Polarization and Issue Selection in Electoral Campaigns

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February 22, 2023

Abstract

The strategy that candidates use to emphasize certain policy issues during electoral campaigns is a critical aspect of electoral competition. In this paper, we contribute to the research on electoral competition by developing a model of issue selection in electoral campaigns. We investigate whether issues that voters are more polarized on or issues that political parties are more polarized on are more likely to be advertised during electoral contests. Our findings show that candidates have greater incentives to advertise issues on which political parties are more polarized rather than issues on which voters hold polarized policy positions. This analysis provides a theoretical basis for developing a better understanding of the content of campaign communication on ideological issues.

Polarization has become an increasingly important topic of public and scholarly interest (Fiorina et al., 2006). While polarization has many facets, the policy issues on the public agenda at any given time are likely to be a key determinant. This is because some policy issues are more divisive than others, and whether issues on which voters are more ideologically divided or more ideologically congruent are part of the political discourse can make a difference. In this paper, we link the scholarship on polarization with the agenda-setting perspective of electoral competition by examining whether issues in which voters are more polarized or issues on which parties are more polarized are more likely to be promoted on the electoral agenda when accounting for party competition over which issues to emphasize.

A significant body of literature shows that parties selectively emphasize various policy issues to influence citizens to prioritize those considerations when casting their votes (Druckman, Jacobs, and Ostermeier, 2004; Iyengar and Kinder, 1987; Riker, 1996). As noted by Donald Stokes (1963, 372), "the skills of political leaders... consist partly in knowing what issue dimensions... can be made salient by suitable propaganda." This strategizing over issue selection has been widely documented in numerous electoral contests in various countries, including the United States, Canada, the United Kingdom, Spain, Germany, and Japan (Aldrich and Griffin, 2003; Budge and Farlie, 1983; Druckman, Kifer, and Parkin, 2009; McCombs, 2004; Laver and Hunt, 1992; Ward et al., 2015).

While scholars have made great strides in understanding the factors that determine which policy issues political parties prioritize in electoral campaigns, an important question remains unanswered: do parties have stronger incentives to focus on issues that divide voters or on those that divide the parties themselves? Despite extensive theoretical and empirical research in this area, our understanding of this fundamental aspect of electoral competition remains limited.

To address this question, we propose a simple model of electoral competition that explains how political parties strive for electoral support by raising the electoral importance of various policy issues. In our framework, there are two political parties and a spectrum of voters;

both parties and voters have ideal policies in a two-dimensional policy space, and the voters' ideal policies on the two issues are multivariate normal. Parties compete over which issues are electorally important by choosing a vector of advertisements to increase the electoral salience of various issue dimensions to maximize their vote share minus the cost of issue advertisement. Each voter selects the party that is closer to her policy position on the two issues, and the proximity between a voter's and a party's policy position is an aggregate of the difference between the voter's and the party's preferred policy on each issue, weighted by the salience of each issue. The relative salience of each issue is determined by the parties' campaign advertisement choices, which also determine the distribution of the electorate's preference for which party is more electorally desirable.

We show that parties have more incentives to advertise an issue on which parties are more ideologically polarized than on which voters are polarized. We also show that the minority party, the party that, in equilibrium, has the lower vote share, has more incentives to advertise an issue on which parties are more polarized than the majority party, the party that in equilibrium has a higher vote share.

In addition to its contribution to the scholarship on the causes of polarization, this paper also adds to the literature on how strategic competition among political parties shapes the issue agenda in elections (Riker 1986, 1996). The formal literature has examined the conditions under which certain policy issues remain on the electoral agenda (Glazer and Lohmann 1999), the conditions under which parties introduce new policy issues relative to the existing status quo (Colomer and Llavador 2008), the effect of media bias on parties' incentives to promote policy issues (Puglisi 2004), the conditions under which candidates emphasize valence issues on which they have an ex-ante advantage when issue ownership is endogenously determined (Aragones, Castanheira, and Giani 2015), whether parties emphasize similar issues during electoral campaigns (Hammond and Humes 1995; Simon 2002; Dragu and Fan 2016), the manipulation of issue dimensions (Moser, Patty, and Penn 2009), and how changing the salience of issues can affect the winners in elections (Feld, Merrill III, and Grofman 2014). To the best of our knowledge, no existing model has analyzed whether parties have more incentives to advertise issues on which the electorate is more polarized or on which parties are more polarized.

Model

The players are a continuum of voters, whose measure is normalized to 1, and two political parties, A and B. The policy space is multidimensional: there are two issue dimensions, and the set of possible policy choices for each issue i ($i = 1, 2$) is $\mathbb{R}$.

Each party $k \in \{A, B\}$ has an ideal policy, a vector $\mathbf{p}^k = (p_1^k, p_2^k) \in \mathbb{R}^2$, where the i-th element denotes party k's preferred policy on issue i. The parties' most preferred policies on each issue dimension differ; that is, $p_i^A \neq p_i^B$ for $i = 1, 2$. Our model focuses on analyzing how parties compete for votes by raising the salience of various issues. Thus we assume the parties' policy positions to be fixed for the campaign's duration.

Each voter has an ideal policy vector $\mathbf{x} \in \mathbb{R}^2$. The location of the voters' ideal policies follows a multivariate normal distribution. That is, a generic voter's ideal policy is $\mathbf{x}_{2\times1} \sim N(\boldsymbol{\mu}_{2\times1}, \boldsymbol{\Sigma}_{2\times2})$.

Each party $k \in \{A, B\}$ chooses an amount of advertisement for each issue dimension, a vector $\mathbf{a}^k = (a_1^k, a_2^k) \in \mathbb{R}_+^2$ at a cost $C^k(\mathbf{a}^k) = \sum_{i=1}^2 c^k(a_i^k)$. The campaign advertisement can be considered the amount of money, time, and effort parties allocate to emphasize certain policy issues during electoral campaigns to persuade voters that those issues are a governing priority. We assume that the cost function is twice continuously differentiable, $c^{k'}(\cdot) > 0$, $c^{k''}(\cdot) > 0$, $c^k(0) = 0$, $c^{k'}(0) = 0$, $\mathbf{lim}_{a\to\infty} c^k(a) = \infty$ and $\mathbf{lim}_{a\to\infty} c^{k'}(a) = \infty$ for $k \in \{A, B\}$. The objective of each party is to maximize the vote share less the cost of issue advertising. Thus party k's utility is

$$U^k(\mathbf{a}^k; \mathbf{a}^{-k}) = v^k(\mathbf{a}^k; \mathbf{a}^{-k}) - C^k(\mathbf{a}^k),$$

where $\mathbf{a}^k$ is the vector of advertisement on the two issues by party k and v^k is party k's vote share, which will be characterized in the next section.

A voter's preference over policies depends on the difference between the implemented policy and her ideal policy on each issue and on the relative importance the voter puts on each policy issue. Thus the utility of a voter with ideal policy $\mathbf{x}$ is

$$U_v = -\sum_{i=1}^{2} w_i(\mathbf{a})(p_i - x_i)^2, \tag{1}$$

where $\mathbf{a} = (\mathbf{a}^A, \mathbf{a}^B)$, and $w_i(\mathbf{a})$ represents the relative importance the voter puts on issue i. The electoral salience of the two policy issues is affected by the parties' issue advertisement. Specifically, for each issue, i denotes by $a_i = \Sigma_k a_i^k$ the total amount of advertisement issue i receives in an electoral campaign. Because the relative importance of each issue depends on the total advertisement of that respective issue, we can re-express the advertisement vector on the two issues as $\mathbf{a} = (a_1, a_2)$. The issue salience vector then is a function of the parties' advertisement strategy: $\mathbf{w}(\mathbf{a}) = (w_1(\mathbf{a}), w_2(\mathbf{a}))$. We assume that $w_i(\mathbf{a}) = \frac{a_i+\alpha}{a_1+a_2+2\alpha}$, where $\alpha > 0$.

The contest success function $w_i(\mathbf{a})$ encapsulates how the parties' campaign advertisement effort to highlight which issues are more critical translates into the voters' assessments regarding the relative salience of various policy issues. We take a reduced-form model of this process because our main interest is to investigate the issue-selection incentives of the parties. Our approach here is similar to other papers that use contest functions to model how campaign advertisement and spending affect the behavior of voters (Snyder 1989; Baron 1994; Skaperdas and Grofman 1995; Grossman and Helpman 1996). The contest success function can be derived from axiomatic theories of (partially) uninformed voting (Luce 1959), from an inferential process of an (uninformed) audience that observes evidence produced by contestants who seek to persuade the audience of the correctness of their views (Skaperdas and Vaidya 2012) or from a political contest in which parties provide costly information to voters (Gul and Pesendorfer 2012).

More importantly, the critical assumption of contest function $w_i(\mathbf{a})$ that if an issue receives more advertisement than others, the relative electoral importance of that policy issue is higher has garnered significant empirical support. An extensive empirical literature documents that the amount of media coverage or candidate discussion of certain policy issues induces citizens to give more weight to those issues when evaluating candidates (McCombs and Shaw 1972; Iyengar and Kinder 1987; Krosnick and Kinder 1990; Johnston et al. 1992; Ansolabehere and Iyengar 1994; Jacobs and Shapiro 1994; Carsey 2000; Jacoby 2000; Simon 2002; Druckman, Jacobs, and Ostermeier 2004, Bartels 2006), effects that have been shown both in observational and experimental studies. Moreover, such agenda-setting effects have been documented on various policy issues during national and local elections in multiple countries, including the United States, Spain, Germany, and Japan, among others (McCombs 2004). The model builds upon these well-documented empirical patterns to investigate the issue-selection strategies of candidates in electoral contests.

Of course, not all voters are susceptible to agenda-setting effects. Specifically, some voters cast their votes based on party identification, regardless of the parties' campaign message. That is, such partisan voters have allegiance to one party or another, and thus campaign messages and advertisements will have little effect on their voting decision. Notice that we could follow a similar modeling strategy as Baron (1994) and Grossman and Helpman (1996) and model the electorate as consisting of both a fraction of voters who are susceptible to campaign effects and a fraction of voters who are not; such modeling would not affect the forthcoming analysis, and thus we focus our model on those (non-partisan) voters who can be susceptible to campaign effects.

The game unfolds as follows. In the first stage, the parties simultaneously choose their advertisement strategies regarding which issue dimensions to emphasize. The second stage is a standard voting game: each voter decides which party to elect.

Party Competition and Issue Selection

In the voting stage, given the parties' strategies of advertisement and the voters' utility function as defined by expression (1), a voter with ideal policy $\mathbf{x}$ prefers party A over B if and only if

$$\sum_{i=1}^{2} w_i(\mathbf{a})(x_i - p_i^A)^2 < \sum_{i=1}^{2} w_i(\mathbf{a})(x_i - p_i^B)^2,$$

which is equivalent to

$$\sum_{i=1}^{2} w_i(\mathbf{a}) d_i(x_i) > 0, \tag{2}$$

where $d_i(x_i) \equiv (p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2})$.[1]

A party's vote share is the fraction of the electorate that prefers that party over the other party. For example, party A's vote share is:

$$v^A(\mathbf{a}^A, \mathbf{a}^B) = \mathbf{P}(x|\sum_{i=1}^{2} w_i(\mathbf{a})(x_i - p_i^A)^2 < \sum_{i=1}^{2} w_i(\mathbf{a})(x_i - p_i^B)^2),$$

which is equivalent to

$$v^A(\mathbf{a}^A, \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\sum_{i=1}^{2} w_i(\mathbf{a}) d_i(x_i) > 0). \tag{3}$$

Similarly, the vote share of party B is

$$v^B(\mathbf{a}^A, \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\sum_{i=1}^{2} w_i(\mathbf{a}) d_i(x_i) < 0). \tag{4}$$

Expressions (3) and (4) show that the vector $\mathbf{d}(\mathbf{x}) \equiv \{d_i(x_i)\}_{i=1}^2$ is an important determinant of a party's vote share; the parameter $d_i(x_i)$ is a measure of whether and by how much a voter with ideal policy $\mathbf{x}$ prefers party A over party B on issue i. Because the distribution

[1] The inequality is equivalent to $\sum_{i=1}^2 w_i(\mathbf{a})(p_i^A - p_i^B)(2x_i - (p_i^A + p_i^B)) = 2\sum_{i=1}^2 w_i(\mathbf{a})(p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2}) > 0$, which is the same as $\sum_{i=1}^2 w_i(\mathbf{a})(p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2}) > 0$.

of voters' policy positions follows a multivariate normal distribution, the distribution of $\mathbf{d}(\mathbf{x})$ is also multivariate normal. That is, $\mathbf{d}(\mathbf{x}) \sim N(\boldsymbol{\nu}^A, \boldsymbol{\Lambda})$ where $\nu_i^A = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$ is the i-th element of vector $\boldsymbol{\nu}^A$, $\lambda_{ij} = (p_i^A - p_i^B)(p_j^A - p_j^B)\sigma_{ij}$ is the (i,j)-th entry of the variance-covariance matrix $\boldsymbol{\Lambda}$, and in particular $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$ is the i-th element of the diagonal entries of $\boldsymbol{\Lambda}$. Similarly, we have $\nu_i^B = (p_i^B - p_i^A)(\mu_i - \frac{p_i^A + p_i^B}{2})$. For simplicity of exposition, in the subsequent analysis, we use the notation $\nu_i = \nu_i^A$ where indexing by party is not important and use the notation ν_i^k for $k \in \{A, B\}$ where party indexing is necessary.

Given that $\mathbf{d}(\mathbf{x})$ follows a normal distribution, we can rewrite party A's and party B's vote share as follows:

$$v^A(\mathbf{a}^A, \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\mathbf{w}(\mathbf{a}) \cdot \mathbf{d}(\mathbf{x}) > 0) = \Phi\left(\frac{\sum_{i=1}^2 w_i(\mathbf{a})\nu_i}{[w_1(\mathbf{a})^2\lambda_{11} + w_2(\mathbf{a})^2\lambda_{22} + 2w_1(\mathbf{a})w_2(\mathbf{a})\lambda_{12}]^{\frac{1}{2}}}\right), \tag{5}$$

and

$$v^B(\mathbf{a}^A, \mathbf{a}^B) = 1 - v^A(\mathbf{a}^A, \mathbf{a}^B)$$

where $\Phi(\cdot)$ is the cdf of standard normal distribution.[2]

The parameters ν_i and λ_{ij} are central to our analysis regarding the issue-selection strategies of parties. The parameter ν_i measures party A's electoral popularity on policy issue i. A positive ν_i implies that a majority of the voters prefers party A over party B on issue i; that is, party A has an advantage on issue i, and a bigger ν_i implies a bigger electoral

[2]The calculation is as follows:

$$v^A(\mathbf{a}^A, \mathbf{a}^B) = \mathbf{P}(\mathbf{x}|\mathbf{w}(\mathbf{a}) \cdot \mathbf{d}(\mathbf{x}) > 0) = \mathbf{P}(\mathbf{x}|\frac{\mathbf{w}(\mathbf{a}) \cdot \mathbf{d}(\mathbf{x}) - \mathbf{w}(\mathbf{a}) \cdot \boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}} > -\frac{\mathbf{w}(\mathbf{a}) \cdot \boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}}) =$$

$$= 1 - \Phi\left(-\frac{\mathbf{w}(\mathbf{a}) \cdot \boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}}\right) = \Phi\left(\frac{\mathbf{w}(\mathbf{a}) \cdot \boldsymbol{\nu}}{\sqrt{\mathbf{w}'(\mathbf{a})\boldsymbol{\Lambda}\mathbf{w}(\mathbf{a})}}\right) = \Phi\left(\frac{\sum_{i=1}^2 w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^2\sum_{j=1}^2 w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij}]^{\frac{1}{2}}}\right)$$

$$= \Phi\left(\frac{\sum_{i=1}^2 w_i(\mathbf{a})\nu_i}{[w_1(\mathbf{a})^2\lambda_{11} + w_2(\mathbf{a})^2\lambda_{22} + 2w_1(\mathbf{a})w_2(\mathbf{a})\lambda_{12}]^{\frac{1}{2}}}\right).$$

advantage for party A relative to party B on issue i. Conversely, a negative ν_i implies that a majority of the voters prefers party B over party A on policy issue i; that is, party B has electoral advantage on issue i, and a bigger $-\nu_i$ implies a bigger electoral advantage for party B relative to party A on policy issue i.[3] How a party's electoral popularity is aggregated across the two policy issues is determined by the salience of each issue dimension, $w_i(\mathbf{a})$. Thus we can think of $\sum_{i=1}^{2} w_i(\mathbf{a})\nu_i$ (the numerator in expression (5)) as a measure of party A's electoral popularity on the two policy issues.

The parameter λ_{ii} measures the electoral heterogeneity regarding which party is more desirable on issue i.[4] A larger (smaller) λ_{ii} connotes a higher (lower) electoral heterogeneity regarding which party is more desirable on policy issue i. The parameter λ_{ij} for $I \neq j$ can be considered a measure of the correlation of a party's electoral popularity between issue 1 and issue 2. Therefore we can think of $[w_1(\mathbf{a})^2\lambda_{11} + w_2(\mathbf{a})^2\lambda_{22} + 2w_1(\mathbf{a})w_2(\mathbf{a})\lambda_{12}]^{\frac{1}{2}}$ (the denominator in expression (5)) as a measure of electoral heterogeneity regarding which party is more desirable on the two policy issues. Notice that the electoral heterogeneity regarding which party is more desirable consists of the sum of the electoral heterogeneity regarding which party is more desirable on each issue and of the correlations of a party's electoral popularity across the two issues. The salience of each issue dimension, $w_i(\mathbf{a})$, determines how λ_{ii} and λ_{12} are aggregated across the n policy issues.

Given the parties' vote shares previously described, party A's optimization problem is

$$\mathbf{max}_{\mathbf{a}^A \in \mathbb{R}_+^2} \Phi \left(\frac{\sum_{i=1}^{2} w_i(\mathbf{a})\nu_i}{[w_1(\mathbf{a})^2\lambda_{11} + w_2(\mathbf{a})^2\lambda_{22} + 2w_1(\mathbf{a})w_2(\mathbf{a})\lambda_{12}]^{\frac{1}{2}}} \right) - C^A(\mathbf{a}^A).$$

Note that the strategy space in our model is compact, even though we formulate the advertisement of each party as chosen from $\mathbb{R}_+^2$. This is because $\Phi \left(\frac{\sum_{i=1}^{2} w_i(\mathbf{a})\nu_i}{[\sum_{i=1}^{2}\sum_{j=1}^{2} w_i(\mathbf{a})w_j(\mathbf{a})\lambda_{ij}]^{\frac{1}{2}}} \right) \leq 1$ for all $\mathbf{a}^A$, but $C^A(\mathbf{a}^A) = \sum_{i=1}^{2} c^A(a_i^A)$ and $c^A(\infty) = \infty$. Therefore there exists $\bar{a}^A > 0$

[3]Since μ_i is the center of x_i, $\nu_i = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$ is the center of $d_i(x_i)$, the distribution of how much voters prefer party A over party B on issue i.

[4]Since σ_{ii}, the variance of x_i, measures the variance of voters' ideal positions on issue dimension i, the parameter $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$ is the variance of $d_i(x_i) = (p_i^A - p_i^B)(x_i - \frac{p_i^A + p_i^B}{2})$.

such that the above optimization problem is equivalent to maximizing the same objective function by choosing $\mathbf{a}^A \in \mathbf{S}^A \equiv [0, \bar{a}^A]^2$. Similarly define $\mathbf{a}^B \in \mathbf{S}^B \equiv [0, \bar{a}^B]^2$, and the same argument applies for party B's optimization problem, defined as

$$\mathbf{max}_{\mathbf{a}^B \in \mathbb{R}_+^2} \left[1 - \Phi\left(\frac{\sum_{i=1}^2 w_i(\mathbf{a})\nu_i}{[w_1(\mathbf{a})^2\lambda_{11} + w_2(\mathbf{a})^2\lambda_{22} + 2w_1(\mathbf{a})w_2(\mathbf{a})\lambda_{12}]^{\frac{1}{2}}}\right)\right] - C^B(\mathbf{a}^B),$$

which is equivalent to maximizing the same objective function by choosing $\mathbf{a}^B \in \mathbf{S}^B$.

In the spatial model of electoral competition, equilibria in pure strategy do not generally exist in a multidimensional policy space because the continuity condition necessary for the existence of such an equilibrium is satisfied only under very restrictive conditions. To overcome the continuity problem, scholars have developed probabilistic voting models (Coughlin 1992), where citizens vote according to probability functions based on their preferences and, as a result, equilibria in a multidimensional space exist provided that the parties' utility functions satisfy a concavity condition which is typically assumed. Our setup here is similar to the probabilistic voting model as the continuity condition is satisfied in our framework because a party's strategy is to choose an amount of advertisement on each issue, and each party's utility function is continuous in the advertisement strategies. Moreover, each party's utility function is concave in its own strategy if the cost function is sufficiently convex. The following proposition states conditions on the cost function to ensure that it is sufficiently convex so that a party's utility is concave in its own strategy.

For $k \in \{A, B\}$, denote k's vote share by $v^k(a_1, a_2) = \Phi(\frac{\sum_{i=1}^2 (a_i+\alpha)\nu_i^k}{(\sum_{i=1}^2 \sum_{j=1}^2 (a_i+\alpha)(a_j+\alpha)\lambda_{ij})^{\frac{1}{2}}})$, where $a_i = a_i^A + a_i^B$ for $i = 1, 2$. Also let $v_{ij}^k \equiv \frac{\partial^2 v^k(a_1,a_2)}{\partial a_i \partial a_j}$. Since v_{ij}^k is continuous for all i and j and $\mathbf{S}^k$ is compact for all k, for $i = 1, 2$ define $m_{ii}^k \equiv \mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} v_{ii}^k(\mathbf{a}^A, \mathbf{a}^B)$. Similarly, define $m_{12}^k \equiv \mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} |v_{12}^k(\mathbf{a}^A, \mathbf{a}^B)|$. The next proposition states conditions on the cost function to ensure that a pure strategy Nash equilibrium exists.

Proposition 1. *A sufficient condition for the game to have a Nash equilibrium in pure* *strategies is* $c^{k''}(a) > \mathbf{max}\{m_{11}^k, m_{22}^k\} + m_{12}^k$ *for all* a.

In the remainder of our analysis, we characterize the issue-selection incentives of parties in a pure strategy equilibrium. First, we show that the parties will advertise different policy issues. Intuitively, if both parties were to announce some issue i in equilibrium, the optimization problems of parties imply that the vote shares of both parties increase in the advertisement on issue i. However, because increasing one party's vote share means decreasing the vote share of the other party, both parties' objective functions cannot increase simultaneously in the advertisement on issue i. Thus we have the following result:

Proposition 2. *The two parties do not advertise the same policy issue in a pure strategy equilibrium (that is,* $a_i^{A*}a_i^{B*} = 0$ *for all* $i = 1, 2$*).*

Polarization and Issue Selection

In this section, we investigate whether parties have incentives to advertise issues on which there are no ideological differences between parties (i.e., $p_i^A = p_i^B$) or issues on which there are no ideological differences among voters (i.e., $\sigma_{ii} = 0$). The following proposition shows neither party has incentives to advertise an issue on which $p_i^A = p_i^B$.

Proposition 3. *Neither party advertises an issue on which there are no ideological differences between parties (that is, if* $p_i^A = p_i^B$*, then* $a_i^{k*} = 0$ *for* $k \in \{A, B\}$*).*

Proposition 3 follows immediately from the observation that when $p_i^A = p_i^B$, then $\lambda_{ii} = 0$ (there is no voters' disagreement regarding which party is more desirable), $\lambda_{ij} = 0$, and $\nu_i = 0$ (neither party has electoral advantage on issue i) since $\lambda_{ii} = (p_i^A - p_i^B)^2\sigma_{ii}$, $\lambda_{ij} = (p_i^A - p_i^B)(p_j^A - p_j^B)\sigma_{ij}$ and $\nu_i = (p_i^A - p_i^B)(\mu_i - \frac{p_i^A + p_i^B}{2})$. If a party were to advertise such an issue, the advertisement does not affect that party's vote share, regardless of what the other party does, but it is costly. As a result, a party is strictly better off not advertising such an issue.

Similarly, we can analyze whether parties have incentives to advertise an issue i on which there are no ideological differences among voters, $\sigma_{ii} = 0$, which implies that $\lambda_{ii} = 0$ (note that, even though $\lambda_{ii} = 0$, $\sigma_{ii} = 0$ does not imply $\nu_i = 0$). We have the following result:

Proposition 4. *A party advertises an issue on which there are no ideological differences among voters if that party has electoral advantage on that issue (i.e. if $\sigma_{ii} = 0$ and $\nu_i \neq 0$, then $a_i^{k*} > 0$ for $k \in \{A, B\}$ such that $\nu_i^k > 0$).*

Proposition 4 suggests that issues on which there are no ideological differences among voters but on which parties have different policy positions are likely to be advertised during electoral campaigns. Propositions 3 and 4 suggest that issues with no ideological differences among voters are more likely to be advertised than issues with no ideological differences between parties.

Next, we investigate the strategies of parties regarding which issues to advertise when the issues only differ in terms of the ideological difference of the parties. That is, we want to assess the parties' incentives regarding which of the two issues is more likely to emphasize when the issues only differ in terms of the electoral heterogeneity regarding which party is more desirable.

For this analysis, we label the party with the higher equilibrium vote share as *the majority party* and the party with the lower equilibrium vote share as *the minority party*. Notice that since the two issues are such that $\nu_1 = \nu_2$, the party with an electoral advantage on both issues will be the election's winner and, thus, the majority party in equilibrium. The following proposition shows that, in any pure strategy Nash equilibrium, it's impossible for the majority party to advertise the issue with higher heterogeneity and for the minority party to advertise the issue with lower heterogeneity.

Proposition 5. *If $\nu_i = \nu_j$ and $\lambda_{ii} > \lambda_{jj}$, then it's not possible to have $a_i^{k*} > 0$ and $a_j^{l*} > 0$, where $k \in \{A, B\}$ is the majority party and $l \in \{A, B\}$ is the minority party.*

Proposition 5 suggests that the minority party has incentives to promote issues on which parties' positions are further apart and thus more polarized. In other words, the election's loser has augmented incentives, for example, to put on the electoral agenda issues on which parties are more polarized.

Conclusions

In this paper, we have developed a model of electoral competition in which parties compete for electoral support by raising the electoral salience of various position issues to analyze whether parties have more incentives to promote policy issues on which voters are more polarized or issues on which parties are more polarized. Our results show that parties have greater incentives to advertise an issue on which they are more ideologically polarized than on which voters are polarized. Furthermore, we find that the minority party has more incentives to promote issues on which parties are more polarized than the majority party.

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Appendix

Proof of Proposition 1. For $k \in \{A, B\}$, party k's objective function is given by

$$U^k(\mathbf{a}^A, \mathbf{a}^B) = v^k(\mathbf{a}^A, \mathbf{a}^B) - C^k(\mathbf{a}^k)$$

$$= \Phi\left(\frac{\sum_{i=1}^{2}(a_i^A + a_i^B + \alpha)\nu_i^k}{(\sum_{i=1}^{2}\sum_{j=1}^{2}(a_i^A + a_i^B + \alpha)(a_j^A + a_j^B + \alpha)\lambda_{ij})^{\frac{1}{2}}}\right) - \sum_{i=1}^{2} c^k(a_i^k)$$

Note that the strategy space in our model is compact, even though we formulate the advertisement of each party as chosen from $\mathbb{R}_+^n$. This is because $v^k(\mathbf{a}^A, \mathbf{a}^B) \leq 1$ for all $\mathbf{a}^k$, but $C^k(\mathbf{a}^k) = \sum_{i=1}^{2} c^k(a_i^k)$ and $c^k(\infty) = \infty$. Therefore for each party $k \in \{A, B\}$, there exists $\bar{a}^k > 0$ such that the above optimization problem is equivalent to maximizing the same objective function by choosing $\mathbf{a}^k \in [0, \bar{a}^k]^2$. Let us denote by $\mathbf{S}^k$ the strategy space of party k where $\mathbf{S}^k = [0, \bar{a}^k]^2$.

Given that the action space is compact and convex, two conditions are sufficient for the existence of a pure strategy equilibrium: each party's utility function is continuous in the parties' strategies, and each party's utility function is concave in its own strategy.[5] The continuity condition is satisfied in our framework because a party's strategy is to choose an amount of advertisement on each issue, and each party's utility function is continuous in the advertisement strategies. Moreover, each party's utility function is concave in its own strategy if the cost function is sufficiently convex.

In this context, U^k is concave in $\mathbf{a}^k$ if the Hessian of U^k is negative definite for all $\mathbf{a}^k \in \mathbf{S}^k$ and all $\mathbf{a}^{-k} \in \mathbf{S}^{-k}$. For player k, the $(i, i) - th$ element of the Hessian of U^k is $U_{ii}^k = v_{ii}^k - c^{k''}$ and the $(i, j) - th$ element is $U_{ij}^k = v_{ij}^k$ for $i \neq j$. The Hessian is negative definite iff all of its n leading principal minors alternate in sign, with odd order being negative and even order being positive. In a scenario with two issues, the Hessian of U^k is negative definite iff

[5] In fact, quasi-concavity of each party's utility function suffices.

$$U_{11}^k = v_{11}^k - c^{k''} < 0$$

and

$$U_{11}^k U_{22}^k - (U_{12}^k)^2 = (v_{11}^k - c^{k''})(v_{22}^k - c^{k''}) - (v_{12}^k)^2 > 0$$

Since v_{ij}^k is continuous for all i and j and $\mathbf{S}^k$ is compact for all k, for $i = 1, 2$ define $m_{ii}^k \equiv \mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} v_{ii}^k(\mathbf{a}^A, \mathbf{a}^B)$. Similarly, define $m_{12}^k \equiv \mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} |v_{12}^k(\mathbf{a}^A, \mathbf{a}^B)|$.

A sufficient condition for the above two inequalities to hold is for the c^k function to be sufficiently convex. For example, let $c^{k''}(a) > \mathbf{max}\{m_{11}^k, m_{22}^k\} + m_{12}^k$ for all a. An example for such a cost function is $c^k(a) = \frac{\theta}{2} a^2$ where $\theta > |\mathbf{max}\{m_{11}^k, m_{22}^k\} + m_{12}^k|$.

To show that this condition on the cost function suffices for U^k to be concave, notice that for any $\mathbf{a}^A \in \mathbf{S}^A$ and any $\mathbf{a}^B \in \mathbf{S}^B$, we have $U_{11}^k(\mathbf{a}^A, \mathbf{a}^B) < 0$ because

$$U_{11}^k(\mathbf{a}^A, \mathbf{a}^B) = v_{11}^k(\mathbf{a}^A, \mathbf{a}^B) - c^{k''}(a_1^k) < m_{11}^k - (\mathbf{max}\{m_{11}^k, m_{22}^k\} + m_{12}^k) \le -m_{12}^k \le 0,$$

since $m_{12}^k \equiv \mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} |v_{12}^k(\mathbf{a}^A, \mathbf{a}^B)| \ge 0$ by construction.

Also, for any $\mathbf{a}^A \in \mathbf{S}^A$ and any $\mathbf{a}^B \in \mathbf{S}^B$, we have $U_{11}^k(\mathbf{a}^A, \mathbf{a}^B) U_{22}^k(\mathbf{a}^A, \mathbf{a}^B) - (U_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2 > 0$ because

$$U_{11}^k(\mathbf{a}^A, \mathbf{a}^B) U_{22}^k(\mathbf{a}^A, \mathbf{a}^B) - (U_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2$$

$$= [v_{11}^k(\mathbf{a}^A, \mathbf{a}^B) - c^{k''}(a_1^k)][v_{22}^k(\mathbf{a}^A, \mathbf{a}^B) - c^{k''}(a_2^k)] - (v_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2$$

$$= [c^{k''}(a_1^k) - v_{11}^k(\mathbf{a}^A, \mathbf{a}^B)][c^{k''}(a_2^k) - v_{22}^k(\mathbf{a}^A, \mathbf{a}^B)] - (v_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2$$

Since $c^{k''}(a_1^k) > \mathbf{max}\{m_{11}^k, m_{22}^k\} + m_{12}^k \ge v_{11}^k(\mathbf{a}^A, \mathbf{a}^B)$, $c^{k''}(a_1^k) - v_{11}^k(\mathbf{a}^A, \mathbf{a}^B) > 0$. Similarly $c^{k''}(a_2^k) - v_{22}^k(\mathbf{a}^A, \mathbf{a}^B) > 0$. Therefore

$$[c^{k''}(a_1^k) - v_{11}^k(\mathbf{a}^A, \mathbf{a}^B)][c^{k''}(a_2^k) - v_{22}^k(\mathbf{a}^A, \mathbf{a}^B)] - (v_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2$$

$$> (m_{12}^k)(m_{12}^k) - (v_{12}^k(\mathbf{a}^A, \mathbf{a}^B))^2$$

$$= [\mathbf{max}_{\mathbf{a}^A \in \mathbf{S}^A, \mathbf{a}^B \in \mathbf{S}^B} |v^k_{12}(\mathbf{a}^A, \mathbf{a}^B)|]^2 - (v^k_{12}(\mathbf{a}^A, \mathbf{a}^B))^2 \geq 0$$

Therefore for $k \in \{A, B\}$, $U^k(\mathbf{a}^A, \mathbf{a}^B)$ is concave in $\mathbf{a}^k$ if $c^{k''}(a) > \mathbf{max}\{m^k_{11}, m^k_{22}\} + m^k_{12}$ for all a.

□

Proof of Proposition 2. Suppose to the contrary that there exists a pure strategy Nash equilibrium $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ such that for some issue i, $a^{A*}_i > 0$ and $a^{B*}_i > 0$. Let $a^*_i = a^{A*}_i + a^{B*}_i$ denote the equilibrium total advertisement on issue i.

Party A's vote share is a function of the total advertisement each issue receives, namely $v^A(\mathbf{a}^A, \mathbf{a}^B) = v^A(a_1, a_2)$, where $a_i = a^A_i + a^B_i$ denotes the total advertisement on issue i for $i = 1, 2$. Similarly, party B's vote share is a function of the total advertisement each issue receives because there are only two parties, $v^B(a_1, a_2) = 1 - v^A(a_1, a_2)$.

Since $a^{A*}_i > 0$, A's maximization problem implies that $\frac{\partial v^A(a^{A*}_i + a^{B*}_i, a^*_{-i})}{\partial a^A_i} - c^{A'}(a^{A*}_i) = 0$, and since $\frac{\partial v^A(a^{A*}_i + a^{B*}_i, a^*_{-i})}{\partial a^A_i} = \frac{\partial v^A(a^*_i, a^*_{-i})}{\partial a_i}$, we have $\frac{\partial v^A(a^*_i, a^*_{-i})}{\partial a_i} = c^{A'}(a^{A*}_i) > 0$.

Since $v^B(a^*_1, a^*_2) = 1 - v^A(a^*_1, a^*_2)$, we have $\frac{\partial v^B(a^*_1, a^*_2)}{\partial a_i} = -\frac{\partial v^A(a^*_1, a^*_2)}{\partial a_i}$, that is $\frac{\partial v^B(a^*_i, a^*_{-i})}{\partial a_i} = -\frac{\partial v^A(a^*_i, a^*_{-i})}{\partial a_i}$. Furthermore $\frac{\partial v^B(a^{A*}_i + a^{B*}_i, a^*_{-i})}{\partial a^B_i} = \frac{\partial v^B(a^*_i, a^*_{-i})}{\partial a_i}$, so $\frac{\partial v^B(a^{A*}_i + a^{B*}_i, a^*_{-i})}{\partial a^B_i} = -\frac{\partial v^A(a^*_i, a^*_{-i})}{\partial a_i} < 0$.

Therefore $\frac{\partial v^B(a^{A*}_i + a^{B*}_i, a^*_{-i})}{\partial a^B_i} - c^{B'}(a^{B*}_i) < 0$. That is, party B's objective function is strictly decreasing in a^B_i at $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$. Since $a^{B*}_i > 0$, a^{B*}_i is not the optimal choice for party B, and thus we have a contradiction.

□

Proof of Proposition 4. Since $\sigma_{ii} = 0$, $\lambda_{ii} = (p^A_i - p^B_i)^2 \sigma_{ii} = 0$. Also, $\sigma_{ii} = 0$ implies $\sigma_{ij} = 0$ for $j \neq i$ and hence $\lambda_{ij} = (p^A_i - p^B_i)(p^A_j - p^B_j)\sigma_{ij} = 0$. Since $\nu_i \neq 0$, without loss of generality let $\nu_i > 0$ (that is, $\nu^A_i > 0$).

Suppose to the contrary that there exists a pure strategy equilibrium $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ such that $a^{A*}_i = 0$. Then given B's equilibrium advertisement, A's vote share if A chooses advertisement $\mathbf{a}^A \equiv (a^A_i, a^{A*}_{-i})$ is

$$v^A(\mathbf{a}^A, \mathbf{a}^{B*}) = \Phi(\frac{(a_i^A + a_i^{B*} + \alpha)\nu_i + (a_{-i}^* + \alpha)\nu_{-i}}{(0 + (a_{-i}^* + \alpha)^2 \lambda_{(-i)(-i)})^{\frac{1}{2}}}).$$

The first-order derivative of A's vote share with respect to a_i^A at 0 is

$$\frac{\partial}{\partial a_i^A} v^A(\mathbf{a}^A, \mathbf{a}^{B*})_{|a_i^A=0} = \frac{\phi(\frac{\bar{\nu}}{\bar{\lambda}^{\frac{1}{2}}})\nu_i}{\bar{\lambda}^{\frac{1}{2}}},$$

where $\phi(\cdot)$ is the standard normal pdf, and we denote for simplicity by $\bar{\nu} \equiv (a_i^{B*} + \alpha)\nu_i + (a_{-i}^* + \alpha)\nu_{-i}$ and by $\bar{\lambda} \equiv (a_{-i}^* + \alpha)^2 \lambda_{(-i)(-i)}$.

Since $\phi(\frac{\bar{\nu}}{\bar{\lambda}^{\frac{1}{2}}}) > 0$, $\nu_i > 0$, and $\bar{\lambda}^{\frac{1}{2}} > 0$, we have $\frac{\partial}{\partial a_i^A} v^A(\mathbf{a}^A, \mathbf{a}^{B*})_{|a_i^A=0} > 0$.

Also, since $c^{A'}(0) = 0$, therefore $\mathbf{a}^{A*}$ where $a_i^{A*} = 0$ cannot be A's best response to B's equilibrium advertisement $\mathbf{a}^{B*}$. Therefore, $(\mathbf{a}^{A*}, \mathbf{a}^{B*})$ where $a_i^{A*} = 0$ cannot be an equilibrium. Hence we have a contradiction.

□

Proof of Proposition 5. Without loss of generality, let $\nu_i = \nu_j = \nu > 0$, then A is the majority party. The case where $\nu < 0$ hence B is the majority party, is analogous. Also without loss of generality let $\lambda_{11} > \lambda_{22}$. Therefore we want to show that it's not possible to have a pure strategy Nash equilibrium in which $a_1^{A*} > 0$ and $a_2^{B*} > 0$.

Suppose to the contrary that there is an equilibrium in which $a_1^{A*} > 0$ and $a_2^{B*} > 0$. From Proposition 2 we have $a_1^{B*} = 0$ and $a_2^{A*} = 0$. B's equilibrium utility, denoted by u^{B*}, is

$$u^{B*} = \left[1 - \Phi\left(\frac{\sum_{i=1}^2 w_i(\mathbf{a}^*)\nu_i}{[\sum_{i=1}^2 \sum_{j=1}^2 w_i(\mathbf{a}^*) w_j(\mathbf{a}^*)\lambda_{ij}]^{\frac{1}{2}}}\right)\right] - C^B(\mathbf{a}^{B*})$$

$$= \left[1 - \Phi\left(\frac{\sum_{i=1}^2 (a_i^* + \alpha)\nu}{[\sum_{i=1}^2 \sum_{j=1}^2 (a_i^* + \alpha)(a_j^* + \alpha)\lambda_{ij}]^{\frac{1}{2}}}\right)\right] - \sum_{i=1}^2 c^B(a_i^{B*})$$

$$= \left[1 - \Phi\left(\frac{[(a_1^{A*} + \alpha) + (a_2^{B*} + \alpha)]\nu}{[(a_1^{A*} + \alpha)^2\lambda_{11} + (a_2^{B*} + \alpha)^2\lambda_{22} + 2(a_1^{A*} + \alpha)(a_2^{B*} + \alpha)\lambda_{12}]^{\frac{1}{2}}}\right)\right] - c^B(0) - c^B(a_2^{B*}).$$

But party B has a profitable deviation by choosing advertisement vector $\mathbf{a}^{B'}$ where

$a_1^{B'} = a_2^{B*} > 0$ and $a_2^{B'} = 0$. Namely, the alternative advertisement for B is to switch its advertisement on the two issues. In this case, the total advertisement on issue 1 is $a_1^{A*} + a_2^{B*}$, and the total advertisement on issue 2 is 0. B's utility from this alternative advertisement vector $\mathbf{a}^{B'}$, denoted by $u^{B'}$, is

$$u^{B'} = \left[1 - \Phi\left(\frac{[(a_1^{A*} + a_2^{B*} + \alpha) + (0 + \alpha)]\nu}{[(a_1^{A*} + a_2^{B*} + \alpha)^2\lambda_{11} + \alpha^2\lambda_{22} + 2(a_1^{A*} + a_2^{B*} + \alpha)\alpha\lambda_{12}]^{\frac{1}{2}}}\right)\right] - c^B(a_2^{B*}) - c^B(0).$$

Compared to the expression for u^{B*} above, the only part that $u^{B'}$ differs from u^{B*} is the denominator in the standard normal cdf function. In particular, $u^{B'} > u^{B*}$ because the denominator in u^{B*} is smaller than the denominator in $u^{B'}$. The derivation is as follows:

$$(a_1^{A*} + a_2^{B*} + \alpha)^2\lambda_{11} + (\alpha)^2\lambda_{22} + 2(a_1^{A*} + a_2^{B*} + \alpha)\alpha\lambda_{12}$$

$$-(a_1^{A*} + \alpha)^2\lambda_{11} - (a_2^{B*} + \alpha)^2\lambda_{22} - 2(a_1^{A*} + \alpha)(a_2^{B*} + \alpha)\lambda_{12}$$

$$= (a_2^{B*})^2(\lambda_{11} - \lambda_{22}) + 2\alpha a_2^{B*}(\lambda_{11} - \lambda_{22}) + 2(a_1^{A*})(a_2^{B*})(\lambda_{11} - \lambda_{12}) > 0$$

because $\lambda_{11} - \lambda_{22} > 0$, and $|\lambda_{12}| \leq \sqrt{\lambda_{11}\lambda_{22}} < \lambda_{11}$ where the first inequality is due to Cauchy-Schwartz inequality, and thus $\lambda_{11} - \lambda_{12} > 0$.

Therefore,

$$[(a_1^{A*} + a_2^{B*} + \alpha)^2\lambda_{11} + \alpha^2\lambda_{22} + 2(a_1^{A*} + a_2^{B*} + \alpha)\alpha\lambda_{12}]^{\frac{1}{2}}$$

$$> [(a_1^{A*} + \alpha)^2\lambda_{11} + (a_2^{B*} + \alpha)^2\lambda_{22} + 2(a_1^{A*} + \alpha)(a_2^{B*} + \alpha)\lambda_{12}]^{\frac{1}{2}}.$$

Therefore, $u^{B'} > u^{B*}$, B has a profitable deviation from $\mathbf{a}^{B*}$ to $\mathbf{a}^{B'}$, and hence the original advertisement strategy cannot be an equilibrium.

□