Optimal Contracts to Prevent Corruption in Auctions

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July 25, 2022

Abstract

We develop a model of a sealed-bid first-price auction where the auctioneer acting on behalf of the seller may be involved in corruption. She is offered a bribe from one of the participants and can allow him to resubmit his bid. We show that the seller often wants to offer the auctioneer a remuneration scheme that depends on the collected revenue and prevents the auctioneer from accepting the bribe even if the legal anti-corruption institutions are weak. We consider both limited and unlimited liability as well as rational and myopic bidders.

Keywords: Auctions; Bribery; Corruption Prevention; Optimal Contract.

JEL Classification Codes: D44, D73, D86, K42.

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1 Introduction

> Corruption is an insidious plague that has a wide range of corrosive effects on societies. It undermines democracy and the rule of law, leads to violations of human rights, distorts markets, erodes the quality of life and allows organized crime, terrorism and other threats to human security to flourish.

So begins the *United Nations Convention Against Corruption (UNCAC)*[1], as written in 2003, and as for today ratified by 183 countries. It shows that corruption is a big concern in the entire world. And this convention engages all parties to fight against corruption. However, it is still a widespread practice, and not all countries are equally effective in their fight against corruption. Despite an international agreement that corruption needs to be fought, despite countries committing to having legal bodies that aim to fight against corruption, the Corruption Perception Index published by Transparency International suggests that we are still far from preventing corruption, especially in Africa, Latin America, Eastern Europe, and Asia.

In particular, corruption in auctions is a widely spread practice. The Chartered Institute of Building, a UK professional association of the construction industry, reported in 2013 that 35% of the respondents had been offered a bribe at least once, and 38% had come across cartel activity in the UK construction industry on at least one occasion, although the UK appears to be one of the less corrupt countries according to the Transparency International CPI[2].

In this paper, we study contracting mechanisms that help to prevent corruption in auctions when the legal institutions are not strong enough. We consider a situation

[1]http://www.unodc.org/documents/treaties/UNCAC/Publications/Convention/08-50026_E.pdf

[2]Check http://www.ciob.org/media-centre/news/no-decrease-corruption-survey-claims-bribery-act-ineffective for the full report

where the seller delegates a sale of a single indivisible good to the auctioneer. If the seller does not need an auctioneer, then the problem of corruption is less relevant because the seller can maximize her revenue using the prominent Myerson's optimal auction (Myerson 1981). Thus, the moral hazard problem arising from the fact that the auctioneer is not the residual claimant of the collected revenue is a crucial part of our model.

We assume that the auctioneer runs a sealed-bid first-price auction. Corruption can take place between the auctioneer and one of the bidders who can give a bribe to the auctioneer in order to see the other bids and to adjust his own bid. We assume that there exists a legal framework aimed at preventing corruption. As we have already mentioned, such a legal framework seems to be ineffective in many countries. Hence, we assume that in addition to the legal anti-corruption framework, the seller can also try to prevent corruption herself by offering a remuneration scheme to the auctioneer that specifies her payment depending on the collected revenue. We show that a properly chosen remuneration scheme can effectively prevent corruption at no cost for the seller, even without any legal framework. The seller proposes a compensation scheme $w(R)$ that pays the auctioneer a fraction of the difference between the realized auction revenue R and the expected revenue of the no corruption case $\mathbb{E}R^{NC}$, i.e., $w(R) = \alpha(R - \mathbb{E}R^{NC})$.

Next, we consider the more complicated case when the auctioneer is protected by limited liability. It does not allow to punish the auctioneer when the collected revenue is low. Thus, the seller has to construct a compensation scheme that pays non-negative wage to the auctioneer in any case, i.e. $w(R) \geq 0$ for any revenue R. We show that the seller, despite having to pay only positive wages, still can often prevent corruption (and finds this profitable) even if the institutions are weak. It is easier to prevent corruption when other bidders are fully rational and become less aggressive in the bidding when they are aware of the fact that one of the bidders can engage in corruption with the auctioneer. However, even if the bidders are myopic and bid with the same aggression, the seller still is often able to prevent corruption when the legal framework alone is not sufficient.

There is already a wide literature on corruption, exploring the ways it can occur and how it can be prevented, e.g., Laffont and Tirole (1989), Laffont and Tirole (1991), Acemoglu and Verdier (2000). Burguet *et al.* (2016) provide a thorough review of the literature on corruption. Corruption in auctions is theoretically studied in Celentani and Ganuza (2002), Burguet and Che (2004), Compte *et al.* (2005), Menezes and Monteiro (2006), Burguet and Perry (2007), Koc and Neilson (2008), Arozamena and Weinschelbaum (2009), Burguet and Perry (2009), Lengwiler and Wolfstetter (2010), and Knyazev (2020). This papers mostly use a positive approach describing what can happen if corruption takes place. We complement the literature on corruption in auctions more from the contract theory perspective using a normative approach. Our main question is not what happens if corruption takes place, rather, how the auctioneer has to be compensated in order to prevent corruption.

We model corruption through the leakage of bid information by the auctioneer in exchange for a bribe. Using the data from Russian procurement auctions, Andreyanov et al. (2018) show that more than 10% of auctions were affected by bid leakage. Thus, their findings support our way of modeling corruption. Their empirical test uses the fact that the preferred bidder waits till the end of the auction to get the information about other bids. Hence, he is often the last bidder chronologically. In our paper, we do not try to detect corruption. Instead, we characterize how the seller can provide incentives to not engage in corruption even if corruption is not detectable by the seller.

The closest to our paper is Arozamena and Weinschelbaum (2009). They study the behavior of bidders in a first-price auction held by the auctioneer on behalf of the seller. The main difference is that they assume that corruption takes place anyway and characterize an equilibrium in this case. Based on their results, we make further steps, first, by endogenizing corruption via the optimal decision of whether to offer and accept the bribe by the bidder and the auctioneer respectively and, second, showing how it can be prevented by the seller. Thus, our model complements the analysis of Arozamena and Weinschelbaum (2009). To our knowledge, it is a first attempt to study the contracting between the seller and the auctioneer in the presence of corruption.

While we concentrate on corruption in auctions that takes place between a seller and one of the buyers, there is an important literature on the collusion among buyers, e.g., Porter and Zona (1993), Kawai and Nakabayashi (2022), and Kawai *et al.* (2022). In our paper, we do not consider this type of collusion and concentrate only on the collusion between the seller and one of the buyers.

2 Setting

There is one seller who hires an auctioneer to sell a good using a first-price auction. There are N bidders, each bidder has an independent private value v_i coming from some continuously differentiable distribution F with the support $[0, \overline{v}]$. All parties are considered to be risk-neutral. The outside utility of all bidders and the auctioneer is zero.[3].

[3] If the outside option of the auctioneer is strictly positive we may assume that there is some additional fixed compensation that just covers that outside option. It does not qualitatively change our results.

2.1 No Corruption

The analysis in the case when there is no corruption is completely standard. Denote $b_i(v_i)$ as the bid of a bidder i with value v_i. If there is no corruption, it is well-known (e.g., see Krishna 2009) that the best strategy for each bidder is to bid according to the same bidding function:

$$b_i(v_i) = \beta(v_i) := \mathbb{E}[Y_{(N-1)}|Y_{(N-1)} < v_i], \tag{1}$$

where $Y_{(N-1)}$ is the highest of $N-1$ independently drawn values. Let R^{NC} be a random variable denoting the ex-post revenue without corruption. Then, $R^{NC} = \beta(\max\{v_j\}_{j=1}^N)$. Thus, the expected revenue is

$$\mathbb{E}R^{NC} = \mathbb{E}_{\mathbf{v}}[\mathbb{E}[Y_{(N-1)}|Y_{(N-1)} < \max\{v_j\}_{j=1}^N]]. \tag{2}$$

Each participant's ex-post utility is:

$$\begin{aligned} U^{NC}(v_i) &= v_i - \beta(v_i) \text{ if } v_i = \max\{v_j\}_{j=1}^N, \\ &= 0 \text{ otherwise.} \end{aligned} \tag{3}$$

This leads to the ex interim utility of each participant $\mathbb{E}(U^{NC}(v_i)) = F^{N-1}(v_i)(v_i - \beta(v_i))$, which, in turn, implies that the ex ante utility of each bidder before learning his private value is

$$\mathbb{E}U^{NC} = \mathbb{E}_{v_i}F^{N-1}(v_i)(v_i - \beta(v_i)).$$

The auctioneer's utility is normalized to zero. The seller's utility is equal to the expected revenue.

2.2 Corruption

Now, we assume that bidder N can give a bribe to the auctioneer before knowing his value for the good in order to get the information about the other participants' bids before submitting his own. The timing is as follows. Bidder N decides on the size of a bribe $B \in \mathbb{R}_+$ offered to the auctioneer. He can also choose not to offer any bribe at all. Then all bidders learn their values and, afterward, bid according to their bidding strategies. If bribe B is accepted by the auctioneer, bidder N can observe all other bids and submit his bid afterward. In the end, all bids are revealed, and the bidder with the highest bid wins and pays his own bid. We say that corruption takes place if the auctioneer accepts a bribe and allows bidder N to see other bids.

If the auctioneer does not accept a bribe, bidder N cannot learn the opponents' bids, and the analysis is the same as before. However, if the bribe is accepted, the game that bidders play changes. Besides that, the behavior of honest bidders may depend on whether they are aware of the fact of corruption or not. We start our analysis with the case of myopic bidders.

3 Myopic Bidders

For now, assume that other bidders are not aware of corruption and do not change their bidding behavior if corruption takes place. Thus, every bidder except bidder N still follows bidding strategy $\beta(v_i)$. Bidder N, however, will bid according to the following equilibrium strategy[4]:

$$
\begin{aligned}
b_N(v_N, \mathbf{v}_{-N}) &= \beta(\max\{v_i\}_{i=1}^{N-1}) \text{ if } v_N \geq \beta(\max\{v_i\}_{i=1}^{N-1}), \\
&= 0 \text{ if } v_N < \beta(\max\{v_i\}_{i=1}^{N-1}),
\end{aligned}
$$

where $\mathbf{v}_{-N}$ is a vector of opponents' bids that bidder N can use in his strategy. Thus, bidder N matches the value of the highest bid if his own value v_N is above this bid[5]. The expected revenue of the seller in the case of corruption with myopic bidders R^C is thus the expectation of $\beta(\max\{v_i\}_{i=1}^{N-1})$:

$$
\mathbb{E}R^C = \mathbb{E}\beta(\max\{v_i\}_{i=1}^{N-1}).
$$

The ex-post utility of bidder N is

$$
\begin{aligned}
U_N^C(v_N, \mathbf{v}_{-N}) &= v_N - \beta(\max\{v_i\}_{i=1}^{N-1}) \text{ if } v_N > \beta(\max\{v_i\}_{i=1}^{N-1}), \\
&= 0 \text{ otherwise.}
\end{aligned}
$$

Let $\phi(b)$ denote the inverse function of β. Then, the ex interim utility of bidder N equals $\mathbb{E}_{\mathbf{v}_{-N}}(U_N^C(v_N,, \mathbf{v}_{-N})) = \int_0^{\phi(v_N)}(v - \beta(v))(N-1)F^{N-2}(v)f(v)dv$, which implies the following expected utility of bidder N from participating in the auction where he can see the bids of the opponents:

$$
\mathbb{E}U_N^C = \mathbb{E}_{v_N} \int_0^{\phi(v_N)} (v - \beta(v))(N-1)F^{N-2}(v)f(v)dv.
$$

[4]This is not a unique equilibrium strategy. If $v_N < \beta(\max\{v_i\}_{i=1}^{N-1})$,bidder N could make any bid below $\beta(\max\{v_i\}_{i=1}^{N-1})$ and still lose an auction. Thus, all equilibrium strategies are outcome equivalent.

[5]We assume here that bidder N gets the good in the case of a tie. However, bidder N could make himself a strict winner by bidding ε more.

Bidder N naturally has a higher expected utility if he can see the opponents' bids in the case when they are myopic, i.e., $\mathbb{E}U_N^C(v_N) - \mathbb{E}U^{NC}(v_N) > 0$ for any $v_N \in (0, \overline{v}]$. Indeed, other bidders do not change their behavior, and bidder N obtains a chance to mimic the highest bid. This implies that the difference between ex-ante utilities is also positive as it represents the expectation of the difference of ex-interim utilities, i.e.

$$\mathbb{E}U_N^C - \mathbb{E}U^{NC} > 0.$$

Thus, the highest bribe $\overline{B}$ that bidder N could offer to the auctioneer is equal to this expected information rent that he can get from learning the opponents' values. Thus,

$$\overline{B} = \mathbb{E}U_N^C - \mathbb{E}U^{NC}.$$

4 Legal Anti-Corruption Framework

Corruption is illegal by nature. 183 countries ratified the UNCAC stating that "*Each State Party shall [...] develop and implement or maintain effective, coordinated anti-corruption policies*". It can be, for example, the implementation of a fine and the creation of an entity aimed at detecting corruption, firing and putting into prison a corrupt bureaucrat, prohibiting a corrupt politician from participating in future elections, etc.

We suppose that a country offers a legal framework consisting of a fine f that an auctioneer has to pay when she is caught on accepting a bribe. The auctioneer who accepts a bribe is detected with probability q. Thus, the auctioneer pays $p = qf$ on average when she engages in corruption. The auctioneer has to choose whether to accept a bribe of bidder N with the risk of being caught or to reject a bribe. She accepts it as long as this bribe is larger than the expected punishment, i.e., $B \geq p$. Therefore, to be an effective way to prevent corruption, we need that the largest bribe the auctioneer could get from one participant does not exceed the expected punishment p. Thus, we get the following requirement on p in order to prevent corruption with an anti-corruption policy:

$$p > \overline{B}.$$

5 Optimal Contracts to Prevent Corruption

The government measures enable to prevent corruption to a certain extent, depending on the intensity of the punishment, reflected in f, whenever corruption is detected, and the quantity of effort put in place to detect it, reflected in q. In many countries, the institutions are weak, so corruption is not detected with sufficiently high probability. Even if it is detected, the involved parties can escape the prosecution which leads to low values of p. In this section, we show that the seller can often prevent the auctioneer from accepting a bribe by proposing a proper compensation scheme to the auctioneer even when the institutions are weak. Thus, this section deals with the legal institutions that do not prevent corruption, i.e., $p < \overline{B}$.

The seller wants to design a payment scheme for the auctioneer so that she has no incentives to accept the bribe. We assume that the seller cannot observe whether corruption takes place or not and can condition the auctioneer's payment w only on the observable and verifiable revenue collected in the auction. Thus, the seller specifies a payment scheme as a function $w : \mathbb{R}_+ \to \mathbb{R}$ mapping any collected revenue R to the payment $w(R)$ that the auctioneer receives.

Then, the auctioneer rejects the bribe offered by bidder N if her expected payment in the case of no corruption $\mathbb{E}(w(R^{NC}))$ is larger than the expected payment in the case of corruption $\mathbb{E}(w(R^{C}))$ plus the received bribe B minus the expected punishment p, i.e.,

$$\mathbb{E}(w(R^{NC})) \geq \mathbb{E}(w(R^{C})) + B - p.$$

The seller wants to minimize the expected payment to the auctioneer subject to the constraint that the seller does not accept even the highest bribe which could be offered in equilibrium, i.e., $\overline{B}$. In addition to that, the compensation scheme should satisfy the auctioneer's participation constraint when there is no corruption. Thus, the problem of the seller can be written as

$$\min_{w(.)} \mathbb{E}(w(R^{NC})) \tag{4}$$

$$s.t. \quad \mathbb{E}(w(R^{NC})) \geq \mathbb{E}(w(R^{C})) + \overline{B} - p \qquad \text{(IC)} \tag{5}$$

$$\mathbb{E}(w(R^{NC})) \geq 0 \qquad \text{(PC)} \tag{6}$$

A solution to this problem is given below.

Proposition 1 *The following compensation scheme is the optimal scheme that completely prevents corruption for any punishment level $p \geq 0$:*

$$w^*(R) = \alpha(R - \mathbb{E}R^{NC}),$$

for any $\alpha \geq \frac{\overline{B}-p}{\mathbb{E}R^{NC}-\mathbb{E}R^{C}}$.

Proof: See Appendix.

Intuitively, it rewards the auctioneer when the collected revenue is above the expectation and punishes her when the collected revenue is below the expectation, where α is a fraction of this difference that the auctioneer receives. If share α of the auctioneer is sufficiently large, she has more incentives to collect more revenue than to accept a bribe. Notice that for low values of punishment, in particular for $p = 0$, share α should be above one. However, it does not mean that the seller has to pay more. The expected cost of proposing such a compensation scheme is zero for the seller for any value of α. Indeed,

$$\mathbb{E}(w^*(R^{NC})) = \mathbb{E}[\alpha(R^{NC} - \mathbb{E}R^{NC})] = \alpha\mathbb{E}R^{NC} - \alpha\mathbb{E}R^{NC} = 0.$$

Thus, by offering the proper incentive scheme the seller can prevent corruption even in the absence of any legal anti-corruption framework.

6 Limited Liability

The optimal compensation scheme proposed in the previous section has to punish the auctioneer when the collected revenue is below the expected revenue. However, such a contract may often not be feasible in real-life situations. For example, the auctioneer can be protected by a law that guarantees that, in any circumstances, she can only receive money for her job. In our model, this can be incorporated by the following limited liability constraint:

$$w(R) \geq 0 \text{ for any } R \in \mathbb{R}_+ \quad \text{(LLC)} \tag{7}$$

It limits the ability of the seller to prevent corruption. The aim of this section is to understand whether contracts under limited liability can still help prevent corruption. Remember that a legal anti-corruption framework alone can prevent corruption if and only if $p > \overline{B}$. Hence, a contract can be useful only if it can decrease this cutoff. Thus, we want to find all values of p such that there exists a compensation scheme $w(R)$

that satisfies PC, IC, and LLC constraints and, at the same time, is still profitable for the seller, that is, the expected payment to the auctioneer is less than the expected loss due to corruption. Denote this set of values of p by Π. Notice that PC is implied by LLC because the latter one is the ex-post version of the former one. Then,

$$\Pi = \{p \in \mathbb{R}_+ : \exists w(R) : \tag{8}$$

$$\mathbb{E}(w(R^{NC})) \geq \mathbb{E}(w(R^{C})) + \overline{B} - p, \qquad \text{(IC)} \tag{9}$$

$$w(R) \geq 0 \text{ for any } R \in \mathbb{R}_+, \qquad \text{(LLC)} \tag{10}$$

$$\mathbb{E}(w(R^{NC})) \leq \mathbb{E}(R^{NC}) - \mathbb{E}(R^{C})\} \qquad \text{(Profitability)} \tag{11}$$

We are interested in finding the lowest level of punishment $\underline{p}$ such that the seller prevents corruption, i.e., $\underline{p} = \inf \Pi$.

Proposition 2 *The lowest level of punishment $\underline{p}$ such that the seller prevents corruption with myopic bidders under the limited liability is*

$$\underline{p} = \overline{B} - \frac{\mathbb{E}(R^{NC}) - \mathbb{E}(R^{C})}{N}.$$

Proof: See Appendix.

Notice that $\underline{p} < \overline{B}$. Thus, even in the limited liability case, it is possible to prevent corruption with the help of a proper compensation scheme when the legal anti-corruption framework alone is not sufficient. To prevent corruption, the seller can propose a compensation scheme that pays more for high values of the realized revenue. Since the bidders do not change their behavior if they are myopic, the realized revenue is always weakly higher if there is no corruption because there are more competitive bidders. Thus, the revenue distribution in the no corruption case first-order stochastically dominates the revenue distribution in the corruption case.

Hence, if the seller proposes a compensation scheme that pays more for higher revenue, such a scheme creates additional incentives for the auctioneer not to be engaged in corruption. The greatest lower bound $\underline{p}$ on the legal punishment needed to support such a payment scheme is achieved when the compensation is heavily skewed to the very high levels of revenue. This provides the highest incentives for the auctioneer to be honest.

7 Fully Rational Bidders

Now, assume that bidders are fully rational, i.e., they understand that there is a collusion agreement between the auctioneer and bidder N that allows him to see the bids of other bidders. Denote by $\beta^C(v)$ the symmetric equilibrium bidding strategy of any of the honest bidders in this case. Arozamena and Weinschelbaum (2009) show how honest bidders change their bidding behavior if they anticipate corruption and characterize β^C. They show that this bidding strategy can be either more or less aggressive than the bidding strategy β used in the auction without corruption. Such a comparison depends on distribution F. The purpose of this section is to discuss how our analysis and results change when bidders are fully rational.

It turns out that the analysis of contracting without any limited liability constraints remains the same. Thus, the statement of Proposition (1) remains true. The only change is that new values for $\mathbb{E}R^C$ and $\overline{B}$ should be used there. These new values reflect the fact that honest bidders use $\beta^C(v)$ instead of $\beta(v)$. Thus, corruption can still be prevented for any level of exogenous punishment p.

However, contracting under limited liability differs substantially when bidders are aware of corruption. Arozamena and Weinschelbaum (2009) show that, for some value distributions, bidders become more aggressive in the presence of corruption.

This can sometimes even lead to increased revenue if corruption takes place. In such cases, corruption is not harmful but beneficial for the seller. Hence, she does not want to prevent it. Hence, to make the problem interesting, we concentrate on the cases where the expected revenue in the presence of corruption still goes down and the seller wants to prevent it. Thus, we make the following assumption which, as shown in Arozamena and Weinschelbaum (2009), is sufficient for bidders to use less aggressive bidding.

Assumption 1 *$F(v)$ is log-concave and $\frac{F(v)}{f(v)}$ is strictly convex*[6][7].

Then, the following result is due to Arozamena and Weinschelbaum (2009).

Proposition 3 *Under Assumption 1, bidders are less aggressive when corruption takes place, i.e., $\beta^C(v) < \beta(v)$ for all $v \in (0, \overline{v}]$.*

Then, we can show the following result.

Proposition 4 *If bidders become less aggressive under corruption, the lowest level of punishment $\underline{p}^l$ such that the seller prevents corruption under limited liability is*

$$\underline{p}^l = \overline{B} - (\mathbb{E}(R^{NC}) - \mathbb{E}(R^C)).$$

Proof: See Appendix.

Notice that $\underline{p}^l < \underline{p}$, i.e., if bidders become less aggressive under corruption, the seller can prevent corruption for lower levels of punishment compared to the case of myopic bidders. The idea here is that the seller can use the fact that bidding

[6]Log-concavity holds for most standard distributions. See Arozamena and Weinschelbaum (2009) for the discussion.

[7]If $\frac{F(v)}{f(v)}$ is linear, the results are the same as for myopic bidders because Arozamena and Weinschelbaum (2009) show that the bidders do not change their behavior in this case.

supports are different in corruption and no corruption cases. Namely, since $\beta^C(\overline{v}) < \beta(\overline{v})$, bids above $\beta^C(\overline{v})$ can appear only if there is no corruption. Thus, if the seller proposes a compensation scheme that rewards the auctioneer only in those cases when $R > \beta^C(\overline{v})$, this provides strong incentives for the auctioneer not to be engaged in corruption.

8 Conclusion

We construct a model of corruption in auctions and analyze the compensation schemes that can be used to prevent corruption. We find that by using an appropriate additional compensation scheme, the seller can often prevent corruption when the government alone cannot.

In this paper, we consider a static framework. However, it would be interesting to explore dynamic frameworks and the effects that reducing corruption at a given time might have from a long-term perspective. Andvig and Moene (1990) argue that "corruption corrupts", that is, the more agents are corrupt in society, the more susceptible this society is to being corrupted. Thus, it could be profitable for a seller to fight corruption at a cost in the beginning in order to reduce the culture of corruption in the future.

In our setting, there is a privileged participant that can "benefit" from corruption. It would be interesting to investigate an endogenously favored participant that is approached by the auctioneer, who, after observing the bids, makes an offer of resubmission to the bidder who could potentially give her the highest bribe, as Lengwiler and Wolfstetter (2010) do. In our framework, it would be more difficult to prevent this kind of corruption.

A APPENDIX: Proofs

A.1 Proof of Proposition 1

First, we compute the expected payment to the auctioneer according to $w^*(.)$ if there is no corruption:

$$\mathbb{E}(w^*(R^{NC})) = \mathbb{E}[\alpha(R^{NC} - \mathbb{E}R^{NC})] = \alpha\mathbb{E}R^{NC} - \alpha\mathbb{E}R^{NC} = 0.$$

Thus, for any value of α, PC (6) is binding for $w^*(.)$, and the payment to the auctioneer achieves the highest lower bound. We only need to check for which values of α the proposed compensation scheme $w^*(.)$ satisfies IC (5). Plugging $w^*(.)$ to (5), we need that the following inequality should be fulfilled:

$$\begin{aligned} 0 &\geq \mathbb{E}[\alpha(R^C - \mathbb{E}R^{NC}) + \overline{B} - p \iff \\ \alpha &\geq \frac{\overline{B} - p}{\mathbb{E}R^{NC} - \mathbb{E}R^C}. \end{aligned}$$

Thus, $w^*(.)$ indeed prevents corruption at the lowest possible cost.

A.2 Proof of Proposition 2

Expressing $\mathbb{E}(w(R^{NC}))$, $\mathbb{E}(w(R^{C}))$, $\mathbb{E}(R^{NC})$, $\mathbb{E}(R^{C})$ and plugging them to (8), we can rewrite Π as

$$\Pi = \{p \in \mathbb{R}_+ : \exists w(R) :$$

$$\int_0^{\overline{v}} w(\beta(x))NF(x)^{N-1}f(x)dx \geq \int_0^{\overline{v}} w(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - p \quad (12)$$

$$w(x) \geq 0 \text{ for any } x \in \mathbb{R}_+, \quad (13)$$

$$\int_0^{\overline{v}} w(\beta(x))NF(x)^{N-1}f(x)dx \leq \mathbb{E}R^{NC} - \mathbb{E}R^{C}\} \quad (14)$$

Step 1 (Binding IC). First, notice that IC (12) should be binding for $p = \underline{p}$. Indeed, otherwise, we can find a lower value $p = \widetilde{p} < \underline{p}$ that still satisfies (12). Since a change in p does not affect any other constraints, we have that $\widetilde{p} \in \Pi$. Thus, we obtain a contradiction to $\underline{p} = \inf \Pi$.

Step 2 (Binding Profitability). Second, we argue that profitability constraint (14) should be binding for $p = \underline{p}$. Suppose, by contradiction, that, for $p = \underline{p}$, there exists compensation scheme w_0 that satisfies (8) such that (14) is not binding, i.e.,

$$\int_0^{\overline{v}} w_0(\beta(x))NF(x)^{N-1}f(x)dx < \mathbb{E}R^{NC} - \mathbb{E}R^{C}.$$

Then, define compensation scheme $w_1(R)$ as follows

$$w_1(R) = w_0(R) + \varepsilon R \text{ for any } R.$$

Then, for sufficiently small ε, the following is true:

$$\begin{aligned}
&\int_0^{\overline{v}} w_1(\beta(x))NF(x)^{N-1}f(x)dx = \\
= \; &\int_0^{\overline{v}} w_0(\beta(x))NF(x)^{N-1}f(x)dx + \varepsilon \underbrace{\int_0^{\overline{v}} \beta(x)NF(x)^{N-1}f(x)dx}_{\mathbb{E}R^{NC}} \geq \\
&\int_0^{\overline{v}} w_0(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - \underline{p} + \varepsilon\mathbb{E}R^{NC},
\end{aligned}$$

$$\begin{aligned}
&\int_0^{\overline{v}} w_1(\beta(x))(N-1)F(x)^{N-2}f(x)dx = \\
= \; &\int_0^{\overline{v}} w_0(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \varepsilon \underbrace{\int_0^{\overline{v}} \beta(x)(N-1)F(x)^{N-2}f(x)dx}_{\mathbb{E}R^{C}}.
\end{aligned}$$

Combining these observations,

$$\begin{aligned}
&\int_0^{\overline{v}} w_1(\beta(x))NF(x)^{N-1}f(x)dx \geq \\
\geq \; &\int_0^{\overline{v}} w_0(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - \underline{p} + \varepsilon\mathbb{E}R^{NC} = \\
&\int_0^{\overline{v}} w_1(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - \underline{p} + \varepsilon\mathbb{E}R^{NC} - \varepsilon\mathbb{E}R^{C}.
\end{aligned}$$

Thus, for sufficiently small ε, compensation scheme $w_1(R)$ should satisfy the following:

$$\begin{aligned}
\int_0^{\overline{v}} w_1(\beta(x))NF(x)^{N-1}f(x)dx &> \int_0^{\overline{v}} w_1(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - \underline{p}, \\
w_1(x) &\geq 0 \text{ for any } x \in \mathbb{R}_+, \\
\int_0^{\overline{v}} w_1(\beta(x))NF(x)^{N-1}f(x)dx &< \mathbb{E}R^{NC} - \mathbb{E}R^{C},
\end{aligned}$$

where the first inequality follows from $\mathbb{E}R^{NC} > \mathbb{E}R^{C}$. Then, it is possible to find $\widetilde{p} < \underline{p}$ such that $w_1(R)$ satisfies (12), (13) and (14), which contradicts to the fact that $\underline{p} = \inf \Pi$.

Step 3 (Equivalent Problem). Plugging binding (14) to binding (12), we have that $\underline{p}$ is the lowest value of p such that there exists $w(R) \geq 0$ such that the following two inequalities hold

$$\begin{aligned}
p &= \int_0^{\overline{v}} w(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - (\mathbb{E}R^{NC} - \mathbb{E}R^{C}), \qquad (15) \\
&\int_0^{\overline{v}} w(\beta(x))NF(x)^{N-1}f(x)dx = \mathbb{E}R^{NC} - \mathbb{E}R^{C}.
\end{aligned}$$

Hence, the problem of finding $\underline{p}$ is equivalent to

$$\begin{aligned}
&\min_{w(.)} \int_0^{\overline{v}} w(\beta(x))(N-1)F(x)^{N-2}f(x)dx \\
&s.t. \int_0^{\overline{v}} w(\beta(x))NF(x)^{N-1}f(x)dx = \mathbb{E}R^{NC} - \mathbb{E}R^{C}.
\end{aligned}$$

Step 4 (Solving Problem). Dividing the objective by $(N-1)$ and subtracting

$\int_0^{\overline{v}} w(\beta(x))F(x)^{N-1}f(x)dx = \frac{\mathbb{E}R^{NC}-\mathbb{E}R^C}{N}$, we get the following problem:

$$\min_{w(.)} \int_0^{\overline{v}} w(\beta(x))F(x)^{N-2}f(x)(1-F(x))dx \tag{16}$$
$$s.t. \int_0^{\overline{v}} w(\beta(x))F(x)^{N-1}f(x)dx = \frac{\mathbb{E}R^{NC}-\mathbb{E}R^C}{N}.$$

Denote $w^*(.)$ as the solution to problem (16). Since $1-F(x)>0$ for any $x<\overline{v}$, the minimum is attained by setting $w^*(R)=0$ for any $R<\beta(\overline{v})$. Hence,

$$\int_0^{\overline{v}} w^*(\beta(x))(N-1)F(x)^{N-2}f(x)dx = \int_0^{\overline{v}} w^*(\beta(x))(N-1)F(x)^{N-1}f(x)dx \tag{17}$$
$$= \frac{N-1}{N}(\mathbb{E}R^{NC}-\mathbb{E}R^C), \tag{18}$$

where the last equation follows from (16). Using (17) and plugging value of $\overline{B}$ in (15), we can find $\underline{p}$:

$$\underline{p} = \int_0^{\overline{v}} w^*(\beta(x))(N-1)F(x)^{N-2}f(x)dx + \overline{B} - (\mathbb{E}R^{NC}-\mathbb{E}R^C) =$$
$$= \frac{N-1}{N}(\mathbb{E}R^{NC}-\mathbb{E}R^C) + \overline{B} - (\mathbb{E}R^{NC}-\mathbb{E}R^C) = \overline{B} - \frac{\mathbb{E}R^{NC}-\mathbb{E}R^C}{N}.$$

A.3 Proof of Proposition 4

Step 1 (possibility of $\mathbb{E}(w(R^C))=0$). Since $\beta^C(v)<\beta(v)$ for any $v\in(0,\overline{v}]$ when bidders are less aggressive, the bidding supports are different in the case of corruption and no corruption, i.e., $\beta^C(\overline{v})<\beta(\overline{v})$. Thus, any bid $b\in(\beta^C(\overline{v}),\beta(\overline{v})]$ can only appear if there is no corruption. Hence, if the seller offers any compensation scheme w such that $w(R)>0$ only if $R>\beta^C(\overline{v})$ and $w(R)=0$ for any $R\leq\beta^C(\overline{v})$, then the expected compensation in the case of corruption is zero, i.e., $\mathbb{E}(w(R^C))=0$. Hence, for any compensation scheme w there exists another compensation scheme $\overline{w}$:

$\mathbb{E}(\overline{w}(R^{NC})) = \mathbb{E}(w(R^{NC}))$ and $\mathbb{E}(\overline{w}(R^C)) = 0$.

Step 2 ($\mathbb{E}(w(R^C)) = 0$ when $p = \underline{p}^l$). Here, we argue that for $p = \underline{p}^l$, any compensation scheme w that satisfies (8) should also satisfy $\mathbb{E}(w(R^C)) = 0$. Suppose to the contrary that for $p = \underline{p}^l$, there exists $w(.)$ that satisfies (8) and $\mathbb{E}(w(R^C)) > 0$. Then, from Step 1, we know that there exists another compensation scheme $\overline{w}$ that satisfies (8) such that $\mathbb{E}(\overline{w}(R^C)) = 0$. Then, (9) implies that there exists $\widetilde{p} < \underline{p}^l$ such that (8) is satisfied for compensation scheme $\overline{w}$. Thus, we have obtained a contradiction to $\underline{p}^l = \inf \Pi$.

Step 3 (Binding IC). Next, IC (9) should be binding for $p = \underline{p}^l$. Indeed, otherwise, we can find a lower value $p = \widetilde{p} < \underline{p}^l$ that still satisfies (9). Since a change in p does not affect any other constraints, we have that $\widetilde{p} \in \Pi$. Thus, we obtain a contradiction to $\underline{p}^l = \inf \Pi$.

Step 4 (Binding Profitability). We argue that profitability constraint (11) should be binding for $p = \underline{p}^l$. Suppose, by contradiction, that, for $p = \underline{p}^l$, there exists compensation scheme w_0 such that (14) is not binding, i.e.,

$$\mathbb{E}(w_0(R^{NC})) < \mathbb{E}(R^{NC}) - \mathbb{E}(R^C)\}.$$

Then, define compensation scheme $w_1(R)$:

$$\begin{aligned} w_1(R) &= w_0(R) + \varepsilon \text{ for } R > \beta^C(\overline{v}), \\ w_1(R) &= w_0(R) \text{ for } R \leq \beta^C(\overline{v}). \end{aligned}$$

For sufficiently small ε, we have

$$\mathbb{E}(w_0(R^{NC})) < \mathbb{E}(w_1(R^{NC})) < \mathbb{E}(R^{NC}) - \mathbb{E}(R^C)\}.$$

However, if there is corruption, w_1 provides the same compensation to the auctioneer as w_0 because they differ only for the revenue values that are beyond the support in the corruption case, i.e.,

$$\mathbb{E}(w_1(R^C)) = \mathbb{E}(w_0(R^C)).$$

Then, it is possible to find $\widetilde{p} < \underline{p}^l$ such that $w_1(R)$ satisfies (9), (10) and (14) which contradicts to the fact that $\underline{p}^l = \inf \Pi$.

Step 5 (Computing $\underline{p}$). From the previous steps, it follows that, for $p = \underline{p}^l$, any compensation scheme w that satisfies (8) should be such that

$$\begin{aligned} \mathbb{E}(w(R^{NC})) &= \mathbb{E}(R^{NC}) - \mathbb{E}(R^C), \\ \mathbb{E}(w(R^C)) &= 0. \end{aligned}$$

Using binding IC, we can determine $\underline{p}^l$ from the binding IC (9):

$$\underline{p}^l = \overline{B} - (\mathbb{E}(R^{NC}) - \mathbb{E}(R^C)).$$

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