990) Theorem 2.16. □

Proof of Lemma 1. (i) Kotz, Kozubowski and Podgórski (2001) Prop. 5.1.1.
(ii) Lemma A.1. □

Proof of Lemma 2. Under delegation to A_2, A_2 will ignore A_1's message as it is uncorrelated to A_2's optimal action which is $a_2 = \theta_2$.

Under delegation to A_1, A_2 sends a message revealing that $\theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)$ and A_1's optimal action is

$$\begin{aligned}
a_1 &= \mathbb{E}\left[X_C + X_1 | \Theta_1 = \theta_1, \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right] \\
&= \theta_1 + \frac{cov(X_C + X_1, \Theta_2)}{var(\Theta_2)} \cdot \mathbb{E}\left[\Theta_2 | \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right] \\
&= \theta_1 + \beta_{12} \cdot \mathbb{E}\left[\Theta_2 | \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right],
\end{aligned}$$

with factor $\beta_{12} = \frac{cov(X_C + X_1, \Theta_2)}{var(\Theta_2)} = \frac{v_C}{v_C + v_2}$.

Under centralization, A_1 and A_2 communicate to H by revealing that $\theta_1 \in [\underline{\theta}_1, \overline{\theta}_1)$ and $\theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)$, respectively. By Lemma 1, this induces H to take the action

$$\begin{aligned}
a_H &= \mathbb{E}\left[X_C + (1-\lambda)X_1 + \lambda X_2 | \Theta_1 \in [\underline{\theta}_1, \overline{\theta}_1), \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right] \\
&= \frac{cov(X_C + (1-\lambda)X_1 + \lambda X_2, \Theta_1)}{var(\Theta_1)} \cdot \mathbb{E}\left[\Theta_1 | \Theta_1 \in [\underline{\theta}_1, \overline{\theta}_1)\right] \\
&\quad + \frac{cov(X_C + (1-\lambda)X_1 + \lambda X_2, \Theta_2)}{var(\Theta_2)} \cdot \mathbb{E}\left[\Theta_2 | \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right] \\
&= \beta_{H1} \cdot \mathbb{E}\left[\Theta_1 | \Theta_1 \in [\underline{\theta}_1, \overline{\theta}_1)\right] + \beta_{H2} \cdot \mathbb{E}\left[\Theta_2 | \Theta_2 \in [\underline{\theta}_2, \overline{\theta}_2)\right],
\end{aligned}$$

with $\beta_{H1} = \frac{cov(X_C+(1-\lambda)X_1+\lambda X_2,\Theta_1)}{var(\Theta_1)} = (1-\lambda)$ and $\beta_{H2} = \frac{cov(X_C+(1-\lambda)X_1+\lambda X_2,\Theta_2)}{var(\Theta_2)} = \frac{v_C+\lambda v_2}{v_C+v_2}$. $\square$

Proof of Lemma 3.

Delegation to $\boldsymbol{A_2}$. A_2's optimal action is $a_2 = \theta_2 = \frac{\sigma_C^2}{\sigma_C^2+\sigma_{\varepsilon_C}^2} s_C + \frac{\sigma_2^2}{\sigma_2^2+\sigma_{\varepsilon_2}^2} s_2$. Since the variables are uncorrelated, A_1's expected payoff is

$$-\mathbb{E}\left[(a_2 - (X_C + X_1))^2\right] = v_C - v_2 - \sigma_C^2 - \sigma_1^2.$$

A_2's expected payoff is

$$-\mathbb{E}\left[(a_2 - (X_C + X_2))^2\right] = v_C + v_2 - \sigma_C^2 - \sigma_2^2.$$

Thus, H's expected payoff can be written as

$$\begin{aligned}&-\lambda\mathbb{E}\left[(a_2 - (X_C + X_2))^2\right] - (1-\lambda)\,\mathbb{E}\left[(a_2 - (X_C + X_1))^2\right]\\ =&v_C + (2\lambda - 1)v_2 - \sigma_C^2 - \lambda\sigma_2^2 - (1-\lambda)\,\sigma_1^2.\end{aligned}$$

Delegation to $\boldsymbol{A_1}$. A_1's optimal action is $a_1 = \beta_{12}\cdot\hat{\mu}_2 + \frac{\sigma_1^2}{\sigma_1^2+\sigma_{\varepsilon_1}^2}\cdot s_1$, with $\beta_{12} = \frac{v_C}{v_C+v_2}$ and some realization $\hat{\mu}_2$ of the conditional expectation μ_2.

Since the variables are uncorrelated, $\mathbb{E}[\mu_2 X_1] = 0$, $\mathbb{E}[\mu_2(X_C+X_2)] = \mathbb{E}[\mu_2^2] = var(\mu_2)$, and by the law of iterated expectations

$$\begin{aligned}\mathbb{E}[\mu_2 X_C] &= \mathbb{E}\left[\mathbb{E}[\mu_2 X_C]\,|\Theta_2 \in [\theta_{2,i-1},\theta_{2,i}]\right] = \mathbb{E}\left[\mu_2\mathbb{E}[X_C|\Theta_2 \in [\theta_{2,i-1},\theta_{2,i}]]\right]\\ &= \mathbb{E}\left[\mu_2\frac{Cov(X_C,\Theta_2)}{Var(\Theta_2)}\mu_2\right] = \frac{v_C}{v_C+v_2}\mathbb{E}\left[\mu_2^2\right] = \beta_{12}\,var(\mu_2).\end{aligned}$$

A_1's expected payoff is

$$-\mathbb{E}\left[(a_1 - (X_C + X_1))^2\right] = (\beta_{12})^2\,var(\mu_2) + v_1 - \sigma_C^2 - \sigma_1^2.$$

A_2's expected payoff is

$$-\mathbb{E}\left[(a_1 - (X_C + X_2))^2\right] = \beta_{12}\,(2-\beta_{12})\,var(\mu_2) - v_1 - \sigma_C^2 - \sigma_2^2.$$

Using equation (3) and rearranging, H's expected payoff can be written as

$$\begin{aligned}
&-\lambda\mathbb{E}\left[\left(a_1-(X_C+X_2)\right)^2\right]-(1-\lambda)\,\mathbb{E}\left[\left(a_1-(X_C+X_1)\right)^2\right]\\
=&\left(2\lambda\beta_{12}+(1-2\lambda)\,\beta_{12}^2\right)\frac{1}{2-\beta_{12}}(v_C+v_2)+(1-2\lambda)\,v_1-\sigma_C^2-\lambda\sigma_2^2-(1-\lambda)\,\sigma_1^2.
\end{aligned}$$

Centralization.

H's optimal action is $a_H=\beta_{H2}\cdot\hat{\mu}_2+\beta_{H1}\cdot\hat{\mu}_1$, with $\beta_{H2}=\frac{v_C+\lambda v_2}{v_C+v_2}$ and $\beta_{H1}=1-\lambda$ and some realizations $\hat{\mu}_2$ and $\hat{\mu}_1$ of the conditional expectations μ_2 and μ_1.

Note that $\mathbb{E}\left[\mu_1\left(X_C+X_2\right)\right]=0$ and $\mathbb{E}\left[\mu_1X_1\right]=\mathbb{E}\left[\mu_1^2\right]=var(\mu_1)$, and recall that $\mathbb{E}\left[\mu_2X_C\right]=\frac{v_C}{v_C+v_2}\,var(\mu_2)$.

A_1's expected payoff is

$$\begin{aligned}
&-\mathbb{E}\left[\left(a_H-(X_C+X_1)\right)^2\right]\\
=&-\left(\beta_{H2}\right)^2\mathbb{E}\left[\mu_2^2\right]-\left(\beta_{H1}\right)^2\mathbb{E}\left[\mu_1^2\right]-\sigma_C^2-\sigma_1^2-2\beta_{H2}\beta_{H1}\mathbb{E}\left[\mu_2\mu_1\right]\\
&+2\beta_{H2}\mathbb{E}\left[\mu_2X_C\right]+2\beta_{H1}\mathbb{E}\left[\mu_1X_1\right]\\
=&\beta_{H2}\left(2\frac{v_C}{v_C+v_2}-\beta_{H2}\right)var(\mu_2)+\beta_{H1}\left(2-\beta_{H1}\right)var(\mu_1)-\sigma_C^2-\sigma_1^2.
\end{aligned}$$

A_2's expected payoff is

$$\begin{aligned}
&-\mathbb{E}\left[\left(a_H-(X_C+X_2)\right)^2\right]\\
=&-\left(\beta_{H2}\right)^2\mathbb{E}\left[\mu_2^2\right]-\left(\beta_{H1}\right)^2\mathbb{E}\left[\mu_1^2\right]-\sigma_C^2-\sigma_2^2-2\beta_{H1}\beta_{H2}\mathbb{E}\left[\mu_1\mu_2\right]\\
&+2\beta_{H1}\mathbb{E}\left[\mu_1\left(X_C+X_2\right)\right]+2\beta_{H2}\mathbb{E}\left[\mu_2\left(X_C+X_2\right)\right]\\
=&\beta_{H2}\left(2-\beta_{H2}\right)var(\mu_2)-\left(\beta_{H1}\right)^2var(\mu_1)-\sigma_C^2-\sigma_2^2.
\end{aligned}$$

Using equation (3) and rearranging, H's expected payoff can be written as

$$\begin{aligned}
&-\lambda\mathbb{E}\left[\left(a_H-(X_C+X_2)\right)^2\right]-(1-\lambda)\,\mathbb{E}\left[\left(a_H-(X_C+X_1)\right)^2\right]\\
=&\left[(1-\lambda)\beta_{H2}\left(2\frac{v_C}{v_C+v_2}-\beta_{H2}\right)+\lambda(\beta_{H2}\left(2-\beta_{H2}\right))\right]var(\mu_2)\\
&+\left[(1-\lambda)\,\beta_{H1}\left(2-\beta_{H1}\right)-\lambda\left(\beta_{H1}\right)^2\right]var(\mu_1)-\sigma_C^2-\lambda\sigma_2^2-(1-\lambda)\,\sigma_1^2\\
=&\frac{1}{2-\beta_{H2}}\beta_{H2}^2(v_C+v_2)+(1-\lambda)^2\,\frac{1}{2-(1-\lambda)}v_1\\
&-\sigma_C^2-\lambda\sigma_2^2-(1-\lambda)\,\sigma_1^2.
\end{aligned}$$

$\square$

Lemma A.2 *There exist three unique cutoffs $\lambda', \lambda'', \lambda''' \in [0, 1)$ satisfying*
(i) H weakly prefers centralization over delegating to A_1 if and only if $\lambda \geq \lambda'$,
(ii) H weakly prefers delegating to A_2 over centralization if and only if $\lambda \geq \lambda''$,
(iii) H weakly prefers delegating to A_2 over delegating to A_1 if and only if $\lambda \geq \lambda'''$.

Proof of Lemma A.2. H's payoff gains under the three protocols of decision making are given in Lemma 3. Note that the payoff gains under delegation to A_1, $\tilde{\pi}_H^{del1}(\lambda)$, and to A_2, $\tilde{\pi}_H^{del2}(\lambda)$, are linear in λ, since β_{12} is independent of λ. The payoff gain under centralization, $\tilde{\pi}_H^{cen}(\lambda)$, is strictly convex in λ, since $2 \geq \beta$, and any expression $\frac{z^2}{k-z}$ is convex iff $k > z$.

(i) At $\lambda = 0$, $\tilde{\pi}_H^{cen}(0) = \tilde{\pi}_H^{del1}(0)$, while at $\lambda = 1$, $\tilde{\pi}_H^{cen}(1) > \tilde{\pi}_H^{del1}(1)$. Linearity of $\tilde{\pi}_H^{del1}(\lambda)$ and strict convexity of $\tilde{\pi}_H^{cen}(\lambda)$ imply that there exists at most one $\lambda \in (0, 1)$ such that $\tilde{\pi}_H^{cen}(\lambda) = \tilde{\pi}_H^{del1}(\lambda)$. If such λ exists define it as λ', otherwise set $\lambda' = 0$.

(ii) At $\lambda = 1$, $\tilde{\pi}_H^{cen}(1) = \tilde{\pi}_H^{del2}(1)$ and moreover, one can show that the derivatives satisfy $\frac{\partial}{\partial\lambda}\left(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del2}(\lambda)\right)\Big|_{\lambda=1} = v_2 \geq 0$. Linearity of $\tilde{\pi}_H^{del2}(\lambda)$ and strict convexity of $\tilde{\pi}_H^{cen}(\lambda)$ imply that there exists at most one $\lambda \in [0, 1)$ such that $\tilde{\pi}_H^{cen}(\lambda) = \tilde{\pi}_H^{del2}(\lambda)$. If such λ exists define it as λ''. Otherwise, if $v_2 > 0$ set $\lambda'' = 0$ and if $v_2 = 0$ set $\lambda'' = 1$.

(iii) For all $\lambda \in [0, 1]$, $\frac{\partial}{\partial\lambda}\left(\tilde{\pi}_H^{del2}(\lambda) - \tilde{\pi}_H^{del1}(\lambda)\right) = \frac{4v_1v_2 + 4v_2^2 + 2v_1v_C}{2v_2 + v_C} > 0$. Solving $\tilde{\pi}_H^{del1}(\lambda) = \tilde{\pi}_H^{del2}(\lambda)$ yields $\lambda = \frac{v_2(2v_2 - v_C) + v_1(2v_2 + v_C)}{4v_1v_2 + 4v_2^2 + 2v_1v_C}$ which can with simple algebra be simplified to $\lambda = \frac{1}{2} - \frac{1}{2}\frac{v_2v_C}{v_1(2v_2 + v_C) + 2v_2^2} \leq \frac{1}{2}$. Thus, if such λ exists with $\lambda > 0$, define it as λ''', otherwise set $\lambda''' = 0$. $\square$

Proof of Proposition 1. By Lemma 3: At $\lambda = 0$, $\tilde{\pi}_H^{cen}(0) = \tilde{\pi}_H^{del1}(0)$. At $\lambda = 1$, $\tilde{\pi}_H^{cen}(1) = \tilde{\pi}_H^{del2}(1)$ and $\frac{\partial}{\partial\lambda}\left(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del2}(\lambda)\right)\Big|_{\lambda=1} \geq 0$. The payoff gains under delegation to A_1, $\tilde{\pi}_H^{del1}(\lambda)$, and to A_2, $\tilde{\pi}_H^{del2}(\lambda)$, are linear in λ and the payoff gain under centralization, $\tilde{\pi}_H^{cen}(\lambda)$, is convex in λ. By Lemma A.2, the winners of the pairwise comparisons of the three protocols of decision-making are characterized by $\lambda', \lambda'', \lambda'''$.

Combining these insights yields that: Delegation to A_1 is optimal for $\lambda \leq \min\{\lambda', \lambda'''\}$. Delegation to A_2 is optimal for $\lambda \geq \max\{\lambda'', \lambda'''\}$. Centralization is optimal for $\lambda = 1$, for $\lambda \in [\min\{\lambda', \lambda'''\}, \max\{\lambda'', \lambda'''\}]$ if the interval is non-empty, and for $\lambda = 0$ if $\lambda'' > 0$ (if $\lambda'' = 0$ delegation to A_2 is optimal for $\lambda = 0$).

The upper envelope is thus characterized by two intersections $\lambda_1 := \min\{\lambda', \lambda'''\}$ and $\lambda_2 := \max\{\lambda'', \lambda'''\}$, which can be equal to 0 and 1. Hence, delegation to A_1 is optimal if and only if $\lambda \in [0, \lambda_1]$ and $\lambda_2 > 0$, centralization is optimal if and only if $\lambda \in [\lambda_1, \lambda_2] \cup \{1\}$ or ($\lambda = 0$ and $\lambda_2 > 0$), and delegation to A_2 is optimal if and only if $\lambda \in [\lambda_2, 1]$. □

Proof of Proposition 2. By Lemma 3, the payoff comparison between delegation to A_1 and delegation to A_2 is equivalent to

$$(1-2\lambda)v_1 = \frac{v_2(v_C - 2v_2(1-2\lambda))}{2v_2 + v_C}.$$

The solution to this indifference is given by $\lambda^{del} = \frac{1}{2} - \frac{1}{2}\frac{v_2 v_C}{v_1(2v_2+v_C)+2v_2^2}$. Taking derivatives yields

$$\frac{\partial \lambda^{del}}{\partial v_C} = -\frac{v_2^2(v_1+v_2)}{(2v_1v_2+2v_2^2+v_1v_C)^2} \leq 0,$$

$$\frac{\partial \lambda^{del}}{\partial v_1} = \frac{v_2v_C(2v_2+v_C)}{2(2v_1v_2+2v_2^2+v_1v_C)^2} \geq 0,$$

$$\frac{\partial \lambda^{del}}{\partial v_2} = \frac{v_C(2v_2^2 - v_1v_C)}{2(2v_1v_2+2v_2^2+v_1v_C)^2} \leq 0 \text{ for } v_1 \leq 2\frac{v_2^2}{v_C}.$$

□

Proof of Proposition 3. For $\lambda = 0$, the coefficients are $\beta_{H2} = \beta_{12} = \frac{v_C}{v_C+v_2}$ and $\beta_{H1} = 1$, and the payoffs given in Lemma 3 simplify to

$$\begin{aligned}
\tilde{\pi}_H^{del2} &= v_C - v_2, \\
\tilde{\pi}_H^{del1} &= v_1 + \beta_{12}^2 \frac{1}{2-\beta_{12}}(v_C+v_2) = v_1 + \frac{v_2^2}{2v_2+v_C}, \\
\tilde{\pi}_H^{cen} &= \beta_{H2}^2 \frac{1}{2-\beta_{H2}}(v_C+v_2) + \frac{1}{2-\beta_{H1}} v_1 = \tilde{\pi}_H^{del1}.
\end{aligned}$$

Comparison of the payoffs shows that centralization is preferred to delegation to A_2 if and only if

$$v_1 + \frac{v_2^2}{2v_2 + v_C} \geq v_C - v_2.$$

Simplification yields the expression in the statement. $\square$

Proof of Proposition 4. For $\lambda = \frac{1}{2}$, the coefficients are $\beta_{H2} = \frac{v_C + \frac{v_2}{2}}{v_C + v_2}$, $\beta_{12} = \frac{v_C}{v_C + v_2}$, and $\beta_{H1} = \frac{1}{2}$, and the payoffs given in Lemma 3 simplify to

$$\begin{aligned}
\tilde{\pi}_H^{del2} =& v_C, \\
\tilde{\pi}_H^{del1} =& \frac{1}{2 - \beta_{12}} \beta_{12}(v_C + v_2) = \frac{v_C(v_C + v_2)}{2v_2 + v_C}, \\
\tilde{\pi}_H^{cen} =& \beta_{H2}^2 \frac{1}{2 - \beta_{H2}} (v_C + v_2) + \frac{1}{4} \frac{1}{2 - \beta_{H1}} v_1 = \frac{v_1}{6} + \frac{\left(v_C + \frac{v_2}{2}\right)^2}{v_C + v_2 \frac{3}{2}}.
\end{aligned}$$

Comparing the payoffs of delegation to A_1 and delegation to A_2, it is immediate that delegation to A_2 is always preferred.

The payoff of centralization is higher than the payoff of delegation to A_2 if and only if

$$\frac{v_1}{6} + \frac{\left(v_C + \frac{v_2}{2}\right)^2}{v_C + v_2 \frac{3}{2}} \geq \frac{v_C(v_C + v_2)}{2v_2 + v_C}$$

which is equivalent to the expression in the statement. $\square$

Proof of Proposition 5. Case $v_C = 0$.

Consider the payoff difference between centralization and delegating to A_2. The difference can be written as

$$\begin{aligned}
\left(\tilde{\pi}_H^{cen} - \tilde{\pi}_H^{del2}\right)\Big|_{v_C=0} &= \frac{(1-\lambda)^2}{1+\lambda} v_1 + \frac{\lambda^2}{2-\lambda} v_2 + (1 - 2\lambda) v_2 \\
&= (1-\lambda) \left(\frac{(1-\lambda)}{1+\lambda} v_1 + \frac{2 - 3\lambda}{2-\lambda} v_2 \right).
\end{aligned}$$

At $\lambda = 0$, the difference is $v_1 + v_2$. At $\lambda = 1$, the difference is zero. For $v_1 = 0$, the difference simplifies to $\frac{2-3\lambda}{2-\lambda}(1-\lambda)v_2$ and thus $\lambda_2 = \frac{2}{3}$. Taking the limit for $v_1 \to \infty$ yields $\lambda_2 = 1$. For $v_2 = 0$, the difference simplifies to $\frac{(1-\lambda)^2}{1+\lambda}v_1$ and thus $\lambda_2 = 1$. Taking the limit for $v_2 \to \infty$ yields $\lambda_2 = \frac{2}{3}$.

Apply the implicit function theorem to the term in brackets, to see that λ_2 is monotonic. Note that the derivatives of the term are: with respect to v_1 equal to $\frac{(1-\lambda)}{1+\lambda} > 0$; with respect to v_2 equal to $\frac{2-3\lambda}{2-\lambda} < 0$ iff $\lambda > \frac{2}{3}$; and with respect to λ equal to $2\left(\frac{1}{(1+\lambda)^2}v_1 + \frac{2}{(2-\lambda)^2}v_2\right) > 0$. Thus λ_2 is increasing in v_1 and λ_2 is decreasing in v_2 for $\lambda > \frac{2}{3}$. Monotonicity implies that $\lambda_2 \in [\frac{2}{3}, 1]$.

Case $v_C = \infty$.

Since $\tilde{\pi}_H^{cen}(0) = \tilde{\pi}_H^{del1}(0)$, it holds that $\lambda_2 > 0$ if and only if $\tilde{\pi}_H^{cen}(0) > \tilde{\pi}_H^{del2}(0)$. In the limit,

$$\lim_{v_C \to \infty}(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del2}(\lambda)) = (1-\lambda)\frac{(v_1(1-\lambda) - v_2(1+\lambda))}{(1+\lambda)}.$$

For $\lambda = 0$, this reduces to $v_1 - v_2$. Thus, $\lambda_2 > 0$ for $v_1 > v_2$.

The solution to $\lim_{v_C \to \infty}(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del2}(\lambda)) = 0$ is $\lambda_2^{C=\infty} = \frac{v1-v2}{v2+v1}$. The derivatives of $\lambda_2^{C=\infty}$ with respect to v_1 and v_2 are $\frac{2v_2}{(v_2+v_1)^2} \geq 0$ and $-\frac{2v_1}{(v_2+v_1)^2} \leq 0$, respectively.

Comparison of $\lambda_2^{C=0}$ and $\lambda_2^{C=\infty}$.

The payoff difference between centralization minus delegation to A_2 simplifies for $v_C = 0$ to

$$(1-\lambda)\left(\frac{(1-\lambda)}{1+\lambda}v_1 + \frac{2-3\lambda}{2-\lambda}v_2\right)$$

and for $v_C = \infty$ to

$$(1-\lambda)\frac{v_1(1-\lambda) - v_2(1+\lambda)}{(1+\lambda)}.$$

The difference between these expressions is always positive: $\frac{2-3\lambda}{2-\lambda}v_2 - (-v_2) \geq 0$.

For completeness, consider now λ_1.

Case $v_C = 0$. Consider the payoff difference between centralization and delegating to A_1. Note that $\beta_{12} = 0$ and $\beta_{H2} = \lambda$. The difference can be written as

$$\left(\tilde{\pi}_H^{cen} - \tilde{\pi}_H^{del1}\right)\big|_{v_C=0} = \frac{(1-\lambda)^2}{1+\lambda}v_1 + \frac{\lambda^2}{2-\lambda}v_2 - (1-2\lambda)v_1$$
$$= \lambda\left(-\frac{1-3\lambda}{1+\lambda}v_1 + \frac{\lambda}{2-\lambda}v_2\right).$$

At $\lambda = 0$, the difference is zero. At $\lambda = 1$, the difference simplifies to $v_1 + v_2$. For $v_1 = 0$, the difference simplifies to $\frac{\lambda^2}{2-\lambda}v_2$ and thus $\lambda_1 = 0$. The limit for $v_1 \to \infty$ yields $\lambda_1 = \frac{1}{3}$. For $v_2 = 0$, the difference simplifies to $-\frac{1-3\lambda}{1+\lambda}v_1$ and thus $\lambda_1 = \frac{1}{3}$. The limit for $v_2 \to \infty$ yields $\lambda_1 = 0$.

Apply the implicit function theorem to the term in brackets to see that λ_1 is monotonic. Note that the derivatives of the term are: with respect to v_1 equal to $-\frac{1-3\lambda}{1+\lambda} < 0$ iff $\lambda < \frac{1}{3}$; with respect to v_2 equal to $\frac{\lambda}{2-\lambda} > 0$; and with respect to λ equal to $2\left(\frac{2}{(1+\lambda)^2}v_1 + \frac{1}{(2-\lambda)^2}v_2\right) > 0$. Thus λ_1 is increasing in v_1 for $\lambda < \frac{1}{3}$ and λ_1 is decreasing in v_2. Monotonicity implies that $\lambda_1 \in [0, \frac{1}{3}]$.

Case $v_C = \infty$. It holds that $\lambda_1 > 0$ if and only if $\frac{\partial}{\partial\lambda}\left(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del1}(\lambda)\right)\big|_{\lambda=0} < 0$. In the limit,

$$\lim_{v_C\to\infty}(\tilde{\pi}_H^{cen}(\lambda) - \tilde{\pi}_H^{del1}(\lambda)) = \lambda\frac{v_2(1+\lambda) - v_1(1-3\lambda)}{(1+\lambda)}.$$

The derivative with respect to λ equals $v_2 + \frac{v_1(-1+3\lambda(2+\lambda))}{(1+\lambda)^2}$, and at $\lambda = 0$ simplifies to $v_2 - v_1$. Thus, $\lambda_1 > 0$ for $v_1 > v_2$. $\square$

Proof of Lemma 4. The headquarters' payoffs for the three institutions are given in Lemma 3.

(i) The headquarters' payoffs under all institutions are strictly increasing in v_C.

(ii) Under centralization, H's payoff is strictly increasing in A_1's private interest information. The optimal choice of precision is thus $\sigma^2_{\varepsilon_1} = 0$, implying $v_1^* = \sigma_1^2$ for all λ.

The gain from delegating to A_1 is strictly increasing in v_1 for $\lambda < \frac{1}{2}$ and strictly decreasing in v_1 for $\lambda > \frac{1}{2}$. The optimal choice of precision is thus for $\lambda < \frac{1}{2}$, $\sigma^2_{\varepsilon_1} = 0$ implying $v_1^* = \sigma_1^2$, and for $\lambda > \frac{1}{2}$, $\sigma^2_{\varepsilon_1} = \infty$ implying $v_1^* = 0$.

The gain from delegating to A_2 is independent of v_1.

(iii) The gain from delegating to A_2 is strictly decreasing in v_2 for $\lambda < \frac{1}{2}$ and strictly increasing in v_2 for $\lambda > \frac{1}{2}$. The optimal choice of precision is thus for $\lambda < \frac{1}{2}$, $\sigma^2_{\varepsilon_2} = \infty$ implying $v_2^* = 0$, and for $\lambda > \frac{1}{2}$, $\sigma^2_{\varepsilon_2} = 0$ implying $v_2^* = \sigma_2^2$.

The gain from delegating to A_1 is strictly decreasing in v_2 for $\lambda < 1$. The optimal choice of precision is thus $\sigma^2_{\varepsilon_2} = \infty$, implying $v_2^* = 0$ for all λ.

The derivative of H's payoff under centralization with respect to A_2's private interest information is given by $\frac{\partial \tilde{\pi}_H^{cen}}{\partial v_2} = \frac{(v_C + v_2\lambda)(v_2(2-\lambda) - v_C(2-3\lambda))}{(v_C + v_2(2-\lambda))^2}$. The threshold $\hat{\lambda}$ is the solution to $\frac{\partial \tilde{\pi}_H^{cen}}{\partial v_2} = 0$, or equivalently $\hat{\lambda} = \frac{2v_2 + 3v_C - \sqrt{(2v_2)^2 + 4v_2 v_C + (3v_C)^2}}{2v_2}$. For pure private interest information, $v_C = 0$, this yields $\hat{\lambda} = 0$. For infinite common interest information, $v_C \to \infty$, this yields $\hat{\lambda} = \frac{2}{3}$. The optimal precision is $\sigma^2_{\varepsilon_2} = 0$ if $\lambda > \hat{\lambda}$ and $\sigma^2_{\varepsilon_2} = \infty$ if $\lambda < \hat{\lambda}$. □

Proof of Lemma 5. For $\lambda \leq \frac{1}{2}$, by Lemma 4, A_1 learns signal s_1 noise free and A_2 learns s_C noise free and s_2 is pure noise. Then, by Lemma 3, delegation to A_1 is preferred over delegation to A_2 for $\lambda \leq \frac{1}{2}$, which is satisfied by assumption.

For $\frac{1}{2} \leq \lambda \leq \hat{\lambda}^{del1}$, by Lemma 4, A_1's signal is pure noise, A_2 learns s_C noise free and s_2 is pure noise. Then by Lemma 3, delegation to A_2 is preferred over delegation to A_1 for $\lambda \geq \frac{1}{2}$, which is satisfied by assumption.

For $\hat{\lambda}^{del1} \leq \lambda$, by Lemma 4, A_1's signal is pure noise and A_2 learns s_C as well as s_2 noise free. Then by Lemma 3, delegation to A_2 is preferred over delegation to A_1 for $\lambda \geq \frac{1}{2} - \frac{v_C}{4v_2}$, which is satisfied as $\lambda \geq \hat{\lambda}^{del1} > \frac{1}{2} - \frac{v_C}{4v_2}$. □

Proof of Proposition 6. Consider first the threshold λ_1 between centralization and delegation to A_1. By Lemma 5, only the range $\lambda \in [0, 1/2]$ needs to be considered for λ_1. For infinite common interest information $v_C = \infty$, by Lemma 4, under delegation to A_1, A_1 observes a noise free signal and A_2 observes pure noise; likewise under centralization, A_1 observes a noise free signal and A_2 observes pure noise. The comparison is thus identical to the case of exogenous signal precision and $\lambda_1^{endo} = \frac{1}{3} = \lambda_1^{C=\infty}$.

Consider now the threshold λ_2 between centralization and delegation to A_2. By Lemma 5, only the range $[1/2, 1]$ needs to be considered. For infinite common interest information $v_C = \infty$, by Lemma 4, under delegation to A_2, A_2 observes a noise free signal; under centralization, A_1 observes noise free signal, A_2 either observes a noise free signal or pure noise, depending on the size of λ relative to $\hat{\lambda}^{cen}$. By Lemma 3, for these parameter values with $\lambda \geq \hat{\lambda}^{cen}$, the comparison reduces to the comparison for exogenous signal precision. Thus the payoff of centralization minus the payoff of delegation to A_2 is equal to $v_1 \frac{(1-\lambda)^2}{1+\lambda} - v_2(1-\lambda)$, thus $\hat{\lambda}_2^{endo} = \lambda_2^{C=\infty}$.

By Lemma 3, for these parameter values with $\frac{1}{2} \leq \lambda \leq \hat{\lambda}^{cen}$, the payoff of centralization minus the payoff of delegation to A_2 equals $v_1 \frac{(1-\lambda)^2}{1+\lambda} - v_2(2\lambda - 1)$. Calculating the zero yields $\check{\lambda}_2^{endo} = \frac{-v_2 - 2v1 + \sqrt{v_2}\sqrt{9v_2 + 8v_1}}{4v_2 - 2v_1}$.

Comparing the conditions from which the thresholds $\hat{\lambda}_2^{endo}$ and $\check{\lambda}_2^{endo}$ are derived, one can see that $v_1 \frac{(1-\lambda)^2}{1+\lambda} - v_2(1-\lambda)$ is equal to $v_1 \frac{(1-\lambda)^2}{1+\lambda} - v_2(2\lambda - 1)$ if $(1-\lambda) = (2\lambda - 1)$. Hence the expressions are equal for $\hat{\lambda}^{cen} = \frac{2}{3}$. For $\frac{1}{2} \leq \lambda \leq \hat{\lambda}^{cen}$, it holds that $(1 - \lambda) \leq (2\lambda - 1)$, implying that $\hat{\lambda}_2^{endo} \leq \check{\lambda}_2^{endo}$. Thus there is more centralization with endogenous signal precision compared to the case of exogenous signal precision.

$\square$

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