# A comprehensive revealed preference approach to approximate utility maximisation

Paweł Dziewulski

Department of Economics
University of Sussex
pawel-dziewulski.com

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# Part I

# Preliminaries

# Basic notation

- The space of alternatives is denoted by $X$.
- A menu is a non-empty subset $A \subseteq X$.
- A dataset $\mathcal{O}$ is a finite set of pairs $(A, x)$, where $x \in A \subseteq X$.

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# Choice models and rationalisation

A choice model is a correspondence $c : 2^X \rightrightarrows X$ such that $c(A) \subseteq A$.

The choice model $c$ rationalises the dataset $\mathcal{O}$ if

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# Choice monotonicity

Let $\rhd$ be a strict preorder on $X$ (irreflexive & transitive).

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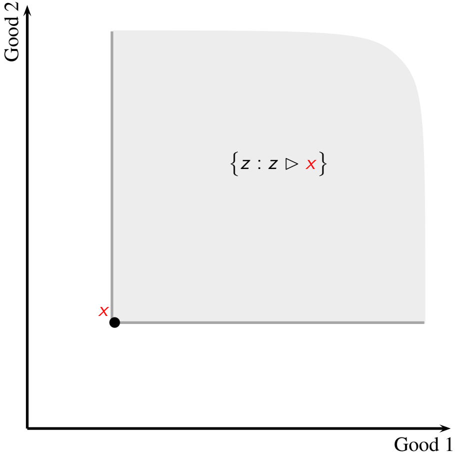
# Choice monotonicity: Examples




$$z \rhd x \quad \text{if} \quad z > x$$

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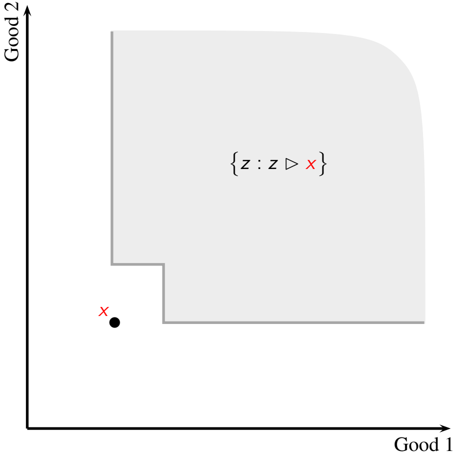


$$z \rhd x \quad \text{if} \quad z \geq x \text{ and } z_i / x_i \geq \lambda_i, \text{ for some } i$$

# Part II

## Classic revealed preference theory

# Utility maximisation

Given a function $u : X \to \mathbb{R}$, the utility maximisation choice model is

$$c(A) := \Big\{ x \in A : u(x) \geq u(y), \text{ for all } y \in A \Big\}.$$

The model $c$ is $\rhd$-monotone iff $x \rhd y$ implies $u(x) > u(y)$.

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# Revealed utility maximisation

Under what conditions on $\mathcal{O}$ there is a function $u : X \to \mathbb{R}$ such that

$$(A, x) \in \mathcal{O} \;\text{ implies }\; x \in c(A),$$

where the correspondence $c$ is $\rhd$-monotone and given by

$$c(A) \;:=\; \Big\{x \in A : u(x) \geq u(y),\ \text{for all } y \in A\Big\}?$$

# Revealed preference relations

- **Directly revealed preference relation:**

$$xR^*y \quad \text{if} \quad (A, x) \in \mathcal{O} \text{ and } y \in A.$$

- **Directly revealed strict preference relation:**

$$xP^*y \quad \text{if} \quad (A, x) \in \mathcal{O} \text{ and } z \rhd y, \quad \text{for some } z \in A.$$

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# Generalised axiom of revealed preference

The dataset $\mathcal{O}$ is rationalisable with utility maximisation only if

$$xR^*y \text{ implies } u(x) \geq u(y), \text{ and } xP^*y \text{ implies } u(x) > u(y).$$

**Definition (GARP)**

There is no sequence of alternatives $z^1, z^2, \ldots, z^n$ such that $z^i R^* z^{i+1}$ or $z^i P^* z^{i+1}$, for all $i = 1, \ldots, (n-1)$, and $z^n P^* z^1$.

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# Part III

# Revealing non-transitive indifferences

# Acyclicity of strict preference

The directly revealed strict preference relation $P^*$ is acyclic if there is no sequence of alternatives $z^1, z^2, \ldots, z^n$ such that

$$z^1 P^* z^2, \; z^2 P^* z^3, \; \ldots \;, \; z^{n-1} P^* z^n, \; \text{and } z^n P^* z^1.$$

# The main result

Take any dataset $\mathcal{O}$ and a strict partial order $\rhd$.*

**Theorem**

*The following statements are equivalent.*

(i) *$P^*$ is acyclic.*

(ii) *There is a utility $u$ and a threshold function $\delta$ such that*

$$c(A) := \Big\{x \in A : u(x) + \delta(z) \geq u(z), \text{ for all } z \in A\Big\},$$

*is $\rhd$-monotone and rationalises the dataset $\mathcal{O}$.*

# Part IV

## Conclusion

# The main take-away

**Approximate utility maximisation = Non-transitive indifferences**

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